

INRAE



Towards a novel quantitative approach for rockfall hazard assessment

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Brief overview

Main objective: providing a few contributions and food for thoughts about theoretical methodologies and current practices in rockfall hazard quantification

Outline:

- Theoretical background and probabilistic approach
- Concepts of partial and global reach probabilities
- Small probabilities' approximation
- Two examples
- Some remarks on the method
- Outlook and perspectives



Initial framework

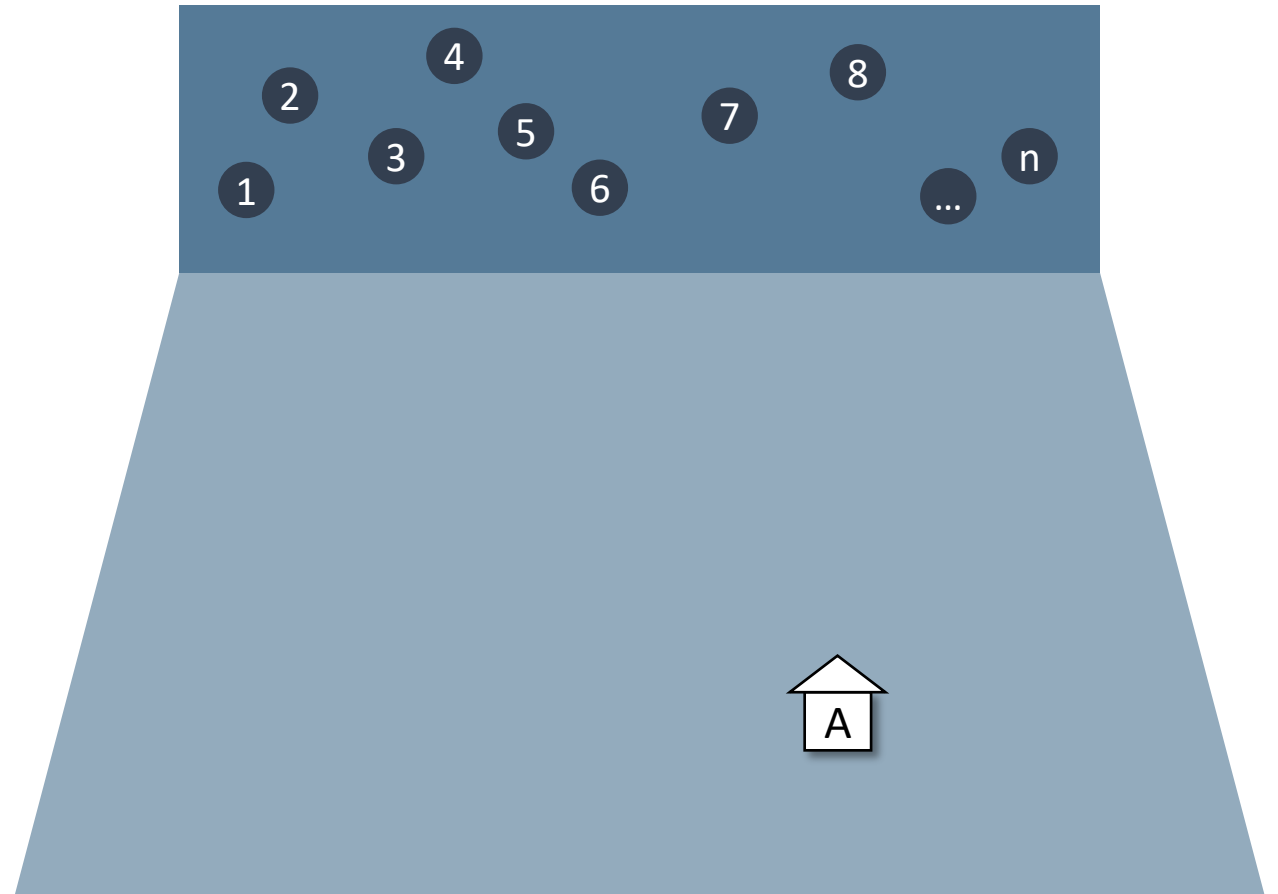
Consider n sources labeled S_1, \dots, S_n

Consider an at-risk element labeled A

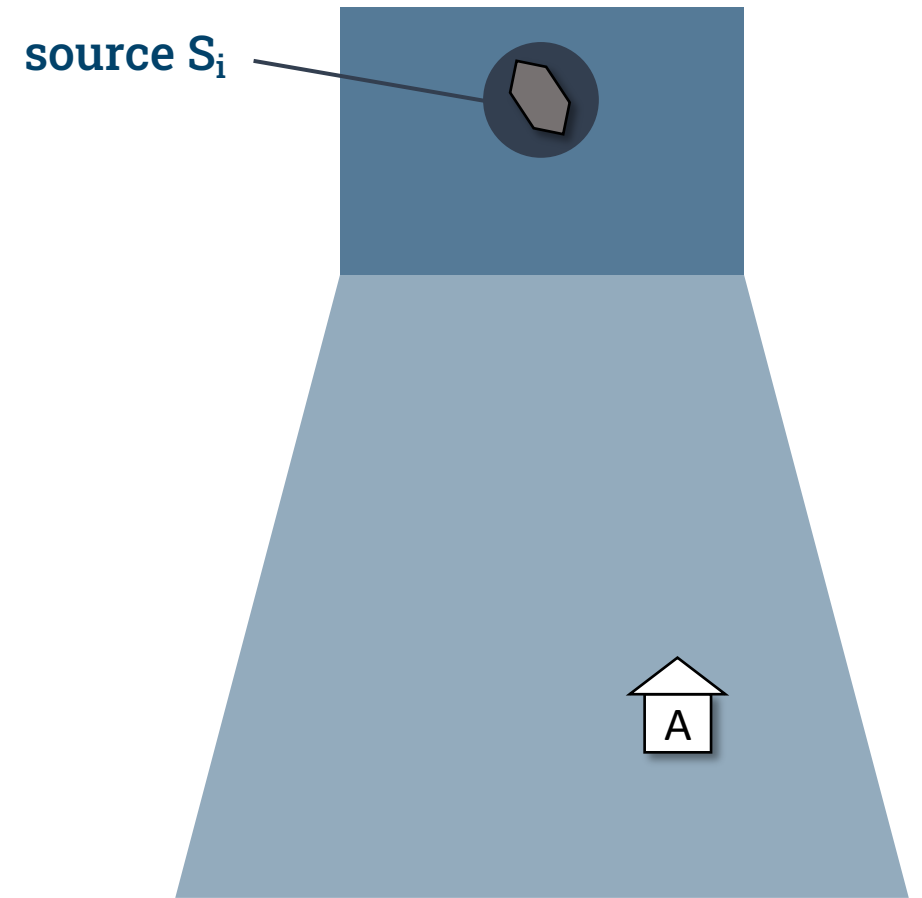
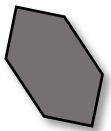
Rock fragmentation is neglected

Set an arbitrary time period T

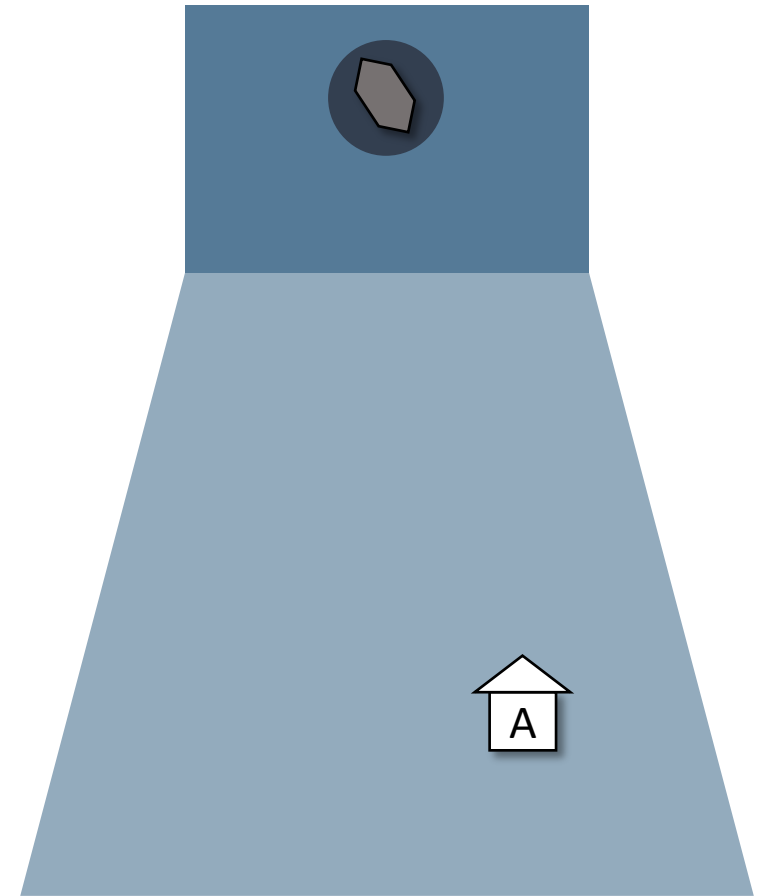
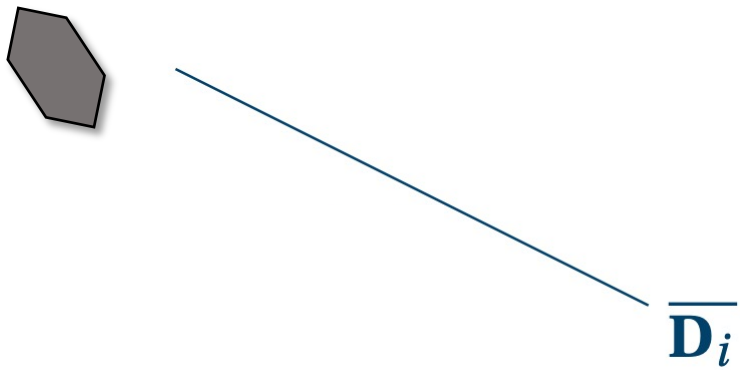
Assume that a source can release one rock only one time (after “activation”, the source is “switched off”)



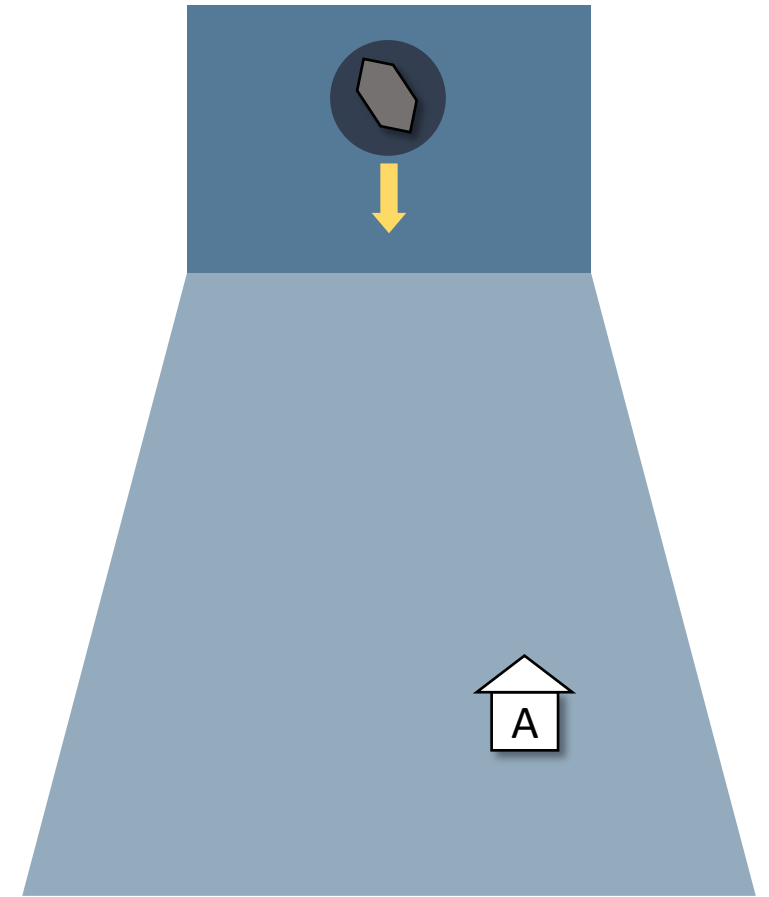
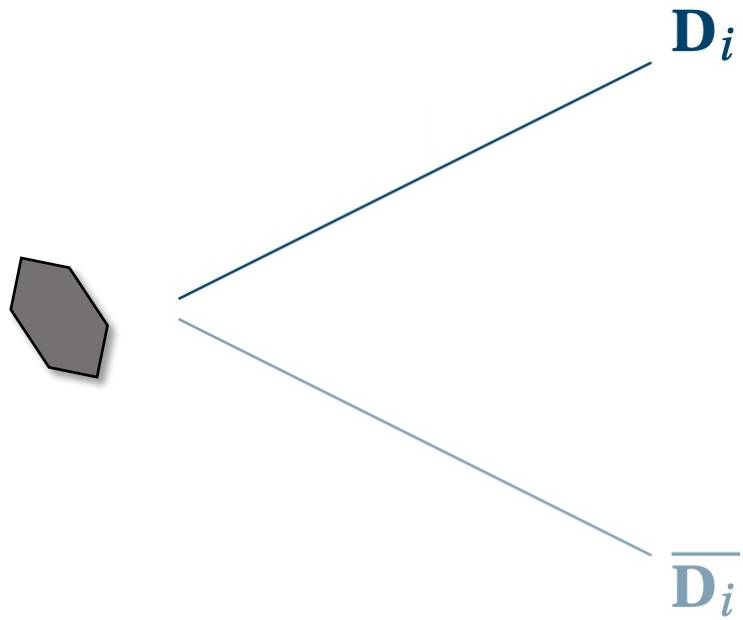
Rockfall as a “random experiment”



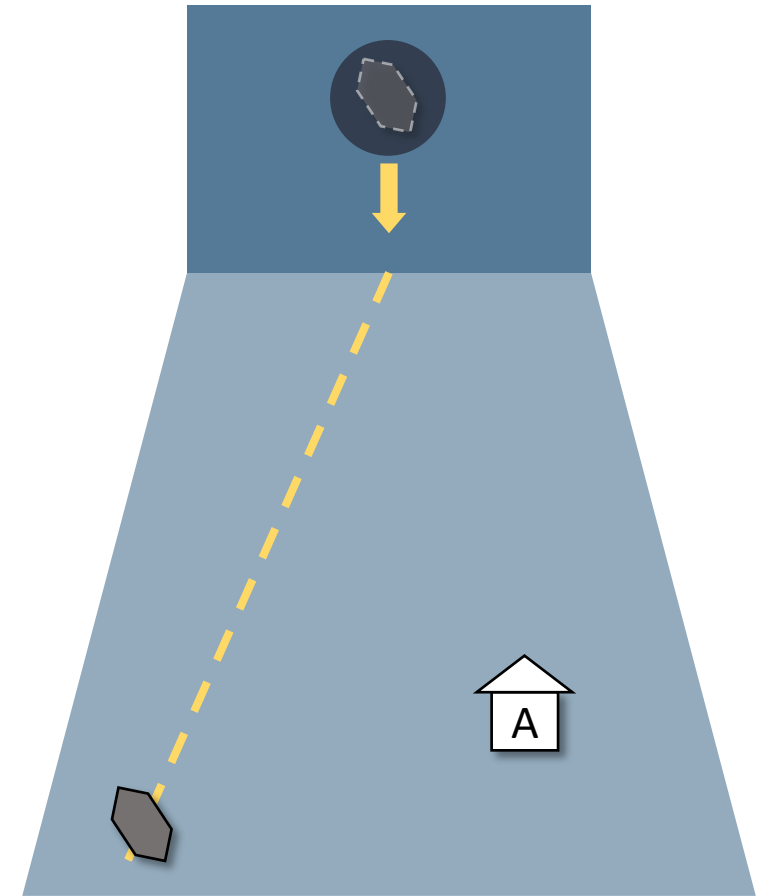
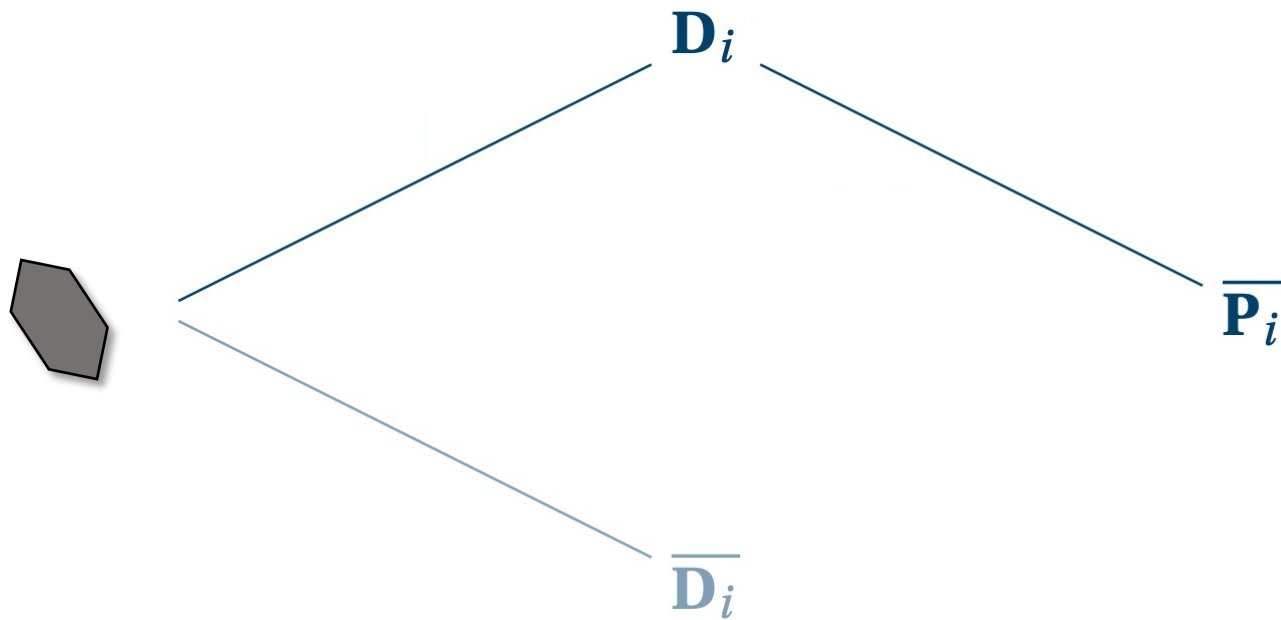
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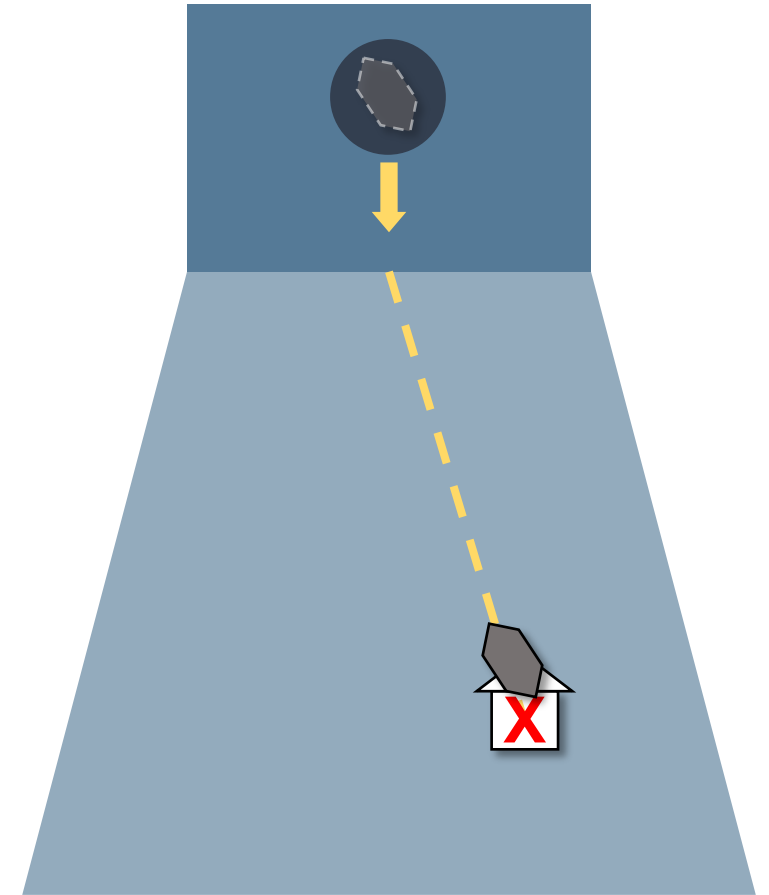
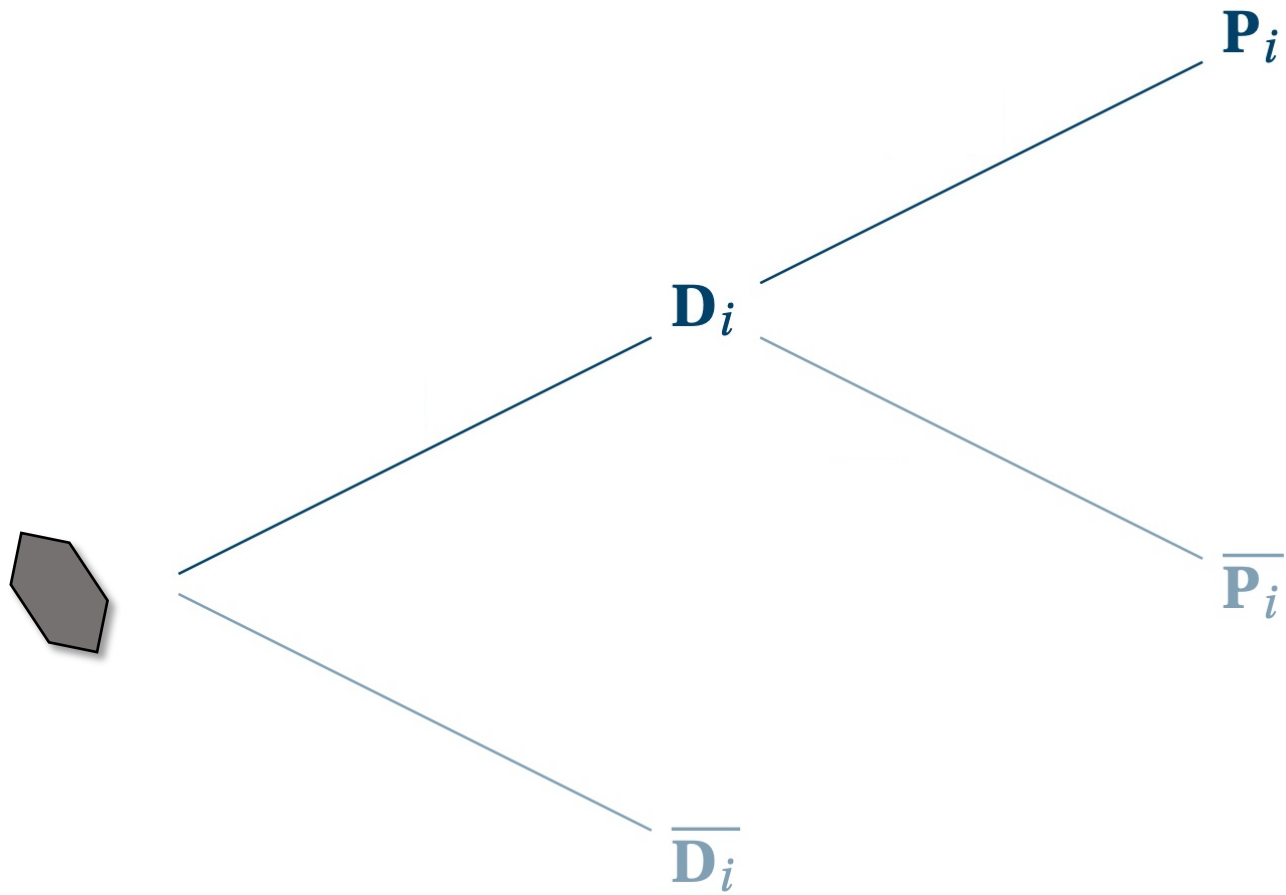
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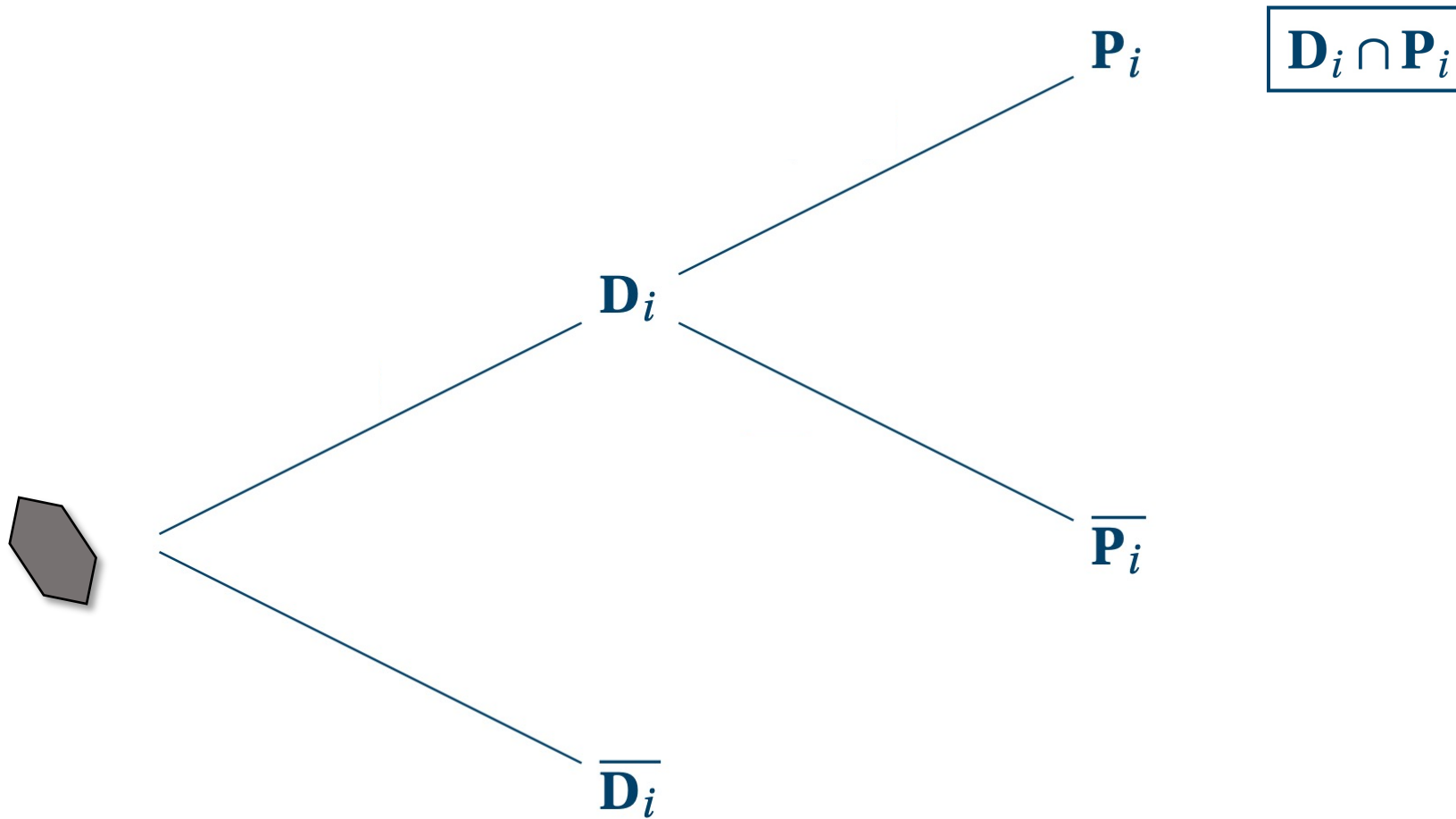
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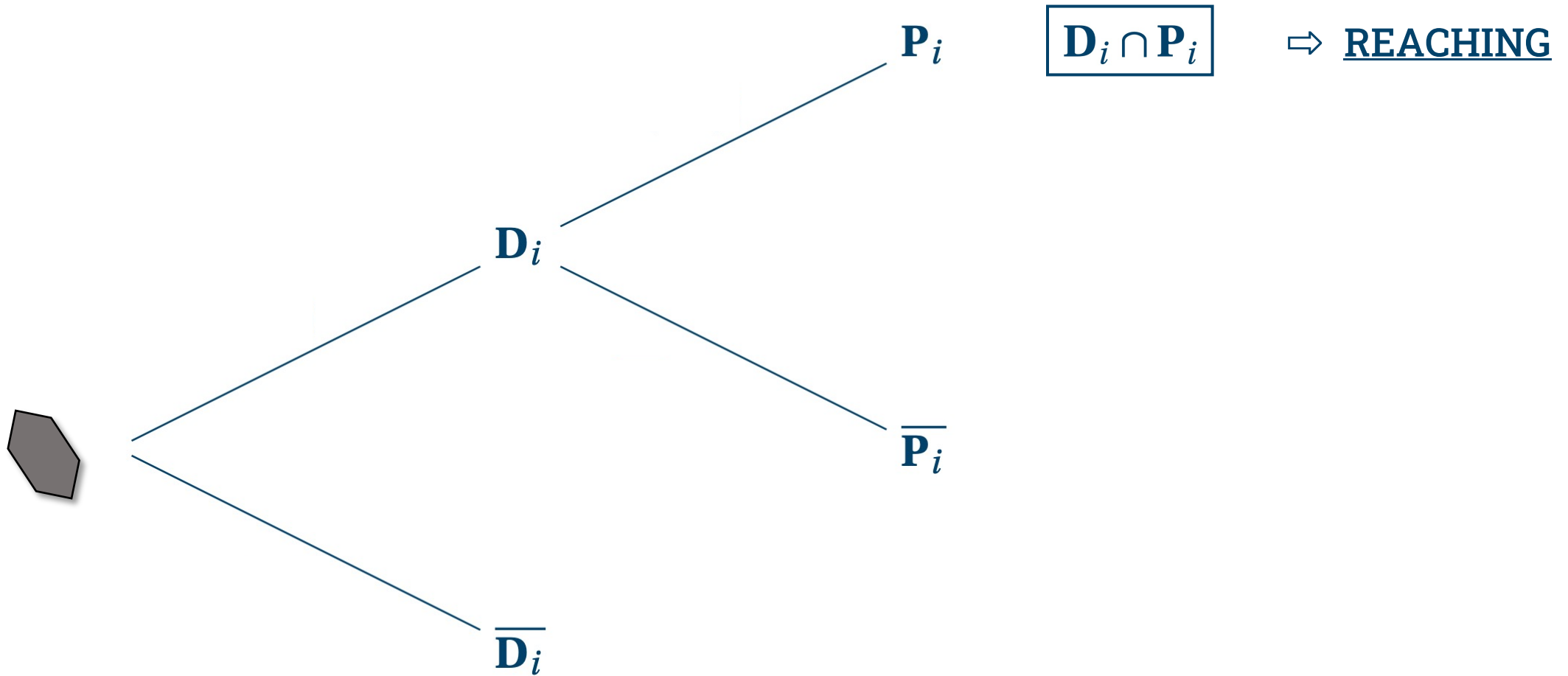
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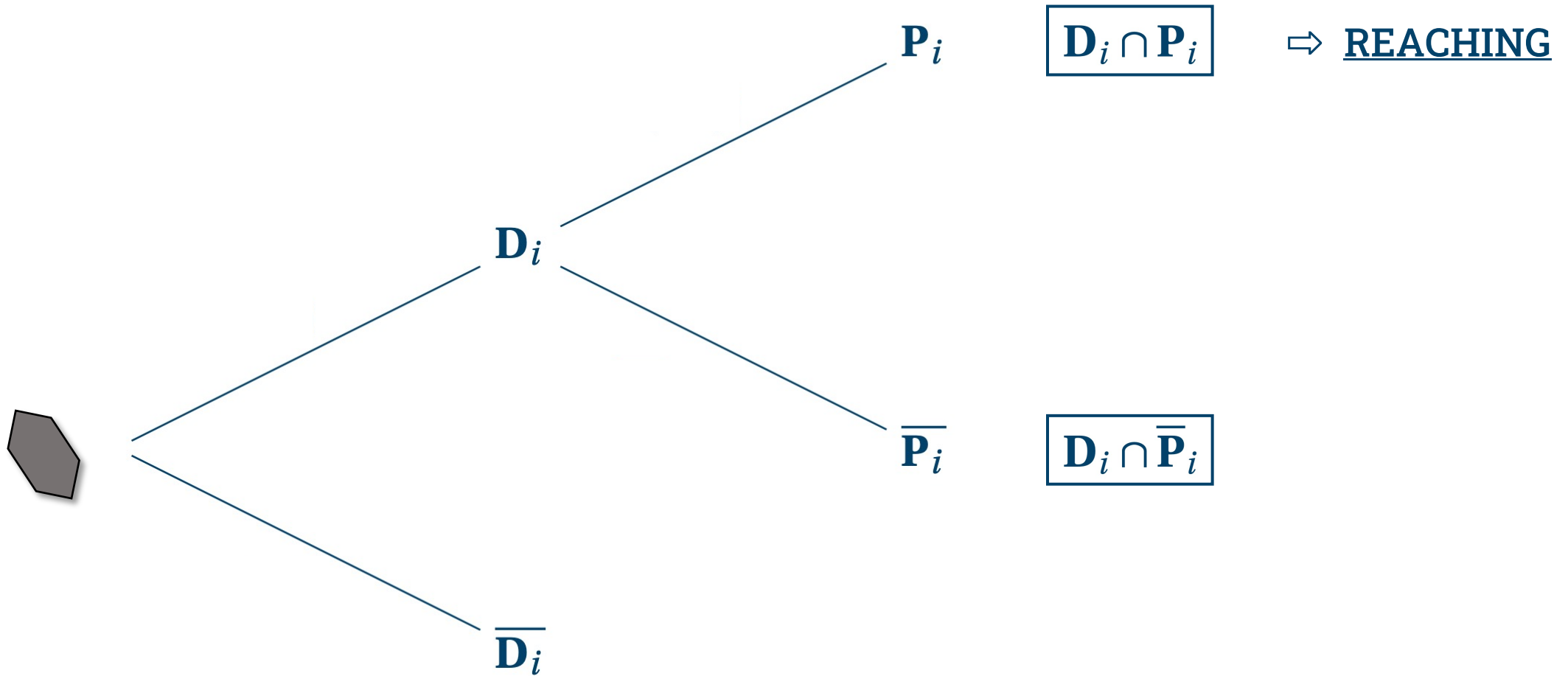
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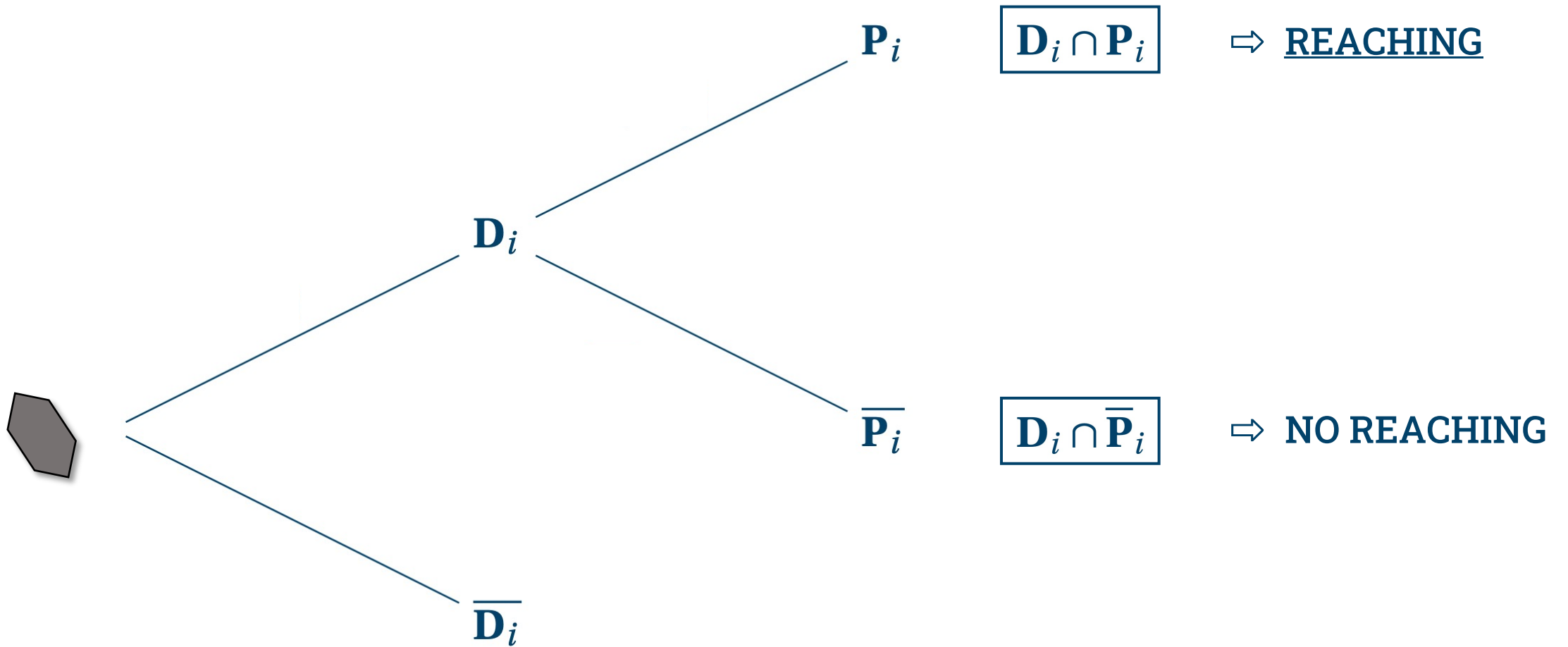
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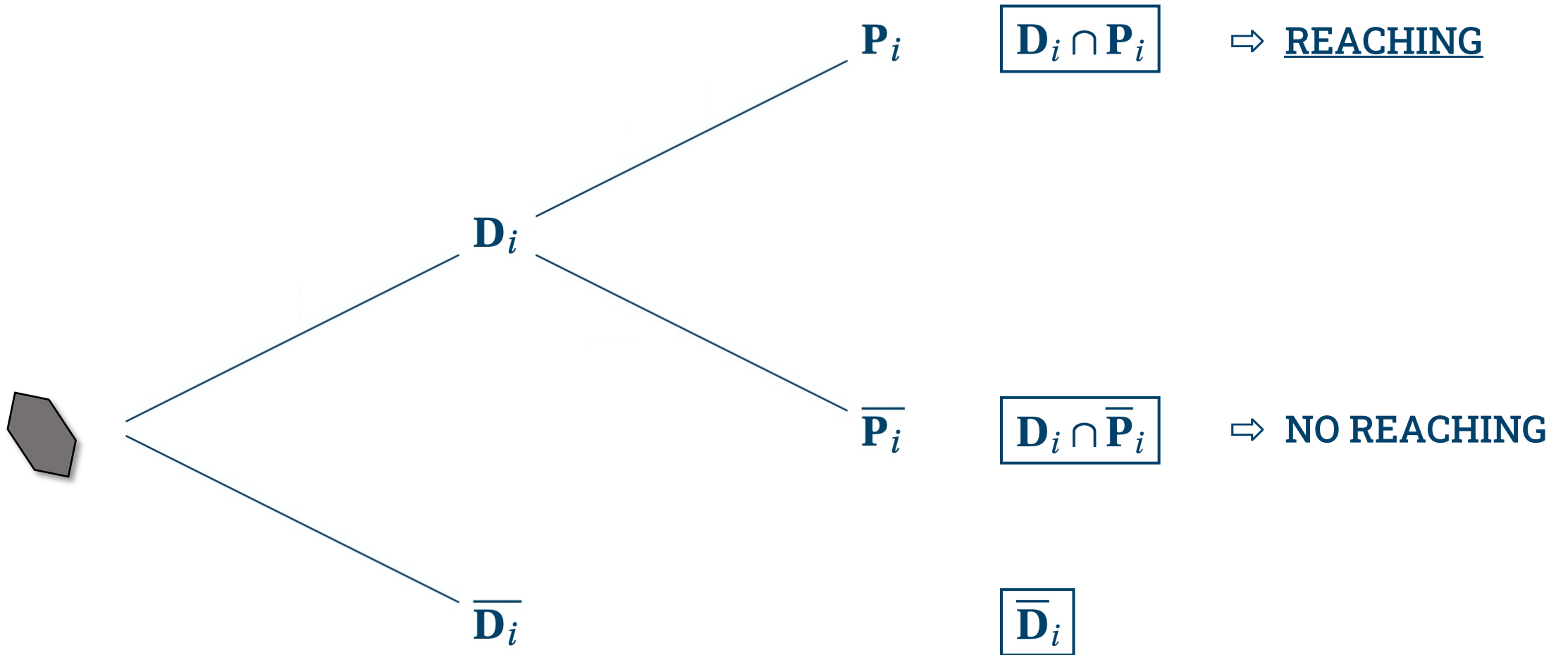
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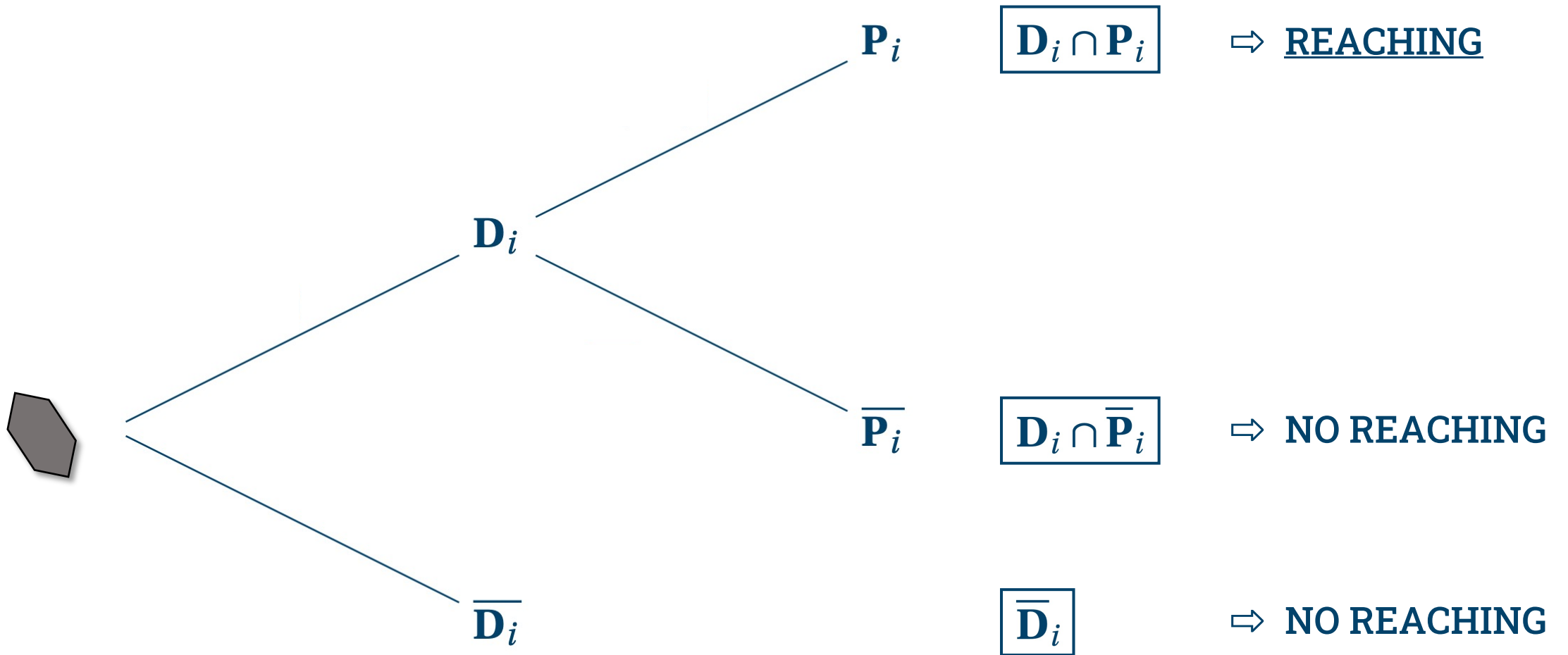
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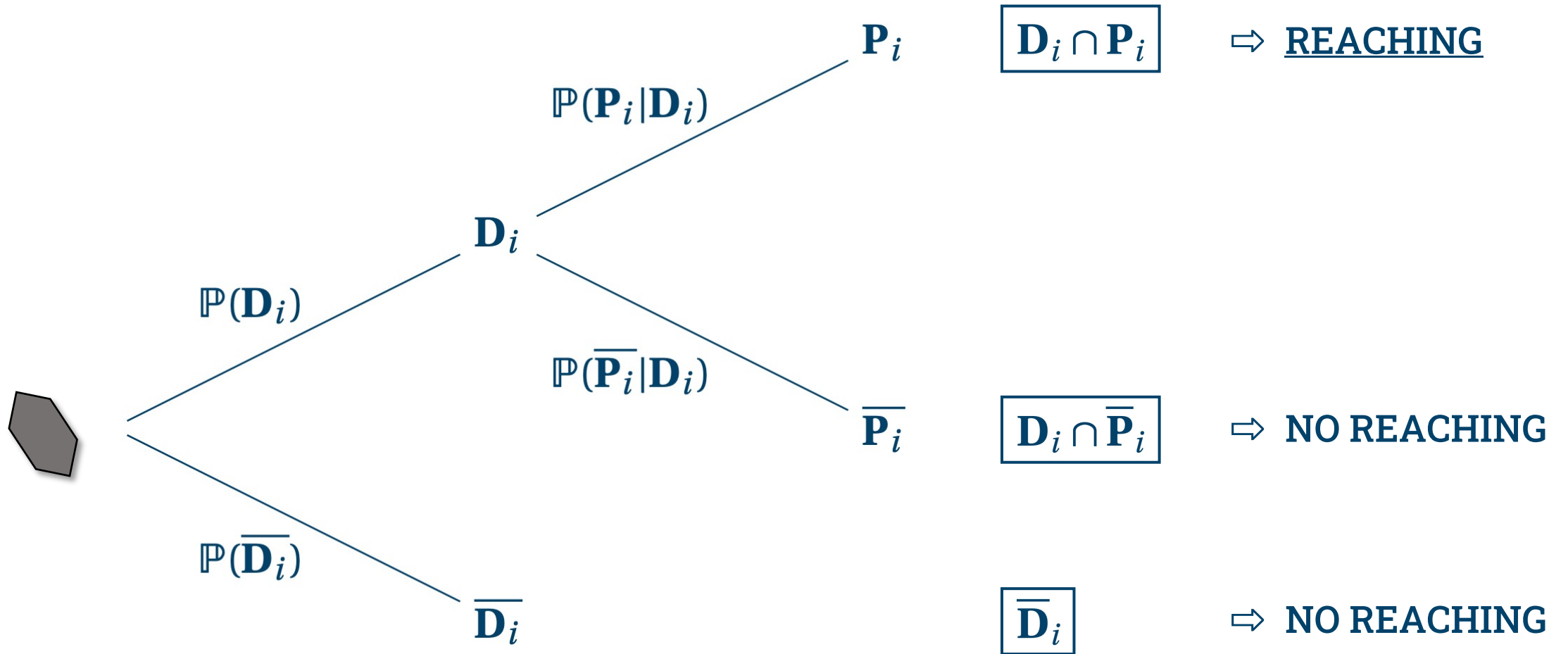
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\Rightarrow aim: expressing \mathbb{P}_r^A as a combination of the $\mathbb{P} [\mathbf{D}_i \cap \mathbf{P}_i] = p_{r,i}^A$

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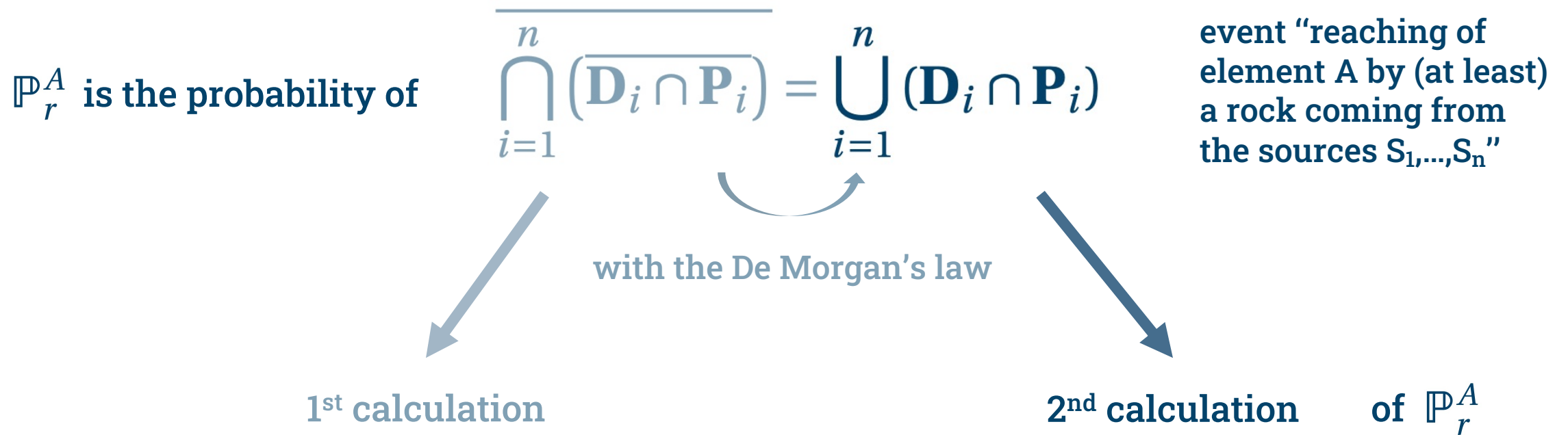
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1st calculation

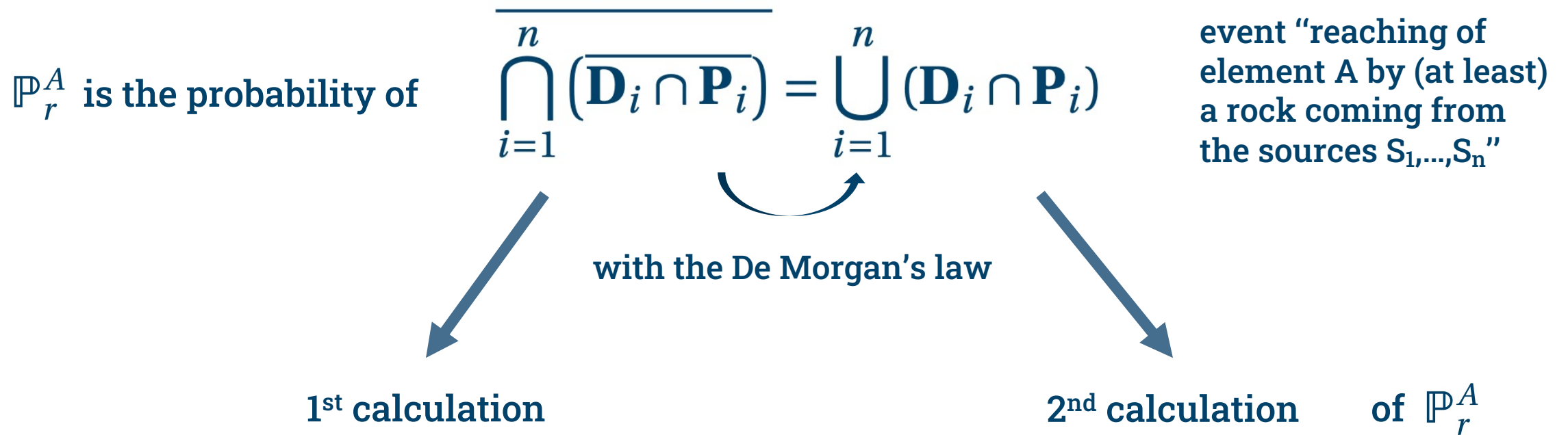
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Small probabilities' approximation

Starting from

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$\Rightarrow \mathbb{P}_r^A \approx \sum_{i=1}^n p_{d,i} p_{p,i}^A$ (small probabilities' approximation)

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Mutual independence:

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(small probabilities' approximation)

Application in operational context: diffuse hazard

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- Discretization of the terrain (DEM):**
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 - propagation probabilities become surface values

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Possibility of procedure:

- identifying source zones on the cliff
- determining a typical rock volume (or volume class) to each source zone
- dividing the source zones into source cells of the same resolution
- assigning a departure probability to each source cells
- estimating all propagation probabilities (with rock propagation simulations)
- computing the field of global reach probability

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Examples of values:

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\Rightarrow if a source zone, divided into 100 source cells, might produce 8 rocks of a given volume class during T , then the departure probability of each source cell in the source zone equals 0.08

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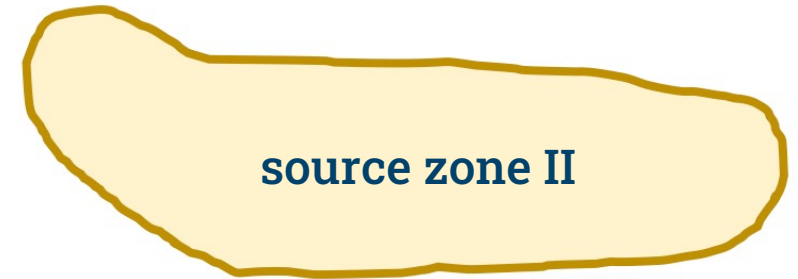
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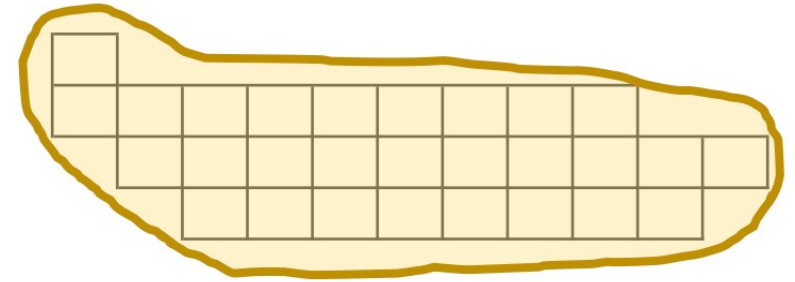
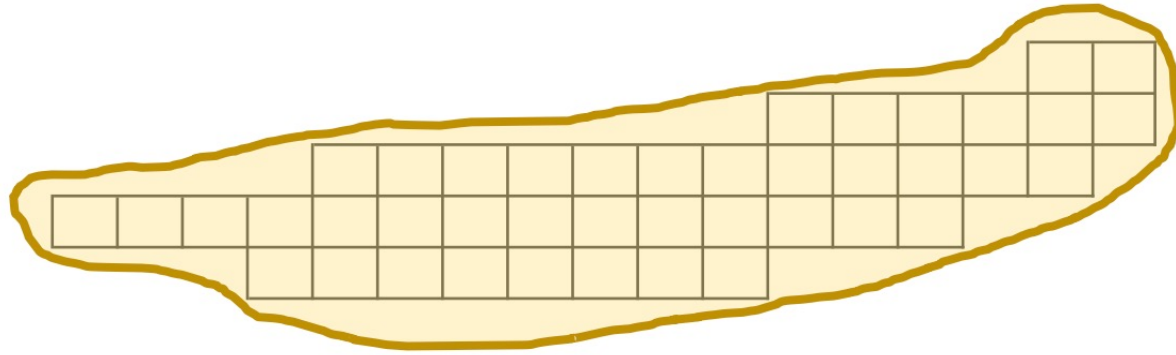
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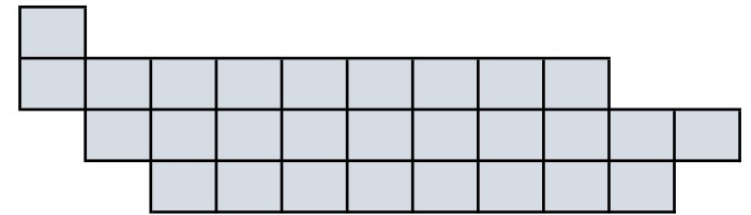
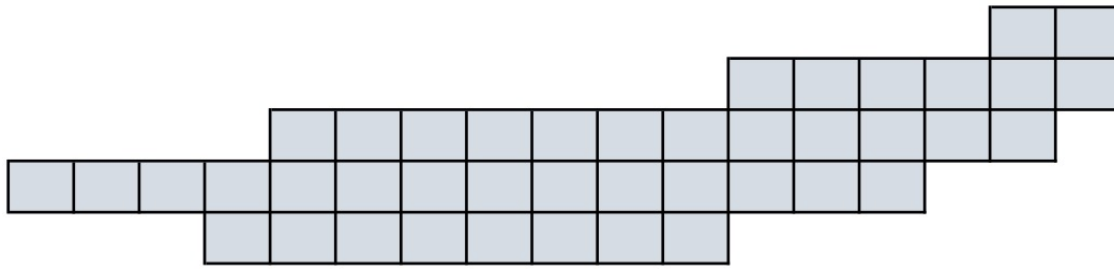
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A virtual example



A

A virtual example

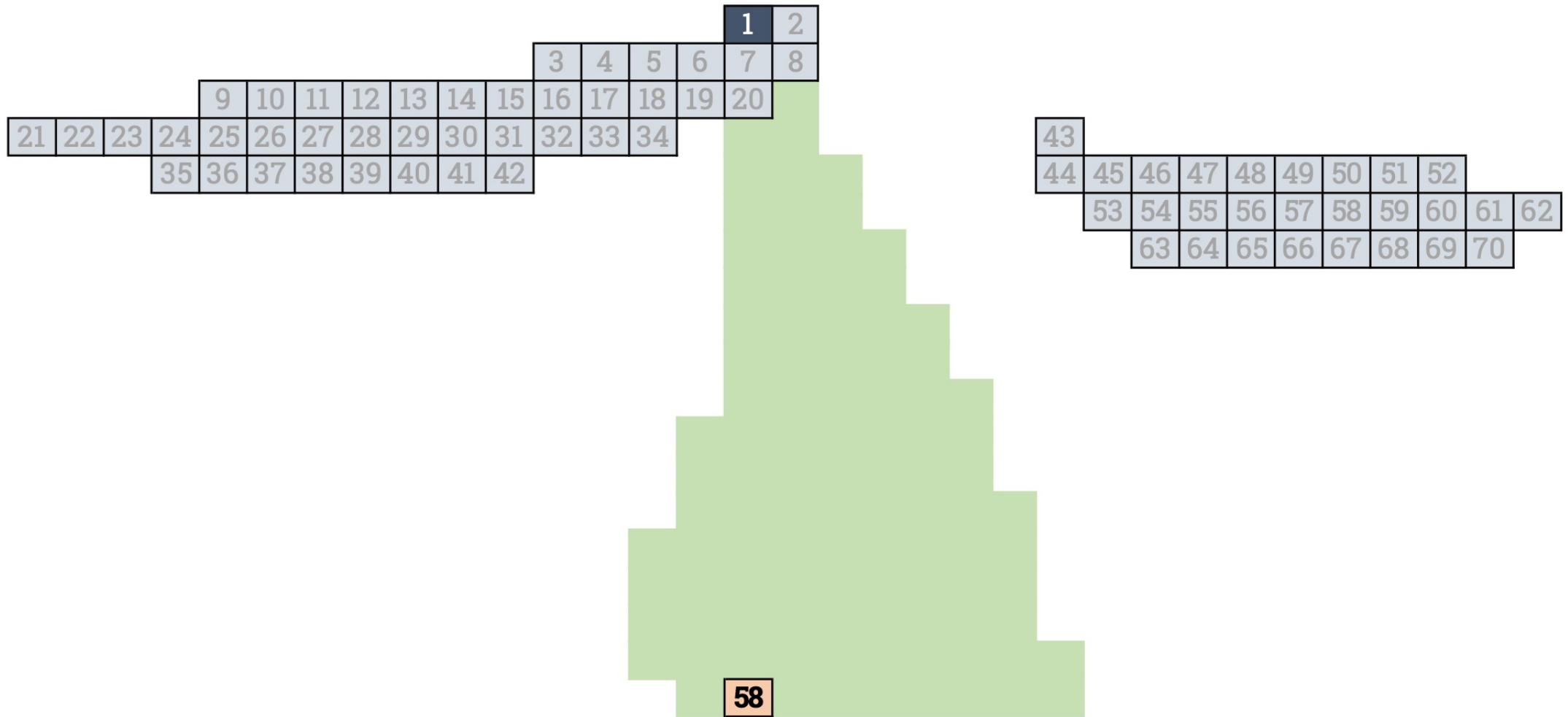
												1	2		
										3	4	5	6	7	8
				9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34		
				35	36	37	38	39	40	41	42				

43															
44	45	46	47	48	49	50	51	52							
		53	54	55	56	57	58	59	60	61	62				
			63	64	65	66	67	68	69	70					

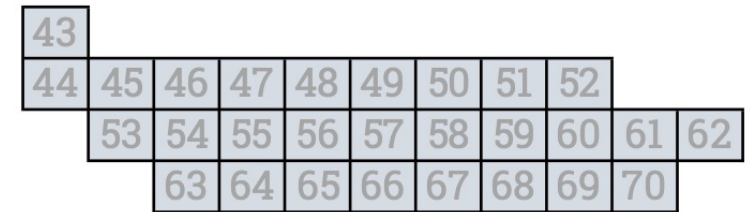
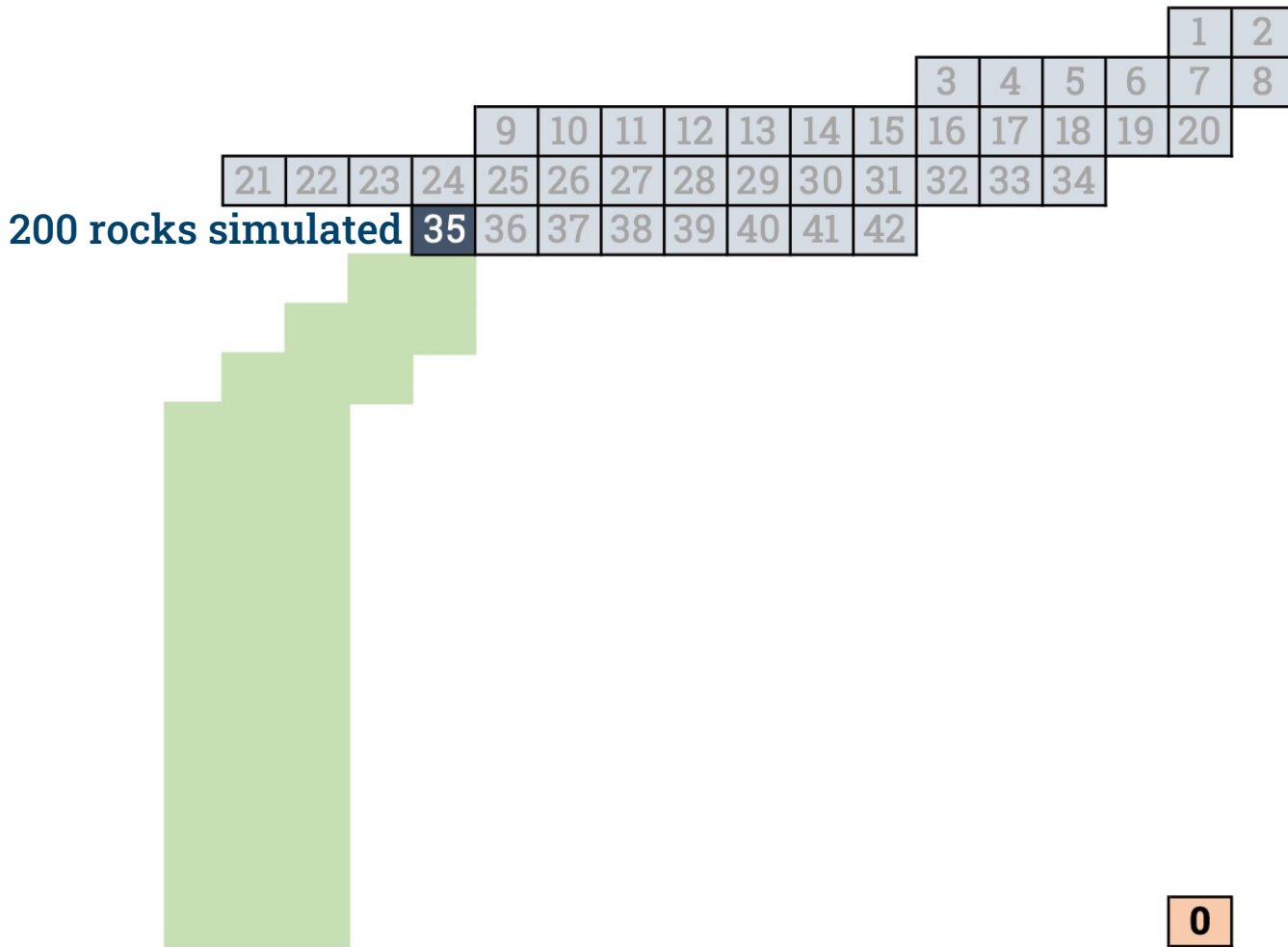
A

A virtual example

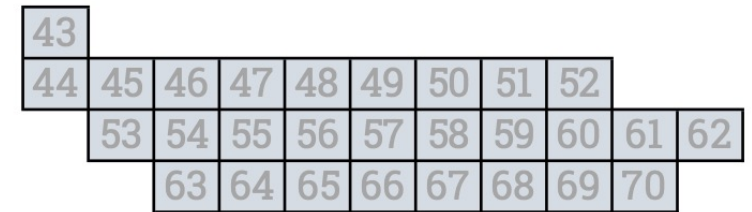
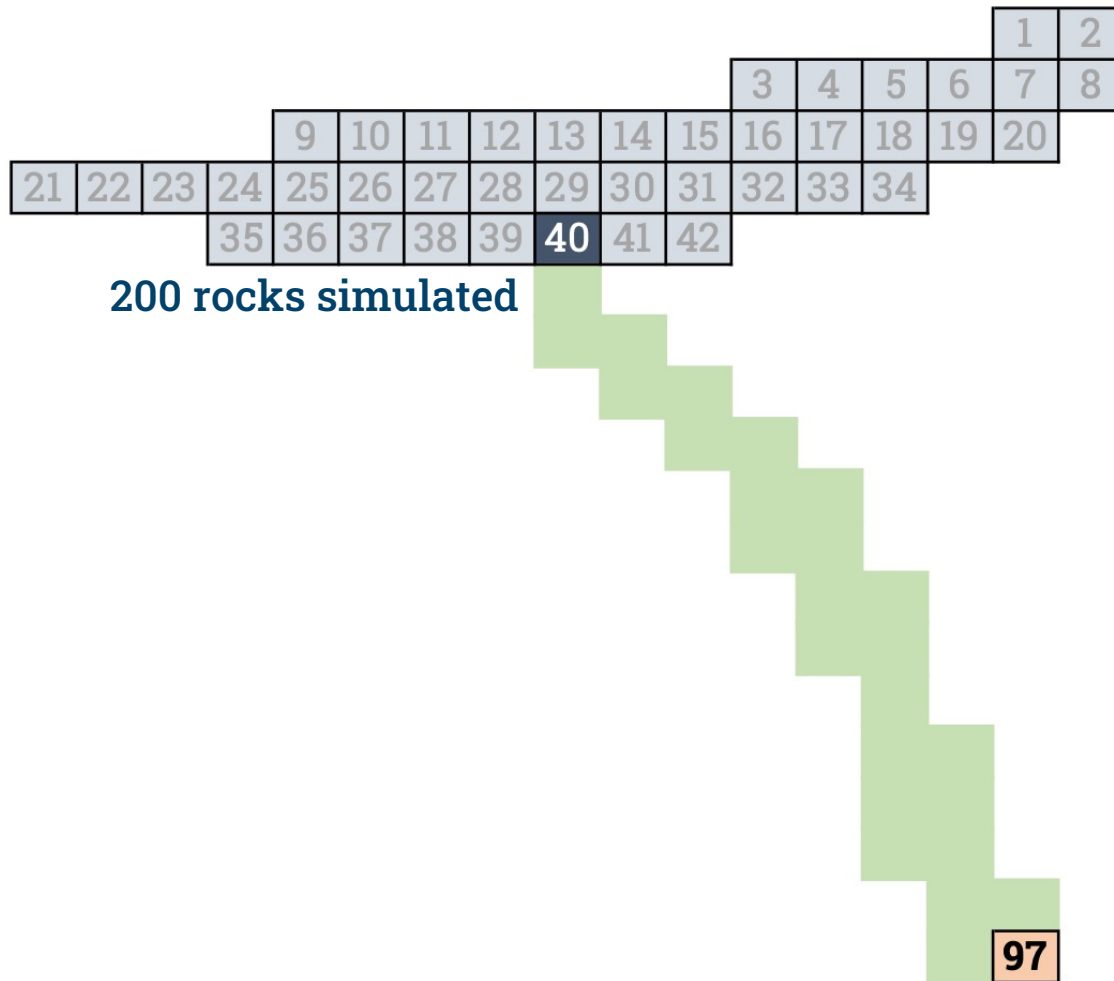
200 rocks simulated



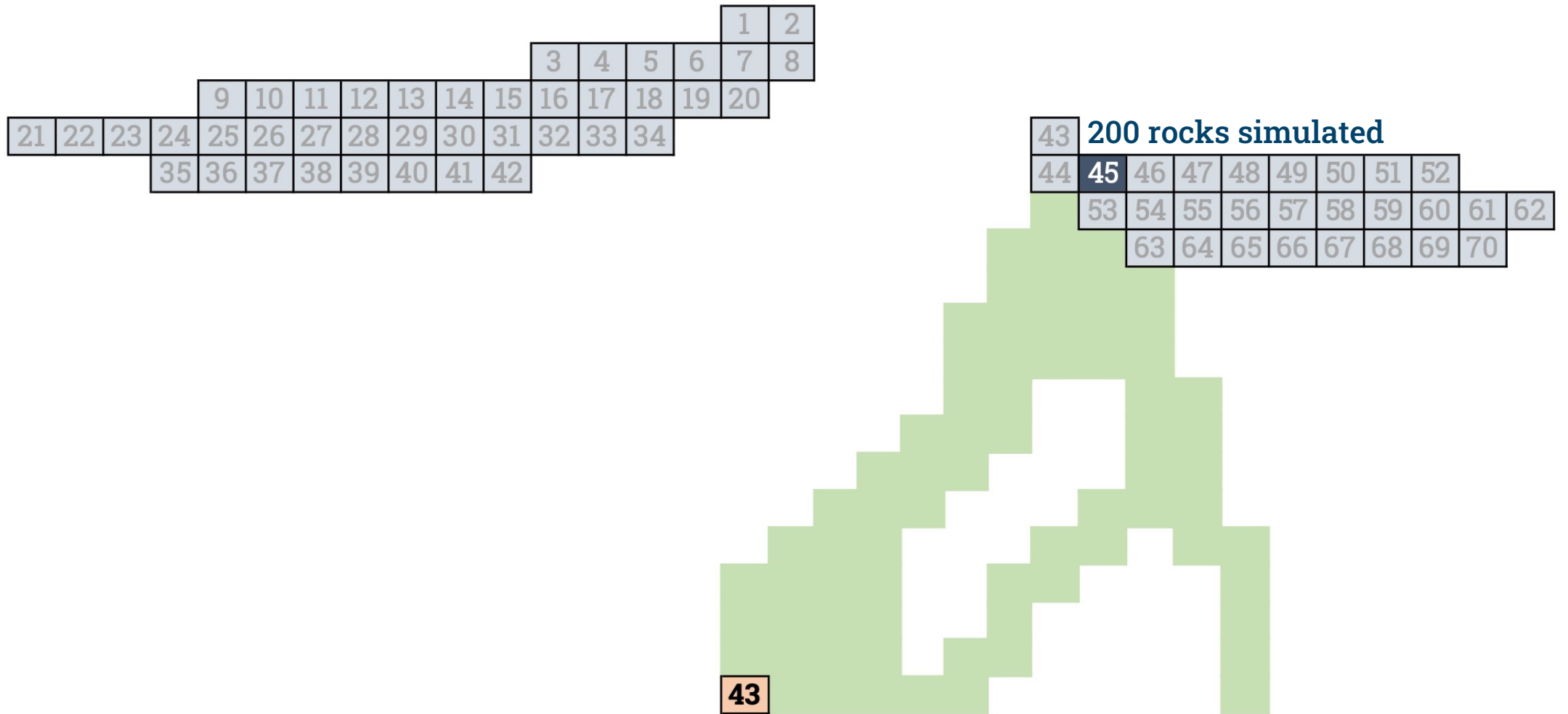
A virtual example



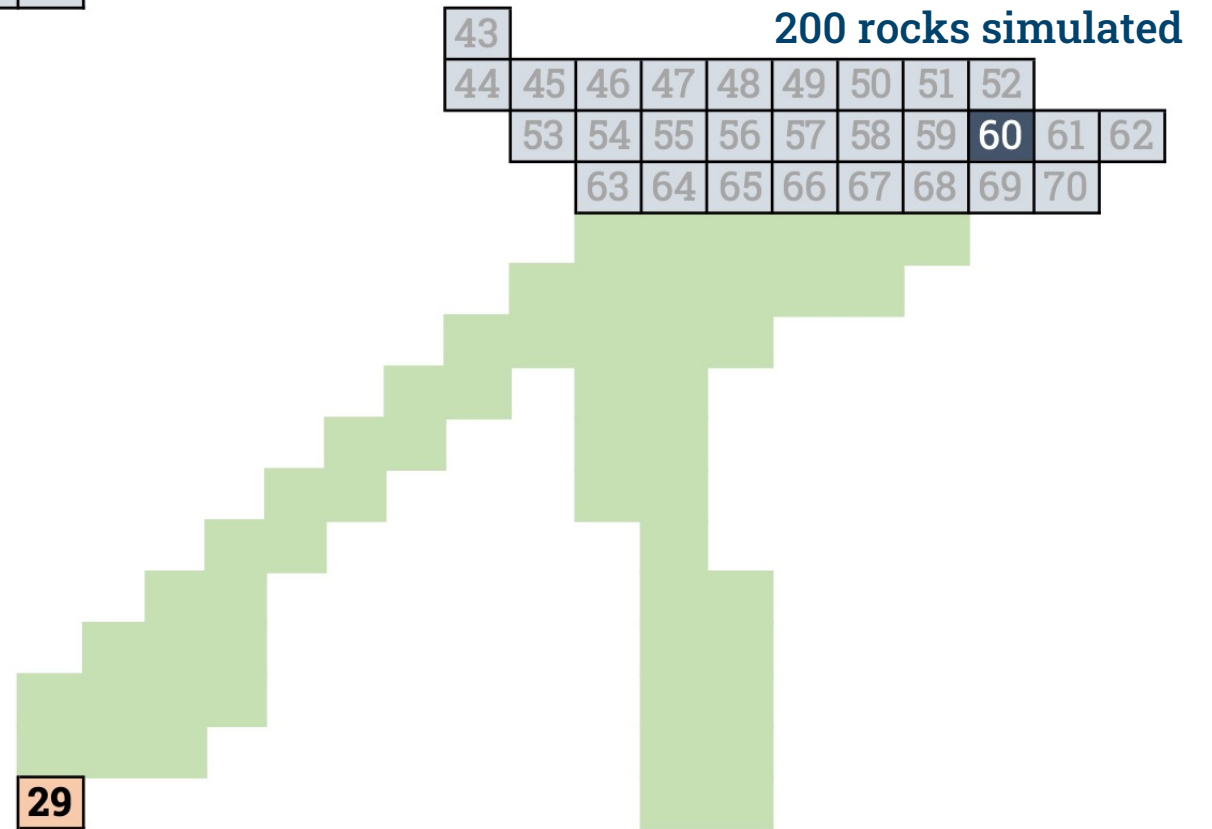
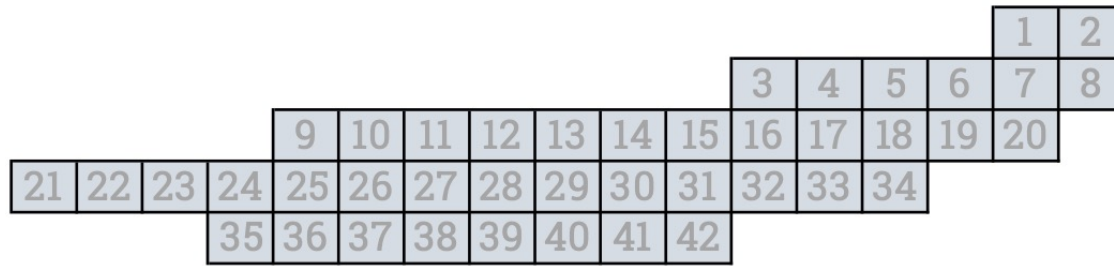
A virtual example



A virtual example



A virtual example



A virtual example

total number of source cells	70 (zone I: 42 zone II: 28)
number of trajectories (rocks) simulated per source cell	200
total number of trajectories (rocks) simulated	14000
departure probability of source cells	zone I: $1/42 \approx 0.0238$ zone II: $0.1/28 \approx 0.0036$

cell	$p_{d,i}$	$N_{b,i}$	$p_{p,i}$	$p_{r,i}$
1	0,023809524	58	0,29	0,006904762
2	0,023809524	63	0,315	0,0075
3	0,023809524	52	0,26	0,006428571
4	0,023809524	54	0,27	0,006428571
5	0,023809524	52	0,26	0,006428571
6	0,023809524	60	0,3	0,007142857
7	0,023809524	59	0,295	0,00702381
8	0,023809524	63	0,315	0,0075
9	0,023809524	5	0,025	0,000595238
10	0,023809524	8	0,04	0,000952381
11	0,023809524	8	0,04	0,000952381
12	0,023809524	21	0,105	0,0025
13	0,023809524	27	0,135	0,003214286
14	0,023809524	79	0,395	0,009404762
15	0,023809524	94	0,47	0,011190476
16	0,023809524	92	0,46	0,010952381
17	0,023809524	53	0,265	0,006309524
18	0,023809524	65	0,325	0,007738095
19	0,023809524	62	0,31	0,007380952
20	0,023809524	60	0,3	0,007142857
21	0,023809524	0	0	0
22	0,023809524	0	0	0
23	0,023809524	0	0	0
24	0,023809524	0	0	0
25	0,023809524	2	0,01	0,000238095
26	0,023809524	6	0,03	0,000714286
27	0,023809524	76	0,38	0,009047619
28	0,023809524	80	0,4	0,00952381
29	0,023809524	99	0,495	0,011785714
30	0,023809524	101	0,505	0,01202381
31	0,023809524	95	0,475	0,011309524
32	0,023809524	87	0,435	0,010357143
33	0,023809524	58	0,29	0,006904762
34	0,023809524	59	0,295	0,00702381
35	0,023809524	0	0	0

cell	$p_{d,i}$	$N_{b,i}$	$p_{p,i}$	$p_{r,i}$
36	0,023809524	0	0	0
37	0,023809524	0	0	0
38	0,023809524	25	0,125	0,00297619
39	0,023809524	76	0,38	0,009047619
40	0,023809524	97	0,485	0,011547619
41	0,023809524	105	0,525	0,0125
42	0,023809524	91	0,455	0,010833333
43	0,003571429	45	0,225	0,000803571
44	0,003571429	40	0,2	0,000714286
45	0,003571429	43	0,215	0,000767857
46	0,003571429	39	0,195	0,000696429
47	0,003571429	45	0,225	0,000803571
48	0,003571429	41	0,205	0,000732143
49	0,003571429	40	0,2	0,000714286
50	0,003571429	40	0,2	0,000714286
51	0,003571429	35	0,175	0,000625
52	0,003571429	30	0,15	0,000535714
53	0,003571429	44	0,22	0,000785714
54	0,003571429	48	0,24	0,000857143
55	0,003571429	45	0,225	0,000803571
56	0,003571429	49	0,245	0,000875
57	0,003571429	39	0,195	0,000696429
58	0,003571429	43	0,215	0,000767857
59	0,003571429	33	0,165	0,000589286
60	0,003571429	29	0,145	0,000517857
61	0,003571429	26	0,13	0,000464286
62	0,003571429	23	0,115	0,000410714
63	0,003571429	24	0,12	0,000428571
64	0,003571429	25	0,125	0,000446429
65	0,003571429	21	0,105	0,000375
66	0,003571429	28	0,14	0,0005
67	0,003571429	19	0,095	0,000339286
68	0,003571429	24	0,12	0,000428571
69	0,003571429	24	0,12	0,000428571
70	0,003571429	23	0,115	0,000410714

A virtual example

total number of source cells	70 (zone I: 42 zone II: 28)
number of trajectories (rocks) simulated per source cell	200
total number of trajectories (rocks) simulated	14000
departure probability of source cells	zone I: $1/42 \approx 0.0238$ zone II: $0.1/28 \approx 0.0036$

Source cell-wise calculation:

$$\mathbb{P}_r^A = 1 - \prod_{i=1}^{70} (1 - p_{d,i} p_{p,i}^A) \quad 0.23463$$

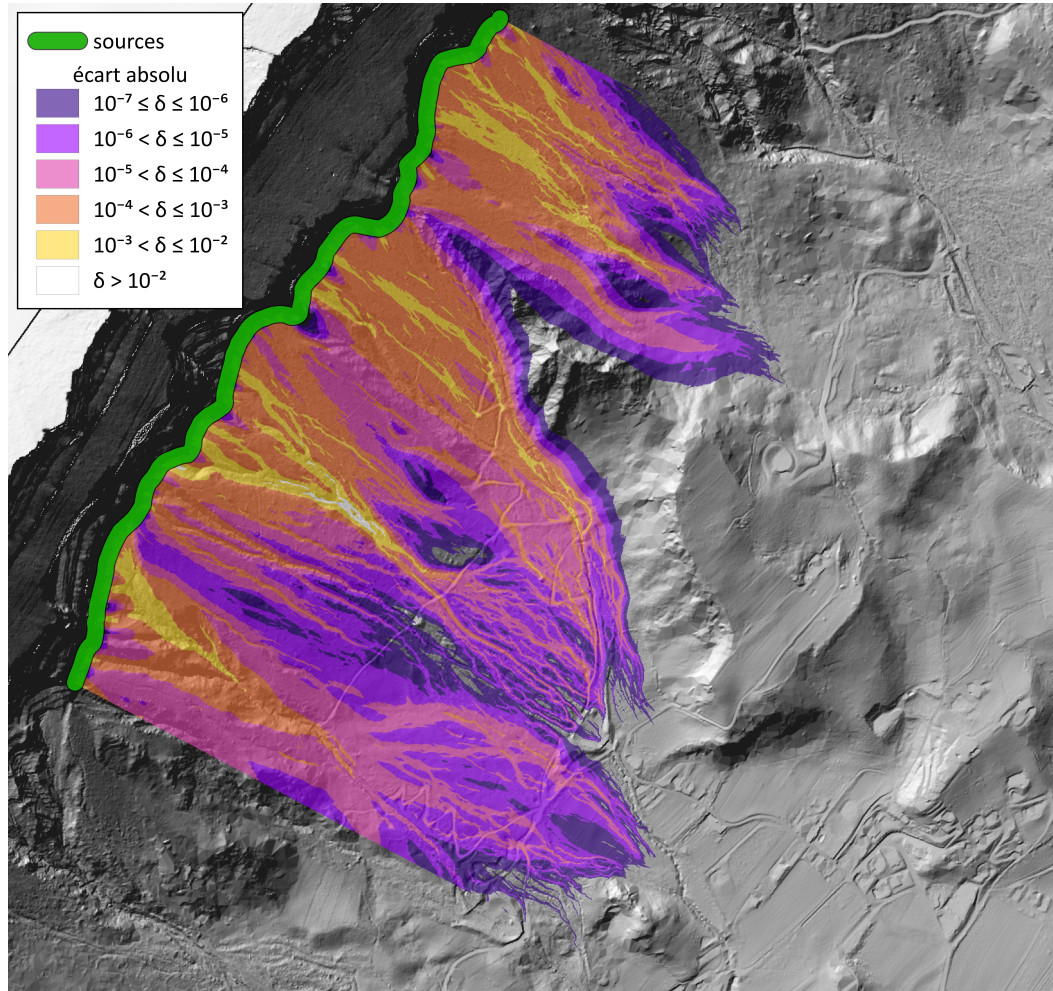
$$\mathbb{P}_r^A \approx \sum_{i=1}^{70} p_{d,i} p_{p,i}^A \quad 0.26628$$

Source zone-wise calculation:

$$\mathbb{P}_r^A = 1 - (1 - p_{d,I} p_{p,I}^A) (1 - p_{d,II} p_{p,II}^A) \quad 0.26199$$

$$\mathbb{P}_r^A = p_{d,I} p_{p,I}^A + p_{d,II} p_{p,II}^A \quad 0.26628$$

The 'Saint-Eynard' example

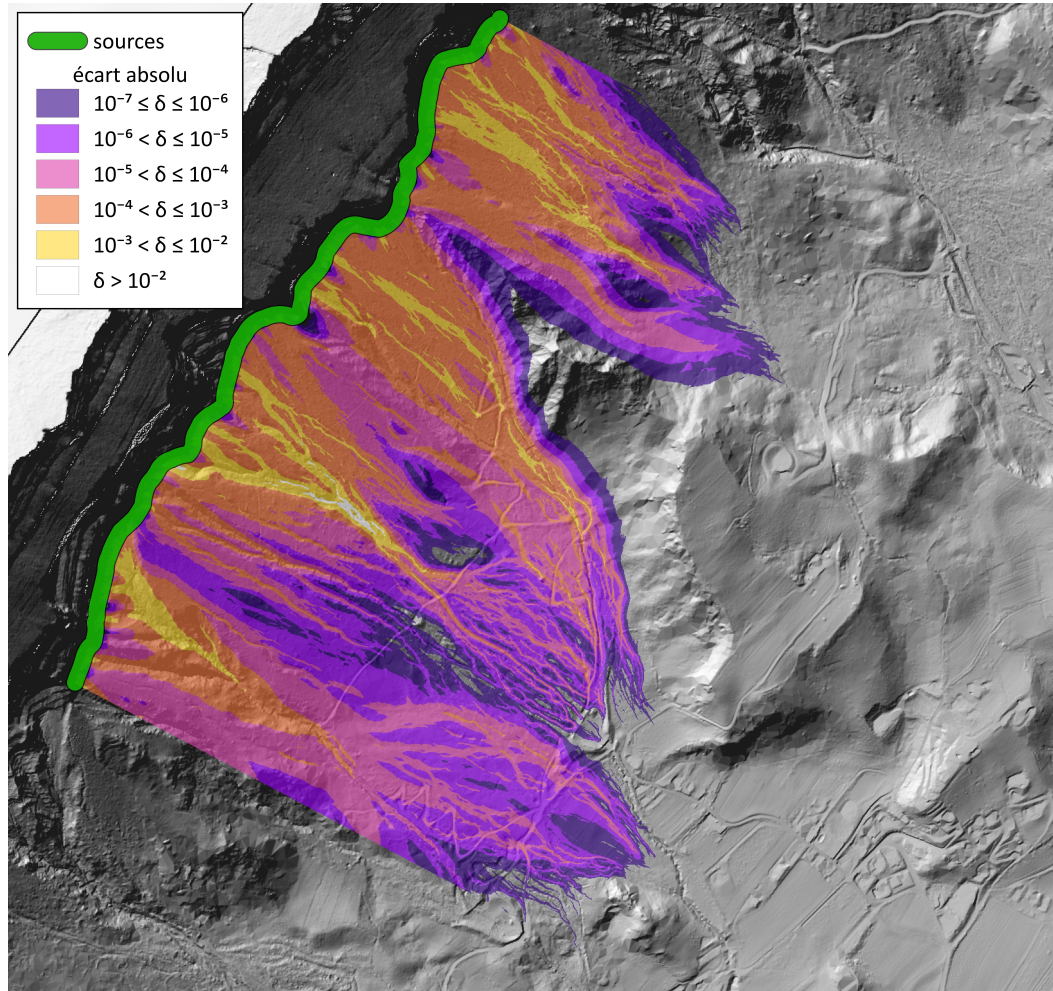


The 'Saint-Eynard' mountain, in the French Alps, near Grenoble

Simulations with Rockyfor3D

Mapping of the absolute gap between the values computed with the exact formula and the approximated formula

The 'Saint-Eynard' example



The 'Saint-Eynard' mountain, in the French Alps, near Grenoble

Simulations with Rockyfor3D

Mapping of the absolute gap between the values computed with the exact formula and the approximated formula

Up to 0.01 of absolute gap

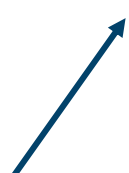
The gap is generally small enough to be absorbed in the logarithmic zoning

Commonly used expression for computing the reach probability

$$P_r = \frac{P_d N_b}{N_t}$$

Commonly used expression for computing the reach probability

reach probability
of the cell A


$$P_r = \frac{P_d N_b}{N_t}$$

Commonly used expression for computing the reach probability

departure probability
of the source zone

$$P_r = \frac{P_d N_b}{N_t}$$

reach probability
of the cell A

Commonly used expression for computing the reach probability

departure probability
of the source zone

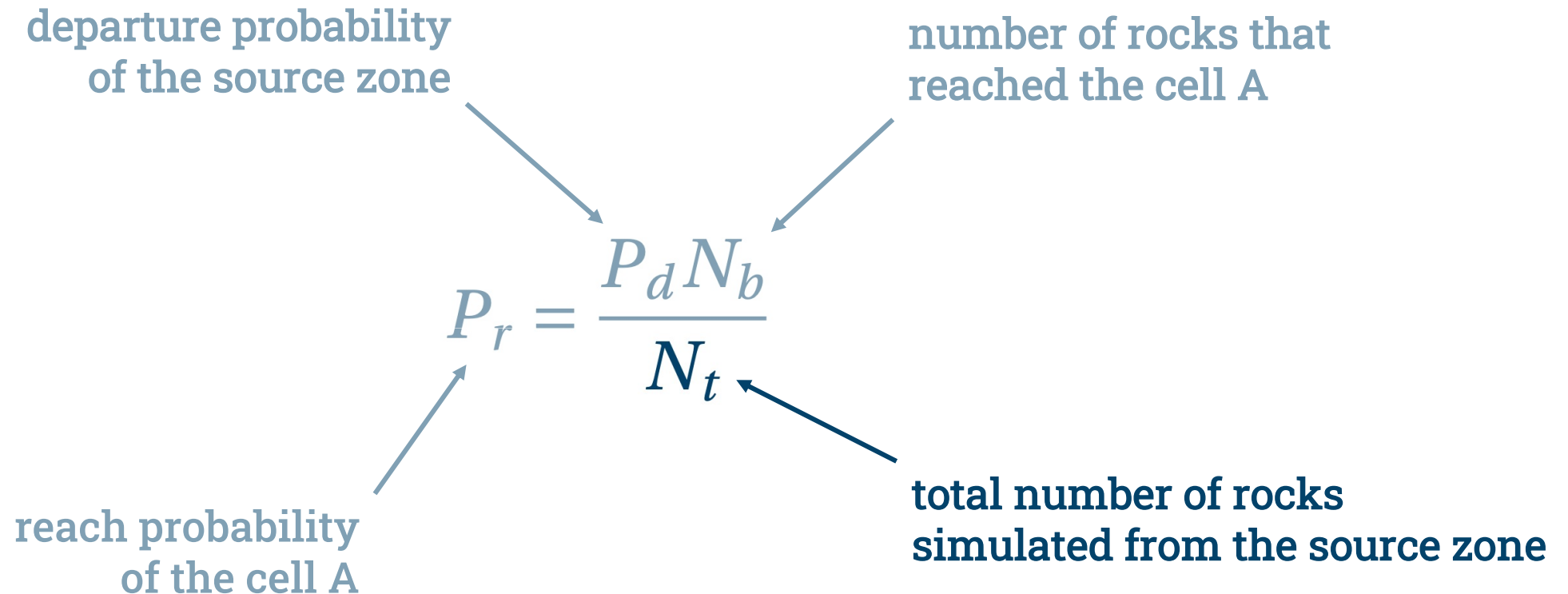
number of rocks that
reached the cell A

$$P_r = \frac{P_d N_b}{N_t}$$

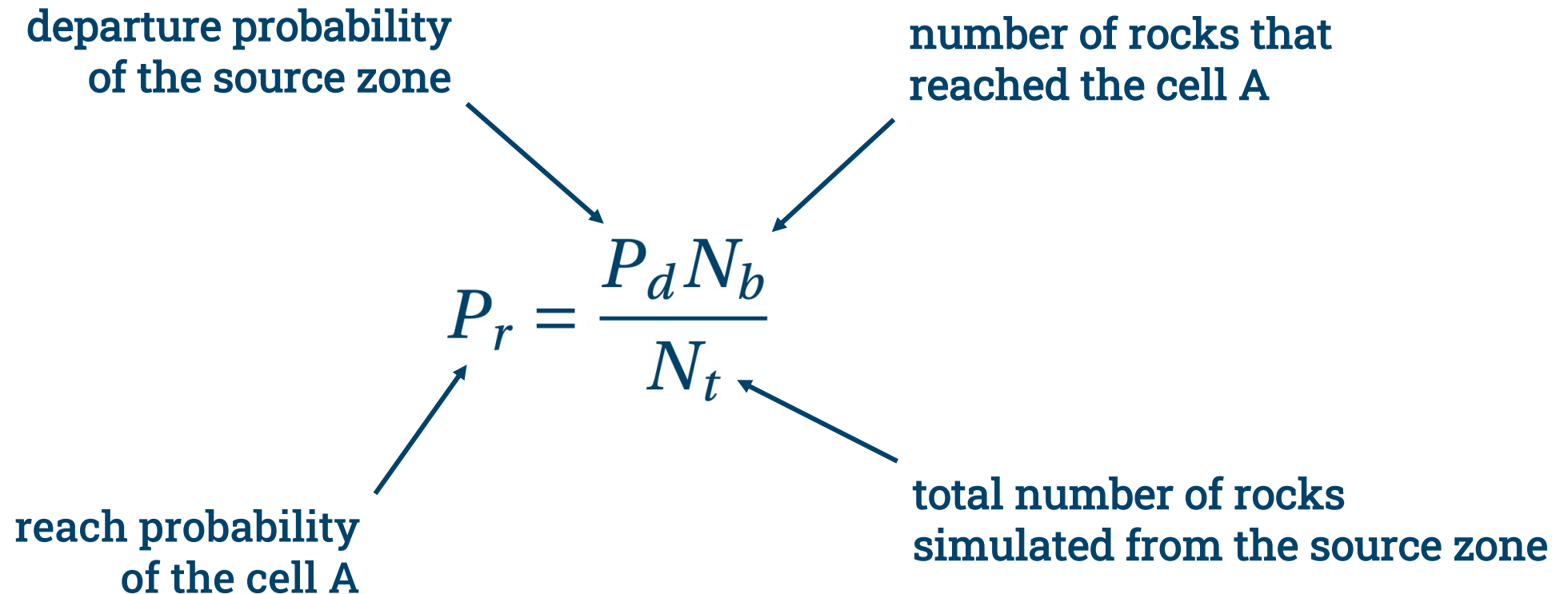
reach probability
of the cell A

The diagram illustrates the equation for reach probability, $P_r = \frac{P_d N_b}{N_t}$. Three arrows point from descriptive text to the variables in the equation: one from 'departure probability of the source zone' to P_d , one from 'number of rocks that reached the cell A' to N_b , and one from 'reach probability of the cell A' to P_r .

Commonly used expression for computing the reach probability



Commonly used expression for computing the reach probability



Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n p_{d,i} \times p_{p,i}^A$$

Start from the small probabilities' approximation

Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n \underbrace{p_{d,i}}_{p_d} \times \underbrace{p_{p,i}^A}_{\frac{N_{b,i}^A}{N_{\text{sim}}}}$$

Assume homogeneous departure probabilities and express the propagation probabilities

Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n \underbrace{p_{d,i}}_{p_d} \times \underbrace{p_{p,i}^A}_{\frac{N_{b,i}^A}{N_{\text{sim}}}} = p_d \sum_{i=1}^n \frac{N_{b,i}^A}{N_{\text{sim}}}$$

Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n \underbrace{p_{d,i}}_{p_d} \times \underbrace{p_{p,i}^A}_{\frac{N_{b,i}^A}{N_{\text{sim}}}} = p_d \sum_{i=1}^n \frac{N_{b,i}^A}{N_{\text{sim}}} = \frac{p_d}{N_{\text{sim}}} \sum_{i=1}^n N_{b,i}^A$$

Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n \underbrace{p_{d,i}}_{p_d} \times \underbrace{p_{p,i}^A}_{\frac{N_{b,i}^A}{N_{\text{sim}}}} = p_d \sum_{i=1}^n \frac{N_{b,i}^A}{N_{\text{sim}}} = \frac{p_d}{N_{\text{sim}}} \underbrace{\sum_{i=1}^n N_{b,i}^A}_{N_{\text{tot}}^A}$$

Recognize the total number of simulated rocks from all sources

Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n \underbrace{p_{d,i}}_{p_d} \times \underbrace{p_{p,i}^A}_{\frac{N_{b,i}^A}{N_{\text{sim}}}} = p_d \sum_{i=1}^n \frac{N_{b,i}^A}{N_{\text{sim}}} = \frac{p_d}{N_{\text{sim}}} \underbrace{\sum_{i=1}^n N_{b,i}^A}_{N_{\text{tot}}^A} = np_d \times N_{\text{tot}}^A \times \frac{1}{nN_{\text{sim}}}$$

Insert the number of sources (n)

Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n \underbrace{p_{d,i}}_{p_d} \times \underbrace{p_{p,i}^A}_{\frac{N_{b,i}^A}{N_{\text{sim}}}} = p_d \sum_{i=1}^n \frac{N_{b,i}^A}{N_{\text{sim}}} = \frac{p_d}{N_{\text{sim}}} \underbrace{\sum_{i=1}^n N_{b,i}^A}_{N_{\text{tot}}^A} = \underbrace{np_d}_{P_d} \times \underbrace{N_{\text{tot}}^A}_{N_b} \times \underbrace{\frac{1}{nN_{\text{sim}}}}_{\frac{1}{N_t}}$$

Define the departure probability of the source zone, the total number of rocks that reached the cell A, and the total number of simulated rocks from the source zone

Commonly used expression for computing the reach probability

This formula is a particular case of the small probabilities' approximation, considering that all sources are part of a unique source zone:

$$\mathbb{P}_r^A \approx \sum_{i=1}^n \underbrace{p_{d,i}}_{p_d} \times \underbrace{p_{p,i}^A}_{\frac{N_{b,i}^A}{N_{\text{sim}}}} = p_d \sum_{i=1}^n \frac{N_{b,i}^A}{N_{\text{sim}}} = \frac{p_d}{N_{\text{sim}}} \underbrace{\sum_{i=1}^n N_{b,i}^A}_{N_{\text{tot}}^A} = \underbrace{np_d}_{P_d} \times \underbrace{N_{\text{tot}}^A}_{N_b} \times \underbrace{\frac{1}{nN_{\text{sim}}}}_{\frac{1}{N_t}} = \frac{P_d N_b}{N_t}$$

A graphical error criterion for the small probabilities' approximation

$$\left| 1 - \prod_{i=1}^n (1 - p_{r,i}^A) - \sum_{i=1}^n p_{r,i}^A \right| \approx \sum_{1 \leq i < j \leq n} p_{r,i}^A p_{r,j}^A$$

A graphical error criterion for the small probabilities' approximation

$$\left| 1 - \prod_{i=1}^n (1 - p_{r,i}^A) - \sum_{i=1}^n p_{r,i}^A \right| \approx \sum_{1 \leq i < j \leq n} p_{r,i}^A p_{r,j}^A$$

$$\sum_{1 \leq i < j \leq n} p_{r,i}^A p_{r,j}^A \leq \binom{n}{2} \max_{1 \leq i \leq n} \{p_{r,i}\}^2 < \frac{n^2}{2} \max_{1 \leq i \leq n} \{p_{r,i}\}^2$$

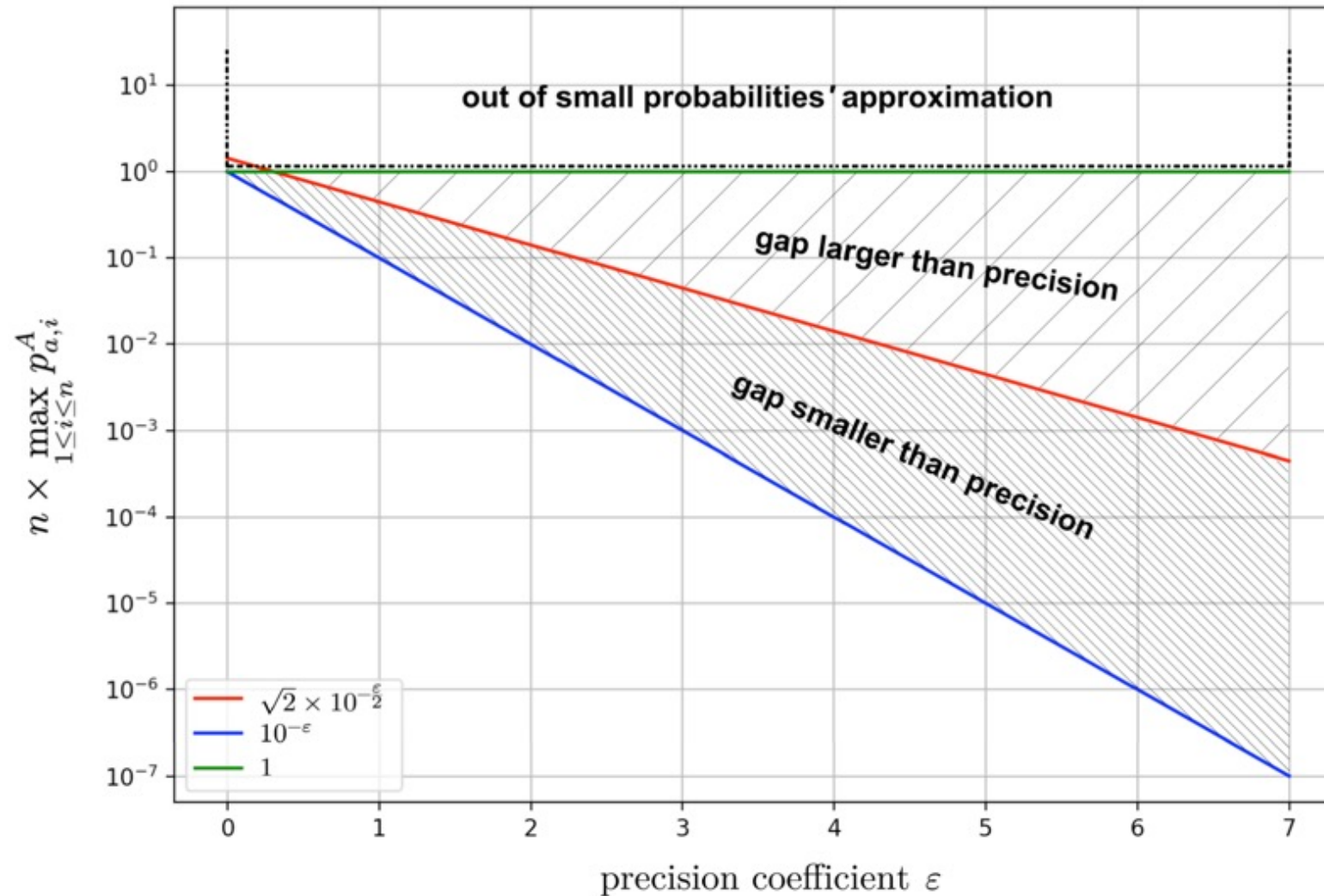
A graphical error criterion for the small probabilities' approximation

$$\left| 1 - \prod_{i=1}^n (1 - p_{r,i}^A) - \sum_{i=1}^n p_{r,i}^A \right| \approx \sum_{1 \leq i < j \leq n} p_{r,i}^A p_{r,j}^A$$

$$\sum_{1 \leq i < j \leq n} p_{r,i}^A p_{r,j}^A \leq \binom{n}{2} \max_{1 \leq i \leq n} \{p_{r,i}\}^2 < \frac{n^2}{2} \max_{1 \leq i \leq n} \{p_{r,i}\}^2$$

$$\frac{n^2}{2} \max_{1 \leq i \leq n} \{p_{r,i}\}^2 < 10^{-\epsilon} \iff n \max_{1 \leq i \leq n} \{p_{r,i}\} < \sqrt{2} \times 10^{-\frac{\epsilon}{2}}$$

A graphical error criterion for the small probabilities' approximation



A few references

Further details available in:

- Rossignol A., Martin R., Bourrier F. (2024), in *Revue française de géotechnique* (in French)
- Rossignol A., Martin R., Bourrier F. (in prep.) (in English)

...and additional references:

- Hantz D., Corominas J., Crosta G.B., Jaboyedoff M. (2021), in *Geosciences*.
- Fell R., Ho K., Lacasse S., Leroi E. (2005), in *Proceedings of the International Conference on Landslide Risk Management*, CRC Press.
- Leroi E., Bonnard C., Fell R., McInnes R. (2005), in *Proceedings of the International Conference on Landslide Risk Management*, CRC Press.
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- Farvacque M., Lopez-Saez J., Corona C., Toe D., Bourrier F., Eckert N. (2020), in *Géomorphologie: relief, processus, environnement*.
- C2ROP (2020), *Caractérisation de l'aléa éboulement rocheux – État de l'art*, Cerema, collection "Connaissances".

Thank you very much for your attention!

I will be happy to answer your questions!

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MINISTÈRE
DE LA TRANSITION
ÉCOLOGIQUE
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