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Do quantities have unique true values? The problem of non-uniqueness in measurement

Fabien GREGIS

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Abstract.

The “problem of non-uniqueness” objects to the idea that an empirical quantity is characterized by a unique “true value”: representational structures involving single-valued quantities in fact conflict with the complexity of the systems under examination. The roots of the problem are tied to the model-based nature of measurement. Metrologists answered it in terms of uncertainty, but erred by providing an epistemic answer to a representational problem. One should rather look at it in terms of the validity and the degree of refinement of the models that are involved in a measurement. I thus propose to draw on approximate truth to reconstruct an alternate understanding of “true value”.

1. Introduction

According to a common realist standpoint, measurement is “an activity aimed at discovering the true value of a specified quantity that exists independently of how we measure it” [1, pp. 368–369]. This appeal to the “true value” of a quantity is not only constitutive of philosophical views on measurement, but is also recurring in measurement textbooks. Indeed, the concept of true value is a central component of scientific theories of experimental error, in particular the probabilistic theories inherited from Gauss and Laplace [2]. It is usually said that measurements are bound to be imperfect, affected by “measurement errors” that make results “inaccurate”, i.e., diverging from the true value of the quantity under examination.¹ When experimenters report measurement results, they must account for this inaccuracy by specifying a margin of error, the “measurement uncertainty”. The main guide on the vocabulary of measurement, the *International Vocabulary of Metrology* (hereinafter VIM for short), provides the summary of a traditional methodological standpoint: “the objective of measurement (...) is to determine an estimate of the true value that is as close as possible to that single true value” [5, p. viii].

Yet, scientists often experienced discomfort in working with the concept of true value of a quantity, which not only proves difficult to define, but also carries a significant metaphysical weight. An example is found in the sudden warning given by precision physicists E. Richard Cohen and Jesse W. DuMond, in the midst of a technical paper:

No one can guarantee that an evaluation of the fundamental constants at a given epoch yields the “true” values. Absolute truth, if these words have meaning, is beyond the realm of physics. [6, p. 540]

In the field of metrology, the science of measurement, this wariness turned into full-fledged criticism, especially in the second half of the twentieth century. Metrologists even alluded to “the catastrophe of the concept of true value” [7, p. 65]. In the wake of this criticism, the current main international guide on measurement uncertainty, the *Guide to the expression of uncertainty in measurement* [8] (hereinafter GUM for short), published in 1993, explicitly avoids the use of the term.²

¹ Accuracy is a polysemic word both in common and scientific usage (see in particular [3], p. 1084). A traditional understanding is “closeness to the truth” ([4], p. 162; [5], p. 21).

² As described by Bich, “the approach based on true value and error was questioned (...) The very terms were almost banned from the literature, and whoever dared to use them was considered suspiciously as a supporter of old ideas” [9, p. 2156]. Giordani and Mari, though, observe that there has been more recently a slight resurgence of the general acceptance of the concept of true value [10].

One may indeed question whether quantities “have” such true values;³ and, if they do, whether we can ever claim to know them; this provides for a mix of arguments that have been directed in metrology against the appeal to “true value”. I have argued in an earlier work that these questions are disjointed and call for their own separate treatment [11]. Drawing from metrologists' own words, I separate between:

- *An epistemic issue: the “problem of unknowability”* [12]. The true value of a given quantity is forever unknown, i.e., unknowable, because one cannot step outside of one’s own cognition to witness reality as it is. It is thus “operationally useless” (as summed up by [10]).
- *A representational issue: the “problem of non-uniqueness”*. A true value ought to be unique, yet in many practical cases it appears that the clause of uniqueness is broken. The traditional system of representation involving single-valued quantities conflicts with the complexity of the empirical systems under examination.

The present article is devoted to the problem of non-uniqueness, which it will approach from the practice-oriented standpoint of metrologists. I will explore its consequences on the metrologists’ conception of a true value of a quantity, and I will defend a model-based framework in which the problem of non-uniqueness does not disqualify “true value” altogether. This is intended as a defense of the concept within an overall (moderate) realist standpoint, but not as a defense of scientific realism itself.

Section 2 states the problem, generalizing it from the example of the Earth’s radius. My main objective will be to clarify in what sense it is a problem and to show that it is intimately tied to the model-based nature of measurement through the crucial metrological concept of “measurand”. Section 3 then recollects and criticizes the way metrologists have suggested to deal with the situation; this will lead to a second crucial concept, the “definitional threshold”. Section 4 suggests a way to bind part of the metrologists’ account together with a realist standpoint based on approximate truth, in order to explore a context in which the validity of the concept of true value of a quantity can be maintained.

2 The problem of non-uniqueness

2.1 Is Earth's radius unique?

The Earth is round. So, it has a radius. A quick search on the web may tell you that this radius is 6 371 km. But in fact, the Earth is *not* round—it is squashed ever so slightly at the poles, for example. So, what does this value refer to? That is, what exactly counts as *the* radius of the Earth? Herein lies a foundational issue. If we ask for *the* value of *the* Earth’s radius, we can only answer in a straightforward way by presupposing that the Earth *has* a definite radius. As soon as we acknowledge that a spherical Earth is itself an approximation, we encounter a problem with the definition of its radius even before we can attempt to measure it.

The radius of a mathematical sphere is found when taking the distance from its center to a point on its surface. Can we apply this to the Earth? Notwithstanding the complexity of identifying the center of the Earth, an even more compelling problem is that the distance from the center to the surface⁴

³ I use the term “have” in quotes to convey a generic meaning as it corresponds to a popular, albeit informal saying: “such quantity has a value of x ”. This already opens up, though, a question central to the philosophy of measurement, namely whether values are assigned to quantities, are representations of quantities, or are expressions of the magnitude of quantities, and so on. The intricacies of this discussion, which encompasses, for example, debates around the now-classical Representational Theory of Measurement (RTM) or realist ontologies of quantities, are left out of this paper.

⁴ “The physical surface of the Earth is the border between the solid or fluid masses and the atmosphere” [13, p. 2.]

depends on the location. The poles are the closest points of its surface to its center, at a distance of approximately 6 357 km. Conversely, scientists have established that Mount Chimborazo in Ecuador is the farthest summit from the Earth’s center, at a distance of about 6 384.4 km.⁵ The difference of roughly 30 km between these two boundary values leads us to wonder why the radius of the Earth would even be given to the nearest kilometer—it appears in fact that there may not be one unique radius of the Earth. At the very least, the term “the radius of the Earth” turns out to be too vague to be attached to a definite length.

Importantly, the origin of this predicament is not epistemic: we are not wondering whether or not we *can exactly know* the radius of the Earth, but whether we can even talk of a radius of the Earth. The issue at stake is not about knowability and is indeed independent of our state of knowledge: perfect knowledge of the shape of the Earth would not make the term “radius of Earth” any less vague; if anything, it would only make us even more aware that it is vague.

An answer to this puzzle could be to make quantity terms more specific. This is, in fact, what geodesists did: based on our knowledge of the shape of the continents and of the Earth’s gravity field (which shapes the surface of the oceans), they were able to design different kinds of *average* radii, built on different reference models of the Earth. The mean Earth model, for example, is “a reference model [that] corresponds to a spherical average of the real Earth” [14, p. 237]; the radius of this average sphere—to which refers the 6 371 km we so often find—is called the mean radius of the Earth. Another model is the reference ellipsoid, more realistic in the sense that it accounts for the flattening of the Earth at its poles;⁶ it defines two radii, “equatorial” and “polar”. The International Earth Rotation and Reference Systems Service (IERS) provides a value for the former, along with its measurement uncertainty [17, p. 18]:

$$a_E = 6\,378\,136.6 (1) \text{ m} \tag{1}$$

This means that geodesists claim knowledge about the equatorial radius with a precision of 10 cm.

Undoubtedly, the “mean radius” and the “equatorial radius” of the Earth are more specific terms than the generic “radius of the Earth”. And yet, they remain vague as well. The shape of the Earth varies with time, for example due to tidal effects, gravity field variations, or redistribution of terrestrial masses [13]. One could even add, stretching the argument to its limits, that it becomes difficult to identify what exactly is the distance between the center of the Earth and a point of its surface, for example because of atomic granularity, thermal agitation, or even quantum indeterminacy (each factor introducing vagueness at a different scale). No matter how many subtleties one can add to this list, they make it challenging to define an absolutely unique length that could be called the radius of the Earth.

Table 1 lists a sample of similar situations that have been pointed out in the scientific and philosophical literature with different kinds of quantities. While most of those examples come from chemistry or physics, an even more extensive review would likely suggest that this predicament concerns many cases of measurement in science, spanning all empirical fields, from physical to social sciences. In the metrological literature, one of the main conclusions has been that “there is not a single true quantity value but rather a set of true quantity values consistent with the definition [of a quantity]” [5, p. 20].

⁵ NOAA’s National Ocean Service, URL=<oceanservice.noaa.gov/facts/highestpoint.html>. Last consulted 2022-11-06.

⁶ For a recent historical account of how geodesists struggled over the development of elliptic models of the Earth in the 18th and 19th centuries, see [15]. See also [16] for an epistemological follow-up, in particular regarding the problem of coordination.

However, the very idea of a non-unique true value seems self-contradictory,⁷ and may appear at first sight to directly violate a classical commitment of realism in measurement, that Hasok Chang called the “principle of single value”: “a real physical property can have no more than one definite value in a given situation” [27, 28]. This apparent contradiction is the starting point of what I call the *problem of non-uniqueness of the true value of a quantity*, which I shall refer to as the “problem of non-uniqueness” throughout this paper. The problem of non-uniqueness has participated in an overall critical stance towards the concept of true value and its use in the formalism in measurement. The *Guide to the expression of uncertainty in measurement* insists on the fact that “a unique ‘true’ value is only an idealized concept” [8, p.50]. In his discussion of French high school physics programs, the physicist Jacques Treiner (who was part of a group commissioned by the Ministry of Education to work on the theme of measurement uncertainties) asserted that this true value “does not exist” [29, p. 10]. Acknowledging that there is an inherent problem with the single-valued account of quantities may provide an incentive to drop the principle of single value and to devise alternative modes of quantification. Indeed, there exists attempts based for example on fuzzy sets [30, 31] or on probabilistic theories of quantification [32, 33, 34] which lead to describing properties of nature with other mathematical structures.

My objective here is not to explore new areas of research, which are certainly fruitful but remain arguably marginal, at least for the moment in a practice-oriented context (as they do not correspond to usual conception of scientists in their day-to-day practice); rather, I wish to better understand why and to what extent there is actually a problem, and what can be done to remedy it without radically shifting approach. Accordingly, the remainder of this section will focus on clarifying the structure of the problem. It now requires refining what is meant by “true value of a quantity”.

⁷ René Dybkær, long involved in the management of the metrology guides, summarizes the situation as follows: “the true magnitude of a quantity can only be represented by a distribution of true values, but definitions of theoretical metrological concepts usually select a true value (...) as a representation of truth” [26, p. 215].

Quantity term	Sources of ambiguities	More specific quantity terms
Length of a sheet of material ^(a)	Shape of the sheet Temperature dependence	Average length Maximum or minimum length Length along a specific axis Length at a specified temperature
Height of a person ^(b)	Daytime compression phenomenon (“For an adult, height varies by approximately one centimeter” from the start to the end of the day) ^(b)	Morning height Evening height Average height over 24h
Population of a country ^(b)	Migrations Deaths, births (≈ 1 every 25 s)	Population at a specific time
Temperature of the triple point of Water ^(c)	Isotopic composition of water	Triple point temperature for: • natural isotopic composition of water • standard isotopic composition of ocean water (“Vienna Standard Mean Ocean Water”) ^(d)
Frequency of a spectral line ^(b)	Half-life of excited atomic states (determines spectral width)	Average frequency and standard deviation
Speed of sound in the air ^(e)	Composition of the air Temperature and pressure conditions	Speed of sound in a homogeneous air of specific composition, at given temperature and pressure conditions ^(f)

Table 1: Examples of instances of the problem of non-uniqueness. (a) is the archetypal example, found under different forms in the GUM, [8], pp. 49–50; [4], p. 170; [18], p. 4; [19]; [20], p. 299; or [21], p. 156. (b) Treated in [22], p. 2. (c) Documented by [23], p. 330. (d) See the BIPM SI brochure [24], p. 184. (e) Given by [20, p. 303] and [25]. (f) [8], p. 49. It could be argued that the second and third examples are of different nature, because they stress a temporal variation of the quantity considered. The point is that the time factor is not accounted for in the definition of the quantity terms. Census tables, for example, will typically record populations by year, while fluctuations are much faster.

2.2 Values of quantities and true values of quantities

Quantification is an operation by which properties are apprehended as quantitative structures⁸ in view of their evaluation. Quantities in a general sense (or *general quantities*), such as mass or length, do not take values. Values are instead meant to characterize *individual* quantities,⁹ such as the length of a given object or the mass of the electron. This characterization requires the introduction of a unit,¹⁰ namely a quantity of the same kind as the quantity under examination, that serves as a common reference. Once a unit has been defined and agreed upon, an individual quantity can then get identified by a quantity value, given as the product of a number and a unit, such as 6 371 km. Following the canonical expression coined by James Clerk Maxwell [37], an individual quantity Q is identified as:

$$Q = \{Q\} \cdot [Q] \quad (2)$$

where $[Q]$ is the measurement unit, $\{Q\}$ is a number (the numerical value of Q in units $[Q]$), and $\{Q\} \cdot [Q]$ is the quantity value. For example, one can write an expression such as:

$$R(\text{Earth}) = 6\,371 \text{ km} \quad (3)$$

This serves two purposes. First, knowledge about the quantity, obtained for example by measurement,¹¹ can be communicated to others since the expression of the quantity is based on the publicly defined unit. Second, the quantity is treated as a mathematical variable¹² that can be involved in mathematical operations, for example in equations of physics.

With equation (2) comes the concept of value of a quantity. What, now, is a “true” value of a quantity? First of all, we must notice that the value is not true in itself, but is said “true” in virtue of the fact that it enters in a true proposition about the quantity. This characterization entices us to distinguish between two uses of equation (2), which can be used either to make prescriptive or descriptive statements. An instruction such as: “consider an object of mass $m = 1 \text{ kg}$ ”, that could be found in a textbook exercise, does not have a truth value in virtue of an actual property of the world. Likewise, an equation such as $L_{\text{std}}(\text{A4}) = 297 \text{ mm}$, summing up the ISO norm 216 which sets that A4 sheets of paper are required to comply with the standard dimension of 297 millimeters long (and 210 millimeters wide), is not made true or false by an actual property of the world. The same goes for the indication of a legal limit, for example a maximum acceptable blood alcohol content. All these are prescriptive uses of

⁸ The question of whether this structure is supposed to be found in the properties themselves, or whether it is attributed to them, or else, again falls within the scope of general discussions of the philosophy of measurement. Those confront various positions, including realism or representationalism for instance.

⁹ Mari and Giordani contrast “general quantities” with “individual quantities” [35], while Tal uses slightly different expressions: “general quantity types”, “specific quantity types” and “specific quantity tokens”. [36, p. 855]

¹⁰ This excludes ordinal quantities (encompassed in the VIM within the superordinate concept “quantity”) which can be ordered “but for which no algebraic operations among those quantities exist” [5, p. 15]. The discussion of this paper does not apply to ordinal quantities.

¹¹ Following [38], I mark a distinction between quantification and measurement, in that measurement is neither necessary nor sufficient for quantification but is nonetheless an epistemically privileged process by which information about empirical entities is acquired.

¹² An intricate question is lurking here about the variables involved in the equations of physics: are they the quantities themselves or their numerical expressions? As [39] showed, physicists became conceptually able to switch from an interpretation of the equations of physics in terms of numbers (the “measures” of the quantities) to an interpretation in terms of quantities themselves—or rather, their “magnitudes”. This relates to an issue already raised by Bertrand Russell about the relevance of two separate concepts of magnitude and quantity [40, chap.19 “The meaning of magnitude”]. In recent discussions, while [35] suggested a way to bypass the concept of magnitude, [41] argued in favor of maintaining a distinction between concrete quantities and magnitudes, the latter being at “an intermediate level of abstraction between objects and numbers” (p.707). Here, I avoid entering into this specific debate.

equation (2). The concept of true value does not apply for this category of propositions, but pertains to a second kind of use of equation (2), when the latter is engaged in a descriptive claim about the empirical world. In this context, introducing the “true value” of a quantity implies, first, that a proposition such as (P) “ $Q = \{Q\} \cdot [Q]$ ” has a truth value; it means, then, that the true value of the quantity is the one for which the proposition (P) is true. In other words,

The true value of a quantity Q is the quantity value $\{Q\} \cdot [Q]$ for which the proposition (P) “ $Q = \{Q\} \cdot [Q]$ ” is true

Sometimes, publications (the GUM, for instance) dispense with the adjective “true” and simply refer to “the value of the quantity”. But this introduces an ambiguity, as there are many different possible kinds of quantity values: “assigned values”, “measured values”, “conventional values”, “theoretical values”, “best values”, “most probable values”, etc., which all have their own specific use. The use of the attribute “true” marks the idea that there is a distinction to be made between the concept of true value and those designated by the previous terms. As a matter of fact, people suspicious of the metaphysical implications of this attribute have sometimes sought to conflate “true value” with one of those less connotative terms,¹³ hoping by the same token to cut loose with any realist commitment and thus dispel the issues it carries with it (there is no conceptual issue, for instance, with the idea of multiple coherent values for a given quantity). In this latter context, the concept of true value, and with it the problem of non-uniqueness, are somewhat downgraded.¹⁴

The problem of non-uniqueness appears in the context of measurement science, because the use of the adjective “true” has commonly been understood in a realist sense. In that case, “true value” cannot be conflated with one of the previous terms, and has a clearly specific function. This is not to say that the epistemological foundations of metrology are, or have to be, strictly realist; in fact, they are often kept implicit, and, when expressed, may change depending on the context or the research traditions. However, the fact that metrologists insisted at will on the “unknowability” of the true value ([9, 44, 45, 46] are few examples among recurring references to unknowability), suggests a prevalence of the realist interpretation, at least when discussing the specific concept of true value—even if the intent is sometimes precisely to criticize it. At the very least, the concept of true value cannot in this context coincide with a coherent value, a best value or a conventional value, since all the latter are determinable on the basis of what agents know, i.e., are deemed “knowable”.

In such a realist context, two propositions (P) $Q = v \cdot [Q]$ and (P') $Q = w \cdot [Q]$ cannot be simultaneously true if $v \neq w$: this is the ground for the principle of single value.

2.3 From reference to models

In reaction to the metrologists’ conclusion that there is a set of true quantity values consistent with the definition of the radius of the Earth, it could be objected that the Earth simply does *not* have a radius. If the quantity term “the radius of the Earth” does not refer to anything in the world, it is neither true nor false to state that its radius is equal to 6 371 km, and the proposition “ $R(\text{Earth}) = x \text{ km}$ ” has no truth value, whatever the real number x . In that sense, there is also no such thing as the true value of

¹³ An example is found in [22] who believe that “theoretical value” or “reference value” should be preferred to “true value”.

¹⁴ Interestingly, metrology guides had introduced in the 1980s the seemingly paradoxical concept of “conventional true value”. This was made to contrast the unknowable true value with a knowable value which, “for a given purpose, may be substituted for the true value”, [42, p. 10]. As [43] suggests, this move was made “plausibly as a means to reduce the conflict between the metaphysical load of the concept of truth and the empirical requirements of measurement” (p. 264). The term “conventional true value” was dropped in the 2008 edition of the VIM.

the radius of the Earth. This is in line with a position defended by Paul Teller, who pointed out what he considers a general “semantic problem of reference failure”:

The world is far too complex for our language to get attached to completely determinate things, in particular, quantities and their value instances (...) Such determinate quantities as there may be fail to get attached to quantity terms, such as ‘time’, ‘mass’, ‘length’, ‘velocity’, ‘temperature’. With no determinate quantities attached to such terms, there are no determinate values for “them” to have. [25, p. 275]¹⁵

Note that Teller himself reminds that he is “not claiming that there are no quantities with exact values in nature, nor, as some antirealists would have it, that the whole idea of ‘things in nature’ is incoherent” [25, p. 275]. Rather he claims that our usual quantity terms do not refer to determinate quantities. Similarly, the core of our present problem is not an ontological concern about the existence of quantities; it is instead a representational issue about the quantity terms that are being used in measurement. One could argue that the quantity term “radius of the Earth” is unsatisfactory, and demand that we should restrict ourselves to sets of quantity terms within which any term unambiguously refers to a determinate quantity characterized by a unique true value. But not only does such a requirement make us more dependent on the validity of strong versions of realism in measurement (and more committed towards the existence of actual quantities in the world), it is also unsure whether we can ever achieve it in practice. Here, Teller’s argument hits critically: the world is too complex for us to proceed that way. An adequate description of the Earth, under these principles, would require not one but a whole galaxy of quantity terms, that would have to be properly identified, in order for them to be so specific that they indeed refer to genuine quantities (provided we can ever find them). A solution would be, at one extreme, to reason only in terms of fundamental constants of physics, for which the VIM claims that they are “considered to have a single true quantity value” [5, p. 20]. But surely, working only with fundamental quantities of physics is straight-out illusory; they are irrelevant to most scientific fields, unless by adhering to an extreme, unreasonable reductionism. Even supposing that the pool of acceptable quantity terms extends beyond the fundamental constants of physics, such a requirement remains very restrictive.

The state of scientific practice is quite different: scientists do not dictate themselves such constraints. Indeed, a counterpoint is to acknowledge that it does make sense for scientists to talk about the length of a sheet of paper, of the distance between Paris and New York, of the population of a country, of the speed of sound in the air, or to use the radius of the Earth as a parameter in some astronomy problems. Despite the limitations of these terms, scientists consider them as suitable parameters for their problems, because they are aware that they do not work on the empirical quantities themselves (whether they exist or not) but on *models of empirical quantities*.¹⁶ Seen from this angle, the problem of non-uniqueness invites us to look at measurements from the model-based standpoint that recently become of paramount importance in the contemporary epistemology of measurement (e.g., [49, 50]). Properties of nature are *being described* using single-valued quantities even when phenomena or objects only approximately comply with their idealized representation— as Teller reminds us, “it is through idealizations that we know the world” [25, p. 294]. The description of the Earth as a round object is the product of an idealization,¹⁷ a simplification designed to “give us a good enough picture to get along

¹⁵ Teller points out reference failures at three different levels: about quantities in general (length, mass, etc. and their dependence on physical theories), about units, and about individual quantities attached to specific objects or phenomena. The situation described in the present article concerns the third case. See [47] for a defense of realism in measurement against Teller’s “semantic anti-realism”.

¹⁶ Duhem had already pinpointed one century ago the idealized nature of the “symbols through which theories represent these realities” [48, p. 169].

¹⁷ Idealization is here understood, following [51], as “a deliberate simplifying of something complicated [...] with

for a wide range of objectives” [25, p. 288]. Idealization serves as an ingredient of a model of the target, itself inserted in a whole hierarchy of models of data, phenomena and theories [52, 53] introduced to mediate between the experiment and the theory [54]. The idea of a radius of the Earth draws from this description, and more complex radii may come with more complex models of the Earth, as seen in section 2.1. This account reiterates the earlier suggestion to mostly stay away from ontological concerns. It is unnecessary for the radius of the Earth to exist in order for scientists to build a spherical model of the Earth. The problem of non-uniqueness questions, in a practice-oriented context, the relevance of a single-valued system of quantitative representation of properties and objects. An ontological account of quantities may bring answers with regard to the validity or representational adequacy of certain models, but is not the main focus.

2.4 Quantities and measurands

Metrologists themselves have acknowledged that the task of identifying the target of the measurement and relating it to a quantity term to which a value can be assigned is a model-based activity [55], in close connection to the fate of a “cornerstone concept” [20] of contemporary metrology: the measurand. The term “measurand” was introduced in the instrumentation literature of the 1950s [56, 57], where it was designed as a shortcut for “the quantity being measured”.¹⁸ Its use spread to metrology after the publication of the first edition of the *International Vocabulary of Metrology* in 1984. Metrologists then pointed out that an experiment does not always directly target the quantity that one wants to measure, and the term “measurand” evolved into “quantity intended to be measured” [5, p. 17]. This new definition leaves open some questions about the nature of measurands [19, 20], especially as it identifies a measurand to a quantity. But this move was accompanied by a deep reflection about the fact that measurands have to be *defined* as a “conceptual construct” [20, p. 300] and not simply pointed out as a quantity that is given to us [60, p. 74].¹⁹

The definition of a measurand involves constructing a model of the target and its interactions with its environment [55, p. 2145], as illustrated by the mean and equatorial radii of the Earth which are based on reference models of the Earth and of its gravity field. The latter task includes reporting what metrologists call influence quantities, i.e., quantities used to describe external conditions that affect the measurement. The definition of the measurand introduces a part of choice, depending on the intentions of the agent, who can take account or not of these influence factors in the measurand model. For example, materials are often subject to thermal expansion; in this context, “the length of object O at 25°C”, “the length of object O at 0°C”, and “the length of object O” are three different measurands. To each influence quantity which is not specified in the definition of a measurand corresponds a degree of freedom in the way an experiment can be realized in order to perform the measurement: the measurand is “multiply realizable” (in the same sense as in Eran Tal’s study of the definition of the standard second, [3]). A measurand is the ingredient of a theoretical model made of abstraction, idealization

a view to achieving at least a partial understanding of that thing”, p. 248. The representation of the Earth by a sphere pertains to a specific type of what McMullin calls “Galilean idealization”, namely “geometrical idealization”.

¹⁸ “A physical quantity, property or condition which is measured”, in the AIBS glossary of instrument terms [58, p. 296]; “a [particular] quantity subjected to measurement” in the first two editions of the VIM [42] [59, p. 12 and p. 20 respectively].

¹⁹ Part of this question pertains to the debate between realism and operationalism, since an option could be to avoid making any model of the target and simply nominate the measurand as the output of a given measurement procedure. [55] also mention “a general option to reduce the requirements as for the measurand idealization is to include in its definition all the environmental conditions and to maintain them implicit (‘the measurand is the electrical resistance of the component in this moment in these conditions, unknown and whatever they are’).” As they point out, the counterpart is that “it prevents projecting the acquired information outside the strict (and unknown) context in which it has been obtained”, p. 2148. It thus makes it difficult to give measurands “transferable meanings” [19, p. 1].

and approximation; borrowing from [25, p. 17], its value describes something about “what it would be in the (...) simplified world characterized by our idealized larger picture”. There are multiple conditions of measurement consistent with its definition, and in that sense, it may be considered, following metrologists, that there are multiple true values consistent with the definition of the measurand.

This provides a model-based framework to understand the metrologists’ idea of a set of true quantity values. Within this framework, metrologists have addressed the problem of non-uniqueness in terms of uncertainty. I now wish to argue that it led them to misconstrue in part the overall issue.

3 The metrologists’ answer: an appeal to uncertainty

The problem of non-uniqueness goes beyond the mere philosophical conundrum and translates into an actual concern for scientific practice. This is because the concept of true value (i) is often summoned to explain the function of measurement and (ii) has an operational role in the management of measurement error and uncertainty. Already in 1947, it was pointed out in the French physics and chemistry teachers’ newsletter that “physicists have achieved the performance of making, in a large number of circumstances, instruments more precise than the quantities they are intended to measure” [61].²⁰ In his 1963 reference article on the assessment of accuracy in measurement, the statistician and metrologist Churchill Eisenhart drew attention to an “intrinsic difficulty at defining the true value” of quantities [4, p. 171], whose meaning depends on the purpose and the scope of the measurements at hand. This acknowledgement, though, was sufficiently secondary to the concerns of scientists that it remained largely ignored. Eisenhart himself did not go beyond some conceptual clarifications, and he pleaded for a terminological change attached to a form of ontological agnosticism:

I hope that the traditional term “true value” will be discarded in measurement theory and practice, and replaced by some more appropriate term such as “target value” that conveys the idea of being the value that one would like to obtain for the purpose in hand, without any implication that it is some sort of permanent constant preexisting and transcending any use that we may have for it. [4, p. 171]

Metrologists only began to tackle the operational side of the problem with the standardization efforts initiated in the late 1970s that converged towards the publication of the *International Vocabulary of Metrology* (in 1984, 1993 and 2008) and the *Guide to the expression of uncertainty in measurement* (in 1993).

3.1 A pragmatic solution: intrinsic and definitional uncertainties

In his analysis, Eisenhart had described the true value as being a “fuzzy concept” [4, p. 171]. The GUM took this thought further: it incorporated the idea, already suggested in the 1980s,²¹ that this fuzziness materializes into an *uncertainty* due to an “incomplete definition” of the measurand [8, p. 4]. Accordingly, “every measurand has [...] an ‘intrinsic’ uncertainty that can in principle be estimated in some way” [8, p. 50]. Intrinsic uncertainty was later renamed “definitional uncertainty”, a term defined in the 2008 edition of the VIM as a “component of measurement uncertainty resulting from the finite amount of detail in the definition of a measurand” [5, p. 25], which is now standard terminology in metrology.

²⁰ I thank Julien Browaeys and Nicolas Decamp for pointing me to this reference.

²¹ See [34] for an overview of the development by which metrologists arrived at this idea during that period.

Simply put, measurement uncertainty is the inclusion of a margin of error in a measurement result, ordinarily expressed by using the symbol \pm . Usual components of uncertainty can typically be due to uncontrolled variability of the experimental outcomes, to a lack of knowledge about the experimental conditions and about the undesired external effects influencing the measurement, to limits in precision and accuracy of the measuring instruments, or to calibration issues. The GUM and VIM's account mainly consists in adding to this list another component of uncertainty, namely intrinsic uncertainty. The main effect of the problem of non-uniqueness is then to increase the measurement uncertainty attached to the result. The list of uncertainty sources provided in the GUM [8, p. 6] makes clear that intrinsic uncertainty was introduced as a genuine component of uncertainty, *of the same kind* as the ordinary measurement uncertainty that metrologists are used to deal with. Indeed, it is suggested that intrinsic uncertainty can be evaluated quantitatively and then be added up ("propagated") to the other components of uncertainty in what metrologists call the "uncertainty budget", so as to derive the total measurement uncertainty attached to the result.²² In this account, the peculiarity of intrinsic uncertainty is not found in its nature but in its origin, since it arises one step before measurement, as a feature linked to the definition of the measurand.

As an interesting outcome, this account presents intrinsic uncertainty as an ultimate limit in accuracy, the "minimum uncertainty with which a measurand can be determined" [8, p. 50]. Indeed, since intrinsic uncertainty is attached to the definition of the measurand, it cannot be reduced by any experimental means. Therefore, even in an ideal case, where experimenters would be able to reduce experimental uncertainties to zero (using perfectly precise and accurate instruments, having total control and complete knowledge of the experimental setup), there would always remain an incompressible component of intrinsic uncertainty.

In sum, the core of the GUM solution is to treat the problem of non-uniqueness not as a foundational issue about quantities, but as a source of uncertainty to be aggregated to the overall budget. By processing the issue in terms of measurement uncertainties, the solution has the strong pragmatic virtue of avoiding any conceptual revolution and staying on familiar grounds. Moreover, it is rather straightforward to implement, provided one is able to evaluate intrinsic uncertainty or maintain it low enough so that its contribution remains insignificant (the latter being the GUM's favorite option). But by the same token, it becomes conceptually dubious: as convenient as it is, the solution intentionally neglects to address the foundational issue that needs being explored *before* one can even talk about uncertainties. There is uncertainty, for sure; but *what is the target we are uncertain about?* I thus argue that the GUM's appeal to measurement uncertainty runs into a major pitfall, because it is based on a category error: "intrinsic uncertainty" is not a genuine uncertainty, despite the efforts to present it as such.

3.2 Intrinsic uncertainty is not a measurement uncertainty

Measurement uncertainty is a quantitative component that scientists include to their report of a measurement result. Let's look for instance at the IERS value for the equatorial radius of the Earth:

$$a_E = 6\,378\,136.6 (1) \text{ m} \tag{4}$$

The measurement uncertainty, given here as 0.1 m, indicates that, in light of the knowledge gathered from experimental and theoretical inquiry, and taking account of the possible errors of measurement, scientists cannot settle on a single value for a_E but only on a set of possible values (usually interpreted probabilistically). The measurement uncertainty thus indicates a dispersion of possible values that can be assigned to the measurand. The idea that there is not one, but multiple possible values for the measurand seems similar to our problem so far. But we need to realize that we are in fact dealing with something different in nature: measurement uncertainty is an indication of the precision of *our*

²² This is made explicit in figure D.2 of the document [8, p. 53].

knowledge, while the problem of non-uniqueness highlights a vagueness in our definition of the measurand, independently of how well we know this measurand.

Intrinsic uncertainty differs from measurement uncertainty with regard to at least two features. Measurement uncertainty (1) is by nature extrinsic to the measurand, since it characterizes not the measurand itself but what one can tell of the measurand; and (2) is epistemic, as it relates to the agent's (or group of agents') *knowledge* of the measurand [62]. By contrast, intrinsic uncertainty (1) is inherent to the measurand and (2) is not epistemic. First, as explained by Baratto, "a convenient procedure is to consider (...) the derived (inherent or definitional) uncertainty as an *intrinsic property* of the measurand" [20, p. 305, my emphasis]. This is made rather explicit in different sentences of the GUM: as explained, "the 'true' value *has* an uncertainty"; or later, "every measurand *has* such an 'intrinsic' uncertainty" [8, p. 50, my emphasis].²³ Secondly, intrinsic uncertainty is not epistemic, since it characterizes a vagueness rooted in the definition of the measurand, independently of any state of knowledge. Whether or not we know that (and how) temperature influences the conductivity of copper wires does not change the fact that the definition of a measurand as "the conductivity of copper" without any reference to the temperature introduces a component of vagueness. In sum, intrinsic uncertainty is not a measurement uncertainty, cannot be evaluated as an uncertainty and cannot be integrated as such in an uncertainty budget. The solution offered by the GUM, which requires to mix up entities of different nature, should not be in principle possible. It seems logical that an epistemic concept such as measurement uncertainty should not enable us to grasp the problem of non-uniqueness if we agree, as defended earlier, that the problem itself is not an issue about our actual state of knowledge.

Interestingly, the term "definitional uncertainty" that was introduced in the VIM as a lexical replacement²⁴ for "intrinsic uncertainty" could allow for a different interpretation, since it refers not to an uncertainty *intrinsic* to the measurand but *resulting* from the measurand definition. Again, a measurand defined as "the conductivity of copper" is vague, as it does not address the property's dependence in temperature. At different temperatures, measurements of such a measurand will yield different outcomes: the vagueness in its definition becomes a source of variability in the measurement outcomes. In principle, one can attempt to monitor this variability by keeping track of the measurement conditions in an appropriate model correlating variations of temperature to variations of conductivity. There will be no loss of information, and no related uncertainty. But if the variability is not controlled, there will be uncertainty. Alternatively, if, say, one measurement is made at a given temperature, but that temperature is omitted in the report of the result, then uncertainty will once again arise when the result is communicated to other agents. We are here in the case described by Ted Vosk and Ashley Emery: "if there is definitional 'uncertainty', the uncertainty is not about the value of the quantity but about the identity of the target itself" [63, p. 201]. In light of brief analysis, it is possible that vagueness in the definition of a measurand may be a factor of actual measurement uncertainty, in connection with the fact that information has been lost in the communication chain. Admittedly, it might be possible to devise a sound formalism to evaluate such uncertainties, call them "definitional" and integrate them into the overall uncertainty budget (see [64, 65] as notable sketches). In that specific sense, "definitional uncertainty" could be considered as a genuine component of measurement uncertainty. However, this hardly relates to what the framework of the GUM and the VIM originally intended. More importantly, this direction neglects once again the conceptual side of the problem of non-uniqueness. It focuses on measurement uncertainty, and thus does not address the problem at its root but only one of its possible epistemic aftermaths.

3.3 An epistemological answer to a representational problem

²³ It should be noted, though, that the GUM is somewhat ambiguous in that it claims right afterwards that intrinsic uncertainty "is the minimum uncertainty *with which* a measurand can be determined" (my emphasis). In my view, this ambiguity shows that the document missed the profound subtlety of the issue.

²⁴ This is made clear in the note 3 of the definition [5, p. 25].

The metrology guides have in common that they insist on dealing with non-uniqueness in terms of measurement uncertainty. The previous clarifications help us understand the inner limits of this account. Focusing on uncertainty is providing an epistemological answer to what is in fact a representational problem, since the problem of non-uniqueness relates to an inadequacy between a mode of representation and the empirical structure of the target, while measurement uncertainty points to a lack of knowledge about that target.

Intrinsic and definitional uncertainties have been introduced with the purpose of continuing to operate under the classical framework, but this classical framework was itself founded upon an assumption of an unequivocally identified unique true value. As a consequence, the VIM and the GUM accounts are ambiguous: while acknowledging that there is not one unique true value of the measurand, they act *as if* there was one. They do as if the consequence of non-uniqueness was only to hinder the knowledge that one can acquire about this unique true value. Yet, on the contrary, if the concept of true value itself is challenged, then it is not even clear what measurement uncertainty is about! Metrologists themselves noticed this drawback, as admitted by the 2010's head of the working group in charge of the maintenance of the VIM, Charles Ehrlich:

[if] there is no essentially unique true value of the measurand [...] the meaning of the PDF curve [the probability distribution describing the measurement result] becomes somewhat obscure [66, p. S151].

According to this analysis, intrinsic and definitional uncertainties may even appear as a subterfuge designed to *avoid* addressing the problem of non-uniqueness frontally. This becomes ever more blatant when one realizes that the GUM does not provide any quantitative method for evaluation of intrinsic uncertainty—nor does it explain how to combine it with other components of measurement uncertainty. As a matter of fact, the GUM swiftly evacuates the problem by narrowing the scope of the document to cases where “the measurand (...) can be characterized by an essentially unique value” [8, p. 1], i.e., cases in which intrinsic uncertainty is negligible to the point that it is not even necessary to evaluate it. In that framework, everything goes indeed as if the measurand was in fact characterized by a unique true value; and the issue is simply dismissed.

Practical as it may be, the response of metrologists to the problem of non-uniqueness cannot be considered satisfactory from a conceptual point of view. I suggest that one should instead look at the descriptive value of the models that are involved in the definition of the measurand. Intrinsic uncertainty could be reintroduced profitably not as a component of uncertainty but as an indication of the level of precision of a given model in accordance to given purposes. I would like to restate the problem accordingly, avoiding in the process the use of the term “uncertainty”.

3.4 Definitional threshold and model evaluation

Even if the Earth is not a perfect sphere, it can be argued that it looks like one when observed from afar. The reason is that the equatorial bulge is small compared to the total size of the Earth, and that other irregularities of the Earth's shape (its relief, for example) become almost imperceptible from that perspective. The existence of relief irregularities, on the contrary, is obvious from the surface itself. Scale seems to be a factor here: it makes it more or less acceptable to base a scientific investigation on a spherical model of the Earth. For an astronomer, it might be good enough, while it would trivially preclude the work of (say) a geologist studying the height of mountain ranges. As simple as it is, this preliminary analysis reminds us once again that it is not the knowledge (and therefore the uncertainty)

that we can get about the objects and phenomena under study that is at stake; instead, the concern is about the relevance of the models involved, namely here of the models used in the definition of the measurand.

[67] have submitted a lexical clarification that can be put to good use in the present situation. They suggested introducing the term “definitional threshold”, defined as the scale “below which the definition of the measurand is no longer valid”. I suggest understanding it (although put slightly differently) as an indication about the vagueness of a given measurand definition resulting from the contrast between the complexity of the target and the limited amount of detail that was brought in the model of the target. Variations in the value of a measurand below the definitional threshold cannot be interpreted because the model is then too coarse. This does not mean that no variation of the target may happen below the scale defined by the definitional threshold, but that the current model(s) in use make us unable to report any of these transformations.

Figure 1 suggests a way to illustrate the definitional threshold in the case of the radius of the Earth. It represents two schematic, fictional, ellipsoidal pseudo-Earths, where the equatorial bulge has been grossly exaggerated to two different degrees. In each case, the schematic Earth is comprised between an inscribed sphere and a circumscribed sphere (both in tight dotted lines) which very roughly delineate a range of acceptable spherical models of the Earth. Of course, a mean sphere (in spaced dotted line) seems intuitively preferable, but the illustration enables us to define, at least as a first attempt, the definitional threshold as the difference in radius between the two boundary spheres (about 30 km for the actual Earth). Obviously, it will be larger in fictional case (a) than in fictional case (b).

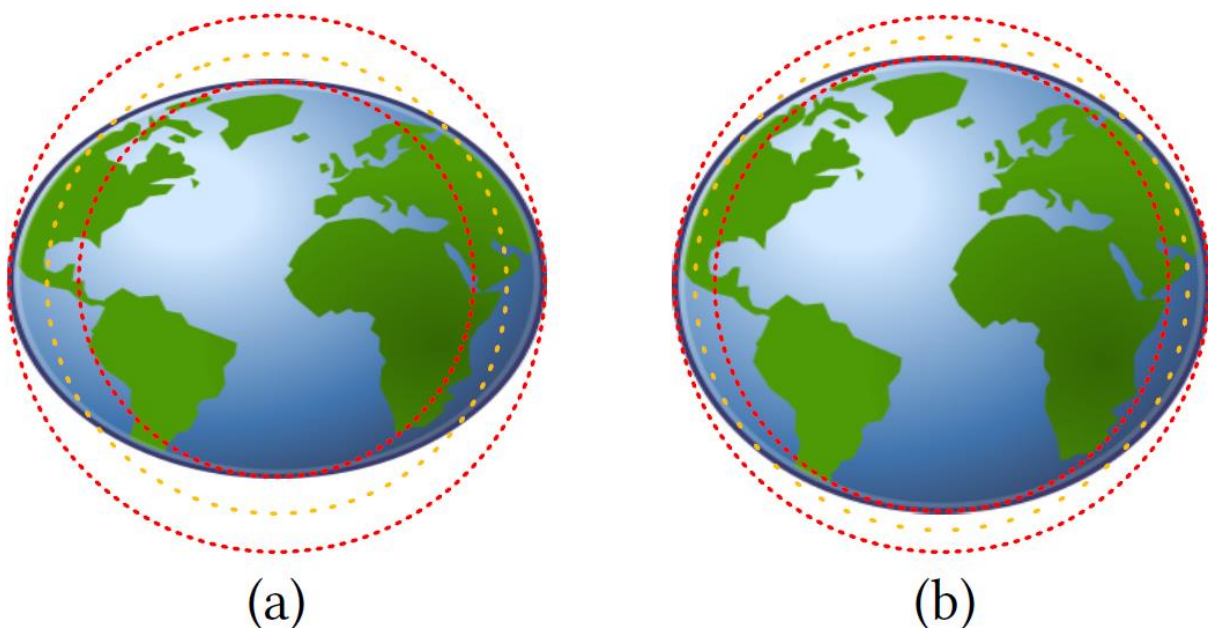


Figure 1: An illustration of the definitional threshold for the radius of the Earth. If the definition of the radius is based on a spherical model of the schematic Earth, there is a definitional threshold, which is larger in case (a) than in case (b). Modeling the target not as a sphere but as an ellipsoid would involve two different radii, to which would correspond no definitional threshold. Since the figure assimilates the schematic Earth to a geometrical figure, an ellipsoid, its mean radius also has no definitional threshold.

(Figure freely adapted from an image published in the public domain by wikimedia commons user Inductiveload)

Actual measurement situations will most often involve more complex factors. Eventually, the definitional threshold tends to increase with the amount of idealization that the model introduces in the representation of its target. A more refined description of the shape of the Earth will result in a lower definitional threshold, which probably explains why geodesists feel justified in providing a value for the equatorial radius of the Earth with a precision of 10 cm (in fact, by claiming this precision, geodesists tacitly assert that the definitional threshold is less than 10 cm). The definitional threshold also tends to increase when influence variables are left free, deliberately or not. A measurand defined as “the electrical conductivity of copper at 25°C” has a lower definitional threshold than “the electrical conductivity of copper” because the influence of temperature is cast away in the latter definition.

Note that the definitional threshold is theory-dependent. While fundamental constants of physics are said to be defined with what is, in our present words, a zero definitional threshold (see section 2.3), their status is provisional and can fluctuate depending on the theory in which these constants are considered [68]. Planck’s constant could be a non-fundamental quantity in candidate unifying theories such as string theory. Claiming that a fundamental constant is *considered* to have a single true quantity value tells something about the status of the quantity in the theories and models in which it is involved.

Definitional threshold borrows both from “definitional uncertainty” by acknowledging the role of the definition of the measurand in the problem at hand, and points to something intrinsic to the measurand, as in the GUM’s “intrinsic uncertainty”. At the same time, it avoids reference to uncertainty, and is thus lexically sounder than the two previous terms. Indeed, I argued earlier that uncertainty is not the answer to the problem of non-uniqueness. Contrarily to measurement uncertainty, an estimate of definitional threshold does not describe a limit of knowledge but the known limits²⁵ (deliberate or not) of the measurand definition. Appealing to the definitional threshold is thus a step in another direction: it suggests looking to the modeling side of measurement. In that context, the definitional threshold is better understood as a parameter allowing for evaluating the suitability of the measurand model for the situation at hand. It is not a component of uncertainty to be evaluated, but a tool used to state the cost, in terms of precision, of choices of modelling. The level of detail provided in the definition of a measurand should be adjusted to the general level of approximation of the models of phenomena in which it is involved, in accordance with the agent’s purposes. Preparing a house plan and assembling the parts of a rocket do not require the same level of precision, and the influence of thermal expansion on the length of material objects may be negligible in the first case, but relevant to the second one.

As I will argue hereafter, the definitional threshold has an operational use: its evaluation can enter into a comparison with an actual uncertainty, allowing for discussing the validity of the models that are being used in the description of the measurement. The next section will be devoted to a more thorough explanation of how the definitional threshold can be used to understand the problem of non-uniqueness, and thus how it enables us to understand the concept of true value in the context of non-uniqueness. As I will conclude, it involves taking the metrologists’ expression “essentially unique true value” seriously, as long as the meaning of “essentially unique” is made clear. I will argue that the concept of an essentially unique true value is an effect of perspective related to the adequacy between the precision of our models and our state of knowledge.

²⁵ The evaluation of the definitional threshold remains of course related to our knowledge: for instance, being unaware of thermal expansion amounts to being unaware about the definitional threshold that may arise if temperature is not specified in a measurand definition. But the difference is that the definitional threshold is not epistemic by nature.

4 Non-uniqueness, true values and approximate truth

One of the directions taken by metrologists was to answer the problem of non-uniqueness by raising the idea of a set of true values, and treating this set as a manifestation of a new source of measurement uncertainty. While I argued that the latter point is a misconstruction and that the answer is not to be found in the evaluation of measurement uncertainty, I would like to lay out a possible epistemological framework for the former idea, suggesting that a *set of true* values of a given quantity is conceivable from the perspective of approximate truth. The argument to follow admits an overall realist context; this is not to exclude any anti-realist counterpoint. Rather, my point is to make sense of the metrological content in realist terms. The purpose of this development is thus not to *plead for* realism as the best account of measurement, but more modestly to defend the consistency of the concept of true value within a realist framework.²⁶ As a consequence, the many ways in which realist accounts of measurement may be challenged will be left out of the scope of this discussion.

4.1 True values and truer values

Is there a true value of the radius of the Earth? In other words, is there a real number x for which the proposition

$$(P_x) R(\text{Earth}) = x \text{ km}$$

is true? The VIM's claim of a multiplicity of true values logically engages a positive answer to this question, but with the caveat that we can at best conceive of a set of multiple true values of $R(\text{Earth})$ corresponding to a *set of true propositions* (P_x). However, this standpoint is met with two main issues: (i) any two propositions of such a set appear to be *mutually contradictory*; and (ii) it remains unclear why there would even be *any* true proposition (P_x).

The latter issue may be addressed first. Let's consider three examples:

The radius of the Earth is 1 meter (P1a)

The radius of the Earth is 1 light-year (P1b)

The radius of the Earth is 6 371 km (P1c)

As we saw, there is a reference issue from the start with the term "radius of the Earth". Hence, the previous propositions cannot straight-out be said either true nor false. Intuitively though, one may still be tempted to conjecture that (P1c) is somehow *closer to the truth* than (P1a) and (P1b)—thus suggesting that 6 371 km is a *truer* value for the radius of the Earth than 1 meter. The previous intuition may be taken as a starting point for the following proposition: if there are indeed "truer" values, then perhaps are there also true value(s) to be closer to. Crucially, then, can we find a way to make sense of (P1c) being somehow truer than (P1a) and (P1b)?

Appealing to closeness to the truth ties up the present problem with a broader and longer-term debate on convergent realism. Closeness to the truth is involved, in particular, in a family of arguments intended to defend a realist position towards scientific theories despite the "pessimistic induction" [70, 71] undermining the possibility to assess the validity of present theories in light of the repeated failures

²⁶ In addition, it is worth noting that the type of realism that may apply to the present account needs not be of a specifically strong form. As an illustration, it may align with Alistair Isaac's concept of "fixed point realism" [69], wherein successful measurements serve as "robust evidence for points of objective fixity in the world" (p.939), but do not automatically lead us to commit to the quantitative reality of objects and laws. I am inclined to agree in particular with Isaac's observation that such a variety of realism about measurement outcomes can be considered "weaker than traditional realism" (p.931).

of the past ones. The key point of closeness to the truth is that it offers to discriminate between false propositions by allowing for degrees of falsity, which Graham Oddie illustrates with the following example:

The proposition *the number of planets in our solar system is 9* may be false, but quite a bit closer to the truth than the proposition that the number of planets in our solar system is 9 billion. (One falsehood may be closer to the truth than another falsehood.) [72]

Closeness to the truth offers an option to assess the value (in realist terms) of theories that are known to be at best approximate (such as classical mechanics) by avoiding a crisp dichotomy between “plain true” and “plain false” propositions that would push all approximate theories in the “plain false” cart.²⁷

Oddie’s example is a simple illustration in which one can easily conceive of a distance to *the* truth, because it is assumed that there is indeed an unambiguous truth (“there are eight planets in our solar system”). Applied to more complex cases, the usual debates about closeness to the truth also typically strive to find an adequate metric for evaluating the distance of a proposition or a theory to *the* truth [72, 73]. We find this pattern again in measurement, where results are said to get more accurate when they lie closer to the true value of the quantity being measured: this is the backbone of what Teller has called “measurement accuracy realism” (and fiercely criticized), where measurement error serves as a metric for the distance to the truth. However, the problem of non-uniqueness piles up on Teller’s criticism by casting doubt on the very possibility to apprehend a unique truth in many ordinary measurement contexts, thus preventing us to define closeness to the truth as straightforwardly as in Oddie’s example. If there is no unique true value to be close or closer to, we cannot devise the idea of a “truer value” in the traditional sense previously outlined, and we certainly cannot use this traditional sense to evaluate (P1a-c) in terms of closeness to the truth. Apprehending the idea of a “truer value” requires another angle of attack.

An option may be found by reminding, as was argued in section 2.4, that scientists are working in a model-based context, on models of quantities. Scientists define measurands as the targets of their measurements within the framework of a model of the system under examination. This reminder compels us to understand propositions (P1a) to (P1c) in the context of a given model of the Earth, for example a spherical model. Thus, we can recast our problem by considering propositions taken as constitutive hypotheses of different spherical models of the Earth:

- | | |
|--|-------|
| The Earth is a sphere of radius 1 meter | (P2a) |
| The Earth is a sphere of radius 1 light-year | (P2b) |
| The Earth is a sphere of radius 6 371 km | (P2c) |

The direct reference issue that surfaced for (P1a-c) disappears here. Supposing that our knowledge in astronomy and geodesy is not completely mistaken, propositions (P2a-c) are each false (the Earth is not a sphere!); but at least *they do have a truth value*, and as such it should be possible to apply to them considerations on truthlikeness. In other words: they are all false, but perhaps some are less false than others—in this case, proposition (P2c) quite clearly looks closer to the truth than its two alternatives. By extension, 6 371 km is then taken as a truer value for the radius of the Earth than 1 m or 1 light-year *in a spherical model context*.

There exists various accounts of truthlikeness, which of course deserve careful discussion with regard to their overall validity, their philosophical underpinnings, and the formalism by which they construct

²⁷ “It is certainly true that with a viable notion of distance from the truth progress in an inquiry through a succession of false theories is rendered possible” [72, section 2].

different metrics for quantifying closeness to the truth.²⁸ Such a discussion cannot be the purpose of the present article, though; instead, those accounts are possible implementations for providing us with a ground to compare propositions such as (P2a-c) in terms of truthlikeness,²⁹ *provided* one takes truthlikeness seriously. This also revives, albeit qualitatively, the initial intent of Popper’s reasoning on verisimilitude, according to which “the more truthlike theory should have (...) more true consequences and less false consequences” [73, section 3.5]: in our case, theoretical structures incorporating a model of the Earth based on (P2c) are expected to entail more truthlike consequences (in cases where the size of the Earth is relevant) than those that would rest on (P2a) or (P2b).

Once again, there should be no confusion here between the truthlikeness of the different propositions and the evaluation of their respective truthlikeness. We believe that proposition (P2b) is closer to the truth than the two other ones because this is what our state of knowledge in astronomy and geodesy suggests. But this is an epistemological matter, and the example taken here is chosen so extreme that it eliminates almost any uncertainty in our judgment. It is chosen as such because the point of the present argument is to explain that accounts of truthlikeness provide us with ways to consider, in a realist framework, that there can be a difference in the truth value of approximate claims about a given measurand. It thus illustrates what it means to consider that there are truer values, *but not* how we can delve into the epistemological issue of determining which values are closer to the truth. As defended in section 2 the nature of the problem of non-uniqueness is not epistemological.

4.2 Truer values and definitional thresholds

The previous framework suggests that some quantity values can be truer than others, and that it remains conceivable for quantity terms such as “the radius of the Earth” to be involved in approximately true propositions, (allegedly) answering earlier objection (ii): “why would there be any true proposition (P_x) $R(\text{Earth}) = x \text{ km}$?”. Now, objection (i) remains: since any two different propositions (P_x) appear to be mutually contradictory, how can we conceive of a set of true quantity values?

Herein lies another decisive point. Even admitting that values attributed to a measurand can get more or less true (or more or less accurate), there nonetheless remains a horizon beyond which their truth value cannot be distinguished. This limit is characterized by the definitional threshold: below the definitional threshold, no value is “truer” than any other. Consider the following claims about the equatorial radius of the Earth:

- | | |
|-----------------------------------|-------|
| $a_E = 6\,378\,136.600 \text{ m}$ | (P3a) |
| $a_E = 6\,378\,136.601 \text{ m}$ | (P3b) |
| $a_E = 6\,378\,137 \text{ m}$ | (P3c) |
| $a_E = 6\,378\,140 \text{ m}$ | (P3d) |

²⁸ Chakravartty gives an overview of the various attempts at formal explicitation of approximate truth from the mid-twentieth century onwards [74, section 3.4]. He lists Karl Popper’s introduction of verisimilitude in his 1963 book *Conjecture and Refutation*, the investigation of the possible worlds approach by Pavel Tichý, Graham Oddie or Ilkka Niiniluoto in the 1970s, or also the hierarchies approach by Jerrold Aronson, Rom Harré or Eileen Cornell Way in the 1990s. Oddie explains some of these approaches in great detail [72].

²⁹ Note that, as [72] points out, “truthlikeness is not just a matter of being close to being true” (section 1.4). That the number of planets in the solar system is a positive integer is true, but not as specific, and therefore not as close to the truth, as saying that this number is 8. Oddie thus distinguishes the “truth factor” and the “content factor” of propositions, both weighing in their respective truthlikeness. In our case we can first consider that we operate at the same content factor for all propositions, in what case the truth factor is the sole discriminant.

All values differ, but not by the same amount, and the possibility to discriminate between their truth value depends on the definitional threshold. Qualitatively speaking, it makes sense to say that a value is truer than another only when they differ by more than the definitional threshold.

This idea is analogous to the way metrologists compare measurement results. When two or more measurements of a given quantity, say a fundamental constant of physics such as the proton radius, are in disagreement, the important point is not so much that the results differ, but whether the disagreement is *significant*; and this significance is evaluated based on a scale factor, the measurement uncertainty. The difference between the two results is only significant if it is at least of the same order of magnitude as the measurement uncertainty. The present situation is alike: the comparison of the truth value of two claims is also based on a scale factor, which this time is the definitional threshold. Suppose that the definitional threshold on the equatorial radius of the Earth is 1 cm: then (P3c) and (P3d) are discernible in terms of truth value, whereas (P3a) and (P3b) are not (again: not because it is impossible for us to *know* such a difference, but because the definition of the measurand is not precise enough to discriminate between the two). According to this reasoning, it is possible to conceive of multiple, different but non-contradictory true values. In the model-based context of measurands and of approximate truth, the multiplicity of true values consistent with the definition of a quantity is not a paradox anymore.

The present analysis leads us to the joint conclusion that: (i) given a measurand definition, it remains possible for some quantity values to be truer than others, meaning that we can get closer to a truth, even though it cannot be identified as a unique “true value”; and (ii) there are truer values only up to a certain scale, characterized by the definitional threshold. It is by combination of these two points that we can make sense (in a realist context) of the metrologists’ idea of “multiple true values” of a measurand in the framework of a given model, of which the measurand is a parameter. The value 6 378 136.6 m is among the set of true values of the equatorial radius of the Earth if it is true that the average shape of the Earth is approximately an ellipsoid with an equatorial radius of 6 378 136.6 m. Non-uniqueness is the expression of the fact that there are other values for which a similar claim is also true.

4.3 Unique true value as an effect of perspective

As we saw, the *Guide to the expression of uncertainty in measurement* framework pleads for a status quo: if the problem of non-uniqueness can be kept marginal, there is no substantial problem and everything can be done as if there was a unique true value of the quantity under examination. Accordingly, in the GUM terms, the document “is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity (...) that “can be characterized by an essentially unique value” [8, p. 1, my emphasis]. The VIM explains what is understood by *essentially unique*:

When the range of the true quantity values believed to represent the measurand is small compared with the measurement uncertainty, a measured quantity value can be considered to be an estimate of an essentially unique true quantity value. [5, pp. 19–20]

The GUM also gives some further explanation which stresses that the “essentially unique” clause is a pragmatic, context-dependent consideration:³⁰

In practice, the required specification or definition of the measurand is dictated by the required accuracy of measurement. The measurand should be defined with sufficient

³⁰ Eisenhart had already emphasized that the refinement with which scientists consider their target “will depend on the purposes for which a determination of the magnitude of the quantity concerned is needed”, [4, p. 170].

completeness with respect to the required accuracy so that for all practical purposes associated with the measurement its value is unique. [8, p. 4]

The “required accuracy of measurement” can be related to another term, “target uncertainty”, defined in the VIM as the “measurement uncertainty specified as an upper limit and decided on the basis of the intended use of measurement results” [5, p. 27], a definition which, by focusing on *intentions*,³¹ reiterates the pragmatic considerations at stake here. This reminds us that modelling is the result of choices from an agent who “actually decides the concepts (...) that she considers appropriate in dependence on her goals” [76, p. 2148]. In other words, the VIM and the GUM both tell us that the definitions of the measurands have to be appropriate to the scale of examination.

I claim that my analysis suggests how to take the idea of an “essentially unique true value” seriously, and sheds light on the conditions of validity of this framework. The main issue with the metrologists’ account, as defended in section 3, is that they transfer the burden of non-uniqueness onto measurement uncertainty. Definitional threshold provides us with a more rigorous analysis tool, in a model-based context, to understand the non-uniqueness of the true value of a quantity. Rephrased in our own terms, the GUM framework corresponds to cases where the definitional threshold is negligible in comparison with measurement uncertainty. If this condition is met, the model is considered to be sufficiently precise for the measurement not to be operationally affected by the problem of non-uniqueness. Conversely, as soon as the definitional threshold turns out to be non-negligible with regard to target uncertainty, the measurands and models need to be refined. By making appropriate choices of idealization, the agent tries to find an adequate trade-off between the tractability of one’s models and their realism.

Two interesting conclusions can be driven from this standpoint. First, while I insisted on the fact that the so-called intrinsic uncertainty is not an actual uncertainty, the evaluation of its counterpart, definitional threshold, still enters into a comparison with measurement uncertainty. This remains qualitative: what matters is to be able to compare definitional threshold and target uncertainty in terms of orders of magnitude. Secondly, the account provided here concurs with the claim made in the GUM and the VIM that the definitional threshold represents a minimum limit in uncertainty; however, it does not reach this conclusion for the same reasons. Metrology documents consider that the total uncertainty budget adds up acquisitional and intrinsic (or definitional) uncertainties: the sum of the uncertainties is of course always greater than the latter component. Instead, according to my present account, claims that have a precision below the definitional threshold do not have any empirical meaning; accordingly, any effort to bring the measurement uncertainty below this threshold does not bring any tangible new knowledge.

In light of this analysis, I conclude that the concept of a unique true value is an effect of perspective. The true value is not a point-value, but it appears so from afar, i.e., when the precision of the measurement remains low with regard to the intrinsic vagueness of the measurand definition. This effect of perspective can be illustrated by revisiting the very common analogy of targets (fig. 2). In this analogy, measurement is compared to aiming at a target, whose center is the true value of the measurand (in an actual measurement situation, one does not know the location of the center). It is usually introduced to explain the difference between precision and accuracy (in fact “trueness” in the vocabulary of metrology [5, p. 21]; an accurate measurement has both good precision and trueness): “accurate measurements are shots that arrive close to the center” [18, p. 95], and measurement is precise if all shots land close to each other, independently of their closeness to the center. Adapted to our problem, fig. 2 illustrates a measurement result by a rifle sight whose radius is the measurement uncertainty. The true value of the measurand is represented by the disk at the center; the gradient indicates that because of non-uniqueness, the target is vague, and the radius of the disk corresponds to the

³¹ This relates to the “intentional character of measurement” that we defended in [75].

definitional threshold. The four situations (a), (b), (c) and (d) illustrate the measurement of the same measurand with a decreasing uncertainty (and an increasing “trueness”, which is only done to keep the figure compact and is not a relevant feature here). In situation (a), measurement uncertainty is much larger than the definitional threshold, and the true value is almost a dot: this is the GUM framework, where the measurement can be characterized by an essentially unique true value. In this analogy, reducing measurement uncertainty then amounts to “zooming in”, and the true value appears more and more blurry. At one point, the model reaches its limits since the measurement threshold is not negligible anymore with regard to measurement uncertainty. If one wants to achieve better precision, a refinement of the models is necessary; in a way, the measurement is too good for the models used.

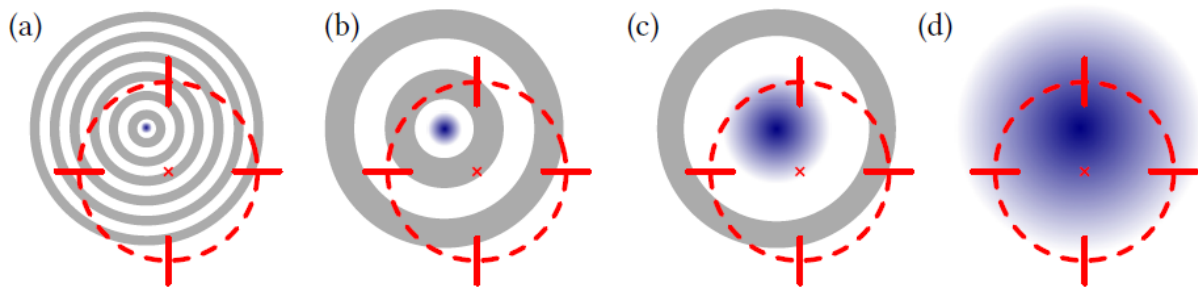


Figure 2: Illustration of the problem of non-uniqueness with the metaphor of targets. Measurement uncertainty is taken by analogy as the size of the sight of a rifle aiming at the true value of the quantity, which is the bullseye of the target, whose radius is the definitional threshold. Figures (a) to (d) represent situations in which the same measurand is being measured with less and less uncertainty. While the true value of the measurand almost appears as a point when the uncertainty is much larger than the definitional threshold, decreasing the uncertainty amounts to zooming in on the target, which makes the non-uniqueness of the true value more and more salient. In the case (a), it is possible to make the center of the rifle closer to the true value; however, in case (d), no further improvement is possible, because the limits of the measurement models are met. This illustrates in familiar metrological terms how, in the context of the problem of non-uniqueness, a unique true value appears as an effect of perspective. It is possible to get truer values, but only to a certain scale.

5 Conclusion

In this paper I argued that there is a sense in which one can continue to think of quantities as being characterized by true values, despite the violation of the principle of single value. It turns out that the direction taken rejoins the framework laid out in the *Guide to the expression of uncertainty in measurement*, which seeks to maintain a traditional form of quantification while limiting the cost of the problem of non-uniqueness. However, it disagrees with the GUM’s resolution of the problem of non-uniqueness in terms of uncertainties. Instead of considering a lack of knowledge about the measurand, the direction chosen invites a reflexive attitude with regard to the validity and the degree of refinement of the models involved in experimental inquiries, thus stressing the model-based nature of the problem.³²

³² The angle taken to address the problem of non-uniqueness, by appealing to approximate truth, resonates with the dynamic account of accuracy that was advocated in [12, 75]. The accuracy of measurements is not to be

It is important to emphasize two points that my argument does not intend to endorse. From a very general standpoint, it does not allege that realism is the (only) appropriate philosophical standpoint for measurement, but rather defends a more modest claim: non-uniqueness does not undermine realism. Moreover, I refrain from asserting that there is one and only valid system of representation of quantities. It is possible, in particular, that the GUM requirements cannot always be met. Scientists have pointed out cases for which it may be impossible to escape from having significant definitional thresholds; in those cases, it is then unsure whether a traditional approach can be maintained and it is possible that scientists have to undertake more significant methodological changes.³³ This specific aspect has been kept out of the scope of this paper, but the developments offered here do not preclude in principle the development of alternative frameworks based for example on fuzzy sets.

The fate of the concept of true value of a quantity illustrates the particular status of metrology, which is not dedicated to the study of a specific class of phenomena, but to the methodological and conceptual foundations of measurement. Metrology is a hybrid of scientific research, methodological issues and standardization efforts, together forming a “hidden infrastructure” of science and technology [78] to which historical, philosophical and sociological questions are an integral part.

considered in a static way, but through the possibilities there are to make future progress – by increasing theoretical and empirical knowledge, and improving models.

³³ Kirkham et al argue that this is an extensive issue in the case of electrical measurements in power systems [77]. Other concerns were voiced about chemistry or biology when the JCGM conducted a survey on the reception of the GUM: see report online, URL=https://www.bipm.org/wg/JCGM/JCGM-WG1/Allowed/sub-committee_5/WG1-SC5-N12-15_JCGM_GUM_Survey_Collated_responses.pdf.

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