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1 **On the mechanism of dynamic response amplification of cable transport**
2 **line: application to the design based on the cable tension signal analysis**

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10 **ABSTRACT**

11 Cables are used for many engineering applications such as transport systems that involve mobile
12 cables suspended between supports for carrying attached vehicles. Because of the inherent cable's
13 flexibility and the modification of dynamics properties during movement, such systems are sensitive
14 to self sustained oscillations under steady-state conditions. To provide an understanding of such
15 vibratory excitation sources and a design method to prevent operating problems, this paper proposes
16 a real case study of a chairlift and the development of an original model. The aim is to highlight the
17 mechanisms responsible for the dynamic response amplification of the cable line. The method is
18 based on a reduction of the complete dynamic model to a single mode which localises the vibration
19 energy along the cable loop. The reduced model thus obtained takes the form of a Mathieu-Hill
20 parametric excitation oscillator. The stability study reveals zones of instability in which the dynamic
21 system response increases. The approach is then validated on the real case study by positioning the
22 chairlift's operating points in the stability diagram according to different configurations. Operating
23 points are determined using the signal of the evolution of the static tension of the cable during the

24 movement of the vehicles. Based on the case study, a general methodology for modifying design
25 parameters is proposed to avoid undesirable line dynamics.

26 INTRODUCTION

27 Cable structures are involved in many engineering fields and applications like bridges, power
28 lines, elevators, offshore platform, robots. Cable element have been the subject of numerous pub-
29 lications in the state of the art because of the multiplicity of models and possible simplifications
30 to describe the mechanical behaviour in statics (Crussels-Girona et al. 2017; Bertrand et al. 2020)
31 and dynamics (Saxon and Cahn 1953; Irvine 1992; Fei et al. 2020; Fei and Danhui 2020; Xu
32 et al. 2022). Cables are flexible structure showing nonlinearities (Rega 2004; Berlioz and Lamar-
33 que 2005; Lacarbonara 2013; Bertrand et al. 2022) due to their slender geometrical characteristics
34 (Marigo 2014). In this paper, the problem of a moving cable assembly is addressed with applica-
35 tion to cable transport systems for which one cable in translation to move suspended masses. The
36 cable transport applications' state of the art is fragmentary since the mechanical studies are gener-
37 ally restricted to a single cable span located between two supports where the cable is supposed to
38 be embedded. Due to the couplings on the global scale of the system, a local approach limits the
39 interpretation of the physical phenomena that the model is able to capture. Analytical works are
40 mainly focused on statics equilibrium using approximate methods (Czitary 1962; Schneigert 1964).
41 More recently, numerical models are generally restricted to the statics or dynamics study of a single
42 cable at rest without translation effects (Portier 1984; Brownjohn 1998; Ntarladima and Gerstmayr
43 2023). (Renezeder et al. 2005; Hurel et al. 2018) studied a mobile cable by justifying a quasi-static
44 approach due to a little increase of the sag observed at usual speed.

45 To the best of the author's knowledge, only few research studies have dealt with pumping os-
46 cillations affecting continuously moving cable transport lines. Pumping phenomenon is a strong
47 vertical oscillation of the cable between two intermediary supports. Portier's (Portier 1985) work is
48 based on the intuition that pumping oscillations can be explained by cable tension pulses introduced
49 at the cable supports by vehicles crossing. Löscher's (Löscher 1997; Engel and Löscher 2005) ex-
50 planation of pumping is based on a resonant interaction between periodically circulating loads and

51 the cable. Mode shape is assumed to be collinear with the static shape and oscillation amplitude
52 calculation is performed using a numerical integration scheme on a grid of speed and inter-vehicle
53 spacing parameters. Renezeder (Renezeder 2006; Renezeder et al. 2006) performs transient calcu-
54 lations on a single-cable transport system modeled by finite elements with a commercial software.
55 Sag oscillations are interpreted as a linear resonance generated by an external excitation. Accord-
56 ing to an alternative approach, but with similar conclusions, Knawa (Knawa-Hawryszków 2017)
57 characterizes the modes of a cable car line and explains pumping oscillations as a linear resonance
58 with a dynamic perturbation generated by the periodic distribution of vehicles. Babaz's (Babaz
59 2016) applied research work focuses on the pumping oscillations of continuously moving cable as
60 a nonlinear phenomenon.

61 The various works available in the literature do not provide a robust design methodology for
62 anticipating the pumping phenomena and for quantifying the influence of design parameters such as
63 static tension or cable speed. Therefore, this paper proposes an original explanation of the pumping
64 oscillations based on a stability study the Mathieu-Hill equation derived from a reduced dynamic
65 model. An application with a case study of a chairlift showing pumping is proposed.

66 Firstly, the pumping oscillation description is given in a general framework and a presentation of
67 the studied cable transport system is made. Then, the experimental protocol used to trigger pump-
68 ing oscillations of the real system is described and the influencing parameters are highlighted. In
69 the light of these initial findings, a heuristic explanation is proposed for the mechanisms that cause
70 pumping oscillations. Secondly, a mathematical modelling is made by writing the dynamic equa-
71 tions of the whole installation considering the overall cable loop and the coupling with system's
72 subcomponents. A modal approach is used, based on the results of a previous work (Bécu et al.
73 2023; Bécu et al. 2024). Vibration modes are calculated accounting for interactions between the
74 cable in translation at variable speed, the supports and the pulleys at the end stations of the instal-
75 lation, with a trace of cable geometric non-linearities through the inclusion of a static correction in
76 the calculation. The mass of the vehicles is assumed to be uniformly distributed along the cable to
77 obtain a model underlying the modal calculation that is homogeneous and does not vary over time.

CASE STUDY PRESENTATION AND PUMPING OSCILLATIONS

In this paper, the cable transport system under study is a chairlift composed of (Figure 1, Table 1):

- A closed loop of track and haul rope of length L [m]. A forward path links the bottom station to the top one, and a return path links the top station to the bottom one.
- Vehicles equidistantly spaced by a length l_d [m]. Vehicles are connected, guided and pulled by the cable.
- Two pulleys in a roughly horizontal plane at both ends of the installation. Pulleys deflect the rope loop. The tension bullwheel at the bottom station ensures mechanical tensioning T [N] via a hydraulic circuit that activates a cylinder. The drive bullwheel at the top station sets the rope loop in motion and is connected to the electric drive.
- An electric motor, a gearbox and a transmission shaft linked to the pulley.
- Towers along the cable path. Intermediate supports maintain an acceptable distance between the cable line and the ground. Two types of support are used, depending on whether the cable is deflected through the support at a positive angle (compression tower) or a negative angle (support tower). The contact between the cable and the structure is made by a sheave assembly equipped with small-diameter pulleys known as rollers. The sheave assembly is an articulated structure that deforms according to the cable geometry.

Pumping is a well-known oscillating phenomenon in the ropeway industry. Cable spans are affected in the frequency range of $[0.1, 1]$ Hz with possible coupling of vehicle pendulum oscillations (Figure 2). Pumping starts and stops at steady-state rope speed V_0 [m/s] for small variations of:

- vehicle load m_v [kg] ;
- cable's speed V_0 [m/s] ;
- spacing l_d [m] between the N_v vehicles on the line.

Pumping is highly sensitive to design parameters such as the static tension T [N] imposed by the

103 hydraulic cylinder, or to operating parameters such as number of passenger and cable speed V_0
104 [m/s]. Pumping is undesirable for a number of reasons prompting manufacturers and operators to
105 ensure that installations are not subject to it, particularly in the context of the development of urban
106 applications of ropeways as a means of mass transit:

- 107 • Dynamic loading cycles cause premature fatigue in the mechanical components of the sheave
108 assembly and tower structures ;
- 109 • Line clearance is reduced by the amplitude of the cable and vehicle dynamic displacement
110 ;
- 111 • Discomfort and apprehension is induced on passengers.

112 The frequency range of pumping oscillations corresponds to the frequencies associated with motion
113 sickness, which in some cases causes kinetosis.

114 **EXPERIMENTAL PROTOCOL TO HIGHLIGHT THE PUMPING**

115 A chairlift line affected by pumping is studied as application case. After several years in ser-
116 vice, the passenger flow was increased by adding new vehicles on the cable loop. The number of
117 vehicles increased from N_v^0 to N_v^1 . The pumping then has appeared in a span at a specific operating
118 speed V_0 [m/s] during pre-commissioning tests. In the absence of a tool able to predicting this dy-
119 namic behaviour in the design phase, and of any precise understanding of the mechanisms behind
120 the activation of pumping, an experimental protocol was set up to empirically study the influence
121 of control parameters (V_0, N_v) ([m/s],[]) to the activation and disappearance of pumping on the
122 chairlift line.

123 The Table 2 summarizes the speed V_0 [m/s] and vehicle number N_v [] configurations tested
124 with a description of oscillation observations in the cable span affected by pumping according to a
125 color code (green: no oscillations, red: strong oscillations):

- 126 • Pumping oscillations occur predominantly for $V_0 = 4.25$ [m/s] with N_v^1 vehicle along the
127 cable with:

- strong and high vertical amplitude oscillations of the cable are observed in the span 8 according to a period $\delta t_c \approx 4.8$ [s] ;
- one mode shape response with one antinode located at mid span ;
- a transverse swaying of the vehicles according to a period $\delta t_v \approx 2.4$ [s] ;
- cable oscillations in phase opposition between the two sides of the track ;
- torque oscillations $\Delta C = C_{max} - C_{min}$ [Nm] in the driving motor C located in the upper station. These torque oscillations reflect the coupling effects of the cable's vertical oscillations on its longitudinal movement which is reflected in speed variations at the driving pulley. Torque is measured via the motor current (DC motor). The driving motor is speed-controlled by a PID corrector integrated into the controller. It explains the disturbance introduced on the motor torque which is an image of the cable's dynamic disorder.

- By increasing the cable speed to 4.5 [m/s] and 5 [m/s], cable and vehicle oscillations gradually disappear (see color code in orange and green in Table 2), in the same way as the oscillations observed on the motor torque (see Table 2).
- Based on this observation, a vehicle was removed from the line such as $N_v = N_v^1 - 1$ [. No dynamic disorder is observed on the line at any cable speed $V_0 \in [4.25, 5]$ [m/s]. Measurements have been made for only the cable speed $V_0 = 4.25$ [m/s] (see Table 2) which showed the absence of any noticeable oscillations in the cable and motor torque compared with the previously tested configuration.

HEURISTIC ANALYSIS OF THE PUMPING

Internal self-excitation

On continuous-motion cable transport system, the equidistant distribution of vehicles along the cable loop introduces a spatial period equal to the inter-vehicle distance $l_d = \frac{L_0}{N_v}$ [m]. When the cable is in steady state movement at a constant speed V_0 [m/s], the movement of vehicles seen from

153 a fixed point on the line is periodic with a passage frequency

$$154 \quad f_e = \frac{V_0}{l_d} = \frac{\Omega}{2\pi}. \quad (1)$$

155 By changing the number of vehicles N_v [] on the line from a value of N_v^0 [] to N_v^1 [], the passage
156 frequency is modified by a relative value

$$157 \quad \Delta f_e = \frac{N_v^1 - N_v^0}{N_v^0}. \quad (2)$$

158 The static state of the cable line also depends on the position of the vehicles. The highly non-linear
159 nature of the static cable line equilibrium problem introduces a strong dependence of the cable
160 tension T_s [N] and sag values, with influence on the cable curvature κ , on the positions successively
161 occupied by the vehicles (see Figure 3).

162 The problem can thus be reduced to a parametric dependence on a geometric parameter l_c [m]
163 describing the set of geometric configurations occupied by the system over time at the origin of a
164 source of periodic internal dynamic disturbance of fundamental frequency f_e [Hz] when the cable
165 is in translational movement. In the case presented here, the Table 3 summarizes the excitation
166 pulsation $\Omega_e = 2\pi f_e$ [rad/s] associated with each line configuration tested.

167 **Geometric coincidence**

168 On single-cable system, when a vehicle is close to an intermediate tower, the clamp that connects
169 it to the cable comes into contact with a zone dedicated to ensuring smooth passage of the vehicle
170 from one side of the support to the other. Technologically speaking, this zone is referred to as a
171 sheave assembly and is made up of a set of small-diameter pulleys interconnected by beams linked
172 by pivots. This local contact alters the force balance, producing a contact resultant \underline{R} which is the
173 sum of the surface contact pressures between the clamp jaws gripping the rope and the roller grooves
174 of the rubber tyre. This contact force, which is absent when the vehicle is outside a support zone,
175 modifies the value of the local cable tension and thus causes a local dynamic disturbance ΔT_s within
176 the moving system at V_0 speed via the static tension T_s . The intensity of the tension disturbance

177 depends on the parameters governing the geometry of the support, the cable, the vehicle load \underline{P} and
 178 the sheave assembly crossing kinematics. From the point of view of the standard, the centripetal
 179 acceleration caused by the geometry of a circle arc-length support of radius R [m] is regulated by
 180 the following criterion,

$$181 \quad \frac{V_0^2}{R} < 2.5 \text{ m/s}^2, \quad (3)$$

182 which limits the dynamic component of the tension disturbance caused by the crossing trajectory.

183 When two vehicles pass simultaneously over the two adjacent supports that separate a cable
 184 span, the local dynamic tension disturbance ΔT_s in the cable is locally amplified by a cumulative
 185 effect resulting in a modified static tension T_s^+ (see Figure 4).

186 **Cable line dynamics**

187 From a dynamic point of view, the pumping oscillations of the cable loop with the vehicles can
 188 be dissociated from the dynamic response of the stations and the supports as the natural frequencies
 189 of these substructures are higher than those observed on the cable line. Under these assumptions,
 190 the dynamics of a cable transport line is described by the mechanical equations of the cable and the
 191 interface with other substructures. A model of a translating cable with an instantaneous speed V_0
 192 and an acceleration A_{V_0} such as $\underline{V}_0 = V_0 \underline{t}$, $\underline{A}_{V_0} = A_{V_0} \underline{A}_{V_0} / \|\underline{A}_{V_0}\|$ is proposed in (Bécu et al. 2023).
 193 The description is based on the cable dynamic displacement

$$194 \quad \underline{U}(S, t) = u(S, t) \underline{t} + v(S, t) \underline{n} + w(S, t) \underline{e}_z, \quad (4)$$

195 expressed according to the curvilinear abscissa $S \in [0, L]$, with L the cable length, in the cable
 196 local frame $(\underline{t}, \underline{n}, \underline{e}_z)$ and given by the tangent vector \underline{t} , the normal vector \underline{n} and the out of plane
 197 vector \underline{e}_z . Keeping only the linear contributions, it leads to the cable dynamic equations in three

198 spatial dimensions:

$$199 \left\{ \begin{array}{l} \mu \ddot{u} + c_u \dot{u} + \frac{T_s}{\|\underline{X}'\|} \kappa (v' + \kappa u) - EA(u'' - \kappa v') - 2\mu V_0 \dot{u}' + \mu V_0^2 u'' + \mu A_{V_0} u' = 0, \\ \mu \ddot{v} + c_v \dot{v} - \frac{T_s}{\|\underline{X}'\|} (v'' + \kappa u') - \kappa EA(u' - \kappa v) - 2\mu V_0 \dot{v}' + \mu V_0^2 v'' + \mu A_{V_0} v' = 0, \\ \mu \ddot{w} + c_w \dot{w} - \frac{T_s}{\|\underline{X}'\|} w'' - 2\mu V_0 \dot{w}' + \mu V_0^2 w'' + \mu A_{V_0} w' = 0. \end{array} \right. \quad (5)$$

200 with c_u, c_v, c_w the damping coefficients, $T_s(S)$ the static cable tension and $\underline{X}(S)$ the static cable
 201 position, \bullet' derivatives with respect to the curvilinear variable S and $\bullet\dot{}$ derivatives with respect to
 202 the time variable t .

203 A mode of vibration is defined as the small undamped in-phase oscillations of the structure
 204 around a static state, which may depend on a prestress load, given by a modal shape

$$205 \underline{\Phi}_k = \Phi_{uk} \underline{t} + \Phi_{vk} \underline{n} + \Phi_{wk} \underline{e}_z \quad (6)$$

206 and a modal frequency

$$207 f_k = \omega_k / 2\pi \quad (7)$$

208 such as the dynamic response is given by variable separation

$$209 \underline{U}(S, t) = q(t) \underline{\Phi}_k(S) = e^{j\omega_k t} \underline{\Phi}_k(S). \quad (8)$$

210 Under several conditions concerning the nature of the mechanical model - linear, time-independent,
 211 undamped, symmetric - forms a basis for the solution space of the underlying dynamic problem
 212 (spectral decomposition). Thus, any solution to the linear dynamics problem of a moving cable
 213 can be approximated by searching for the solution in a subspace spanned by a selection of modes
 214 $\left[\underline{\Phi}_k \right]$ obtained by solving the following modal problem without taking into account time-dependent

215 damping terms:

$$216 \left\{ \begin{array}{l} -\omega_k^2 \mu \Phi_{uk} + \frac{T_s(S)}{\|\underline{X}(S)'\|} \kappa (\Phi'_{vk} + \kappa \Phi_{uk}) - EA (\Phi''_{uk} - \kappa \Phi'_{vk}) + \mu V_0^2 \Phi''_{uk} + \mu A_{V_0} \Phi'_{uk} = 0, \\ -\omega_k^2 \mu \Phi_{vk} - \frac{T_s(S)}{\|\underline{X}(S)'\|} (\Phi''_{vk} + \kappa \Phi'_{uk}) - \kappa EA (\Phi'_{uk} - \kappa \Phi_{vk}) + \mu V_0^2 \Phi''_{vk} + \mu A_{V_0} \Phi'_{vk} = 0, \\ -\omega_k^2 \mu \Phi_{wk} - \frac{T_s(S)}{\|\underline{X}(S)'\|} \Phi''_{wk} + \mu V_0^2 \Phi''_{wk} + \mu A_{V_0} \Phi'_{wk} = 0. \end{array} \right. \quad (9)$$

217 Using a projection, a reduced order model (ROM) is obtained under the following form,

$$218 \quad \ddot{q} + \left(c_k + \Psi_k^1(V_0) \right) \dot{q} + \left(\omega_k^2 + \Psi_k^2(V_0, A_{V_0}) \right) q = 0. \quad (10)$$

219 The Appendix 1 provides details on how to obtain the reduced order model.

220 The advantage of such a reduced order model is that it contains the spatial dynamic information
221 along the system, provided by the mode, in a scalar equation form according to one degree of
222 freedom q .

223 Modal excitation

224 The reduced order model given by Eq. (10) describes the dynamic behaviour of the cable trans-
225 port system when the mode (Φ_k, ω_k) is activated in a privileged way. The projection on one mode
226 allows to describe system dynamics around resonance under the assumption of a low modal cou-
227 pling.

228 In the absence of an external excitation term in the second member of Eq. (80), the dynamic
229 excitation comes from the static tension T_s and static curvature κ evolution when the cable and the
230 vehicles are moving. The tension $T_s(S, l_c)$ and curvature $\kappa(S, l_c)$ depend on the geometric evolution
231 parameter l_c (see subsection 4) which is related to time $l_c = V_0 \times t$ and can be written as follows

$$232 \quad T_s(S, t) = \langle T_s \rangle (S) + \Delta T_s(S, t), \quad (11)$$

$$233 \quad \kappa(S, t) = \langle \kappa \rangle + \Delta \kappa(S, t). \quad (12)$$

235 In other words, Eq. (80) can be rewritten in the non-dimensional generic form of a Mathieu-type

236 equation (Nayfeh and Mook 1979) with a parametric excitation term $a(T)$,

$$237 \quad \ddot{q} + c\dot{q} + (\delta + a(T))q = 0. \quad (13)$$

238 with

$$239 \quad T = \frac{2\pi V_0}{l_d} t, \quad (14)$$

$$240 \quad c = \frac{1}{\Omega^2} (c_k + \Psi_k^1), \quad (15)$$

$$241 \quad \delta = \frac{1}{\Omega^2} (\omega_p^2 (\langle T_s \rangle, \langle \kappa \rangle) + \Psi_k^2), \quad (16)$$

$$242 \quad a(T) = \frac{1}{\Omega^2} \omega_p^2 (\Delta T_s(S, T), \Delta \kappa(S, T)). \quad (17)$$

246 In the following, such a model is used to study the ability of a distributed excitation at fonda-
 247 mental frequency $f = \omega/2\pi$ along the cable line applied in the curvilinear abscissa S_i to generate
 248 a vibration at an observation point S_o .

249 *Modal excitability*

250 The notion of transfer $H(\omega)$ is introduced in modal analysis and establishes the link between a
 251 dynamic harmonic excitation at ω pulsation applied to a structure via input

$$252 \quad inp(S_i, \omega) = F(S_i) e^{j\omega t} \quad (18)$$

253 and the observed structure's linear response

$$254 \quad obs(S_o, S_i, \omega) = \Phi_k(S_o) Q(S_i, \omega) e^{j\omega t} \quad (19)$$

255 according to the mode k such as

$$256 \quad obs(S_o, S_i, \omega) = H(S_i, S_o, \omega) \times inp(S_i, \omega) \quad (20)$$

257 and

$$258 \quad \left(-\omega^2 + j\omega c_k + \omega_k^2\right) Q = \int_0^L \Phi_k(S) \delta(S - S_i) F(S) dS \quad (21)$$

259 One can combine Eq. (20) and Eq. (21) to obtain

$$260 \quad H(\omega) = \frac{\Phi_k(S_0)\Phi_k(S_i)}{-\omega^2 + j\omega c_k + \omega_k^2} \quad (22)$$

261 It reveals modal excitability e_k (Balmès et al. 2004; Balmès 2005) which is the evaluation of the
262 transfer $|H(\omega)|$ at resonance $\omega = \omega_k$,

$$263 \quad e_k = \frac{\Phi_k(S_0)\Phi_k(S_i)}{\omega_k c_k}, \quad (23)$$

264 as a product of the modal controllability term $\Phi_k(S_i)$ and the modal observability $\Phi_k(S_o)$ divided
265 by the damping effect.

266 Because of the parametric nature of excitation in Eq. (13) via the coefficient $a(T)$ depending on
267 the non-dimensional time T , the notion of transfer is lost here. An equivalent notion of modal con-
268 trollability is introduced. In the following, only the disturbance generated by static tension $\Delta T_s(S, T)$
269 variations is taken into account. The static curvature κ is supposed to be time independent. Thus,
270 the parametric excitation term of Eq. 80 can be written

$$271 \quad a(T) = \frac{1}{\Omega^2 \int_0^L (\Phi_{uk}^2 + \Phi_{vk}^2 + \Phi_{wk}^2) \mu dS} \int_0^L \frac{\Delta T_s(S, T)}{\|\underline{X}'\|} \left[\Phi_{uk}^2 \kappa^2 + \Phi_{uk} \Phi'_{vk} \kappa - \Phi_{vk} \Phi'_{uk} \kappa - \Phi_{vk} \Phi''_{vk} - \Phi_{wk} \Phi''_{wk} \right] dS. \quad (24)$$

272 Here, the concept of modal excitability is the ability of the static tension perturbation $\Delta T_s(S, T)$,
273 depending on its spatial distribution, to activate a mode $\Phi_k(S)$. From Eq. (24), the level of para-
274 metric excitation is given by the integral over the rope length $S \in [0, L]$ of the product of $\Delta T_s(S, T)$
275 and the quantity

$$276 \quad \Phi_{uk}^2 \kappa^2 + \Phi_{uk} \Phi'_{vk} \kappa - \Phi_{vk} \Phi'_{uk} \kappa - \Phi_{vk} \Phi''_{vk} - \Phi_{wk} \Phi''_{wk} \quad (25)$$

277 that only depends on the mode shape relative amplitude along the cable. Thus, the level of para-
278 metric excitation $a(T)$ is maximum for a mode that localises the vibratory energy over a span of
279 the cable line where the tension perturbation is also maximum.

280 *Modal amplification*

281 In linear theory, modal amplification quantity

$$282 \quad -\omega^2 + j\omega c_k + \omega_k^2 \quad (26)$$

283 is linked to the resonance phenomenon when the excitation pulsation ω is equal to the mode pul-
284 sation ω_k by minimizing the norm of the denominator in the transfer Eq. (22). Here parametric
285 resonance is discussed in the case of Eq. (13) and an equivalent definition of modal amplification
286 is given.

287 The special feature of Mathieu-type equations Eq. (13) is that a time-periodic excitation term
288 $a(T)$ appears as a coefficient in the differential equation governing the system. The excitation is
289 designated by parametric. Unlike systems with only an external excitation source $F(S_i)$ where a
290 small amplitude does not produce a strong system response except near one of the system's modal
291 frequencies f_k (primary resonance), a small parametric excitation can produce a large system re-
292 sponse if the excitation source frequency f is close to twice one of the system's modal frequencies,
293 such as (Nayfeh and Mook 1979)

$$294 \quad f = 2f_k. \quad (27)$$

295 Melde (Minorsky 1945) was the first to demonstrate experimentally the phenomenon of parametric
296 excitation, using a string stretched between a rigid support and excited longitudinally by the vi-
297 bration of one arm of a tuning fork. For excitation in the longitudinal direction, he observed that,
298 for certain combinations of string linear mass and tension, a strong oscillation is produced in the
299 vertical direction, with a frequency equal to half the excitation frequency f . This is called paramet-
300 ric resonance which corresponds to zones of dynamic instability. More generally, this observation

301 applies to parametric resonance condition of order n such that

$$302 \quad f = n f_k. \quad (28)$$

303 The driving parametric excitation amplitude required to reach the instability zone increases with
304 the order of the parametric resonance.

305 **Heuristic interpretation of the pumping**

306 According to a heuristic approach, the pumping phenomenon affecting continuous-motion cable
307 transport lines can be interpreted as the coupling of strong modal controllability, observability and
308 amplification:

- 309 • The high modal controllability is due to a parametric excitation driven by the cable tension
310 periodic variations generated by the vehicles as they travel. These variations can be ampli-
311 fied by a geometric coincidence of vehicle crossing over supports in a cable span for which
312 the responding mode has a large relative amplitude with respect to the other points of the
313 cable.
- 314 • The high modal observability is due to a responding mode shape that localizes vibrating
315 energy in a specific span.
- 316 • The high modal amplification is caused by a dynamic instability driven by a parametric
317 resonance depending on:
 - 318 • the spectral content of dynamic disturbance of fundamental frequency f ;
 - 319 • a mode of frequency f_k of the cable loop involved in strong modal controllability.

320 **ANALYTICAL STUDY OF THE PUMPING**

321 **Stability analysis**

322 In the first section, it was seen that a parametric instability can cause a dynamic amplification
323 of the cable line response when a mode with an high modal controllability and high modal observ-
324 ability is activated.

325 A stability analysis of the Mathieu-type equation Eq. (13) governing the pumping oscillations
 326 is proposed under the effect of a periodic perturbation in tension. The assumption is made that the
 327 pumping oscillations evolve in a subspace correctly described by a single mode $(\underline{\Phi}_k, \omega_k)$ and that
 328 the dynamic perturbation is uniform and localised in a single span for $S \in [L_1, L_2]$ such as

$$329 \quad \Delta T_s(S, T) = \begin{cases} \Delta T_s(T), & \forall L \in [L_1, L_2], \\ 0, & \forall L \in [0, L_1[\cup]L_2, L]. \end{cases} \quad (29)$$

330 The tension can be written into Fourier series,

$$331 \quad \Delta T_s(T) = \sum_i \Delta T_{si}^c \cos(iT) + \Delta T_{si}^s \sin(iT) = \sum_i |\Delta T_{si}| e^{j\psi_i} e^{jiT} + |\overline{\Delta T}_{si}| e^{-j\psi_i} e^{-jiT}$$

$$332 \quad = \sum_{i=-\infty}^{+\infty} |\Delta T_{si}| e^{j(iT+\psi_i)} = 2 \sum_i |\Delta T_{si}| \cos i(T + \psi_i), \quad (30)$$

333 with $\Delta T_{si}^c = 2|\Delta T_{si}| \cos(\psi_i)$, $\Delta T_{si}^s = -2|\Delta T_{si}| \sin(\psi_i)$. A parameter ϵ is introduced, such as

$$334 \quad \epsilon = \frac{1}{\Omega^2} |\Delta T_s(T)| \frac{1}{\int_0^L (\Phi_{uk}^2 + \Phi_{vk}^2 + \Phi_{wk}^2) \mu \, dS} \int_0^L \frac{1}{\|\underline{X}'\|} \left[\Phi_{uk}^2 \kappa^2 + \Phi_{uk} \Phi'_{vk} \kappa - \Phi_{vk} \Phi'_{uk} \kappa - \Phi_{vk} \Phi''_{vk} - \Phi_{wk} \Phi''_{wk} \right] dS, \quad (31)$$

335 where $|\Delta T_s(T)| = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{\infty} (|\Delta T_{si}^c|^2 + |\Delta T_{si}^s|^2)}$. Without loss of generality, Eq. (13) can be rewritten
 336 in the following form,

$$337 \quad \frac{d^2 q(T)}{dT^2} + \epsilon c \frac{dq(T)}{dT} + (\delta + \epsilon \Delta T^*(T)) q(T) = 0, \quad (32)$$

338 with $\Delta T^*(T) = \frac{\Delta T_s(T)}{|\Delta T_s(T)|}$.

339 Eq. (32) highlights a multi-harmonic $T_s^*(T)$ parametric excitation so that it is called Mathieu-
 340 Hill equation. In order to detect pumping, which corresponds to operation in an unstable zone
 341 where the response q is theoretically unbounded in the absence of non-linear and damping effects,
 342 a stability study of the Mathieu-Hill equation Eq. (32) is proposed using a multi-scale method.

343 Initially, no damping is taken into account on the cable line. It will then be added and modifications
 344 to the instability regions discussed.

345 The parametric excitation term is multi-harmonic,

$$346 \quad \Delta T_s^*(T) = \sum_{i>0} \Delta T_{s_i}^{*c} \cos(iT) + \Delta T_{s_i}^{*s} \sin(iT). \quad (33)$$

347 The multi-scale method consists of developing the dynamic response $q(T)$ as a function of the
 348 parameter ϵ and time T (Nayfeh and Mook 1979):

$$349 \quad T_i = \epsilon^i T, \quad i \in \mathbf{N}, \quad (34)$$

$$350 \quad \frac{d}{dT} = D_0 + \epsilon D_1 + \epsilon^2 D_2 + \dots, \quad \text{with } D_i = \frac{\partial}{\partial T_i}, \quad (35)$$

$$351 \quad q(T) = q_0(T_0, T_1, T_2, \dots) + \epsilon q_1(T_0, T_1, T_2, \dots) + \epsilon^2 q_2(T_0, T_1, T_2, \dots). \quad (36)$$

352 Developments at multiple scales are replaced in Eq. (32),

$$353 \quad (D_0 + \epsilon D_1 + \epsilon^2 D_2 + \dots)^2 (q_0 + \epsilon q_1 + \epsilon^2 q_2 + \dots) + \left[\delta + \epsilon \left(\sum_{i>0} \Delta T_{s_i}^{*c} \cos(iT) + \Delta T_{s_i}^{*s} \sin(iT) \right) \right] (q_0 + \epsilon q_1 + \epsilon^2 q_2 + \dots) \\ 354 \quad = 0. \quad (37)$$

355 The terms at each order of Eq. (37) are identified.

356 At order ϵ^0 , one can get:

$$357 \quad D_0^2 q_0 + \delta q_0 = 0. \quad (38)$$

358 Thus, a solution q_0 is written:

$$359 \quad q_0(T_0, T_1, T_2, \dots) = Q(T_1, T_2, \dots) e^{j\sqrt{\delta}T_0} + \bar{Q}(T_1, T_2, \dots) e^{-j\sqrt{\delta}T_0}, \quad (39)$$

360 where Q does not depend on T_0 .

363 At order ϵ^1 , one can get:

$$364 \quad D_0^2 q_1 + \delta q_1 = -2D_0 D_1 q_0 - \left(\sum_{i>0} \Delta T_{s_i}^{*c} \cos(iT) + \Delta T_{s_i}^{*s} \sin(iT) \right) q_0. \quad (40)$$

365 Resonance of order n is studied, $n \approx \sqrt{\delta} \rightarrow n = \sqrt{\delta} + \epsilon\sigma$, with σ a detuning parameter. By replacing
 366 q_0 by its expression given by the development to order ϵ^0 , the terms of the second member of Eq.
 367 (40) are expressed and given by the Appendix 2.

368 In the dynamic equation at order ϵ , terms of the form $\mathcal{A}e^{j\sqrt{\delta}T_0}$ are resonant terms, i.e. if they
 369 are not zero, $\mathcal{A}(T_1, T_2, \dots) = 0$, the solution q_1 is not bounded. However, we're looking for periodic
 370 solutions according to Floquet's theory, i.e. bounded solutions. \mathcal{A} must therefore be zero: we say
 371 that we're eliminating the secular terms.

372 The resonant terms of the second member are of the kind $e^{j\sqrt{\delta}T_0}$ and are identified as

$$373 \quad -2D_1 Q j\sqrt{\delta} e^{j\sqrt{\delta}T_0} - \Delta T_{s_{2n}}^* \bar{Q} e^{j(2n-\sqrt{\delta})T_0}. \quad (41)$$

because for $i = 2n = 2\sqrt{\delta} + 2\epsilon\sigma$:

$$e^{j(2n-\sqrt{\delta})T_0} = e^{j(2\sqrt{\delta}+2\epsilon\sigma-\sqrt{\delta})T_0} = e^{j2\epsilon\sigma T_0} e^{j\sqrt{\delta}T_0}.$$

374 Identifying $T_1 = \epsilon T_0$, cancelling the resonant terms of Eq. (41) leads to:

$$375 \quad -2D_1 Q j\sqrt{\delta} - \Delta T_{s_{2n}}^* \bar{Q} e^{j2\sigma T_1} = 0. \quad (42)$$

376 The remaining non-resonant terms are:

$$377 \quad \sum_i \Delta T_{s_i}^* Q e^{j(i+\sqrt{\delta})T_0} + \sum_{i \neq 2n} \Delta T_{s_i}^* \bar{Q} e^{j(i-\sqrt{\delta})T_0} + cc, \quad (43)$$

378 with cc the complex conjugate.

379 The particular solution in q_1 is calculated by keeping the non-resonant terms in the second

380 member of Eq. (40),

$$381 \quad D_0^2 q_1 + \delta q_1 = - \sum_i \Delta T_{si}^* Q e^{j(i+\sqrt{\delta})T_0} - \sum_{i \neq 2n} \Delta T_{si}^* \bar{Q} e^{j(i-\sqrt{\delta})T_0} + cc. \quad (44)$$

382 q_1 is sought in the form:

$$383 \quad q_1(T_0, T_1, T_2, \dots) = \sum_i K_{1i}(T_1, T_2, \dots) e^{j(i+\sqrt{\delta})T_0} + \sum_{i \neq 2n} K_{2i}(T_1, T_2, \dots) e^{j(i-\sqrt{\delta})T_0}. \quad (45)$$

384 By substituting,

$$385 \quad \sum_i K_{1i}(-(i+\sqrt{\delta})^2 + \delta) e^{j(i+\sqrt{\delta})T_0} + \sum_{i \neq 2n} K_{2i}(-(i-\sqrt{\delta})^2 + \delta) e^{j(i-\sqrt{\delta})T_0} + cc \\ 386 \quad = - \sum_i \Delta T_{si}^* Q e^{j(i+\sqrt{\delta})T_0} - \sum_{i \neq 2n} \Delta T_{si}^* \bar{Q} e^{j(i-\sqrt{\delta})T_0} + cc. \quad (46)$$

387 By identification, $K_{1i} = \frac{\Delta T_{si}^* Q}{i(i+2\sqrt{\delta})}$, $K_{2i} = \frac{\Delta T_{si}^* \bar{Q}}{i(i-2\sqrt{\delta})}$. As,

$$388 \quad q_1(T_0, T_1, T_2, \dots) = \sum_i \frac{\Delta T_{si}^* Q}{i(i+2\sqrt{\delta})} e^{j(i+\sqrt{\delta})T_0} + \sum_{i \neq 2n} \frac{\Delta T_{si}^* \bar{Q}}{i(i-2\sqrt{\delta})} e^{j(i-\sqrt{\delta})T_0} + cc, \quad (47)$$

389 at order ϵ^2 , one can get:

$$390 \quad D_0^2 q_2 + \delta q_2 = -2D_0 D_2 q_0 - D_1^2 q_0 - 2D_0 D_1 q_1 - \left(\sum_i \Delta T_{si}^* e^{jiT_0} + \overline{\Delta T_{si}^*} e^{-jiT_0} \right) q_1. \quad (48)$$

391 The same procedure is applied to the order ϵ^2 and is given by the Appendix 3.

392 Finally, one can obtain the following equation by eliminating the secular terms at the order ϵ^2 :

$$393 \quad -2j\sqrt{\delta} D_2 Q + \frac{\Delta T_{s2n}^* \sigma \bar{Q}}{\sqrt{\delta}} e^{j2\sigma T_1} - \left(\sum_{m \neq 2n} \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*}}{m(m-2\sqrt{\delta})} + \sum_m \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*}}{m(m+2\sqrt{\delta})} + \frac{\Delta T_{s2n}^* \overline{\Delta T_{s2n}^*}}{4\delta} \right) Q \\ 394 \quad - \left(\frac{\Delta T_{sn}^{*2}}{n(n-2\sqrt{\delta})} + \sum_m \sum_{\substack{l \\ m-l=2n}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*}}{l(l+2\sqrt{\delta})} + \sum_m \sum_{\substack{l \\ l \neq 2n \\ -m+l=2n}} \frac{\overline{\Delta T_{sm}^*} \Delta T_{sl}^*}{l(l-2\sqrt{\delta})} \right) \bar{Q} e^{j2\epsilon\sigma T_0}$$

$$= 0. \quad (49)$$

The complex quantity ΔT_{s2n}^* is expressed as a function of its phase and modulus,

$$\Delta T_{s2n}^* = |\Delta T_{s2n}^*| e^{j\phi_1}. \quad (50)$$

The real term resulting from parametric excitation is denoted by

$$\Gamma_1 = \sum_{m \neq 2n} \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*}}{m(m - 2\sqrt{\delta})} + \sum_m \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*}}{m(m + 2\sqrt{\delta})} + \frac{\Delta T_{s2n}^* \overline{\Delta T_{s2n}^*}}{4\delta}, \quad \Gamma_1 \in \mathbf{R}. \quad (51)$$

The complex term resulting from parametric excitation is denoted by

$$\Gamma_{2R} + j\Gamma_{2I} = |\Gamma_2| e^{j\phi_2} = \frac{\Delta T_{sn}^{*2}}{n(n - 2\sqrt{\delta})} + \sum_m \sum_{\substack{l \\ m-l=2n}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*}}{l(l + 2\sqrt{\delta})} + \sum_m \sum_{\substack{l \\ l \neq 2n \\ -m+l=2n}} \frac{\overline{\Delta T_{sm}^*} \Delta T_{sl}^*}{l(l - 2\sqrt{\delta})}. \quad (52)$$

The secular terms vanishing condition to be imposed for finding non-trivial solutions is given by Eq. (94) that can be rewritten according to Γ_1 and Γ_2 ,

$$-2j\sqrt{\delta}D_2Q + \frac{|\Delta T_{s2n}^*| \sigma \overline{Q}}{\sqrt{\delta}} e^{j(2\sigma T_1 + \phi_1)} - \Gamma_1 Q - |\Gamma_2| \overline{Q} e^{j(2\epsilon\sigma T_0 + \phi_2)} = 0. \quad (53)$$

The two cancellation conditions for secular terms of orders ϵ and ϵ^2 given by Eq. (42) and Eq. (53) result from the expansion to different orders ϵ^n of a reconstituted modulation equation (Luongo et al. 2002):

$$-2j\sqrt{\delta} \frac{dQ}{dT} + \epsilon \left(-1 + \frac{\epsilon\sigma}{\sqrt{\delta}} \right) \overline{Q} |\Delta T_{s2n}^*| e^{j(2\sigma\epsilon T + \phi_1)} - \epsilon^2 \left(\Gamma_1 Q + |\Gamma_2| \overline{Q} e^{j(2\epsilon\sigma T + \phi_2)} \right) = 0. \quad (54)$$

In order to eliminate the time T , the solution is sought in the form $Q = (Q_R + jQ_I) e^{j(\epsilon\sigma T + \frac{\phi_1}{2})}$, i.e. by injecting into Eq. (54),

$$\begin{aligned}
& -2j\sqrt{\delta} \left(\left[\frac{dQ_R}{dT} + j \frac{dQ_I}{dT} \right] e^{j(\epsilon\sigma T + \frac{\phi_1}{2})} + (Q_R + jQ_I) j\epsilon\sigma e^{j(\epsilon\sigma T + \frac{\phi_1}{2})} \right) + \epsilon \left(-1 + \frac{\epsilon\sigma}{\sqrt{\delta}} \right) |\Delta T_{s2n}^*| (Q_R - jQ_I) e^{j(\epsilon\sigma T + \frac{\phi_1}{2})} \\
& - \epsilon^2 \left(\Gamma_1 (Q_R + jQ_I) e^{j(\epsilon\sigma T + \frac{\phi_1}{2})} + (\Gamma_{2R} + j\Gamma_{2I}) (Q_R - jQ_I) e^{j(\epsilon\sigma T + \frac{\phi_1}{2})} \right) = 0. \quad (55)
\end{aligned}$$

The Appendix 4 gives details of how to obtain the conditions to guarantee the existence of a non-trivial solution of the kind $(Q_R, Q_I) = (q_R, q_I) e^{\gamma T}$. A polynomial of degree two in γ is obtained such that the solutions are written in the form,

$$\gamma_{1,2} = \frac{4\sqrt{\delta}\epsilon^2\Gamma_{2I} \pm \sqrt{16\delta C}}{8\delta}, \quad (56)$$

where $C = \epsilon^2 \left[-|\Delta T_{s2n}^*| + 2\sqrt{\delta}\sigma + \frac{\epsilon\sigma|\Delta T_{s2n}^*|}{\sqrt{\delta}} - \epsilon(\Gamma_1 + \Gamma_{2R}) \right] \left[-|\Delta T_{s2n}^*| - 2\sqrt{\delta}\sigma + \frac{\epsilon\sigma|\Delta T_{s2n}^*|}{\sqrt{\delta}} + \epsilon(\Gamma_1 + \Gamma_{2R}) \right]$.

According to the Floquet theory, motion is at the boundary of the instability zone when $\gamma = 0$, i.e.:

$$\epsilon^2\Gamma_{2I} \pm \sqrt{C} = 0, \quad (57)$$

$$\epsilon^4\Gamma_{2I}^2 = C. \quad (58)$$

After further development by replacing the expression of C in Eq. (58), the equation to be solved is:

$$\epsilon^2|\Delta T_{s2n}^*|^2 - 4\delta\sigma^2\epsilon^2 - 4\sqrt{\delta}\sigma\epsilon^3(\Gamma_1 + \Gamma_{2R}) + \frac{\epsilon^4\sigma^2}{\delta} - \epsilon^4(\Gamma_1 + \Gamma_{2R})^2 + \epsilon^4\Gamma_{2I}^2 = 0. \quad (59)$$

Only quantities that imply a product between σ and ϵ with ϵ of order lower than two are considered, so Eq. (59) is written:

$$\epsilon^2|\Delta T_{s2n}^*|^2 - 4\delta\sigma^2\epsilon^2 - \epsilon^4(\Gamma_1 + \Gamma_{2R})^2 + \epsilon^4\Gamma_{2I}^2 = 0. \quad (60)$$

One can obtain:

$$\sigma^2 = \frac{|\Delta T_{s2n}^*|^2}{4\delta} + \frac{\epsilon^2}{4\delta} \left[(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2 \right]. \quad (61)$$

431 An expansion of σ is sought in the form $\sigma = k_1 + \epsilon k_2$, i.e.:

$$432 \quad \sigma^2 = k_1^2 + 2\epsilon k_1 k_2 + \epsilon^2 k_2^2. \quad (62)$$

433 By identification with Eq. (61):

$$434 \quad k_1 = \pm \frac{|\Delta T_{s2n}^*|}{2\sqrt{\delta}}, \quad (63)$$

$$435 \quad k_2 = \pm \frac{\sqrt{(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2}}{2\sqrt{\delta}}. \quad (64)$$

437 Thus, the instability zones of the cable transport line dynamic response are given by:

$$438 \quad \sigma > -\frac{|\Delta T_{s2n}^*|}{2\sqrt{\delta}} - \epsilon \frac{\sqrt{(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2}}{2\sqrt{\delta}}, \quad (65)$$

$$439 \quad \sigma < \frac{|\Delta T_{s2n}^*|}{2\sqrt{\delta}} - \epsilon \frac{\sqrt{(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2}}{2\sqrt{\delta}}. \quad (66)$$

441 Note that $n = \sqrt{\delta} + \epsilon\sigma$, so $n^2 = \delta + 2\sqrt{\delta}\epsilon\sigma + \epsilon\sigma^2$. The boundaries separating zones of stability
442 from zones of instability are expressed as a function of n and δ ,

$$443 \quad n^2 = \delta + \epsilon \left(2\sqrt{\delta} \left[-\frac{|\Delta T_{s2n}^*|}{2\sqrt{\delta}} - \epsilon \frac{\sqrt{(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2}}{2\sqrt{\delta}} \right] \right), \quad (67)$$

$$444 \quad n^2 = \delta + \epsilon \left(2\sqrt{\delta} \left[\frac{|\Delta T_{s2n}^*|}{2\sqrt{\delta}} - \epsilon \frac{\sqrt{(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2}}{2\sqrt{\delta}} \right] \right). \quad (68)$$

446 After simplification, by replacing δ by its expression as a function of ω_p , the pulsation of the mode,
447 and Ω , the fundamental of the parametric excitation,

$$448 \quad n^2 = \frac{\omega_p^2}{\Omega^2} - \epsilon |\Delta T_{s2n}^*| - \epsilon^2 \sqrt{(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2}, \quad (69)$$

449

450

$$n^2 = \frac{\omega_p^2}{\Omega^2} + \epsilon |\Delta T_{s2n}^*| - \epsilon^2 \sqrt{(\Gamma_1 + \Gamma_{2R})^2 + \Gamma_{2I}^2}. \quad (70)$$

451

452

453

Up to now, damping has not been taken into account in stability study. A physical system always has damping, so it is reasonable to study the effect of a damping contribution $c\dot{q}$, reduced to the order ϵ , on the stability zones of the Mathieu-Hill equation Eq. (32),

454

$$\frac{d^2 q(T)}{dT^2} + \epsilon c \frac{dq(T)}{dT} + \left(\delta + \epsilon \Delta T_s^*(T) \right) q(T) = \epsilon f^*(T). \quad (71)$$

455

Details of developments with damping are given in the Appendix 5.

456

457

The boundaries separating zones of stability from zones of instability are expressed as a function of n et $\delta = \frac{\omega_p}{\Omega}$,

458

$$n^2 = \frac{\omega_p^2}{\Omega^2} - \epsilon \sqrt{|\Delta T_{s2n}^*|^2 - c^2 n^2} - \epsilon^2 \sqrt{\left(\Gamma_1 - \frac{c^2}{4} + \Gamma_{2R} \right)^2 + \Gamma_{2I}^2}, \quad (72)$$

459

460

$$n^2 = \frac{\omega_p^2}{\Omega^2} + \epsilon \sqrt{|\Delta T_{s2n}^*|^2 - c^2 n^2} - \epsilon^2 \sqrt{\left(\Gamma_1 - \frac{c^2}{4} + \Gamma_{2R} \right)^2 + \Gamma_{2I}^2}. \quad (73)$$

461

Assuming $c_0 = \epsilon c$,

462

$$n^2 = \frac{\omega_p^2}{\Omega^2} - \sqrt{\epsilon^2 |\Delta T_{s2n}^*|^2 - c_0^2 n^2} - \epsilon^2 \sqrt{\left(\Gamma_1 - \frac{c_0^2}{4\epsilon^2} + \Gamma_{2R} \right)^2 + \Gamma_{2I}^2}, \quad (74)$$

463

464

$$n^2 = \frac{\omega_p^2}{\Omega^2} + \sqrt{\epsilon^2 |\Delta T_{s2n}^*|^2 - c_0^2 n^2} - \epsilon^2 \sqrt{\left(\Gamma_1 - \frac{c_0^2}{4\epsilon^2} + \Gamma_{2R} \right)^2 + \Gamma_{2I}^2}. \quad (75)$$

465

466

467

468

469

Given the frequencies commonly involved in cable transport line oscillations, and the feedback from practical pumping situations which occur at a frequency equal to twice the frequency at which vehicles pass through the spans, $n = 2$ is adopted in the general case. Instability zones are therefore plotted around $\delta = n^2 = 4$ in Figure 5 for two excitation levels $|\Delta T_{s2n}^*|$ and three damping values c_0 .

According to Figure 5, increasing damping has the effect of lifting the instability zone off the

470 δ axis and narrowing its width. Still according to Figure 5, the term $|\Delta T_{s_{2n}}^*|$, which represents the
471 relative amplitude of the second harmonic of the parametric excitation by the tension ΔT_s^* , plays an
472 important role in defining the instability zone: an increase in $|\Delta T_{s_{2n}}^*|$ widens the instability zone.

473 **Analytical interpretation of the pumping**

474 According to an analytical approach, the pumping phenomenon affecting continuous-motion
475 cable transport lines can be interpreted as the consequence of a parametric resonance of the rope
476 loop introduced by the quasi-static tension variations generated by the vehicles translation. Para-
477 metric resonance corresponds to the cable transport line's operating point associated with a zone
478 of instability in the underlying dynamical system governing its behavior. As highlighted in ??, the
479 stability study can be reduced to the positioning of a point in the space of two reduced parameters
480 (δ, ϵ) depending on the physical quantities of the cable transport line. Depending on the position
481 of the operating point relative to the boundary, it is possible to define whether oscillations are likely
482 to be amplified or not. The boundary is given by a plane curve separating a stable zone, for which
483 the cable transport line motion is bounded, from an unstable zone, for which the cable transport
484 line motion is theoretically unbounded. In this way, pumping results in a parametric instability
485 phenomenon that leads to high-amplitude oscillations.

486 **NUMERICAL VALIDATION AND DESIGN APPLICATION**

487 **Pumping mode**

488 Using numerical data from the Table 1 corresponding to the chairlift case study and pumping ob-
489 servations, the first four vibration modes of the line are calculated numerically using an analytical-
490 numerical method described by (Bécu et al. 2023; Bécu et al. 2024) (see Figure 6). The modes
491 couple longitudinal and vertical dynamic of the rope. According to Figure 6, modes at $\omega_1 = 1.36$
492 [rad/s] and $\omega_3 = 1.75$ [rad/s] imply an anti-symmetrical vertical vibration between the two tracks,
493 localised in certain spans, predominating over the longitudinal vibration, which is symmetrical.
494 According to Figure 6,, modes at $\omega_2 = 1.41$ [rad/s] and $\omega_4 = 1.78$ [rad/s] imply a generalized
495 longitudinal vibration over the entire line, outweighing the vertical vibration localised in certain

496 spans. According to Figure 6, modes at $\omega_2 = 1.41$ [rad/s] and $\omega_4 = 1.78$ [rad/s] lose all symmetry
497 due to the different boundary conditions at the two end stations and translational effect of the cable,
498 which are mainly involved in the longitudinal component of the dynamics, introducing a break in
499 symmetry between the two tracks.

500 The mode at $\omega_1 = 1.36$ [rad/s] is identified as the pumping mode observed experimentally for
501 $(V_0, N_v) = (4.25, N_v^1)$ configuration. In fact, this mode localises the energy of the vibration on both
502 tracks in span # 8, with a phase opposition of the oscillations between the two sides (see Figure 7).
503 The modal properties actually display a dependence on the position of the vehicles over a period of
504 travel T . A parametric study carried out with a model that takes into account the punctual nature
505 of vehicle loading on the cable line shows that variations in modal pulsation are of the order of
506 magnitude of $\Delta\omega_1 \approx 0.01\omega_1$.

507 **Tension signal**

508 The signal corresponding to the evolution of the quasi-static tension of the cable line at span # 8
509 is calculated numerically with a quasi-static model of the chairlift line. Numerical data used for the
510 calculation come from the Table 1. The calculation is performed according to two configurations.

511 Firstly, for $(V_0, N_v) = (4.25, N_v^1)$, which corresponds to the pumping experimental configura-
512 tion.

513 Figure 8 shows the evolution of the quasi-static tension T_s in span # 8 as a function of the
514 evolution parameter l_c according to two models with $N_v = N_v^1$. Figure 8 curve in blue is based on
515 a model that considers the tower support as a point. Figure 8 curve in yellow considers the support
516 to be linear and the modification of the vehicle's equilibrium, by contact forces in addition to the
517 effects of the tension, when it is in contact with the tower's sheave assembly. The effect of the
518 interaction between the vehicle grip attached to the cable and the rollers of the sheave assembly can
519 be observed for $l_c \in [22, 26]$ [m] with a transient cable tension decrease. That can be explained by
520 a mechanism of force transfer: a part of the vehicle's weight, which was transmitted to the cable by
521 the tension, is taken up by the support, reducing the corresponding tension value. This last model
522 provides a more realistic description.

523 Figure 9 shows the relationship between the amplitude of the quasi-static tension perturbation
524 and the quasi-static tension averaged over a period of vehicle travel in each of the two previous
525 models with $N_v = N_v^1$. One can note that by considering the contact between the support and the
526 vehicle in the underlying model, the variation in cable tension is amplified as the vehicles pass
527 over the support. For the span # 8, the relative tension perturbation increases by 2.5 % to 5 %.
528 This increase is explained by the geometrical coincidence effect, described in 4, in the span # 8
529 with the inter-vehicle spacing corresponding to $N_v = N_v^1$ and which involves contact forces. In
530 this specific case, contact between vehicle and tower is an important factor in characterizing the
531 excitation source.

532 Figure 10 shows the Fourier's transform of the tension signal over a period of vehicles passing
533 through span # 8 with $N_v = N_v^1$. The left-hand plot shows the transform of the tension signal
534 without considering the support contact. The right-hand plot is obtained from the tension signal
535 with support contact taken into account. The effect of contact is to increase the amplitude of the
536 peaks visible in the spectrum from the second harmonic.

537 Secondly, the same plots are made for the $(V_0, N_v) = (4.25, N_v^1 - 1)$ configuration for which
538 pumping oscillations are no longer observed experimentally.

539 Figure 11 curve in blue is based on a model that considers the tower support as a point. Figure 11
540 curve in yellow considers the support to be linear and the modification of the vehicle's equilibrium
541 by contact. The effect of the interaction between the vehicle grip attached to the cable and the
542 rollers of the sheave assembly can not be longer observed for $N_v = N_v^1 - 1$ since there is no longer
543 any geometric coincidence effect in the span # 8.

544 Figure 12 shows the relationship between the amplitude of the quasi-static tension perturbation
545 and the quasi-static tension averaged over a period of vehicle travel in each of the two previous
546 models with $N_v = N_v^1 - 1$. One can note that there is no longer any significant difference between
547 the two calculation models. For the span # 8, the relative tension perturbation is 1.6 % without
548 considering contact and 1.4 % by considering contact.

549 Figure 13 shows the Fourier's transform of the tension signal over a period of vehicles passing

550 through span # 8 with $N_v = N_v^1 - 1$. The left-hand plot shows the transform of the tension signal
 551 without considering the support contact. The right-hand plot is obtained from the tension signal
 552 with support contact taken into account. The effect of contact does not significantly modify the
 553 amplitude of the peaks visible in the spectrum.

554 **Operating point positioning**

555 The operating points (δ, ϵ) are calculated from the quasi-static tension signal analysis and the
 556 modal calculation corresponding to the two configurations $(V_0, N_v) = (4.25, N_v^1)$ and $(V_0, N_v) =$
 557 $(4.25, N_v^1 - 1)$. These points are placed in the stability diagram given by Figure 14 associated with
 558 the reduced model describing the chairlift's dynamic behavior:

- 559 • for $(V_0, N_v) = (4.25, N_v^1)$, which corresponds to the pumping experimental configuration:
 - 560 • $\omega_1^2 \in [1.67, 2.03] \text{ [s}^{-2}\text{]}$ taking into account an uncertainty interval of $\pm 5 \%$ on the
 561 modal pulsation introduced by model parameter uncertainty, $\Omega^2 = 0.429 \text{ [s}^{-2}\text{]} \rightarrow$
 562 $\delta \in [3.89, 4.73] \text{ []}$ (see solid horizontal interval),
 - 563 • $|\Delta T_s| = 4934 \text{ [N]}$, $|\Delta T_{s,2n}^*| = 0.21 \text{ []} \rightarrow \epsilon = 0.12 \text{ []}$.
- 564 • for $(V_0, N_v) = (4.25, N_v^1 - 1)$, which corresponds to a configuration without pumping:
 - 565 • $\omega_1^2 \in [1.67, 2.03] \text{ [s}^{-2}\text{]}$ taking into account an uncertainty interval of $\pm 5 \%$ on the
 566 modal pulsation introduced by model parameter uncertainty, $\Omega^2 = 0.410 \text{ [s}^{-2}\text{]} \rightarrow$
 567 $\delta \in [4.07, 4.795] \text{ []}$ (see solid horizontal interval),
 - 568 • $|\Delta T_s| = 1725 \text{ [N]}$, $|\Delta T_{s,2n}^*| = 0.19 \text{ []} \rightarrow \epsilon = 0.05 \text{ []}$.

569 The positioning of the chairlift's operating point in the stability diagram confirms that the config-
 570 uration $(V_0, N_v) = (4.25, N_v^1)$ may be unstable, with amplification of the dynamic movement of
 571 the rope line as the system evolves with a steady state translation of vehicles, while the config-
 572 uration $(V_0, N_v) = (4.25, N_v^1 - 1)$ is stable. When the vehicles move along the line, a parametric
 573 study shows that variations in modal pulsation are of the order of magnitude of $\Delta\omega_1 \approx 0.01\omega_1$
 574 which modifies the δ of ± 0.04 . Indeed, Figure 14 shows that the operating point associated with

575 $(V_0, N_v) = (4.25, N_v^1)$ is likely to move into an unstable zone for a value of the damping coefficient
 576 $c_0 \in [0, 0.01]$.

577 **Design application**

578 The designer's aim is to stabilize the pumping dynamics observed for $(V_0, N_v) = (4.25, N_v^1)$.
 579 It has been seen that removing a vehicle, from N_v^1 to $N_v^1 - 1$, stabilizes the line but this is not
 580 necessarily the best solution as it reduces the passenger flow. Several parameters can be modified
 581 a posteriori when pumping is observed on an installation according to different strategies:

- 582 • By reducing the level of the excitation source through the cable speed V_0 : the effect of a
 583 change in speed highlights the notion of critical speed or forbidden speed band that can be
 584 easily entered by the system's operator to avoid pumping.

585 The boundary separating the stable from the instable zones is given by the following
 586 relation:

$$587 (n^2 - \delta)^2 = \epsilon^2 |T_{s,2n}^*|^2 - n^2 c_0^2. \quad (76)$$

588 All other parameters being equal, as the cable speed V_0 changes, the excitation level ϵ re-
 589 mains equal while $\delta = \frac{\omega_1^2}{\Omega^2}$ varies with $\Omega = \frac{2\pi V_0}{l_d}$.

590 In the configuration $(V_0, N_v) = (4.25, N_v^1)$, one can defined a forbidden speed band
 591 $V_{0,f} = [V_{0,f1}, V_{0,f2}]$ corresponding to the operating points inside the interval $[\delta_1, \delta_2]$ when
 592 the operating point is on or inside the boundary of instability:

$$593 \delta_1 = 4 - \sqrt{\epsilon^2 |T_{s,2n}^*|^2 - 4c_0^2}, \quad (77)$$

$$594 \delta_2 = 4 + \sqrt{\epsilon^2 |T_{s,2n}^*|^2 - 4c_0^2}, \quad (78)$$

596 and by considering the variations in modal pulsation which modifies δ of ± 0.04 . A numer-
 597 ical application yields $V_{0,f} = [4.19, 4.31]$ [m/s].

- 598 • With a control of the modal characteristics of the line (Φ_k, ω_k) by moving an intermedi-
 599 ate support or modifying the level of mechanical tension controlled by Δ^2 parameter and

600 imposed on the cable loop by the counterweight or hydraulic actuator.

601 In general, support and cable tension modifications require heavy and invasive interventions on
602 the installation, questioning its initial dimensioning. In another perspective of active damping of
603 pumping instabilities, the role of the spectral content of tension evolution in triggering parametric
604 resonance has been highlighted, opening the door to the study of a control-active system that would
605 mechanically filter out harmful tension harmonics to escape system instabilities.

606 **CONCLUSION AND PERSPECTIVES**

607 This paper proposes an analysis of the pumping affecting single-cable transport lines by studying
608 the cable tension signal as the system evolves over time. Pumping has long been known to occur in
609 the ropeway industry but there is no analytical model for anticipating it in the design phase.

610 The analysis proposed in this paper is based on the case study of a chairlift that is subject to
611 pumping, with tests carried out under real conditions for several configurations. The effect of a
612 change in rope speed and vehicle spacing on the dynamic behaviour of the line is experimentally
613 tested.

614 Based on the experimental data collected on the real system, a heuristic analysis of pumping
615 reveals an internal excitation mechanism generated by the vehicles' travel, amplified by a geometric
616 coincidence affecting the cable tension when two vehicles pass simultaneously over two support
617 sheave assembly of the same span, which excites in a privileged way a mode of the cable line.

618 The analytical interpretation of pumping is based on the study of the Mathieu-Hill equation,
619 which governs the dynamics of the system, under the effect of a parametric perturbation by the
620 periodic tension evolution of the system. Parametric resonances correspond to zones of instability
621 in which oscillations are amplified.

622 A numerical application highlights the pumping mode involved in chairlift line oscillations.
623 The cable tension signal is calculated from the numerical model of the quasi-static evolution of the
624 chairlift line as the vehicles move. The Fourier transform of the signal is used to extract the spectral
625 content and shows the role of contact between the vehicle and the support in the excitation.

626 Finally, in the light of the results, possible approaches are proposed for defining optimal design
 627 parameter, in order to avoid system operating points likely to activate pumping.

628 **Appendix 1: Reduced order model (ROM)**

629 A reduced order model (ROM) is obtained using an inner product, defined on $\mathcal{H}_0([0, L])$ func-
 630 tions. The inner product is applied on the set of linear dynamic equations, expressed in the strong
 631 form Eq. (5), which involve a displacement $\underline{U}(S, t) = q(t)\underline{\Phi}_k(S)$ written as a linear combination
 632 of mode shape functions, and a test function which is evaluated as the mode shape unknown. This
 633 is an equivalent weak formulation of the problem given by Eq. (5) written in strong form, such as

$$\begin{aligned}
 & \int_0^L (\Phi_{uk}^2 + \Phi_{vk}^2 + \Phi_{wk}^2) \mu \, dS \, \ddot{q} \\
 & + \int_0^L \left[c_u \Phi_{uk}^2 \, dS + c_v \Phi_{vk}^2 + c_w \Phi_{wk}^2 - 2\mu V_0 [\Phi'_{uk} \Phi_{uk} + \Phi'_{vk} \Phi_{vk} + \Phi'_{wk} \Phi_{wk}] \right] dS \, \dot{q} \\
 & + \int_0^L \left[\Phi_{uk}^2 \frac{T_s \kappa^2}{\|\underline{X}'\|} + \Phi_{vk}^2 \kappa^2 EA + \Phi_{uk} \Phi'_{vk} \left(\frac{T_s}{\|\underline{X}'\|} + EA \right) \kappa - \Phi_{vk} \Phi'_{uk} \left(\frac{T_s}{\|\underline{X}'\|} + EA \right) \kappa - \Phi_{uk} \Phi''_{uk} EA \right. \\
 & \quad \left. - \Phi_{vk} \Phi''_{vk} \frac{T_s}{\|\underline{X}'\|} - \Phi_{wk} \Phi''_{wk} \frac{T_s}{\|\underline{X}'\|} \right. \\
 & \left. + \mu V_0^2 [\Phi''_{uk} \Phi_{uk} + \Phi''_{vk} \Phi_{vk} + \Phi''_{wk} \Phi_{wk}] + \mu A V_0 [\Phi'_{uk} \Phi_{uk} + \Phi'_{vk} \Phi_{vk} + \Phi'_{wk} \Phi_{wk}] \right] dS \, q = 0. \quad (79)
 \end{aligned}$$

639 The differential equation in time verified by the generalised coordinate q after projection onto the
 640 mode $\underline{\Phi}_k$ is that of a damped harmonic oscillator of ω_k pulsation according to the mode of vibration
 641 definition,

$$\ddot{q} + \left(c_k + \Psi_k^1(V_0) \right) \dot{q} + \left(\omega_k^2 + \Psi_k^2(V_0, A V_0) \right) q = 0. \quad (80)$$

643 such as $\omega_k^2 = \frac{\int_0^L \left[\Phi_{uk}^2 \frac{T_s \kappa^2}{\|\underline{X}'\|} + \Phi_{vk}^2 \kappa^2 EA + \Phi_{uk} \Phi'_{vk} \left(\frac{T_s}{\|\underline{X}'\|} + EA \right) \kappa - \Phi_{vk} \Phi'_{uk} \left(\frac{T_s}{\|\underline{X}'\|} + EA \right) \kappa - \Phi_{uk} \Phi''_{uk} EA - \Phi_{vk} \Phi''_{vk} \frac{T_s}{\|\underline{X}'\|} - \Phi_{wk} \Phi''_{wk} \frac{T_s}{\|\underline{X}'\|} \right] dS}{\int_0^L (\Phi_{uk}^2 + \Phi_{vk}^2 + \Phi_{wk}^2) \mu \, dS}$,

644

$$\Psi_k^2(V_0, A_{V_0}) = \frac{\int_0^L \mu V_0^2 [\Phi''_{uk} \Phi_{uk} + \Phi''_{vk} \Phi_{vk} + \Phi''_{wk} \Phi_{wk}] + \mu A_{V_0} [\Phi'_{uk} \Phi_{uk} + \Phi'_{vk} \Phi_{vk} + \Phi'_{wk} \Phi_{wk}] dS}{\int_0^L (\Phi_{uk}^2 + \Phi_{vk}^2 + \Phi_{wk}^2) \mu dS} \quad (81)$$

646

$$c_k = \frac{\int_0^L [c_u \Phi_{uk}^2 dS + c_v \Phi_{vk}^2 + c_w \Phi_{wk}^2 dS]}{\int_0^L (\Phi_{uk}^2 + \Phi_{vk}^2 + \Phi_{wk}^2) \mu dS} \quad (82)$$

647

648

649

$$\Psi_k^1(V_0) = \frac{\int_0^L -2\mu V_0 [\Phi'_{uk} \Phi_{uk} + \Phi'_{vk} \Phi_{vk} + \Phi'_{wk} \Phi_{wk}] dS}{\int_0^L (\Phi_{uk}^2 + \Phi_{vk}^2 + \Phi_{wk}^2) \mu dS} \quad (83)$$

650 Appendix 2: Multiple scale developments at order ϵ

651 At order ϵ , Eq. (40) is given by:

$$-2D_0 D_1 q_0 = -2D_1 [Q j \sqrt{\delta} e^{j\sqrt{\delta} T_0} - \bar{Q} j \sqrt{\delta} e^{-j\sqrt{\delta} T_0}] = -2D_1 Q j \sqrt{\delta} e^{j\sqrt{\delta} T_0} + 2D_1 \bar{Q} j \sqrt{\delta} e^{-j\sqrt{\delta} T_0}, \quad (84)$$

653

$$\begin{aligned} -\left(\sum_{i>0} \Delta T_{s_i}^{*c} \cos(iT) + \Delta T_{s_i}^{*s} \sin(iT) \right) q_0 &= -\left(\sum_{i>0} \Delta T_{s_i}^* e^{jiT_0} + \overline{\Delta T_{s_i}^*} e^{-jiT_0} \right) \left(Q e^{j\sqrt{\delta} T_0} + \bar{Q} e^{-j\sqrt{\delta} T_0} \right) \\ &= -\sum_i \Delta T_{s_i}^* Q e^{j(i+\sqrt{\delta})T_0} + \Delta T_{s_i}^* \bar{Q} e^{j(i-\sqrt{\delta})T_0} + \overline{\Delta T_{s_i}^*} Q e^{j(-i+\sqrt{\delta})T_0} + \overline{\Delta T_{s_i}^*} \bar{Q} e^{j(-i-\sqrt{\delta})T_0}, \quad (85) \end{aligned}$$

655

$$656 \text{ with } \Delta T_{s_i}^* = \frac{\Delta T_{s_i}^{*c} - j \Delta T_{s_i}^{*s}}{2}$$

657 Appendix 3: Multiple scale development at order ϵ^2

658 By substituting q_0 et q_1 with their expressions given by developments at order ϵ^0 et ϵ :

$$659 -2D_0 D_2 q_0 = -2j\sqrt{\delta} D_2 Q e^{j\sqrt{\delta} T_0} + 2j\sqrt{\delta} D_2 \bar{Q} e^{-j\sqrt{\delta} T_0}, \quad (86)$$

660

$$661 -D_1^2 q_0 = -D_1^2 Q e^{j\sqrt{\delta} T_0} + cc. \quad (87)$$

662 The term $-2D_0D_1q_1$ is not resonant.

$$\begin{aligned}
663 \quad & -\left(\sum_i T_{si}^* e^{jiT_0 + \overline{\Delta T_{si}^*}} e^{-jiT_0}\right) q_1 = -\sum_m \sum_l \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*} Q}{l(l+2\sqrt{\delta})} e^{j(m+l+\sqrt{\delta})T_0} - \sum_m \sum_{\substack{l \neq 2n \\ l \neq m}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*} \overline{Q}}{l(l-2\sqrt{\delta})} e^{j(m+l-\sqrt{\delta})T_0} \\
664 \quad & -\frac{T_{sn}^{*2} \overline{Q}}{n(n-2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} e^{j\sqrt{\delta}T_0} - \sum_m \sum_{\substack{l \neq 2n \\ m-l \neq 2n}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*} \overline{Q}}{l(l+2\sqrt{\delta})} e^{j(m-l-\sqrt{\delta})T_0} - \sum_m \sum_{\substack{l \neq 2n \\ m-l=2n}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*} \overline{Q}}{l(l+2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} e^{j\sqrt{\delta}T_0} \\
665 \quad & -\sum_m \sum_{\substack{l \neq 2n \\ l \neq m}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*} Q}{l(l-2\sqrt{\delta})} e^{j(m-l+\sqrt{\delta})T_0} - \sum_{m \neq 2n} \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*} Q}{m(m-2\sqrt{\delta})} e^{j\sqrt{\delta}T_0} - \sum_m \sum_{l \neq m} \frac{\overline{\Delta T_{sm}^*} \Delta T_{sl}^* Q}{l(l+2\sqrt{\delta})} e^{j(-m+l+\sqrt{\delta})T_0} \\
666 \quad & -\sum_m \frac{\overline{\Delta T_{sm}^*} \Delta T_{sm}^* Q}{m(m+2\sqrt{\delta})} e^{j\sqrt{\delta}T_0} - \sum_m \sum_{\substack{l \neq 2n \\ -m+l \neq 2n}} \frac{\overline{\Delta T_{sm}^*} \Delta T_{sl}^* \overline{Q}}{l(l-2\sqrt{\delta})} e^{j(-m+l-\sqrt{\delta})T_0} - \sum_m \sum_{\substack{l \neq 2n \\ -m+l=2n}} \frac{\overline{\Delta T_{sm}^*} \Delta T_{sl}^* \overline{Q}}{l(l-2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} e^{j\sqrt{\delta}T_0} \\
667 \quad & -\sum_m \sum_l \frac{\overline{\Delta T_{sm}^*} \overline{\Delta T_{sl}^*} \overline{Q}}{l(l+2\sqrt{\delta})} e^{j(-m-l-\sqrt{\delta})T_0} - \sum_m \sum_{l \neq 2n} \frac{\overline{\Delta T_{sm}^*} \overline{\Delta T_{sl}^*} Q}{l(l-2\sqrt{\delta})} e^{j(-m-l+\sqrt{\delta})T_0}. \quad (88)
\end{aligned}$$

668 The secular terms of the equation in ϵ^2 are cancelled, i.e.:

$$\begin{aligned}
669 \quad & -2j\sqrt{\delta}D_2Q - D_1^2Q - \frac{\Delta T_{sn}^{*2} \overline{Q}}{n(n-2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} - \sum_m \sum_{\substack{l \neq 2n \\ m-l=2n}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}^*} \overline{Q}}{l(l+2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} \\
670 \quad & -\sum_{m \neq 2n} \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*} Q}{m(m-2\sqrt{\delta})} - \sum_m \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*} Q}{m(m+2\sqrt{\delta})} - \sum_m \sum_{\substack{l \neq 2n \\ -m+l=2n}} \frac{\overline{\Delta T_{sm}^*} \Delta T_{sl}^* \overline{Q}}{l(l-2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} = 0. \quad (89)
\end{aligned}$$

671 The expressions for D_1Q and D_1^2Q are given by the cancellation condition for secular terms in the
672 ϵ order equation:

$$673 \quad D_1Q = \frac{j}{2\sqrt{\delta}} \Delta T_{s2n}^* \overline{Q} e^{j\sigma T_1} e^{j\sigma T_1}, \quad (90)$$

$$675 \quad D_1^2Q = \left(\frac{j\Delta T_{s2n}^*}{2\sqrt{\delta}} D_1\overline{Q} - \frac{\Delta T_{s2n}^* \overline{Q} \sigma}{\sqrt{\delta}} \right) e^{j2\sigma T_1}, \quad (91)$$

$$677 \quad D_1\overline{Q} = \frac{-j}{2\sqrt{\delta}} \overline{\Delta T_{s2n}^*} Q e^{-j2\sigma T_1}. \quad (92)$$

678 By replacing,

$$679 \quad D_1^2 Q = \frac{\overline{\Delta T_{s2n}} \Delta T_{s2n}^* Q}{4\delta} - \frac{\Delta T_{s2n}^* \overline{Q} \sigma}{\sqrt{\delta}} e^{j2\sigma T_1}. \quad (93)$$

680 By injecting Eq. (93) into the cancellation condition for secular terms Eq. (89),

$$681 \quad -2j\sqrt{\delta} D_2 Q + \frac{\Delta T_{s2n}^* \sigma \overline{Q}}{\sqrt{\delta}} e^{j2\sigma T_1} - \left(\sum_{m \neq 2n} \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}}}{m(m-2\sqrt{\delta})} + \sum_m \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}}}{m(m+2\sqrt{\delta})} + \frac{\Delta T_{s2n}^* \overline{\Delta T_{s2n}}}{4\delta} \right) Q$$

$$682 \quad - \left(\frac{\Delta T_{s2n}^{*2}}{n(n-2\sqrt{\delta})} + \sum_m \sum_{\substack{l \\ m-l=2n}} \frac{\Delta T_{sm}^* \overline{\Delta T_{sl}}}{l(l+2\sqrt{\delta})} + \sum_m \sum_{\substack{l \\ l \neq 2n \\ -m+l=2n}} \frac{\overline{\Delta T_{sm}} \Delta T_{sl}^*}{l(l-2\sqrt{\delta})} \right) \overline{Q} e^{j2\epsilon\sigma T_0} = 0. \quad (94)$$

683 **Appendix 4: Non trivial solution conditions**

684 Real and imaginary parts are separated,

$$685 \quad \begin{cases} 2\sqrt{\delta} \frac{dQ_I}{dT} + 2\sqrt{\delta} Q_R \epsilon \sigma + \epsilon \left(-1 + \frac{\epsilon\sigma}{\sqrt{\delta}} \right) |\Delta T_{s2n}^*| Q_R - \epsilon^2 (\Gamma_1 Q_R + \Gamma_{2R} Q_R + \Gamma_{2I} Q_I) = 0, \\ 2\sqrt{\delta} \frac{dQ_R}{dT} - 2\sqrt{\delta} Q_I \epsilon \sigma + \epsilon \left(-1 + \frac{\epsilon\sigma}{\sqrt{\delta}} \right) |\Delta T_{s2n}^*| Q_I + \epsilon^2 (\Gamma_1 Q_I + \Gamma_{2R} Q_I - \Gamma_{2I} Q_R) = 0. \end{cases} \quad (95)$$

$$686 \quad \begin{cases} 2\sqrt{\delta} \frac{dQ_I}{dT} + \left[\epsilon [2\sqrt{\delta}\sigma + (-1 + \frac{\epsilon\sigma}{\sqrt{\delta}}) |\Delta T_{s2n}^*|] - \epsilon^2 (\Gamma_1 + \Gamma_{2R}) \right] Q_R - \epsilon^2 \Gamma_{2I} Q_I = 0, \\ 2\sqrt{\delta} \frac{dQ_R}{dT} + \left[\epsilon [-2\sqrt{\delta}\sigma + (-1 + \frac{\epsilon\sigma}{\sqrt{\delta}}) |\Delta T_{s2n}^*|] + \epsilon^2 (\Gamma_1 + \Gamma_{2R}) \right] Q_I - \epsilon^2 \Gamma_{2I} Q_R = 0. \end{cases} \quad (96)$$

688 The system Eq. (96) admits solutions in the form $(Q_R, Q_I) = (q_R, q_I) e^{\gamma T}$, i.e. by replacing:

$$689 \quad \begin{cases} (2\sqrt{\delta}\gamma - \epsilon^2 \Gamma_{2I}) q_I + \left[\epsilon (-|\Delta T_{s2n}^*| + 2\sqrt{\delta}\sigma) + \epsilon^2 \left(\frac{\sigma |\Delta T_{s2n}^*|}{\sqrt{\delta}} - \Gamma_1 - \Gamma_{2R} \right) \right] q_R = 0, \\ (2\sqrt{\delta}\gamma - \epsilon^2 \Gamma_{2I}) q_R + \left[\epsilon (-|\Delta T_{s2n}^*| - 2\sqrt{\delta}\sigma) + \epsilon^2 \left(\frac{\sigma |\Delta T_{s2n}^*|}{\sqrt{\delta}} + \Gamma_1 + \Gamma_{2R} \right) \right] q_I = 0. \end{cases} \quad (97)$$

690 Eq. (97) is written in the following matrix form,

$$691 \quad \begin{bmatrix} \epsilon (-|\Delta T_{s2n}^*| + 2\sqrt{\delta}\sigma) + \epsilon^2 \left(\frac{\sigma |\Delta T_{s2n}^*|}{\sqrt{\delta}} - \Gamma_1 - \Gamma_{2R} \right) & 2\sqrt{\delta}\gamma - \epsilon^2 \Gamma_{2I} \\ 2\sqrt{\delta}\gamma - \epsilon^2 \Gamma_{2I} & \epsilon (-|\Delta T_{s2n}^*| - 2\sqrt{\delta}\sigma) + \epsilon^2 \left(\frac{\sigma |\Delta T_{s2n}^*|}{\sqrt{\delta}} + \Gamma_1 + \Gamma_{2R} \right) \end{bmatrix} \begin{pmatrix} q_R \\ q_I \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (98)$$

692 The existence of non-trivial solutions is ensured by canceling the determinant of the system matrix,

$$(2\sqrt{\delta}\gamma - \epsilon^2\Gamma_{2l})^2 = \epsilon^2 \left[-|\Delta T_{s2n}^*| + 2\sqrt{\delta}\sigma + \frac{\epsilon\sigma|\Delta T_{s2n}^*|}{\sqrt{\delta}} - \epsilon(\Gamma_1 + \Gamma_{2R}) \right] \left[-|\Delta T_{s2n}^*| - 2\sqrt{\delta}\sigma + \frac{\epsilon\sigma|\Delta T_{s2n}^*|}{\sqrt{\delta}} + \epsilon(\Gamma_1 + \Gamma_{2R}) \right]. \quad (99)$$

694 Appendix 5: Case with damping

695 The resonant terms of order ϵ are now written:

$$-2D_1Qj\sqrt{\delta} - cQj\sqrt{\delta} - \Delta T_{s2n}\bar{Q} e^{j2\sigma T_1} = 0. \quad (100)$$

697 Similarly, the secular terms of order ϵ^2 can be written as:

$$\begin{aligned} & -2j\sqrt{\delta}D_2Q - D_1^2Q - cD_1Q - \frac{\Delta T_{sn}^{*2}\bar{Q}}{n(n-2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} - \sum_m \sum_{\substack{l \\ m-l=2n}} \frac{\Delta T_{sm}^* \overline{\Delta T_{s1}^*} \bar{Q}}{l(l+2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} \\ & - \sum_{m \neq 2n} \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*} Q}{m(m-2\sqrt{\delta})} - \sum_m \frac{\Delta T_{sm}^* \overline{\Delta T_{sm}^*} Q}{m(m+2\sqrt{\delta})} - \sum_m \sum_{\substack{l \neq 2n \\ -m+l=2n}} \frac{\overline{\Delta T_{sm}^*} \Delta T_{s1}^* \bar{Q}}{l(l-2\sqrt{\delta})} e^{j2\epsilon\sigma T_0} = 0. \end{aligned} \quad (101)$$

700 The expressions for D_1Q and D_1^2Q are given by the cancellation condition for secular terms in the
701 ϵ order equation:

$$D_1Q = -\frac{cQ}{2} + \frac{j}{2\sqrt{\delta}} \Delta T_{s2n}^* \bar{Q} e^{j2\sigma T_1}, \quad (102)$$

$$D_1\bar{Q} = -\frac{c\bar{Q}}{2} - \frac{j}{2\sqrt{\delta}} \overline{\Delta T_{s2n}^*} Q e^{-2j\sigma T_1}, \quad (103)$$

$$D_1^2Q = \frac{c^2Q}{4} - \left(\frac{cj}{2\sqrt{\delta}} + \frac{\sigma}{\sqrt{\delta}} \right) \Delta T_{s2n}^* \bar{Q} e^{j2\sigma T_1} + \frac{\Delta T_{s2n}^* \overline{\Delta T_{s2n}^*} Q}{4\delta}. \quad (104)$$

707 By injecting the expression for D_1^2Q into the cancellation condition for secular terms Eq. (101),
708 one can obtain:

$$-2j\sqrt{\delta}D_2Q - \frac{c^2Q}{2} + \left(\frac{cj}{2\sqrt{\delta}} + \frac{\sigma}{\sqrt{\delta}} \right) |\Delta T_{s2n}^*| e^{j\phi_1} \bar{Q} e^{j2\sigma T_1}$$

$$710 \quad -\frac{\Delta T_{s2n}^* \overline{\Delta T_{s2n}^*} Q}{4\delta} + \frac{c^2 Q}{2} - \frac{cj}{2\sqrt{\delta}} \Delta T_{s2n}^* \overline{Q} e^{j2\sigma T_1} - |\Gamma_2| e^{j\phi_2} \overline{Q} e^{j2\sigma T_1} - \Gamma_1 Q = 0. \quad (105)$$

711 The two cancellation conditions of the secular terms at orders ϵ and ϵ^2 given by Eq. (100) and Eq.
712 (101) result from the development at different orders ϵ^n of:

$$713 \quad -2j\sqrt{\delta} \frac{dQ}{dT} + \epsilon \left(-1 + \frac{\epsilon\sigma}{\sqrt{\delta}} \right) \overline{Q} |\Delta T_{s2n}^*| e^{j(2\sigma\epsilon T + \phi_1)} - \epsilon cj\sqrt{\delta} Q - \epsilon^2 \left[\left(\Gamma_1 - \frac{c^2}{4} \right) Q + |\Gamma_2| \overline{Q} e^{j(2\epsilon\sigma T + \phi_2)} \right] = 0. \quad (106)$$

714 Following the same approach as above, the cancellation condition for the exponent γ leads to the
715 equation:

$$716 \quad \epsilon^2 |\Delta T_{s2n}^*|^2 - 4\delta\sigma^2\epsilon^2 - \epsilon^4 \left(\Gamma_1 - \frac{c^2}{4} + \Gamma_{2R} \right)^2 + \epsilon^4 \Gamma_{2I}^2 - \epsilon^2 c^2 \delta = 0. \quad (107)$$

717 One can obtain:

$$718 \quad \sigma^2 = \frac{|\Delta T_{s2n}^*|^2 - c^2\delta}{4\delta} + \frac{\epsilon^2}{4\delta} \left[\left(\Gamma_1 - \frac{c^2}{4} + \Gamma_{2R} \right)^2 + \Gamma_{2I}^2 \right]. \quad (108)$$

719 Thus, the instability zones of the dynamic response of the cable transport line are given by:

$$720 \quad \sigma > -\frac{\sqrt{|\Delta T_{s2n}^*|^2 - c^2\delta}}{2\sqrt{\delta}} - \epsilon \frac{\sqrt{\left(\Gamma_1 - \frac{c^2}{4} + \Gamma_{2R} \right)^2 + \Gamma_{2I}^2}}{2\sqrt{\delta}}, \quad (109)$$

$$721 \quad \sigma < \frac{\sqrt{|\Delta T_{s2n}^*|^2 - c^2\delta}}{2\sqrt{\delta}} - \epsilon \frac{\sqrt{\left(\Gamma_1 - \frac{c^2}{4} + \Gamma_{2R} \right)^2 + \Gamma_{2I}^2}}{2\sqrt{\delta}}. \quad (110)$$

723 Data Availability Statement

724 Some or all data, models, or code generated or used during the study are proprietary or confi-
725 dential in nature and may only be provided with restrictions

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791 pension bridge based on continual formula method.” *Mechanical Systems and Signal Processing*,
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TABLE 1

Numerical data of the case study	
Cable length L	2236 m
Young's modulus E	130 GPa
Cable diameter d	45 mm
Cable linear mass μ	7.4 kg/m
Static tension Δ^2 parameter	1.9×10^5 N
Static curvature κ parameter	10^{-3} m^{-1}
Pulley radius R	3.05 m

TABLE 2

Vehicle number N_v []	N_v^1			$N_v^1 - 1$		
Cable speed V_0 [m/s]	4.25	4.50	5.00	4.25	4.50	5.00
Line dynamic behavior []						
Driving motor torque oscillation $\frac{C_{max}-C_{min}}{C_{moy}}$ [%]	120	51	40	19	x	x

TABLE 3

Vehicle number N_v []	N_v^1			$N_v^1 - 1$		
Cable speed V_0 [m/s]	4.25	4.50	5.00	4.25	4.50	5.00
Spacing distance l_d [m]	42.0			42.7		
Fondamental excitation pulsation Ω_e [rad/s]	0.655	0.694	0.771	0.640	0.678	0.753

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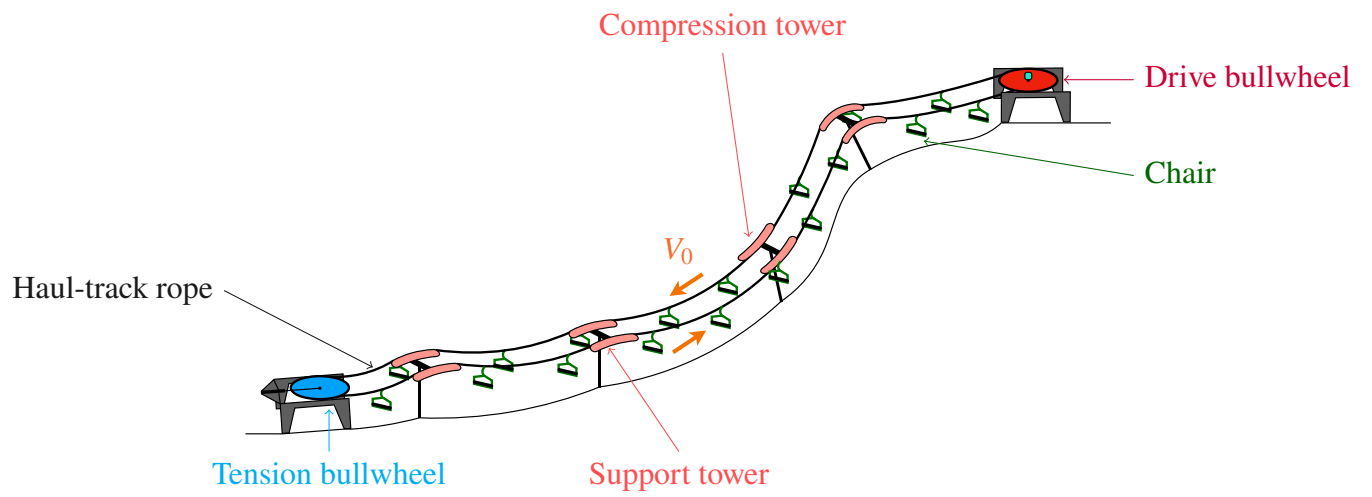


Fig. 1. Chairlift scheme

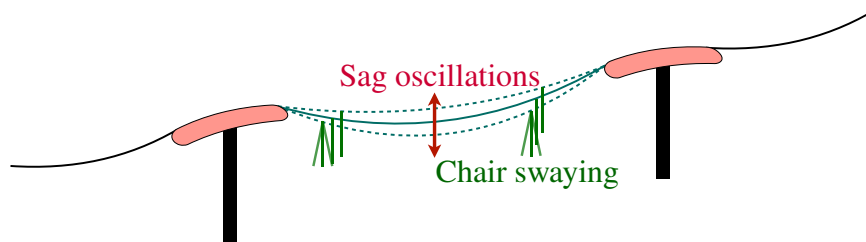


Fig. 2. Pumping scheme

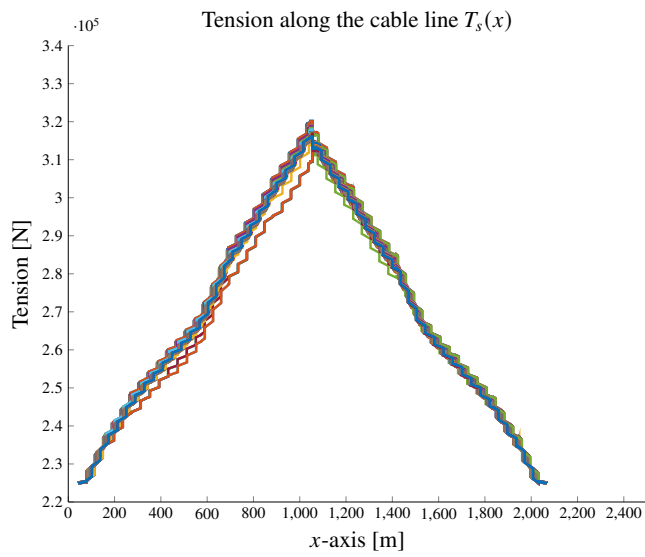


Fig. 3. Tension T_s along the cable line for several vehicle positions

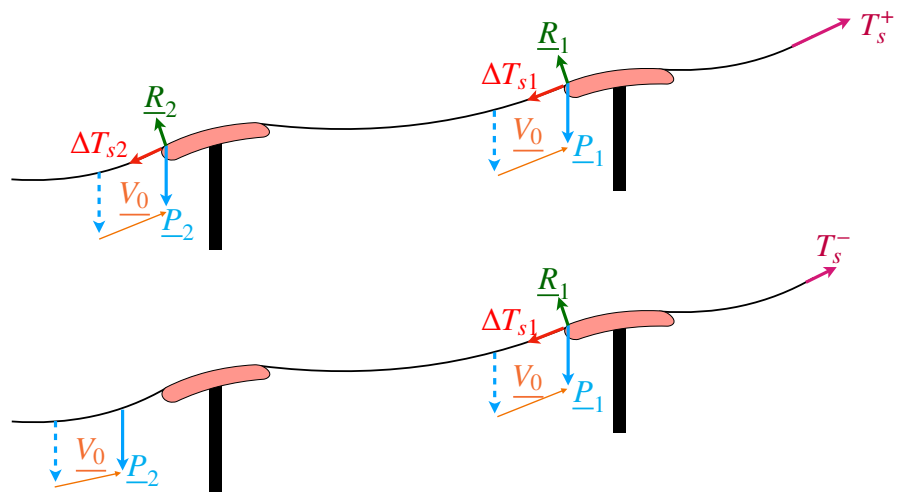


Fig. 4. Geometric cable span and vehicle spacing coincidence scheme

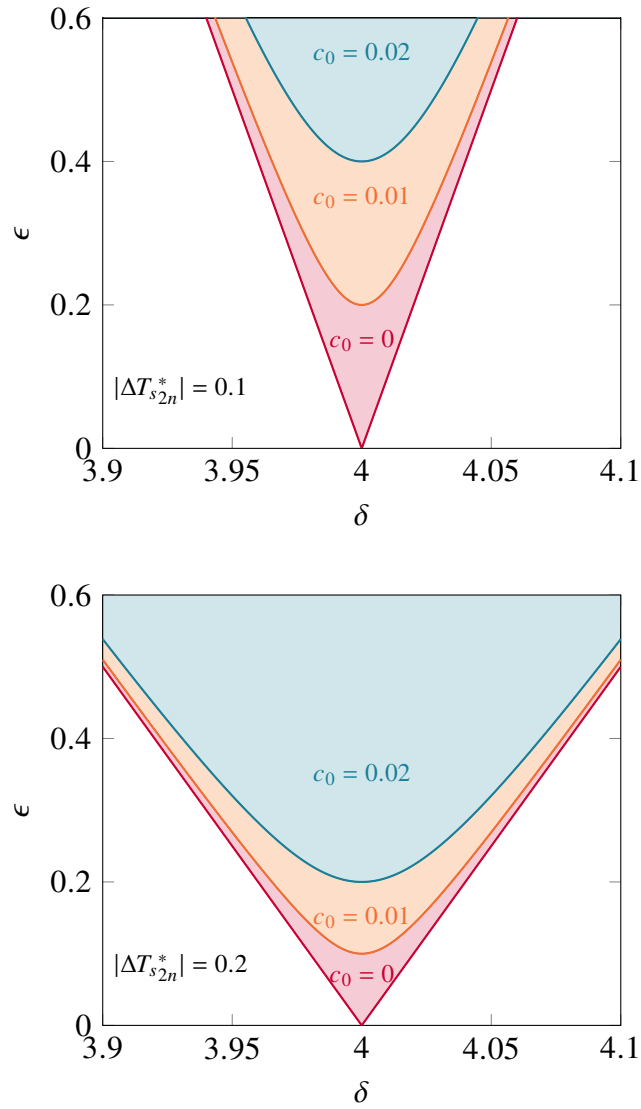


Fig. 5. Colored surfaces delimit areas of cable transport line instability (a) $|\Delta T_{s_{2n}}^*| = 0.1$, (b) $|\Delta T_{s_{2n}}^*| = 0.2$

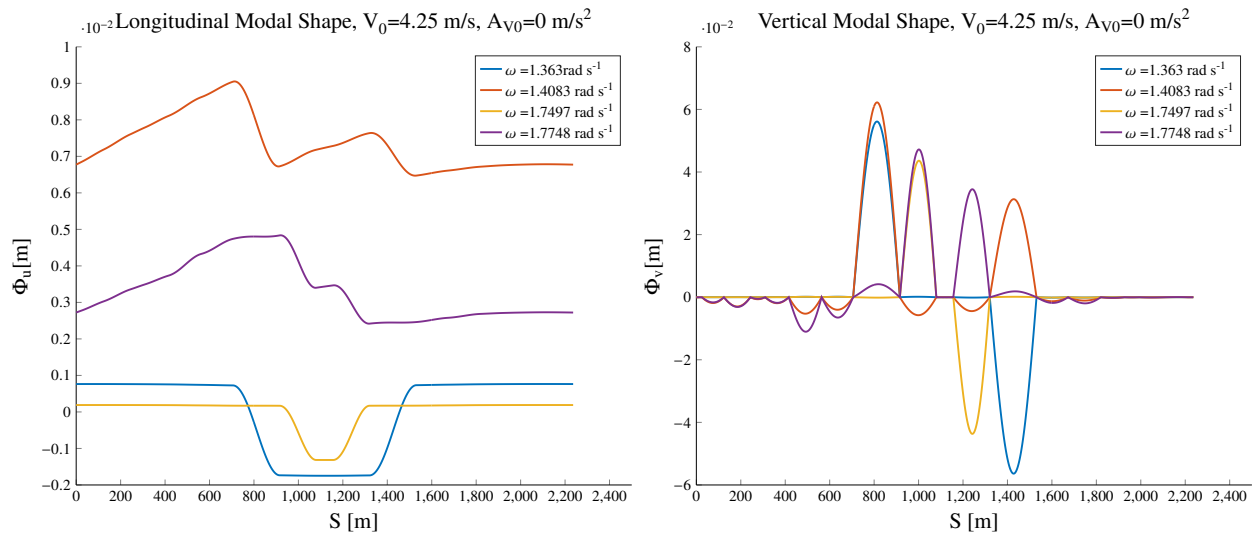


Fig. 6. First chairlift line (a) longitudinal and (b) vertical in-plane modes

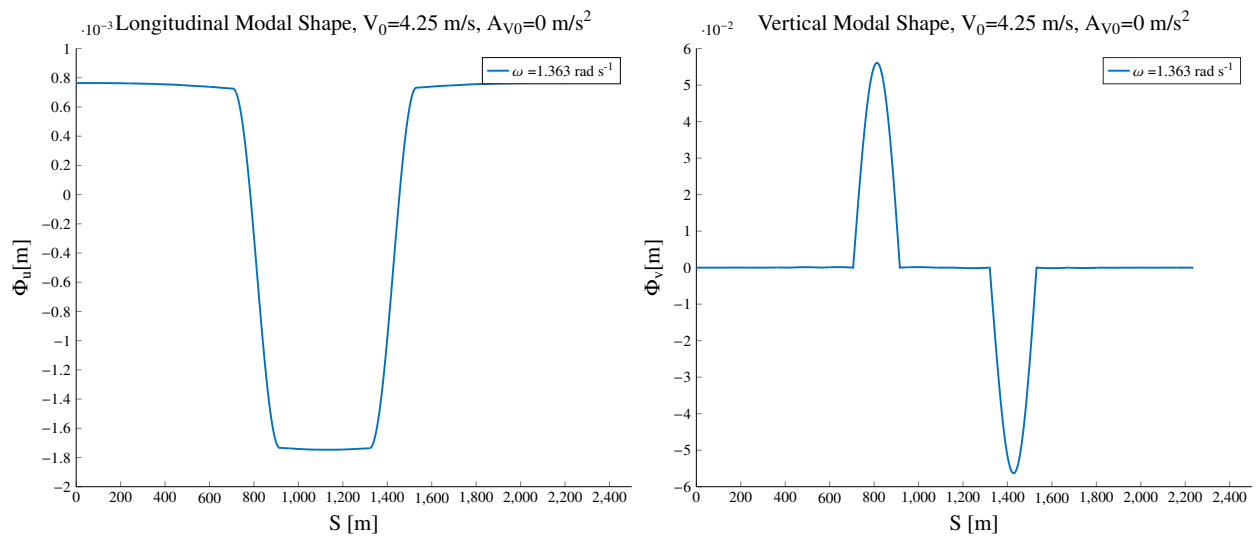


Fig. 7. Chairlift line (a) longitudinal and (b) vertical in-plane mode incriminated in pumping observation

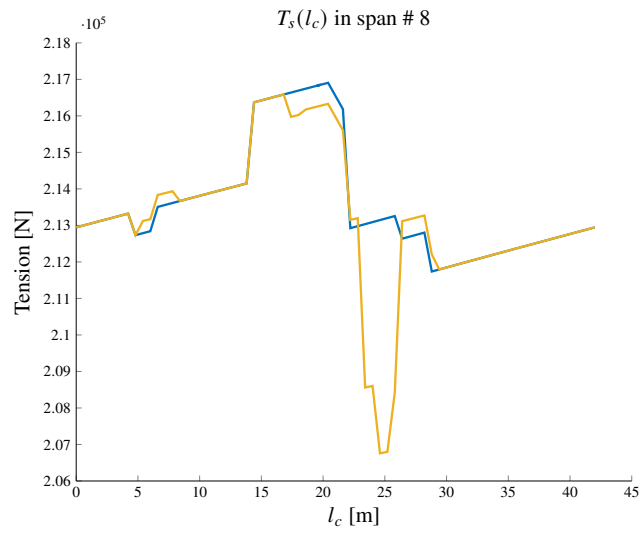


Fig. 8. Tension signal $T_s(l_c)$ in span # 8 for $(V_0, N_v) = (4.25, N_v^1)$ without (blue) and with (yellow) contact modelling between vehicle and tower's sheave assembly

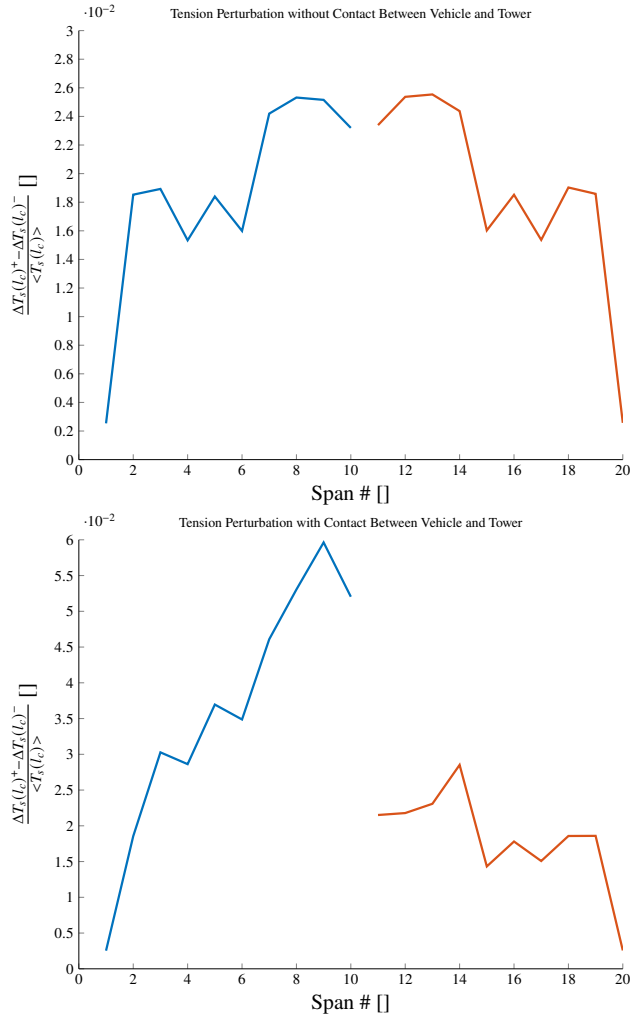
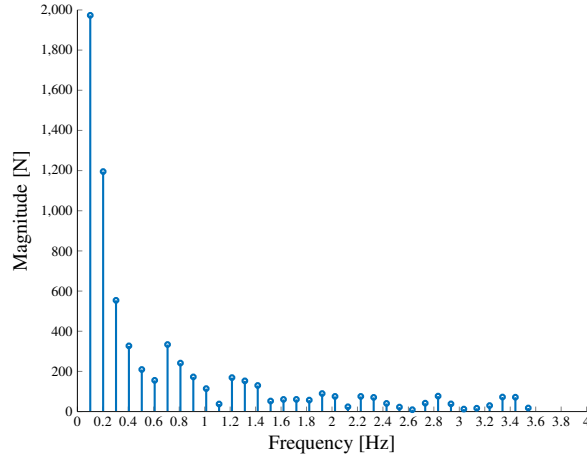


Fig. 9. Tension perturbation $\frac{\Delta T_s(l_c)^+ - \Delta T_s(l_c)^-}{\langle T_s(l_c) \rangle}$ as function of the span number along the chairlift line for $(V_0, N_v) = (4.25, N_v^1)$ without (a) and with (b) contact modelling between vehicle and tower's sheave assembly

Fourier Transform of the tension perturbation $\Delta T_s(T)$ in span # 8 without contact model



Fourier Transform of the tension perturbation $\Delta T_s(T)$ in span # 8 with contact model

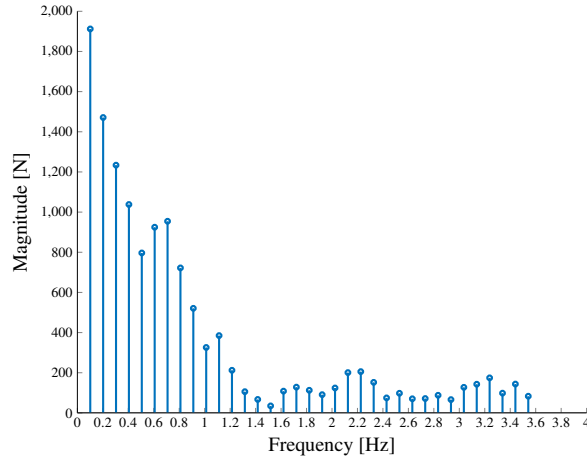


Fig. 10. Fast Fourier Transform of the tension $T_s(T)$ for $(V_0, N_v) = (4.25, N_v^1)$ without (a) and with (b) contact modelling between vehicle and tower's sheave assembly

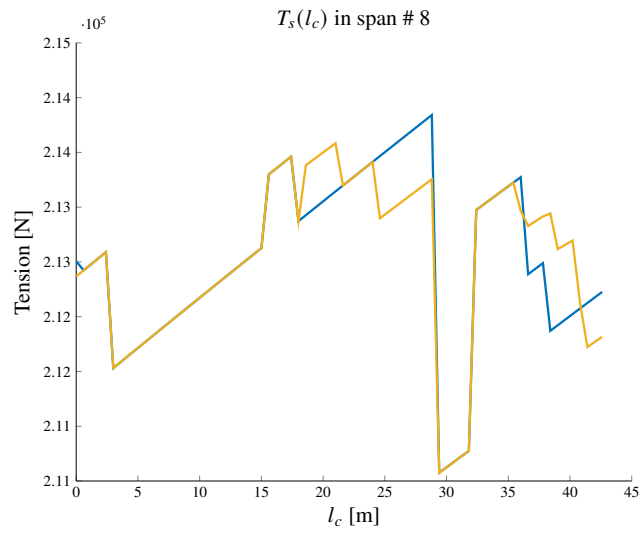


Fig. 11. Tension signal $T_s(l_c)$ in span # 8 for $(V_0, N_v) = (4.25, N_v^1 - 1)$ without (blue) and with (yellow) contact modelling between vehicle and tower's sheave assembly

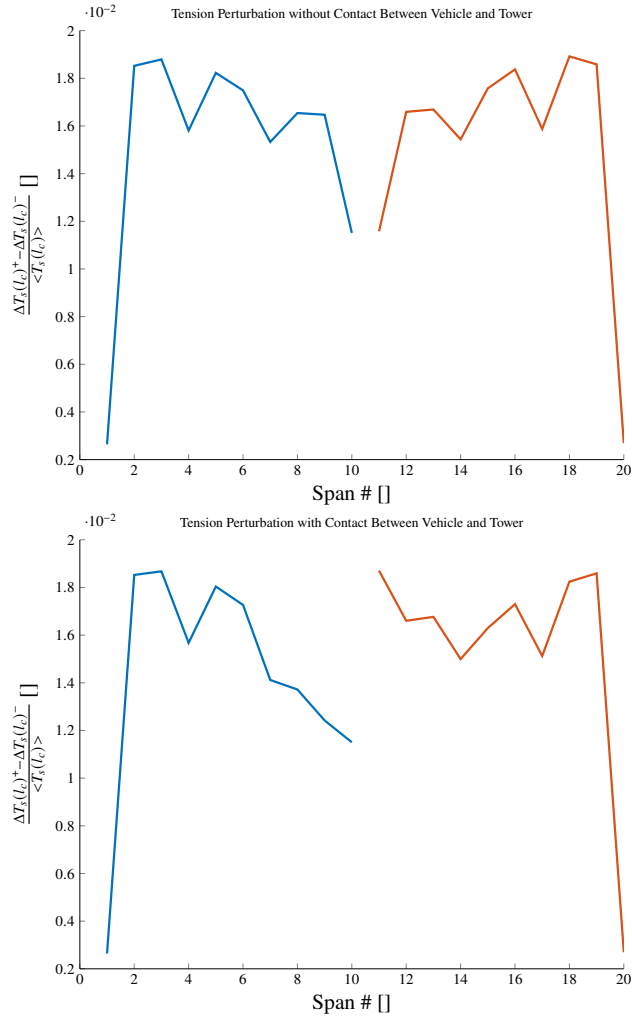
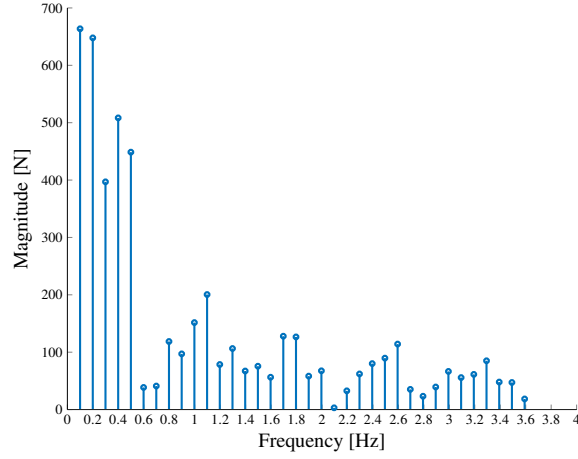


Fig. 12. Tension perturbation $\frac{\Delta T_s(l_c)^+ - \Delta T_s(l_c)^-}{\langle T_s(l_c) \rangle}$ as function of the span number along the chairlift line for $(V_0, N_v) = (4.25, N_v^1 - 1)$ without (a) and with (b) contact modelling between vehicle and tower's sheave assembly

Fourier Transform of tension perturbation $\Delta T_s(T)$ in span # 8 without contact model



Fourier Transform of tension perturbation $\Delta T_s(T)$ in span #8 with contact model

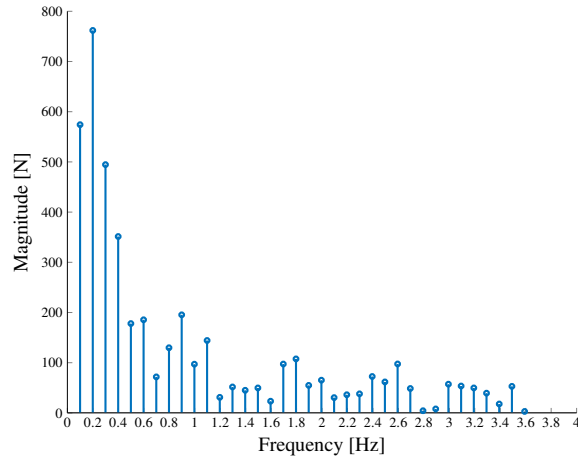


Fig. 13. Fast Fourier Transform of the tension $T_s(T)$ for $(V_0, N_v) = (4.25, N_v^1 - 1)$ without (a) and with (b) contact modelling between vehicle and tower's sheave assembly

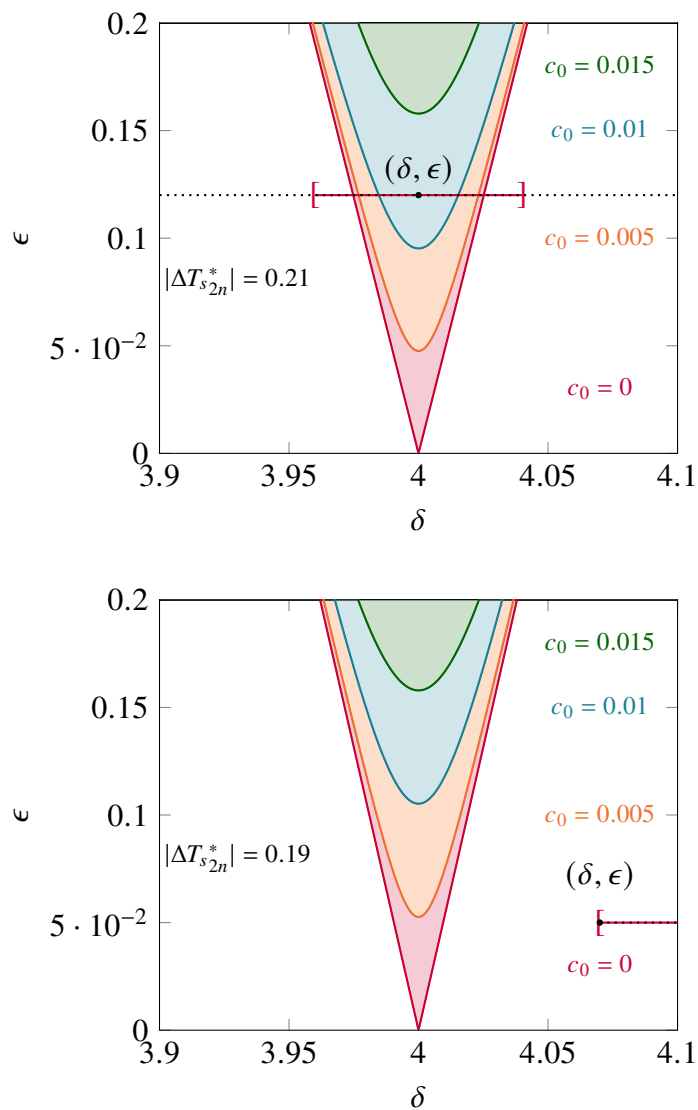


Fig. 14. Colored areas correspond to unstable zones plotted according to (a) $|\Delta T_{s_{2n}}^*| = 0.19$, (b) $|\Delta T_{s_{2n}}^*| = 0.21$