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Neural prediction of Lagrangian drift trajectories on the sea surface

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8 ABSTRACT: We simulate Lagrangian drift on the sea surface and investigate deep learning ap-
9 proaches to address the shortcomings of current model-based and Markovian approaches, particu-
10 larly concerning error propagation and computational complexity. We present a novel deep learn-
11 ing framework, referred to as DriftNet, inspired by the Eulerian Fokker-Planck representation of
12 Lagrangian dynamics. Through numerical experiments for simulated and real drift trajectories on
13 the sea surface, we illustrate the effectiveness of DriftNet compared to existing state-of-the-art
14 schemes. We also delve into the influence of diverse geophysical fields, whether derived from
15 models or observations, used as inputs by DriftNet on drift simulation. Our objective is to assess
16 the amount of dynamic information required to accurately simulate realistic trajectories.

17 SIGNIFICANCE STATEMENT: We propose a novel Deep Learning model, DriftNet, for con-
18 ditional generation of Lagrangian trajectories on the sea surface. Our model is based on Eulerian
19 Fokker-Planck formalism and can be conditioned by multiple geophysical fields. We highlight the
20 overall over-performance of DriftNet compared to the baseline model-based and learning-based
21 approaches. We put in evidence the capacity of the proposed method to extract pertinent infor-
22 mation from various geophysical fields, both from modeled and observed data. We highlight the
23 significant impact of the observed sea surface height when combined to sea surface currents in the
24 quality of the generation of Lagrangian trajectories.

25 **1. Introduction**

26 The modeling and simulation of Lagrangian drift on the sea surface holds relevance for various
27 applications such as the tracking of plastics and other debris Maximenko et al. (2012), the study
28 of algae and plankton dynamics Son et al. (2015), or the prediction of future locations of drifting
29 objects that is crucial for search and rescue operations Breivik et al. (2013). Furthermore, the
30 analysis of Lagrangian drifts serves as a valuable tool for diagnosing ocean numerical models in
31 their capability for reproducing small-scale dynamics Barron et al. (2007). However, reproducing
32 realistic Lagrangian trajectories on the sea surface poses a significant scientific challenge within
33 operational oceanography Röhrs et al. (2021).

34 From a methodological standpoint, we can distinguish three main categories of approaches
35 for Lagrangian drift simulation: model-based approaches Liubartseva et al. (2018); Zambianchi
36 et al. (2017), probabilistic data-driven schemes Visser (2008), and more recent deep learning tech-
37 niques Botvynko et al. (2023); Jenkins et al. (2023). Model-based approaches involve a sequen-
38 tial advection process based on known sea surface velocity fields Lange and van Sebille (2017).
39 However, small errors in the underlying velocity fields or velocity fields without very fine spa-
40 tial resolution may lead to unrealistic Lagrangian trajectories Callies et al. (2021). This often
41 impedes their operational usefulness due to significant uncertainties in the forecasting and recon-
42 struction of sea surface dynamics, especially regarding mesoscale ocean dynamics Prants et al.
43 (2017). Due to their sequential nature, these schemes also face scalability challenges when simu-
44 lating large ensembles of drift trajectories. By contrast, probabilistic data-driven schemes usually
45 leverage first-order Markovian models and naturally account for uncertainties in the drift process.

46 However, they are primarily suited for relatively coarse space-time resolutions. Extending these
47 schemes to capture fine-scale patterns poses a challenge Fine et al. (1998). For these two first cat-
48 egories, the simulation of the Lagrangian drift relies on location-wise velocities at each time step,
49 which may only be applicable for smooth velocity fields. Deep Learning has recently emerged as
50 a novel class of numerical tools for the learning-based simulation of movement patterns. We may
51 cite applications to pedestrians Korbmacher and Tordeux (2022), seabirds Roy et al. (2022), cars
52 Jiang et al. (2019) or maritime vessels Nguyen and Fablet (2024). These applications leverage
53 state-of-the-art recurrent neural networks, such as Long Short-Term Memory (LSTM) networks
54 Yu et al. (2019), as well as deep generative schemes Goodfellow et al. (2020); Roy et al. (2022);
55 Ma et al. (2019); Gan et al. (2020); Julka et al. (2021); Paz et al. (2021). Most of these studies can
56 be regarded as neural extensions of probabilistic Markovian schemes.

57 In this context, we present an innovative Deep Learning framework, referred to as DriftNet, for
58 simulating Lagrangian trajectories at the sea surface. The proposed neural model takes as input
59 geophysical fields characterizing the ocean dynamics in a spatio-temporal region and produces
60 the associated Lagrangian trajectory. Our approach is fully-convolutional and features a spatially-
61 explicit latent representation, drawing inspiration from Eulerian Fokker-Planck representations of
62 drift processes Botvynko et al. (2023). From numerical simulation datasets, we demonstrate Drift-
63 Net to outperform state-of-the-art neural schemes for the simulation of Lagrangian trajectories,
64 including when considering degraded sea surface velocities. The application to real sea surface
65 drifters' datasets further supports the relevance of DriftNet to improve the simulation of Lagra-
66 gian drift on the sea surface compared with model-driven simulations. This last experiment also
67 illustrates how DriftNet can exploit additional data sources, besides sea surface velocities, here
68 satellite-derived sea surface height fields.

69 This paper is organized as follows: section 2 focuses on the problem description; we introduce
70 DriftNet in section 3 and the considered experimental setup in section 4; section 5 details our
71 results and section 6 discusses key aspects of this study.

72 **2. Problem statement**

73 In the context of Lagrangian fluid dynamics¹, one may describe a flow by following the motion
74 of an ensemble of individual particles Davis (1991). In order to describe the flow in the Lagrangian

¹From now on, the Lagrangian notations are presented with \rightarrow -symbol, and the Eulerian notations are in **bold**.

75 framework, an ordinary differential equation defines the particle's trajectory :

$$\frac{\partial \vec{r}(\vec{r}_{i_0}, t)}{\partial t} = \vec{v}(\vec{r}_{i_0}, t) = \mathbf{u}(\vec{r}(t), t) \quad (1)$$

76 where $\vec{r}(\vec{r}_{i_0}, t)$ and $\vec{v}(\vec{r}_{i_0}, t)$ are respectively the position and velocity at time t of the particle situated
77 at \vec{r}_{i_0} at $t = 0$, and $\mathbf{u}(\vec{r}(t), t)$ is the Eulerian velocity of the underlying flow given position $\vec{r}(t)$ and
78 time t . Classical model-driven approaches simulate Lagrangian drift dynamics through explicit
79 time and space integration methods such as Runge-Kutta 4. Thus, the integration of eq.(1) leads
80 to :

$$\vec{r}(\vec{r}_{i_0}, t + \Delta t) = \vec{r}(\vec{r}_{i_0}, t) + \int_t^{t+\Delta t} \vec{v}(\vec{r}_{i_0}, \tau) d\tau \quad (2)$$

81 The sequential nature of this equation introduces important propagation of errors when the under-
82 lying velocities are not accurately known Callies et al. (2021).

83 Interestingly, we can also derive an Eulerian formulation of Lagrangian dynamics. Through
84 Fokker-Planck formalism Visser (2008), we can generalize eq.(2) to the time propagation of the
85 Probability Density Function (PDF) of the moving particle as follows:

$$\frac{\partial}{\partial t} p_{\vec{r}}(x, t) = -\frac{\partial}{\partial x} [\mu(x, t) p_{\vec{r}}(x, t)] \quad (3)$$

86 where x represents spatial position in the Eulerian framework, t the time, $p_{\vec{r}}(x, t)$ the PDF of the
87 particle at position \vec{r} and $\mu(x, t)$ a drift field. In our case, this drift term relates to the underlying
88 Eulerian velocity field $\mathbf{u}(x, t)$. Solving eq.(3) relies on a time integration scheme from the initial
89 PDF $p_{\vec{r}}(x, t = 0) = p_{\vec{r}_{i_0}}(x, 0)$.

90 Here, we rely on this Eulerian Fokker-Planck representation of Lagrangian dynamics to explore
91 convolutional neural architectures for the simulation of individual drift trajectories.

92 3. DriftNet for Lagrangian drift simulation

93 This section introduces the mathematical framework and the architecture of the proposed neural
94 network model, called DriftNet, along with the considered learning scheme.

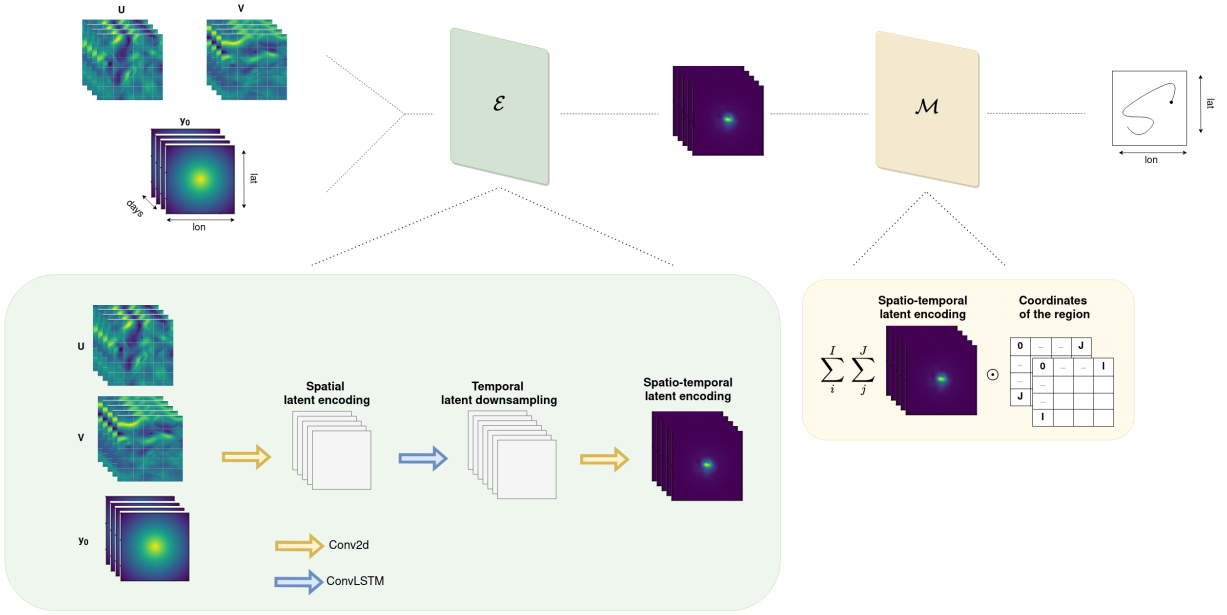


FIG. 1: **DriftNet architecture:** The input geophysical fields over 9-day (here zonal \mathbf{U} and meridional \mathbf{V} components of the velocity field) coupled to the initial spatio-temporal positional encoding \mathbf{y}_0 are first passed to the encoding block \mathcal{E} and output a spatio-temporal latent encoding. The second block of DriftNet \mathcal{M} maps this latent representation to a 1D trajectory, *i.e.* a time series of positions on the sea surface.

95 *a. Proposed neural scheme*

96 Given a spatial domain of interest \mathcal{D} defined as a regularly-gridded domain of size $I \times J$, we
 97 define the associated sequence of velocity fields $\mathbf{u} = \{\mathbf{u}_{t_0}, \mathbf{u}_{t_0+\Delta}, \dots, \mathbf{u}_{t_0+K\Delta}\}$ from time t_0 to $t_0 +$
 98 $K\Delta$, where Δ is the time sampling and K the number of time steps, and the trajectory of a particle as
 99 a time series of locations $\vec{r} = \{\vec{r}_{t_0}, \vec{r}_{t_0+\Delta}, \dots, \vec{r}_{t_0+K\Delta}\}$ in \mathcal{D} from t_0 to $t_0 + K\Delta$, where \vec{r}_{t_0} corresponds
 100 to the initial position of the particle.

101 Inspired by Fokker-Plank eq.(3), we consider the following latent representation for a simulated
 102 Lagrangian drift \vec{r} :

$$\begin{cases} \mathbf{y} = \mathcal{E}(\mathbf{u}, \mathbf{y}_0) \\ \vec{r} = \mathcal{M}(\mathbf{y}) \end{cases} \quad (4)$$

103 where $\mathbf{y} = \{\mathbf{y}_{t_0}, \mathbf{y}_{t_0+\Delta}, \dots, \mathbf{y}_{t_0+K\Delta}\}$ is a space-time-explicit latent embedding of \vec{r} , and $\mathbf{y}_0 =$
 104 $\{\mathbf{y}_{0,t_0}, \mathbf{y}_{0,t_0+\Delta}, \dots, \mathbf{y}_{0,t_0+K\Delta}\}$ some initial encoding of the initial position \vec{r}_{t_0} . In particular, we define
 105 \mathbf{y}_{0,t_0} by assigning to each spatial pixel of the grid a value corresponding to the normalized distance

106 to the initial position of the particle eq.(5).

$$d = -\sqrt{\sum_{x \in \mathcal{D}} (\vec{r}_{t_0} - x)^2} \quad (5)$$

$$\mathbf{y}_{0,t_0} = \frac{d + |\min d|}{\max(d + |\min d|)} \quad (6)$$

107 while the other time steps of \mathbf{y}_0 are initialized as all-ones matrix of the same dimension as \mathcal{D} .

108 The operator \mathcal{E} computes a latent embedding \mathbf{y} given velocity conditions \mathbf{u} and initial repre-
 109 sentation \mathbf{y}_0 , as introduced in eq.(3). And, the operator \mathcal{M} maps latent representation \mathbf{y} to the
 110 targeted Lagrangian drift \vec{r} . In other words, it maps a multi-dimensional tensor to a time series of
 111 locations in \mathcal{D} . The analytical expression of the operator \mathcal{M} for a given time step t grounds on the
 112 Hadamard product between the spatially-explicit encoding x of the space \mathcal{D} and \mathbf{y}_t field:

$$\mathcal{M} = x \cdot \mathbf{y}_t \quad (7)$$

113 where $\mathbf{y}_t \in \mathbf{y}$, a sample of \mathbf{y} at a time step $t \in [t_0, \dots, t_{0+K\Delta}]$.

114 Regarding the operator \mathcal{E} , we consider the architecture sketched in Figure 1 and composed of:

- 115 1) a 2D convolutional layer increasing the number of channels from 3 (zonal and meridional
 116 components of \mathbf{u} coupled to \mathbf{y}_0) to 64, followed by a LeakyReLU activation function.
- 117 2) 2D convolutional LSTM block Shi et al. (2015).
- 118 3) 2D convolutional layer decreasing the number of channels from 64 to 2, followed by 2D
 119 Softmax function.

120 Since our model is mainly based on 2D convolutional layers, we expect the resulting architecture
 121 to capture relevant information at different space-time scales, and not only in a point-wise manner,
 122 as it would be the case for the straightforward implementation of Fokker-Plank representation
 123 eq.(3). Thus, our network can leverage non-local physical information explaining the particle's
 124 motion and processes it to propagate the latent representation of particle's positions through time.

125 Interestingly, the proposed architecture easily generalizes to multi-source input data. Gridded
 126 fields such as SSH fields with the same space-time resolution as the sea surface velocities convey
 127 relevant information on sea surface dynamics. Stacking SSH and sea surface velocities, we may

128 apply the same neural architecture, just increasing the number of channels of the input fields. The
 129 same applies for a prior on the diffusivity from the initial location of a particle in the field, that can
 130 be also included by stacking with the temporal evolution of the initial condition \mathbf{y}_0 .

131 *b. Learning scheme*

132 We consider a supervised training of the proposed neural architecture according to the following
 133 two losses:

- 134 1. the Mean Square Error (MSE) between the reference and simulated trajectories for the K -step
 135 simulation of N_T particles

$$\mathcal{L}_{MSE} = \frac{1}{N_T K} \sum_{i=0}^{N_T} \sum_{j=0}^K (\vec{r}_R(\vec{r}_{0,i}, j\Delta) - \vec{r}_S(\vec{r}_{0,i}, j\Delta))^2 \quad (8)$$

136 where \vec{r}_R and \vec{r}_S are the vector of positions of the reference and simulated trajectories respec-
 137 tively.

- 138 2. Liu index between the reference and simulated trajectories Liu and Weisberg (2011)

$$\mathcal{L}_{Liu} = \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{\sum_{j=1}^K d_{ij}}{\sum_{j=1}^K l_{ij}} \quad (9)$$

139 where

$$d_{ij} = \sqrt{(\vec{r}_R(\vec{r}_{0,i}, j\Delta) - \vec{r}_S(\vec{r}_{0,i}, j\Delta))^2} \quad (10)$$

140 is the Euclidean distance between the reference and the simulated trajectories number i at
 141 time step j and l_{ij} is the length of reference trajectory i between the initial position and the
 142 position at time step j .

143 Overall, the training loss \mathcal{L} is a weighted sum: $\mathcal{L}_{Liu} = \alpha \cdot \mathcal{L}_{MSE} + \beta \cdot \mathcal{L}_{Liu}$. From cross-validaton
 144 experiments, we set α and β to 0.2 and 0.8. Using Pytorch², our learning setup relies on Adam
 145 optimizer with a learning rate of 5e-3 over 750 epochs.

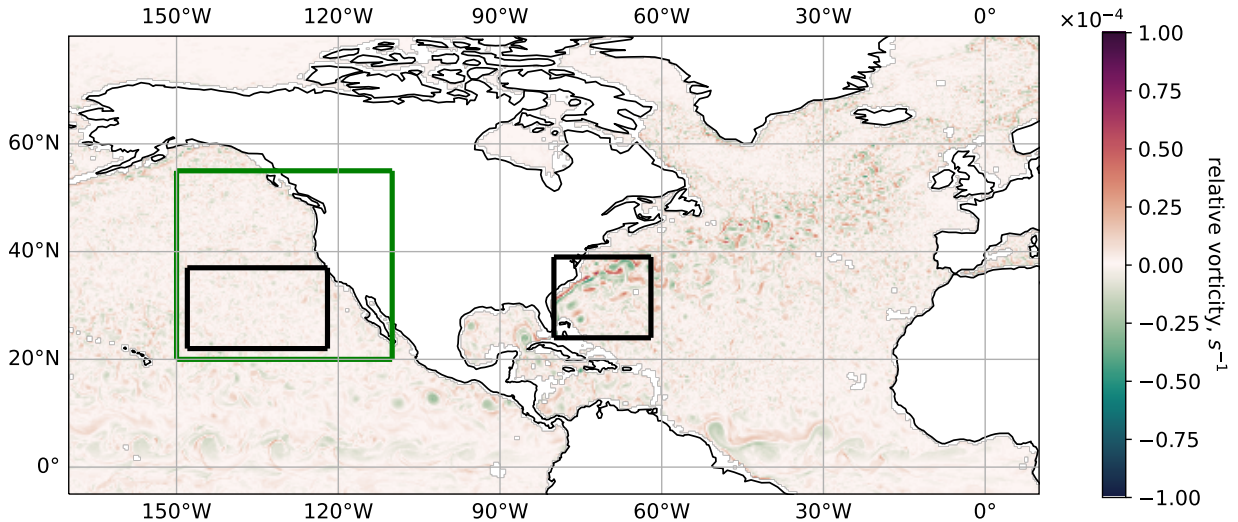


FIG. 2: **Study cases delineation:** Relative vorticity of North East Pacific and North Atlantic oceans from E1 simulation of 01/10/2015. The two black boxes indicate the North East Pacific and Gulf Stream case study regions for benchmarks B1 and B2. The green box defines the North East Pacific region of study for benchmark B3.

146 4. Experimental workflow

147 Section 4.a details the two case-study regions considered because of their contrasting turbulent
 148 activity: the Gulf Stream and the North-East Pacific region, see Figure 2. We describe the different
 149 datasets used in our experiments in section 4.b, see Table 1, and the metrics used to evaluate
 150 performance in section 4.c. In section 4.d we present the benchmarked deep learning models. We
 151 perform three different experiments: two relying on numerical simulation datasets such that the
 152 full sea surface velocity field is known, and a last experiment dealing with real drifters in the sea
 153 surface. These three benchmarks are explained in section 4.e.

154 a. Case-study Regions

155 **The Gulf Stream** is a Western boundary current system characterized by a fast flowing jet that
 156 originates in the Gulf of Mexico and flows into the North Atlantic Gula et al. (2015); Dewar
 157 and Bane (1989), becoming a part of it’s anticyclonic subtropical gyre Talley (2011). Surface
 158 velocity can exceed $2 \text{ m} \cdot \text{s}^{-1}$ and the current is characterized by high eddy variability. Exhibiting
 159 strong velocities for hundreds of kilometers, the Gulf Stream transports warm saline water from
 160 lower latitudes north-eastward. The Gulf Stream plays a crucial role in redistributing heat and

²DriftNet code is available at <https://github.com/CIA-Oceanix/DriftNet>

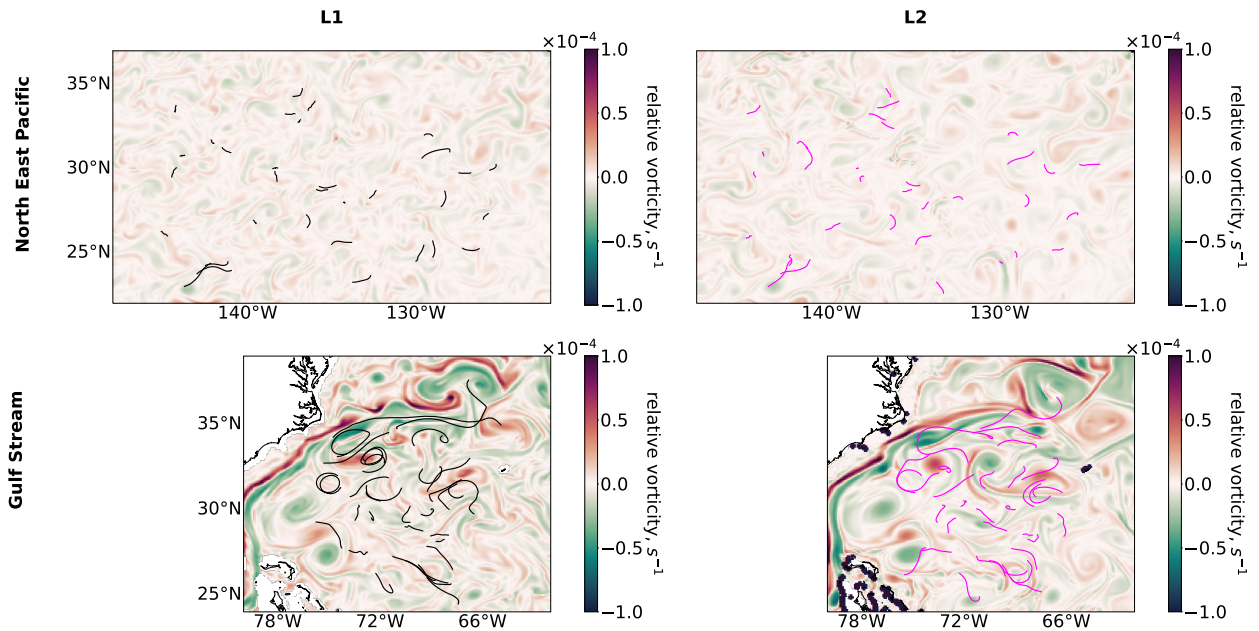


FIG. 3: **Examples of trajectories:** Trajectories from dataset L1 and dataset L2 superimposed respectively to the E1 and E2 relative vorticity fields of 2015/01/01.

161 influencing climate patterns and the surrounding marine ecosystem Gula et al. (2015), Wills et al.
 162 (2016).

163 **The North East Pacific** is a vast oceanic region extending from the western coast of North
 164 America to the central Pacific. It is influenced by various ocean currents, including the California
 165 Current and North Pacific Current. The California Current System (CCS) is an eastern boundary
 166 of the slow ($0.1 \text{ m} \cdot \text{s}^{-1}$) eastward North Pacific Current. It is a cold, nutrient-rich current that
 167 flows southward along the Western coast of North America Checkley and Barth (2009); Aua
 168 et al. (2011), impacting marine ecosystems and fisheries. The CCS is an upwelling region due to
 169 Ekman transport, spinning up eddies westward offshore. It has a mean surface velocity of $0.4 - 0.8$
 170 $\text{m} \cdot \text{s}^{-1}$. Due to the baroclinic instability of the coastal upwelling current, the CCS contains high
 171 mesoscale eddy activity, associated with the highly variable sea-surface height Talley (2011).

172 *b. Datasets*

173 Our study involves three different Eulerian datasets used as input data for the benchmarked
 174 models:

TABLE 1: Summary description of the Eulerian and Lagrangian datasets used in the current study: *Source* makes reference to the used numerical or satellite product, *Variable* indicates the physical quantity of interest, *Period* the studied years. The spatial and temporal resolutions of each product are also indicated. SSC - sea surface currents. SSH - sea surface height.

<i>Name</i>	<i>Source</i>	<i>Variable</i>	<i>Period</i>	<i>Spatial resolution</i>	<i>Temporal resolution</i>
Eulerian					
E1	Nature Run	SSC and SSH	2015	1/12°	1 day
E2	OSSE-based assimilated free run	SSC	2015	1/12°	1 day
E3	Operational assimilated reanalysis GLORYS12	SSC	1992 - 2020	1/12°	1 day
E3	DUACS	SSH	1992 - 2020	1/4°	1 day
Lagrangian					
L1	Ocean Parcels simulation on E1 SSC	Position	2015	-	6 hours
L2	Ocean Parcels simulation on E2 SSC	Position	2015	-	6 hours
L3	CMEMS drifters	Position	1992 - 2020	-	6 hours
L4	Ocean Parcels simulation on E3 SSC	Position	1992 - 2020	-	6 hours

- 175 • **Eulerian dataset E1:** This Eulerian dataset contains the sea surface current (SSC) velocity
176 and sea surface height (SSH) fields. Both fields are obtained from a high-resolution free
177 simulation of the NEMO ocean model without any data assimilation Benkiran et al. (2021b).
178 Those are regularly-gridded products with a horizontal spatial resolution of 1/12° and a daily
179 time resolution. This study involves the entire year of 2015.

- 180 • **Eulerian dataset E2** This dataset contains the SSC fields from a data assimilated run, which
181 dynamically reconstructs the ocean state using pseudo observations from E1 which mimic
182 the current observational network (satellite altimetry tracks). Those fields are at the same
183 1/12° and daily space-time resolution as the E1 dataset. As in the previous E1 dataset, we
184 equivalently use the complete 2015 year.

- 185 • **Eulerian Real-world dataset E3:** this real-world dataset combines the SSC from the oper-
186 ational ocean reanalysis GLORYS12 Lellouche et al. (2021) and the optimally-interpolated
187 altimetry-derived SSH product DUACS Pujol et al. (2016). The GLORYS12 velocity fields
188 are provided on the regular grid with spatial resolution of 1/12° and temporal resolution of
189 1 day. We re-interpolate the SSH fields on the same 1/12° regular grid at daily temporal
190 resolution.

191 For each individual trajectory, the considered spatial domain of the Eulerian conditional fields is
192 equal to, $4^\circ \times 4^\circ$ window for the North East Pacific and $10^\circ \times 10^\circ$ for the Gulf Stream. The size of
193 the domains corresponds to the furthest displacement from a given particle's deployment location
194 in each case study region.

195 Our benchmarks involve four different Lagrangian datasets:

196 • **Lagrangian Dataset L1:** this dataset is composed of 9-day trajectories of virtual particles
197 advected by the sea surface velocities from dataset E1 using Ocean Parcels, a state-of-the-art
198 tool for model-based Lagrangian drift simulation Lange and van Sebille (2017). Around 20
199 particles were randomly seeded every day across the domain and the studied period, amount-
200 ing to a total of 7900 particles.

201 • **Lagrangian Dataset L2:** this dataset contains 9-day trajectories of virtual particles advected
202 by the SSC of the E2 dataset with Ocean Parcels. It contains 7900 particles deployed at the
203 exact same locations and times as in the Lagrangian Dataset L1 described above, see Figure 3.

204 • **Lagrangian Dataset L3** This dataset contains trajectories from CMEMS drifters Etienne
205 et al. (2023) in the North East Pacific region, from 1992 to 2020. Those drifters are a part
206 of the Surface Velocity Program (SVP) and are built of 1-m long floating buoy attached to a
207 “drogue” deployed at 15 meters depth in order to reduce the windage slip Lumpkin et al.
208 (2017). Drifters positions are subject to a quality control and editing procedures before
209 being operational. Subsequently, trajectory positions undergo regular reinterpolation using
210 the Kriging technique at 6-hour intervals Etienne et al. (2023). For this study, we refer to
211 drogued-only drifters which are representative of the ocean currents at 15 meters depth and
212 are widely used in various application domains Lumpkin et al. (2012); Koszalka et al. (2011);
213 Lumpkin and Flament (2013); Kaplan et al. (2005); Lumpkin and Garzoli (2011).

214 This dataset contains 12.785 9-day trajectories in the North East Pacific case-study region,
215 green box in Figure 2. These trajectories are obtained by dividing the whole drifter trajectories
216 of the region in 9-day long non-overlapping segments.

217 • **Lagrangian Dataset L4:** this dataset contains 9-day trajectories of virtual particles advected
218 by the velocity fields from E3 with Ocean Parcels. We use the same initial deployment posi-
219 tion and time of the trajectories as in dataset L3.

220 *c. Performance metrics*

221 As evaluation metrics, we consider

- 222 • the mean Euclidean distance at the last time step (9-th day) in kilometers, that we denote D :

$$D = \frac{1}{N_T} \cdot \sum_{i=0}^{N_T} d_{i_{9\Delta}} \quad (11)$$

223 with N_T the number of trajectories and $d_{i_{9\Delta}}$ defined in eq.(10) with $j = 9$ days.

- 224 • the Mean Liu index \mathcal{L}_{Liu} without units, see eq.(9).
- 225 • the Mean absolute error between Lagrangian Time Scales $\Delta\mathcal{T}$ Krauß and Böning (1987), in
- 226 days:

$$\Delta\mathcal{T} = \frac{1}{N_T} \cdot \sum_{i=0}^{N_T} |\mathcal{T}_{R_i} - \mathcal{T}_{S_i}| \quad (12)$$

227 where \mathcal{T}_{R_i} and \mathcal{T}_{S_i} are respectively the Lagrangian time of the reference and simulated i -th
228 trajectory defined as:

$$\mathcal{T} = \int_{t_0}^{t_0+9\Delta} R_{\vec{v}}(\tau) d\tau \quad (13)$$

229 where $R_{\vec{v}}(\tau) = \frac{1}{9\Delta} \cdot \sum_{t=0}^9 \vec{v}(t\Delta + \tau)\vec{v}(t\Delta)$ is the autocorrelation function of the Lagrangian
230 velocity of a given particle.

231 These three metrics are computed between the reference and generated trajectories and averaged
232 over the whole ensemble of studied trajectories.

233 *d. Benchmarked deep learning models*

234 To evaluate the performance of DriftNet, we compare it to the following CNN and LSTM base-
235 lines inspired by Zheng et al. (2022); Ma et al. (2019):

- 236 • Convolutional Neural Network (CNN): a 2D convolutional block varying the number of chan-
237 nels from 27 (\mathbf{u} and \mathbf{y}_0 concatenated along the time dimension) to $8 \rightarrow 16 \rightarrow 32 \rightarrow 35$. Each

convolution layer has a kernel of size 5 and is followed by a LeakyReLU activation function with a slope of 0.2 for negative inputs. The output of this convolutional block is introduced by a reshape, where the two spatial dimensions of initial 3d matrix are flattened together, and the output is passed to a fully connected layer with 256 and 128 neurons decreasing the mentioned spatial dimension to 2.

- Long Short-Term Memory (LSTM): a 5-layer LSTM block with hidden size 96, followed by a ReLU activation function, then two fully connected layers with a ReLU in between. Each dense layer contains 864, and 128 neurons respectively, reducing the latent space to 2·35. This output is then reshaped so to contain the spatial dimension 2 and temporal one 35.

We also explore different configurations of DriftNet. In order to investigate the impact of each component of the model on the evaluation metrics, we modify its inner architecture, see section 3. a and Figure 1, and assess the performance of the following three architectures:

- DriftNet without the ConvLSTM block in the encoding operator \mathcal{E} ;
- DriftNet without the CNN block in the encoding operator \mathcal{E} ;
- a DriftNet-Unet architecture, referred to as DriftNet_U: in this architecture, we replace the CNN block in the encoding operator \mathcal{E} by a U-net block. The latter is composed of three 2d Conv-MaxPool-ReLU layers with kernel size 3 and channels evolving from 27 to 256 and three 2d ConvTranspose-Upsample-ReLU layers with kernel size 3 and channels evolving from 256 to 27.

e. Benchmarks

This section presents the experimental setups that we employ to assess the performance of DriftNet in both virtual and real-case scenarios.

Benchmark B1 This benchmark B1 aims to assess the reliability and accuracy of learning-based schemes in reproducing Lagrangian drift trajectories given fully-known underlying dynamics. This benchmark uses the Eulerian velocity field extracted from the dataset E1 and the trajectories from the Lagrangian Dataset L1. The resulting dataset is randomly splitted into train (80%), validation (10%) and test (10%) sets. DriftNet is fed with E1 velocity fields and trained to

265 reproduce reference trajectories from the L1 dataset. For benchmarking purposes, we consider the
266 CNN and LSTM baselines trained following the same strategy as DriftNet.

267 **Benchmark B2** This benchmark aims to assess from simulation data the ability to predict accu-
268 rately Lagrangian drift trajectories when the sea surface velocities are imperfectly known. We use
269 E2 velocity fields and L1 trajectories as reference. Similarly to Benchmark B1, we randomly split
270 the resulting dataset into train (80%) validation (10%) and test (10%) datasets. For benchmarking
271 purposes, we consider the state-of-the-art model-based method corresponding to trajectory dataset
272 L2 as well as the CNN and LSTM baselines trained following the same strategy as DriftNet.

273 Benchmark B2 also assesses the ability of DriftNet to exploit multi-source geophysical data.
274 Here, we complement the data with the Sea Surface Height (SSH) from Dataset E1. In this con-
275 figuration, Driftnet is fed with E2 velocity fields and E1 SSH fields to reproduce L1 trajectories.

276 **Benchmark B3** This benchmark addresses a real-case scenario with real drifters' trajectories
277 and operational ocean reanalyses. We use E3 velocity and SSH fields and real trajectories from L3
278 dataset as reference. Similarly to Benchmark B1, we randomly split the resulting dataset into train
279 (70%) validation (10%) and test (20%) datasets.

280 For benchmarking purposes, we consider the state-of-the-art model-based method correspond-
281 ing to trajectory dataset L4 as well as the CNN and LSTM baselines trained following the same
282 strategy as DriftNet. This benchmark assesses a DriftNet using solely E3 velocity fields and a
283 DriftNet using jointly E3 velocity and SSH fields.

284 5. Results

285 This section reports our numerical experiments for the three benchmarks introduced in the pre-
286 vious section, namely benchmark B1 (section 5.a), benchmark B2 (section 5.b) and benchmark
287 B3 (section 5.c).

288 *a. Results for Benchmark B1*

289 DriftNet presents the best performance for all metrics and significantly outperforms the CNN
290 and LSTM baselines, see Table 2. For the North East Pacific, the separation distance of 5.3km after
291 9 days is of around one order of magnitude smaller than the baseline CNN and LSTM models.
292 Similar performances are noted for Liu index, 0.06, and the error between Lagrangian time scales,

TABLE 2: **Performance of DriftNet for Benchmark B1:** we report the performance metrics for DriftNet and classical learning-based approaches for benchmark B1 in the two study regions: North East Pacific and Gulf Stream. The evaluation metrics are the last time step mean separation distance (D), mean Liu index (\mathcal{L}_{Liu}) and mean absolute error between Lagrangian time scales ($\Delta\mathcal{T}$). Bold indicates the best performance. We refer the reader to section 4.d for the details of the benchmarked architectures.

Method	North East Pacific			Gulf Stream		
	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days
DriftNet	5.3	0.06	0.14	91.1	0.26	1.17
CNN	43	0.5	1.46	151.85	0.63	4.03
LSTM	30.6	0.4	3.36	188	0.77	4.42

0.14. For the Gulf Stream, the overall accuracy is lower (separation distance after 9 days of 91.1km, Liu index 0.26 and Lagrangian time scale error 1.17), but still DriftNet outperforms the baselines. These experiments support the proposed Eulerian Fokker-Plank-inspired architecture.

DriftNet simulation error is larger for the Gulf Stream region. This is consistent with the resolution error in encoding initial positions, as evidenced in Callies et al. (2021). Here, Lagrangian trajectories simulated with an error corresponding to the resolution of the grid (*i.e.*, $1/12^\circ$) leads to a separation distance after 9 days of 20km (resp. 94km) for the North East Pacific region (resp. the Gulf Stream region). The larger sea surface velocities and sharper associated spatial gradients in the Gulf Stream region explains the larger separation distance after 9 days.

- **Impact of DriftNet components** we compare DriftNet in it’s original configuration to three variations presented in section 4.d namely a configuration combining two commonly used U-Net models separated by one ConvLSTM block, a second one containing the ConvLSTM block only, and finally one composed of 2D convolutional layers only.

Removing the CNN or LSTM blocks from the DriftNet architecture impacts the simulation performance, see Table 3. The separation distance increases from 5.3km to 42.1km (resp. 36.2km) for the North East Pacific region when we remove the LSTM (resp. CNN) block. Similarly, the U-Net version of the DriftNet does not lead to a better simulation performance. This likely relates to some overfitting in our training schemes due to the greater complexity of this UNet configuration. Same qualitative results are found in the Gulf Stream region.

TABLE 3: **Ablation study of DriftNet**: we compare the metrics of different configurations of DriftNet for benchmark B1: namely, the reference DriftNet, DriftNet_U with a U-net block instead of CNN, a DriftNet without the CNN block and DriftNet without ConvLSTM. The evaluation metrics are the last time step mean separation distance (D), mean Liu index (\mathcal{L}_{Liu}) and mean absolute error between Lagrangian time scales ($\Delta\mathcal{T}$). Bold indicates best performance.

<i>Method</i>	North East Pacific			Gulf Stream		
	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days
DriftNet	5.3	0.06	0.14	91.1	0.26	1.17
DriftNet _U	16.1	0.21	1.79	96	0.32	0.9
DriftNet without CNN	36.2	0.39	0.32	163	0.68	1.7
DriftNet without ConvLSTM	42.1	0.37	1.45	177	0.73	2.84

TABLE 4: **Impact of spatial resolution on DriftNet performance** Performance of DriftNet in function of the spatial resolution of the input Eulerian velocity field for benchmark B1 in both study regions: North East Pacific and Gulf Stream. Each line corresponds to a different spatial resolution of the input E1 field. The evaluation metrics are the last time step mean separation distance (D), mean Liu index (\mathcal{L}_{Liu}) and mean absolute error between Lagrangian time scales ($\Delta\mathcal{T}$). Bold indicates best performance.

<i>Resolution</i>	North East Pacific			Gulf Stream		
	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days
$1/12^\circ \times 1/12^\circ$	5.3	0.06	0.14	91.1	0.26	1.17
$1/6^\circ \times 1/6^\circ$	8.7	0.15	0.17	102.3	0.35	1.23
$1/3^\circ \times 1/3^\circ$	17.9	0.27	0.28	109.4	0.47	0.91
$1/2^\circ \times 1/2^\circ$	28.2	0.40	0.99	115.9	0.46	1.05

- **Impact of the resolution of the conditioning velocity fields.** Coarsening the resolution of the conditioning fields leads to degrading the performance, see Table 4. For both regions, the impact is more significant in the mean separation distance and the Liu index, while Lagrangian time scale estimation does not seem to be impacted by the degradation in the underlying resolution. Resolution is thus an important characteristic of the input fields.

317 *b. Results for Benchmark B2*

318 Table 5 illustrates the performance of the simulation of Lagrangian trajectories using degraded
319 velocity fields instead of the true velocity fields. Similarly to Benchmark B1, we compare DriftNet
320 to CNN and LSTM baselines. Here we also benchmark DriftNet with respect to the Lagrangian
321 simulations with Ocean Parcels Lange and van Sebille (2017).

TABLE 5: **Performance metrics for Benchmark B2 experimental framework.** We report the performance metrics for DriftNet and baseline models, all supplied with velocity fields from E2 and trained to generate trajectories from reference Dataset L1 in the two study regions: North East Pacific and Gulf Stream. The evaluation metrics are the last time step mean separation distance (D), mean Liu index (\mathcal{L}_{Liu}) and mean absolute error between Lagrangian time scales ($\Delta\mathcal{T}$). Bold indicates the best performance.

<i>Method</i>	North East Pacific			Gulf Stream		
	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days
Ocean Parcels	53.6	0.87	0.1	168.3	0.75	0.04
DriftNet	45.4	0.7	1.96	154.6	0.63	1.82
CNN	51.7	0.72	2.12	184.1	0.67	2.48
LSTM	47.9	0.69	4.35	163.2	0.67	2.1

322 DriftNet presents smaller D 45.4 km (154.6 km) and $\Delta\mathcal{T}$ than the other learning-based models,
 323 and smaller D , \mathcal{L}_{Liu} than Ocean Parcels, thus consistently outperforming the baseline approaches.
 324 The \mathcal{L}_{Liu} for the North East Pacific region is of 0.7 and is smaller than the most baseline ap-
 325 proaches, except the LSTM model (0.69) slightly outperforming the proposed method. While in
 326 the Gulf Stream the \mathcal{L}_{Liu} (0.63) is smaller than all the other approaches. The Lagrangian time
 327 scale error is larger for the proposed method than for the baseline Ocean Parcels.

328 **Impact of auxiliary variables** Table 6 assesses how the availability of the SSH fields could
 329 improve the simulation of Lagrangian drift trajectories. Whereas Lagrangian model eq.(1) solely
 330 depends on the sea surface velocities, deep learning schemes can easily extend to multiple condi-
 331 tioning fields. Among the observed satellite-derived tracers on the sea surface, the SSH explicitly
 332 seems appealing as it informs the geostrophic component of sea surface currents Taqi et al. (2019);
 333 Ballarotta et al. (2022). In an idealized setting, we evaluate the potential added value of the true
 334 SSH fields provided as inputs to DrifNet either in place of E2 velocity fields or to complement
 335 them, see Table 6.

336 These results emphasize the synergistic potential of SSH observations and E2 velocity fields
 337 compared with using only one of these two input data. In the North East Pacific region, D is
 338 reduced by over 55%, \mathcal{L}_{Liu} by over 60% and $\Delta\mathcal{T}$ by over 45%. Similarly in the Gulf Stream,
 339 we report a relative gain greater than 25% for D , 30% for \mathcal{L}_{Liu} and 20% for $\Delta\mathcal{T}$. These re-
 340 sults are in agreement with the ability of observation-based SSH products to capture geostrophic
 341 dynamics Ballarotta et al. (2022), while data-assimilation-based velocity fields can reveal the

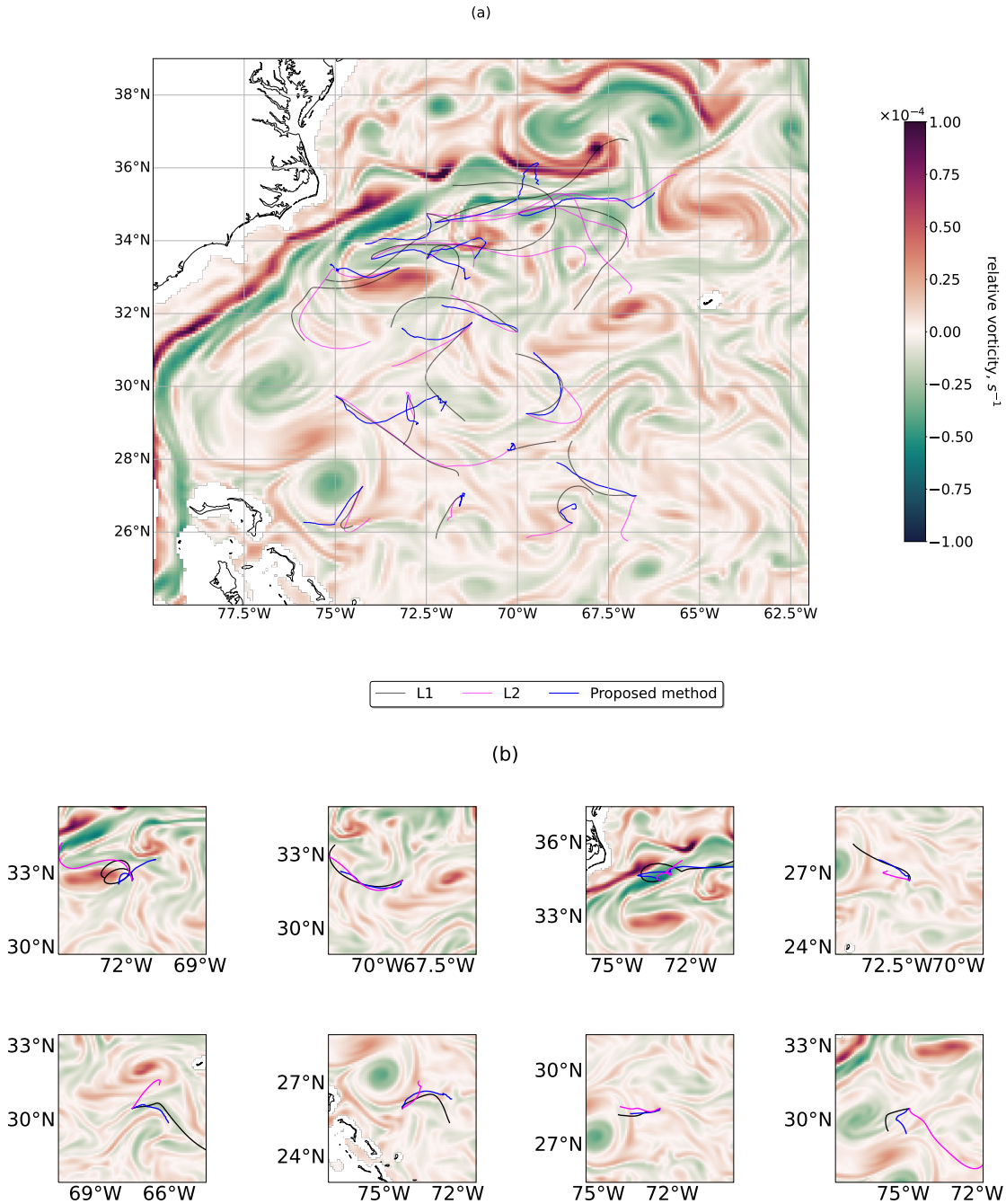


FIG. 4: Trajectories for Benchmark B2 in the Gulf Stream Lagrangian trajectories from Dataset L2 are depicted in magenta, trajectories simulated with DriftNet conditioned with SSC from E2 and SSH from E1 in blue, and trajectories from Dataset L1 in black. For better visual appreciation panel a) shows 16 randomly picked trajectories for the whole region while b) shows 8 random trajectories zoomed. All superimposed to the mean relative vorticity field of E1.

342 ageostrophic sea surface dynamics. The latter are expected to be significant for horizontal scales

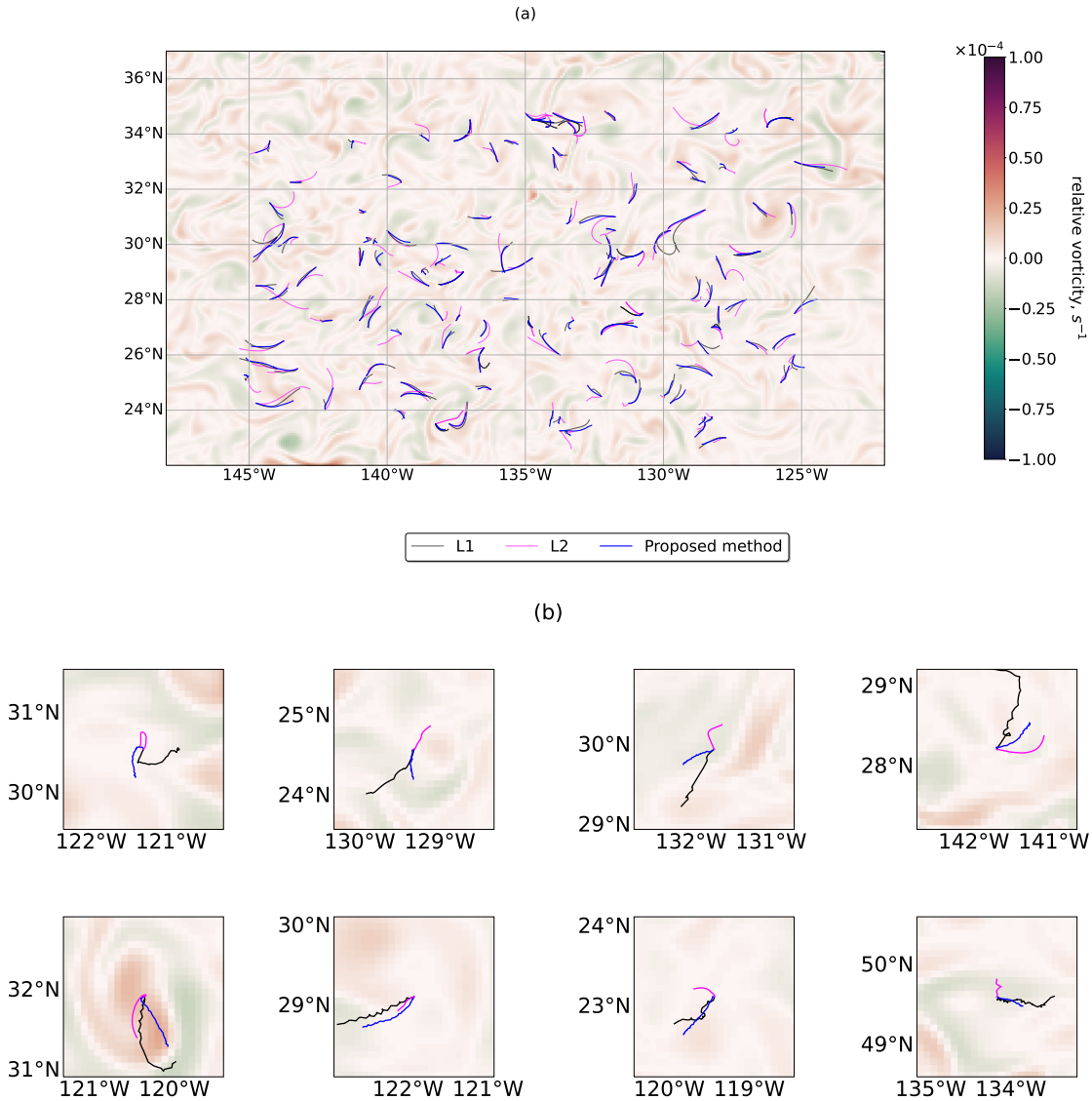


FIG. 5: **Trajectories for Benchmark B2 in the North East Pacific** Lagrangian trajectories from Dataset L2 are depicted in magenta, trajectories simulated with DriftNet conditioned with SSC from E2 and SSH from E1 in blue, and trajectories from Dataset L1 in black. a) shows 128 randomly picked trajectories for the whole region while b) shows 8 random trajectories zoomed. All superimposed to the mean relative vorticity field of E1.

343 below a few hundreds of kilometers, especially in Western Boundary currents such as the Gulf
 344 Stream Gula et al. (2015).

345 For both regions the trajectories are visualised at the Figure 4 and Figure 5 which show that
 346 DriftNet generates trajectories closer to the ground truth from L1 than the baseline L2.

TABLE 6: **Synergistic use of SSH observations and data-assimilation-based velocity fields in Benchmark B2.** We benchmark DriftNet schemes using as inputs data-assimilation-based velocity fields from dataset E2 and idealized perfectly-observed Sea Surface Height (SSH) fields (i.e., SSH fields from dataset E1). We illustrate performance in the two study regions: North East Pacific and Gulf Stream. The evaluation metrics are the last time step mean separation distance (D), mean Liu index (\mathcal{L}_{Liu}) and mean absolute error between Lagrangian time scales ($\Delta\mathcal{T}$). Bold indicates the best performance.

Method(Variable)	North East Pacific			Gulf Stream		
	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days
DriftNet (U, V)	45.4	0.71	1.96	157	0.73	1.84
DriftNet (SSH)	31.2	0.32	1.32	160.3	0.7	1.41
DriftNet (U, V, SSH)	22.2	0.29	0.73	127	0.52	1.44

TABLE 7: **Performance metrics for Benchmark B3.** Performance of different Lagrangian simulation schemes for Benchmark B3 which involves real data. For each scheme, we detail the considered input data: namely the velocity fields U and V from the GLORYS12 reanalysis Lellouche et al. (2018) possibly complemented by satellite-derived optimally-interpolated SSH fields Pujol et al. (2016). We report the performance metrics in the two study regions: North East Pacific and Gulf Stream. The evaluation metrics are the last time step mean separation distance (D), mean Liu index (\mathcal{L}_{Liu}) and mean absolute error between Lagrangian time scales ($\Delta\mathcal{T}$). Bold indicates the best performance.

North East Pacific			
Method(Variable)	D , km	\mathcal{L}_{Liu}	$\Delta\mathcal{T}$, days
Ocean Parcels (U, V)	73.77	0.65	2.2
DriftNet (U, V)	62.9	0.55	0.71
CNN (U, V)	68.64	0.57	0.32
LSTM (U, V)	64.3	0.55	2.1
DriftNet (U, V, SSH)	60.2	0.52	0.04

347 *c. Results for Benchmark B3*

348 In this real case situation, DriftNet outperforms the classical advection method, see Table 7
349 and Figure 6. Compared to the model-based baseline, the mean separation distance of 62.9km
350 is reduced by around 15%, the Liu index of 0.55 by 15%. Unlike previous B2 experiment, the
351 Lagrangian time scale error of 0.71 is significantly reduced. As in benchmark B2, we observe a
352 significant impact of the observed SSH fields in the quality of the simulation of the Lagrangian
353 trajectories. The combination of GLORYS12 velocities and satellite-derived SSH fields leads to
354 a reduction of the mean separation distance by over 4% compared to the baseline DriftNet and

355 by over 18% compared to the baseline model-based advection method. The Liu index is also
356 reduced by more than 5% compared to the DriftNet baseline and by over 20% compared to the
357 baseline advection method. The Lagrangian time scale error is significantly reduced, by over
358 94% compared to the DriftNet baseline and by over 98% compared to the baseline model-based
359 approach.

360 **6. Conclusions and Discussion**

361 In this study, we presented a novel deep learning framework, called DriftNet, for the condi-
362 tional simulation of Lagrangian drift trajectories on the sea surface. We drew inspiration from the
363 Fokker-Planck equation to explore a spatially-explicit Eulerian latent representation of the trajec-
364 tories in the proposed neural architecture. We evaluate DriftNet on three different benchmarks
365 using synthetic and real-world datasets. Our experiments support the relevance of DriftNet to ad-
366 vance Lagrangian drift simulations on the sea surface and better exploit available reanalysis and
367 observation datasets.

368 DriftNet consistently demonstrated a better performance than state-of-the-art model-based and
369 learning-based approaches, with respect to the following Lagrangian metrics: mean separation
370 distance, Liu index and Lagrangian time scale error between the ground-truth trajectories and
371 the simulated ones. When DriftNet is provided with fully known velocity fields from Nature
372 Run simulation (Benchmark B1), it reduces the overall error by more than 80% compared to the
373 state-of-the-art learning-based approaches. Moreover, when DriftNet is provided with degraded
374 velocity fields from OSSE-based assimilated simulation (Benchmark B2), it reduces the overall
375 error by over 15% compared to conventional point-wise schemes. These results are consistent for
376 the two case-study regions, even if they are characterized by notably different dynamical regimes.
377 Interestingly, we draw similar conclusions between synthetic and real datasets.

378 The performance of DriftNet exhibits variabilities across the two different regions. It likely un-
379 derscores the importance of region-specific adaptations in trajectory simulation models. As such,
380 the deployment of DriftNet on a global scale may require retraining the model to accommodate
381 the diverse dynamics in various oceanic regions Miao et al. (2023). While we expect the proposed
382 neural architecture to be generic, fine-tuning its parameters to align with the characteristics of a
383 given region seems crucial for the relevance of the simulations. This approach allows for the incor-

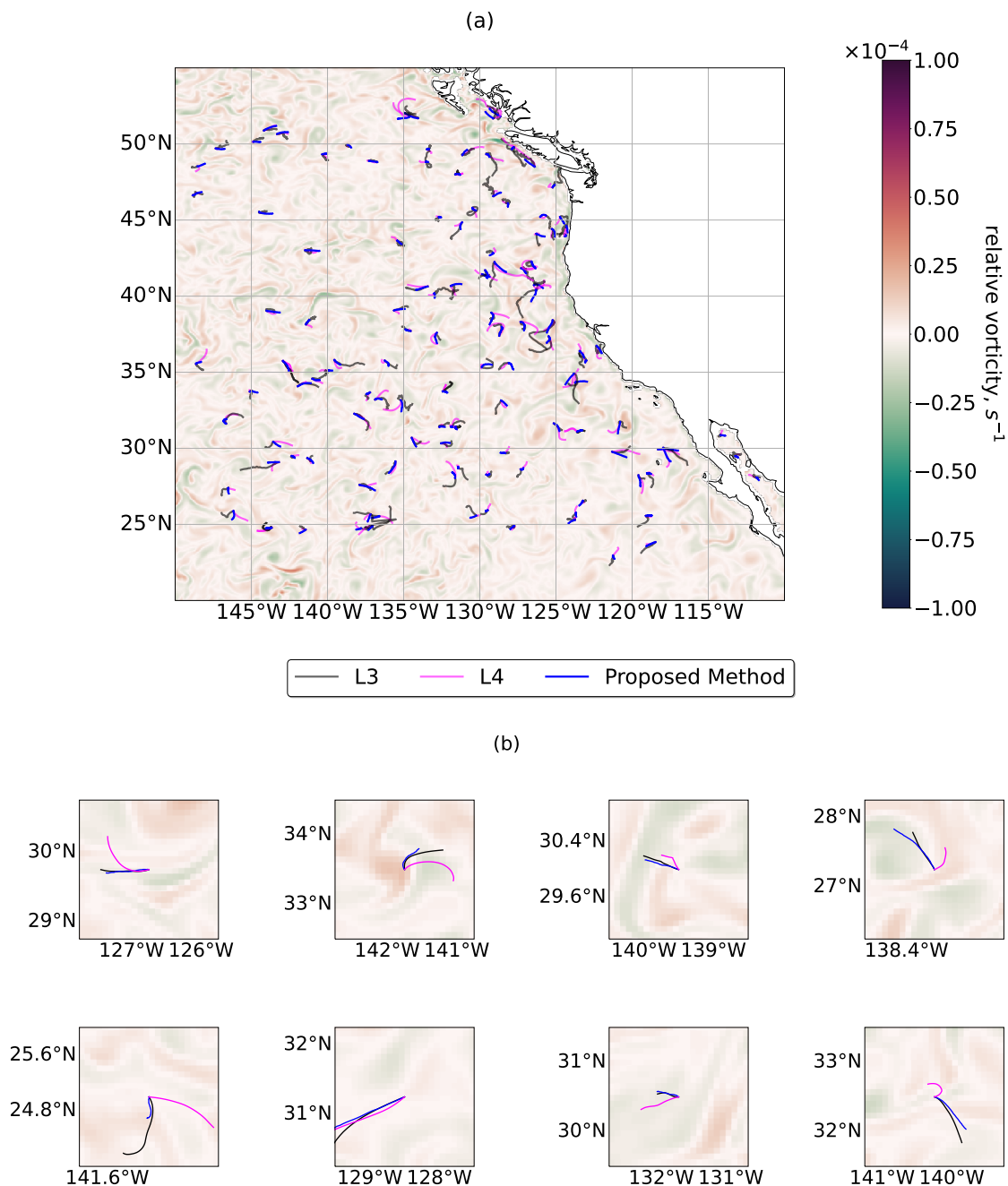


FIG. 6: **Examples of simulated trajectories for Benchmark B3:** we depict real drifters trajectories in black (L3), trajectories simulated with Ocean Parcels using GLORYS12 velocity fields in magenta (L4) and trajectories simulated with DriftNet using both GLORYS12 velocity fields and satellite-derived optimally-interpolated SSH fields in blue. Panel (a) illustrates trajectories from the whole region. Panel (b) zoom on a subset of randomly-selected trajectories. All superimposed to the mean relative vorticity of the GLORYS12 velocity fields.

384 poration of region-specific features, thereby enhancing the model’s predictive capabilities across
385 diverse geographical regions. Furthermore, leveraging transfer learning techniques to capitalize
386 on the knowledge gained from training in one region to inform model adjustments in others holds
387 promising avenue. By iteratively refining DriftNet through region-specific training and fine-tuning
388 iterations, we hope to develop a robust trajectory simulation tool capable of capturing the intricacies
389 of oceanic dynamics on a global scale.

390 Our benchmarks highlight the relevance of the SSH fields to improve the prediction of La-
391 grangian drifts on the sea surface both in synthetic and real case-studies, *i.e.* Benchmarks B2
392 and B3. We report a significant improvement for all the considered metrics when we comple-
393 ment reanalysis velocity fields with SSH fields. The SSH informs the geostrophic component of
394 the sea surface currents, which is important for the two case-study regions for horizontal scales
395 above 100km Checkley and Barth (2009); Thomas and Joyce (2010); Johns et al. (1989). As ocean
396 reanalysis datasets can usually retrieve sea surface currents for horizontal scales above a few hun-
397 dred of kilometers Benkiran et al. (2021a); Lellouche et al. (2021), the learning schemes likely
398 benefit from the SSH fields to recover a more robust representation of the sea surface dynamics
399 and improve the simulation of Lagrangian drift on the sea surface. The advent of wide-swath
400 satellite altimeters Benkiran et al. (2021b); Tchonang et al. (2021), possibly combined with neural
401 mapping schemes Martin et al. (2023); Beauchamp et al. (2022); Fablet et al. (2023), will lead
402 to a better reconstruction of satellite-derived SSH fields, which could in turn result in improved
403 learning-based Lagrangian simulations for real drifters on the sea surface.

404 Beyond the exploraton of satellite-derived SSH fields, the use of additional geophysical variables
405 to enhance trajectory generation offers another interesting approach to improve the simulation
406 performance. Future studies could investigate other variables such as Sea Surface Temperature
407 Ciani et al. (2020), Ocean Color Yang et al. (2015); Liu et al. (2017), Winds Cucco et al. (2016);
408 Solanlki et al. (2001) or Waves Tang et al. (2007) to refine trajectory simulations and gain deeper
409 insights into oceanic processes. By leveraging these variables, the proposed method could not only
410 improve trajectory accuracy but also facilitate comprehensive analysis of the underlying ocean
411 dynamics.

412 Inspired by established initiatives such as WeatherBench Rasp et al. (2020) and OceanBench
413 Johnson et al. (2024), we propose a new benchmark for a standardized assessing and comparing

414 Lagrangian trajectory simulation models <https://github.com/CIA-Oceanix/DriftNet>. Through pro-
415 posed datasets and evaluation metrics, this framework seeks to foster collaborative frameworks
416 involving ocean scientists and ML practitioners. In this context, future work could explore new
417 evaluation metrics and extend the proposed benchmarks to a broader array of oceanic regions.

418 From a methodological point of view, it seems appealing to extend the DriftNet to a probabilistic
419 framework with a view to sampling ensembles of realistic drift trajectories. Conditional Genera-
420 tive Adversarial Networks (GAN) Roy et al. (2022) naturally arise among the promising solutions.
421 Other generative models such as diffusion models Tashiro et al. (2021) could also be explored. Re-
422 garding real-time operational applications, such as search and rescue operations, or iceberg drifts,
423 such generative schemes would also naturally apply to the short-term forecasting of Lagrangian
424 trajectories on the sea surface using sea surface conditions issued from short-term forecasts. This
425 research direction would likely benefit from the development of neural ocean forecasts Wang et al.
426 (2024); Xiong et al. (2023) to advance model-based operational forecasts Drévilion et al. (2008);
427 Benkiran et al. (2024).

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437 *Data availability statement.* The data used in this study, including ocean currents and sea surface
438 height are openly accessible through the Copernicus Marine Environment Monitoring Service
439 (CMEMS). The data from the B1 and B2 benchmarks are subject to access restriction, one should
440 refer to Mercator Ocean International in order to claim the access rights. Additionally, the code
441 implemented for the evaluation of the methods discussed in this article is available as an open-
442 source project on GitHub: <https://github.com/CIA-Oceanix/DriftNet>.

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