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1 Towards digitally programmed nonlinear electroacoustic  
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6 **Abstract**

Passive acoustic foams have limitations in efficiently reducing low-frequency noise below 800 Hz due to the associated wavelengths for reasonable material thickness. To address this challenge, passive or semi-active resonators are commonly employed solutions. Linear resonators are very efficient within a narrow frequency bandwidth. However, nonlinear oscillators can broaden this bandwidth, but activation of their nonlinear responses typically requires high excitation amplitudes, beyond human hearing tolerance amplitudes. Furthermore, the specific type of nonlinearity in such devices is often predetermined by the inherent properties of the resonator. In this study, we employ a novel digital control algorithm, allowing to activate the nonlinear response of the electroacoustic resonator at low excitation amplitudes. This algorithm, which relies on real-time integration, facilitates the creation of nonlinear resonators featuring polynomial or diverse non-polynomial nonlinearities within the range of low amplitudes. The nonlinear control is carried out on a loudspeaker equipped with a microphone. Our research highlights the potential to create nonlinear resonators with different, versatile and programmable behaviors. Unprecedented non-polynomial nonlinear behaviors are experimentally exhibited, we consider cubic, piece-wise linear, and logarithmic nonlinearities. These behaviors are implemented and compared to a semi-analytic model to control an acoustic mode of a tube under conditions of low excitation amplitudes and frequencies.

7 *Keywords:* Programmable, Low amplitude, Electroacoustic Resonator,  
8 Noise Control, Cubic nonlinearity, piece-wise nonlinearity, logarithmic  
9 potential nonlinearity, non-polynomial nonlinearities

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## 10 **1. Introduction**

11 Addressing noise and vibrations has remained a central objective of re-  
12 search efforts for a significant period of time [1]. To solve this particular  
13 matter, passive, active, and hybrid categories have been considered for miti-  
14 gating vibrations and noises across many fields such as mechanics, aeronau-  
15 tics, and acoustics. In acoustics, passive noise control stands as the most  
16 widely employed approach. Indeed, acoustic foams are commonly used in  
17 the building industry. However, their absorption capabilities are limited to  
18 the frequencies above 1 kHz [2]. The efficiency of acoustic foams diminishes  
19 at lower frequencies due to their damping mechanism [3, 4]. To overcome  
20 these limitations, passive resonators are widely used such as Helmholtz res-  
21 onators. These resonators are very efficient on a narrow frequency bandwidth  
22 focused around their resonating frequency. With the intention of finding so-  
23 lutions to these limitations, active noise cancellation technology emerged in  
24 1878 with Rayleigh’s experiment involving the superposition of sound fields  
25 using two synchronized tuning forks [5]. Lueg [6] wrote the first patent on  
26 noise cancellation using a wave in phase opposition with an incident wave.  
27 Then, active noise cancellation has been rapidly developed, supported by a  
28 substantial volume of articles published on the subject matter [7, 8, 9, 10].  
29 However, active noise cancellation demands a substantial amount of energy  
30 to be efficient in spatial zones. Another active noise control method has  
31 been created and lies on Impedance Control (IC), with the original idea from  
32 Olson and May [11]. The IC consists of adapting the boundary conditions  
33 of a material or a device - usually a loudspeaker - to choose the absorption  
34 and reflection conditions to be applied on the incident wave by the material.  
35 Unlike active noise cancellation, this approach doesn’t introduce additional  
36 waves, rendering the system acoustically passive. Building upon Olson and  
37 May’s work [11], Guicking [12, 13] extended the IC concept to electroacoustic  
38 absorbers composed of a loudspeaker collocated to a microphone at normal  
39 incidence in a Kundt’s tube. Furtoss and Thenail [14, 15] designed an IC  
40 system that created a quarter wavelength resonator using an accelerometer  
41 and a microphone to target specific impedances. Subsequently, the studies

42 focused on the active complex impedance control (e.g. see [16]) where the  
43 optimal impedance is targeted. The following works oriented to create a dis-  
44 tributed network of electroacoustic absorber cells with a loudspeaker control  
45 based on the advection equation [17, 18, 19]. Following this, a model inver-  
46 sion control of the electroacoustic absorber has been created by targeting the  
47 optimal acoustic impedances at both normal and grazing incidences, with  
48 and without flow [20, 21, 22, 23]. Recently, a new kind of electroacoustic  
49 absorber has been designed using the concept of plasmacoustic metalayers to  
50 achieve broadband sound control [24].

51 Nonlinear phenomena in acoustics are not employed a lot for noise control,  
52 unlike mechanics where such behaviors are extensively explored and used for  
53 their benefits [25]. Indeed, nonlinear absorbers present faster decreases for  
54 transient regimes [26], or can feature targeted energy transfer phenomenon  
55 which consists of an irreversible energy transfer from a primary system to  
56 a nonlinear absorber. Furthermore, nonlinear systems can be designed to  
57 be efficient on large frequency bandwidths [26]. Extensive researches have  
58 been conducted on nonlinear behaviors, including the duffing type oscilla-  
59 tors [27, 28], piece-wise oscillators [29, 30, 31, 32, 33, 34], or vibroimpact  
60 oscillators [35, 36], all aimed at mitigating vibrations. These studies have  
61 sparked interest in applying nonlinear principles to acoustic absorbers [37].  
62 However, the activation of nonlinear phenomena is usually constrained by  
63 an amplitude threshold rarely reached. Indeed, this threshold is situated at  
64 amplitudes that pose the risk of permanently damaging a human ear. Bellet  
65 [38] implemented a visco-elastic membrane with an amplitude of 148 dB that  
66 behaves as a pure cubic oscillator, and showed multiple nonlinear phenom-  
67 ena, with both stationary and transient regimes, along with efficient control  
68 of the coupled acoustic mode. Gourdon [39, 40] placed a Helmholtz resonator  
69 in its nonlinear regime, above 138 dB and showed interesting noise reduc-  
70 tion in the coupled tube. Bitar [41] attempted to create a passive nonlinear  
71 electroacoustic absorber with an analogic circuit. Finally, Guo [42] created  
72 a nonlinear electroacoustic absorber at hearing amplitudes around 95 dB  
73 thanks to an additional microphone in the back cavity of the loudspeaker  
74 giving the displacement of the membrane at each time step. This additional  
75 measurement is needed, as frequency-domain approaches cannot perform the  
76 creation of nonlinear behaviors, and permits the calculation of a polyno-  
77 mial nonlinear current to be added to a linear current implemented through  
78 frequency-domain approaches. In this study, unprecedented non-polynomial  
79 nonlinear behaviors are exhibited through an innovative method to digitally

80 program nonlinear Electroacoustic Resonator (ER) at low excitation ampli-  
81 tudes, without additional microphones in the back cavity of the loudspeaker.  
82 It is a real-time based method briefly presented in Section 2 and detailed in  
83 [43]. Subsequently, the ER is weakly coupled to an acoustic mode of a tube  
84 which is modeled in Section 3. Finally, experimental results are presented  
85 for multiple original programmed nonlinear behaviors and compared to the  
86 model in Section 4. The good correlation between numerical and experimen-  
87 tal results confirms the achievement of nonlinear dynamics involving cubic,  
88 piece-wise, and logarithmic restoring force functions, and demonstrates the  
89 potentialities of such nonlinear behaviors for noise reduction and the control  
90 of acoustic modes.

## 91 **2. The Electroacoustic Resonator**

### 92 *2.1. The concept*

93 The underlying concept is to create an acoustic resonator with a loud-  
94 speaker collocated to a microphone, and equipped with a feedback con-  
95 trol that runs by a dedicated processor. The study aims at creating pro-  
96 grammable nonlinear resonators at low excitation amplitudes. The classical  
97 approach which relies on the Z-transform and the infinite impulse response  
98 experimental implementation [44] is not suitable for nonlinear resonators im-  
99 plementation due to its frequency-based approach. As a result, an innovative  
100 and efficient real-time-based method has been created to address this chal-  
101 lenge. Assuming that the loudspeaker is positioned on its first mode, the  
102 algorithm carries out a pressure-based current-driven control of the mem-  
103 brane displacement. Indeed, it retrieves the measured pressure coming from  
104 the microphone and estimates the corresponding electrical current to send  
105 into the loudspeaker coil. Once the electrical current is delivered, the loud-  
106 speaker behaves as the programmed nonlinear resonator. Further details on  
107 the algorithm can be found in the next section, and in [43, 45].

### 108 *2.2. Programming of the Electroacoustic Resonator*

109 The algorithm considers the pressure as an input and the electrical current  
110 as an output. It estimates the electrical current based on the measured  
111 pressure to achieve the desired behavior of the loudspeaker. Given the low  
112 operational frequencies and amplitudes, the membrane is regarded as both  
113 rigid and planar. The loudspeaker is placed on its first mode, modeled as a  
114 single-degree-of-freedom oscillator.

115 The dynamic model of the loudspeaker's first mode is a classical mass-spring-  
 116 damper:

$$M_0\ddot{u}_m(t) + R_0\dot{u}_m(t) + K_0u_m(t) = S_dp(t) - Bli(t) \quad (1)$$

117 where  $\dot{\bullet}$  represents the time derivative of the variable.  $u_m$  denotes the dis-  
 118 placement of the membrane of the loudspeaker.  $M_0$ ,  $R_0$ , and  $K_0$  are the  
 119 modal parameters of the loudspeaker's first mode without any applied cur-  
 120 rent.  $Bli$  is the Laplace force, while  $B$  is the intensity of the magnetic field,  
 121 and  $l$  the coil's length.  $S_dp$  stands for the force induced by the incident acous-  
 122 tic waves, while  $S_d$  represents the effective area of the loudspeaker membrane  
 123 [46]. The desired behavior equation can be written as:

$$M_d\ddot{u}_m(t) + R_d\dot{u}_m(t) + F_{\text{NL}}(t, u_m) = S_dp(t) \quad (2)$$

124 where  $M_d$ ,  $R_d$ , and  $K_d$  denote the desired modal parameters of the loud-  
 125 speaker's first mode, and  $F_{\text{NL}}$  stands for a chosen nonlinear restoring forcing  
 126 function. However, also a linear part of the stiffness of the loudspeaker should  
 127 be taken into account in  $F_{\text{NL}}$  for stability issues. It is due to the impossibility  
 128 to reach a zero stiffness system. It should be mentioned that in this study  
 129 we suppose that  $F_{\text{NL}}$  is a nonlinear restoring forcing function i.e.  $F_{\text{NL}}(t, u_m)$ ,  
 130 but generally speaking it can be  $F_{\text{NL}}(t, u_m, \dot{u}_m, \ddot{u}_m)$ . The algorithm is now  
 131 explained using Eqs. (1) and (2).

132 The algorithm is structured around two main tasks. The first task consists  
 133 of the determination of the desired displacement, speed, and acceleration of  
 134 the loudspeaker's membrane. To achieve this result, at time step  $t_n$ , the  
 135 algorithm uses the measured pressure to integrate Eq. (2) with a numerical  
 136 scheme. This real-time resolution predicts the desired displacement  $u_m$  and  
 137 speed  $\dot{u}_m$  for the next time step  $t_{n+1}$  and keeps the values in memory. At  
 138 time step  $t_{n+1}$ , the algorithm uses the prediction of the desired displacement  
 139 and speed done at time step  $t_n$  along with the newly measured pressure at  
 140 time step  $t_{n+1}$  to estimate the desired acceleration  $\ddot{u}_m$  using Eq. (2). Then,  
 141 the second task is devoted to the calculation of the electrical current to send  
 142 into the loudspeaker. From Eq. (1), the controller to enforce the desired  
 143 displacement is given by:

$$i(t) = \frac{S_d}{Bl} \left( p(t) - \left( \frac{M_0}{S_d}\ddot{u}_m + \frac{R_0}{S_d}\dot{u}_m + \frac{K_0}{S_d}u_m \right) \right) \quad (3)$$

144 At time step  $t_{n+1}$ , and employing Eq. (3), the electrical current is computed  
 145 using the previously determined desired displacement, speed and accelera-

146 tion, and the measured pressure.

147

### 148 3. Considered System

#### 149 3.1. Principle and experimental set-up

150 This section aims at modeling the experimental acoustic set-up in order  
 151 to predict the system behavior. The experimental set-up has been designed  
 152 such that it can be modeled as a two-degrees-of-freedom system [38]. The  
 153 model of the experiment is presented in Fig. 1 and has been widely studied  
 154 in previous works [38, 37, 39]. It is composed of a cylindrical reduced section  
 155 tube linked to the Electroacoustic Resonator through a cylindrical coupling  
 156 box. The system is excited through an external loudspeaker (LS) placed at  
 157 the opposite end of the tube. The microphone placed in the reduced section  
 158 tube is employed to verify the noise attenuation performance achieved by  
 159 our nonlinear ER, and it does not play a role in the programming. In the  
 160 following parts, each segment of the experimental set-up will be explained in  
 detail and will be modeled.

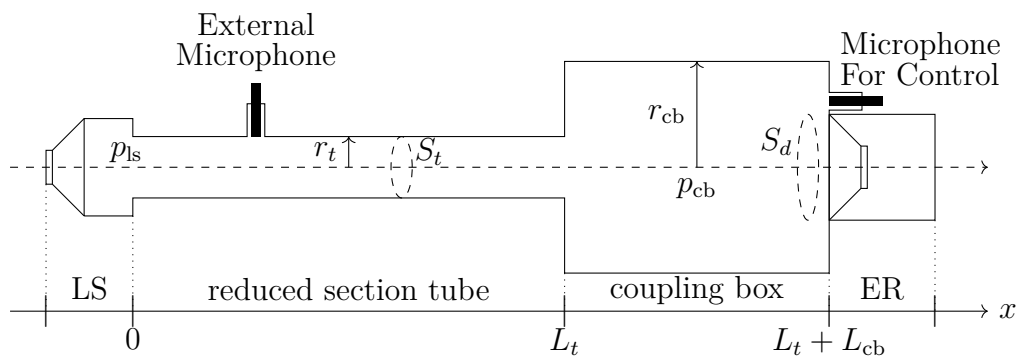


Figure 1: Scheme of the experimental set-up

161

#### 162 3.2. Analytical modeling

163 This model assumes that waves are plane. The section of the largest tube  
 164 of the experiment allows the first transverse modes to appear above 2 kHz.  
 165 It ensures plane waves below this excitation frequency.

166 *3.2.1. The acoustic source model*

167 One of the innovations introduced in this work is the functionality of  
 168 the proposed system at low excitation amplitude. As previously mentioned,  
 169 acoustic resonators usually need to be excited at high excitation amplitudes  
 170 to activate their nonlinear behaviors [40, 39, 38]. These sound pressure levels  
 171 range from 120 dB to 160 dB depending on the characteristics of the device.  
 172 The working frequency is chosen to bound the frequencies of the reduced  
 173 section tube's first acoustic mode: from 350 Hz to 800 Hz. The acoustic  
 174 source is modeled as an incident pressure at  $x = 0$  (see Fig. 1), named  $p_{\text{is}}$ .

175 *3.2.2. The coupling box model*

176 The goal of introducing the coupling box into the system is to maintain  
 177 a weak coupling between the primary system to be controlled (the reduced  
 178 section tube) and the added system (the ER). To achieve such a condition  
 179 through the coupling box, its section is chosen to be large enough compared  
 180 to the ones of the narrowed tube section and the membrane of the ER. Ad-  
 181 ditionally, the length of the coupling box is kept relatively short compared  
 182 to the wavelengths of the working frequencies to maintain a constant pres-  
 183 sure within the box. Using the mass conservation equation, one can express  
 184 the pressure in the coupling box, denoted  $p_{\text{cb}}$ , as a function of its volume  
 185 variation:

$$p_{\text{cb}}(t) = -\rho_0 c_0^2 \frac{dV_{\text{cb}}(t)}{V_{\text{cb},0}} \quad (4)$$

186 where  $V_{\text{cb},0}$  represents the initial volume of the coupling box,  $\rho_0$  is the air  
 187 density, and  $c_0$  is the velocity of the sound in the air. The variation of volume  
 188  $dV_{\text{cb}}$  can be described in terms of displacements of the membrane of the ER  
 189 and of the reduced section tube air mass. Equation (4) reads:

$$p_{\text{cb}}(t) = k_b \left( S_t u_a(t) - S_d u_m(t) \right) \quad (5)$$

190 with  $k_b = \frac{\rho_0 c_0^2}{V_{\text{cb},0}}$ .

191 *3.2.3. The model of the reduced section tube*

192 The reduced-section tube is considered with boundary conditions at each  
 193 end of the tube that ensures the continuity of the displacements. These  
 194 conditions are given in Eq. (11). From the mass conservation equation, one



195 can easily get the relation between the pressure  $p$  and the displacement of  
 196 the air particles denoted as  $u_{\text{air}}$ :

$$p(x, t) = -\rho_0 c_0^2 \frac{\partial u_{\text{air}}}{\partial x}(x, t) \quad (6)$$

197 The Euler equation for a plane wave without flow reads:

$$\rho_0 \frac{\partial^2 u_{\text{air}}}{\partial t^2} = -\frac{\partial p}{\partial x} \quad (7)$$

198 By replacing Eq. (6) in Eq. (7), one can obtain:

$$\rho_0 \frac{\partial^2 u_{\text{air}}}{\partial t^2} = \rho_0 c_0^2 \frac{\partial^2 u_{\text{air}}}{\partial x^2} \quad (8)$$

199 Let  $u_{\text{air}}^*(x, t)$  be a virtual displacement field that is kinematically admissible,  
 200 continuous, and differentiable with respect to  $x$  and  $t$ . Multiplying Eq. (8) by  
 201  $u_{\text{air}}^*(x, t)$  and integrating over the volume yields the variational formulation  
 202 of Eq. (8):

$$\int_0^{L_t} \rho_0 \frac{\partial^2 u_{\text{air}}}{\partial t^2} S_t u_{\text{air}}^* dx = \int_0^{L_t} \rho_0 c_0^2 \frac{\partial^2 u_{\text{air}}}{\partial x^2} S_t u_{\text{air}}^* dx \quad (9)$$

203 After an integration by parts and using Eq (6) at  $x = 0$ , and  $x = L_t$ , this  
 204 yields:

$$\begin{aligned} \int_0^{L_t} \rho_0 \frac{\partial^2 u_{\text{air}}}{\partial t^2} S_t u_{\text{air}}^* dx &= - \int_0^{L_t} \rho_0 c_0^2 S_t \frac{\partial u_{\text{air}}}{\partial x} \frac{\partial u_{\text{air}}^*}{\partial x} dx \\ &\quad - S_t u_{\text{air}}^*(L_t, t) p_{\text{cb}} + S_t u_{\text{air}}^*(0, t) p_{\text{ls}} \end{aligned} \quad (10)$$

205 Let us suppose that the response of the system is dominated by its first mode,  
 206 with the boundary conditions:

$$\begin{cases} u_{\text{air}}(0, t) = -u_{\text{ls}}(t) \\ u_{\text{air}}(L_t, t) = u_{\text{cb}}(t) \end{cases} \quad (11)$$

207 where the displacement is positive when the air goes out of the tube. We use  
 208 a Rayleigh-Ritz method considering that:

$$u_{\text{air}}(x, t) = u_a(t) \Phi_1(x) = u_a(t) \left( -\cos\left(\frac{\pi x}{L_t}\right) \right) \quad (12)$$

209 where  $\Phi_1$  is the shape function (which represents also the first acoustic mode  
 210 of the tube), and  $u_a$  stands for the modal coordinates. The same considera-  
 211 tions are applied to the virtual displacement. Equation (10) yields to:

$$\left(\frac{\rho_0 S_t L_t}{2}\right)\ddot{u}_a + \left(\frac{\rho_0 c_0^2 S_t \pi^2}{2L_t}\right)u_a = -S_t(p_{cb} + p_{ls}) \quad (13)$$

212 We aim to have a model that closely represents the reality. However, the  
 213 experiments revealed the need to model a damping coefficient. The choice of  
 214 linear viscous damping is arbitrary. In typical structural or fluid-structure  
 215 dynamics problems [47], it is often assigned a viscous term to synthesize  
 216 the multi-physical phenomena contributing to the modal dissipation (such  
 217 as mechanical damping and visco-thermal losses). The corresponding modal  
 218 viscous coefficient is then usually estimated experimentally [48]. This for-  
 219 malism is kept in the reduced model by introducing a viscosity contribution  
 220 in Eq (13), proportional to  $c_a$ . This parameter is obtained by experimental  
 221 measurements and a brief study of its influence on the numerical results is  
 222 presented in Appendix A.2. We obtain:

$$m_a \ddot{u}_a + c_a \dot{u}_a + k_a u_a = -S_t(p_{cb} + p_{ls}) \quad (14)$$

223 The modal mass  $m_a$  and modal stiffness  $k_a$  of the first acoustic mode of the  
 224 reduced section tube reads:

$$\begin{cases} m_a = \frac{\rho_0 S_t L_t}{2} \\ k_a = \frac{\rho_0 c_0^2 S_t \pi^2}{2L_t} \end{cases} \quad (15)$$

### 225 3.2.4. Coupled equations of the two-degrees of freedom model

226 Each segment of the overall system has been described individually. Equa-  
 227 tions of the coupled systems can be constructed by incorporating boundary  
 228 conditions. The considered ER model is given by the desired behavior equa-  
 229 tion Eq. (2) in the tube with the compatibility condition  $p = p_{cb}$ :

$$M_d \ddot{u}_m(t) + R_d \dot{u}_m(t) + F_{NL}(t, u_m) = S_d p_{cb}(t) \quad (16)$$

230 Equations (14) and (16) yield the system of equations:

$$\begin{cases} m_a \ddot{u}_a + c_a \dot{u}_a + k_a u_a = -S_t(p_{cb} + p_{ls}) \\ M_d \ddot{u}_m(t) + R_d \dot{u}_m(t) + F_{NL}(t, u_m) = S_d p_{cb}(t) \end{cases} \quad (17)$$

231 Introducing Eq. (5) into Eq. (17) yields the weakly coupled two-degree-of-  
 232 freedom system of equations:

$$\begin{cases} m_a \ddot{u}_a + c_a \dot{u}_a + k_a u_a + S_t (k_b (S_t u_a - S_d u_m)) = -S_t p_{\text{ls}} \\ M_d \ddot{u}_m + R_d \dot{u}_m + F_{\text{NL}}(t, u_m) - S_d (k_b (S_t u_a - S_d u_m)) = 0 \end{cases} \quad (18)$$

233 We set  $\gamma = k_b S_t S_d$  and  $\alpha = S_d / S_t$ . The system can be further rearranged as:

$$\begin{cases} m_a \ddot{u}_a + c_a \dot{u}_a + k_a u_a + \frac{\gamma}{\alpha} (u_a - \alpha u_m) = -S_t p_{\text{ls}} \\ M_d \ddot{u}_m + R_d \dot{u}_m + F_{\text{NL}}(t, u_m) + \gamma (\alpha u_m - u_a) = 0 \end{cases} \quad (19)$$

234 It is worth noting that the weak coupling in this system appears to be non-  
 235 reciprocal. Nevertheless, this is a consequence of formulating the equations  
 236 in terms of forces. The formulation in terms of pressure, which is the com-  
 237 mon acoustic approach, results in a reciprocal coupling. In fact, the system  
 238 of equations can be expressed using the acoustic parameters  $m_a / S_t$ ,  $c_a / S_t$ ,  
 239  $k_a / S_t$ ,  $M_d / S_d$  and  $R_d / S_d$ :

$$\begin{cases} \frac{m_a}{S_t} \ddot{u}_a + \frac{c_a}{S_t} \dot{u}_a + \frac{k_a}{S_t} u_a + \frac{\gamma}{S_d} (u_a - \alpha u_m) = -p_{\text{ls}} \\ \frac{M_d}{S_d} \ddot{u}_m + \frac{R_d}{S_d} \dot{u}_m + \frac{1}{S_d} F_{\text{NL}}(t, u_m) + \frac{\gamma}{S_d} (\alpha u_m - u_a) = 0 \end{cases} \quad (20)$$

240 The coupling between the mechanical behavior of the ER, and the acoustic  
 241 behavior of the tube allows one to choose either of the formulations. In the  
 242 following, the formulation in terms of forces defined Eq. (19) is employed.

## 243 4. Experimental results

### 244 4.1. Experimental set-up

245 The experimental set-up is described in Fig. 1 and shown in Fig. 2. The  
 246 values of the parameters of the system are presented in Table 1. The ER  
 247 is linked to a D-Space MicroLabBox DS1202 device which serves for pro-  
 248 gramming and for the control of the loudspeaker displacement. The device  
 249 operates with a sampling frequency of 50 kHz. The external excitation is pro-  
 250 duced by an external loudspeaker. The acoustic excitation lasts 60 seconds  
 251 in the form of a linear chirp from 350 Hz to 800 Hz with either increasing  
 252 or decreasing frequencies. Experiments are carried out with the coupled ER

$r_t$	$1.45 \times 10^{-2}$ m
$S_t$	$6.61 \times 10^{-4}$ m <sup>2</sup>
$L_t$	0.204 m
$r_{cb}$	$5.0 \times 10^{-2}$ m
$L_{cb}$	0.125 m
$S_d$	$1.3 \times 10^{-3}$ m <sup>2</sup>

Table 1: Values of parameters of the system

$M_0$	$3.89 \times 10^{-4}$ kg
$R_0$	$2.63 \times 10^{-1}$ kg.s <sup>-1</sup>
$K_0$	$4.34 \times 10^3$ kg.s <sup>-2</sup>
$Bl/S_d$	$136.7$ kg.m <sup>-1</sup> .A <sup>-1</sup> .s <sup>-2</sup>

Table 2: Values of the modal parameters of the first mode of the loudspeaker when the current is set to zero.

253 and with a rigid termination. All results are confronted with those obtained  
254 from numerical integration of system equations. The modal parameters of  
255 the ER loudspeaker are detailed in Table 2. Desired parameters of the target  
256 dynamics are  $\mu_M$ ,  $\mu_R$ , and  $\mu_K$ , defined as follows:

$$\begin{cases} M_d = \mu_M M_0 \\ R_d = \mu_R R_0 \\ K_d = \mu_K K_0 \end{cases} \quad (21)$$

257

258 To illustrate the functionality of the programming at low excitation ampli-  
259 tudes, a variety of nonlinear behaviors are exploited and subjected to exper-  
260 imental testing.

261 To begin with, a cubic stiffness force is considered. It is a classic case  
262 that has been extensively studied across multiple domains [27, 26, 38]. An-  
263 other nonlinear behavior considered is a piece-wise linear restoring forcing  
264 function, which has also been widely studied in fields such as mechanics  
265 [29, 31, 32, 33, 49]. However, its implementation in acoustics remains un-  
266 precedented and is implemented here. Finally, a logarithmic potential func-  
267 tion is also studied, which leads to a non-polynomial restoring function. This  
268 force has shown the capability to induce targeted energy transfer as demon-  
269 strated by Gendelman in [50], and is experimentally implemented here for  
270 the first time. Each nonlinear behavior is compared to an optimized linear

271 resonator, whose behavior resembles a Tuned Mass Damper in mechanics.  
 272 However, it is worth noting that the linear oscillator is optimized, while the  
 273 nonlinear resonator is not. Optimization of such nonlinear behavior for noise  
 274 reduction would need an additional and complete study, and will be addressed  
 275 in future research. Notice that the presented behaviors can be replicated at  
 276 lower or higher excitation amplitudes in the limits of the sensors and actu-  
 277 ator capabilities. It is possible thanks to the ability to adjust  $\beta_{\text{NL}}$  which  
 278 controls the threshold for nonlinear behavior activation [43]. A demonstra-  
 279 tion is carried out showcasing a measure at higher excitation amplitudes with  
 the logarithmic potential restoring force in Fig. 13.

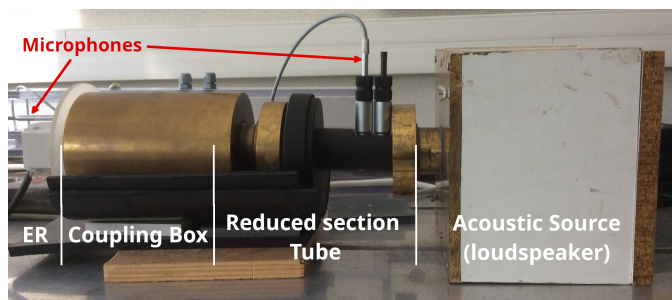


Figure 2: Experimental set-up

280

## 281 4.2. The cubic restoring forcing function

### 282 4.2.1. Design

283 As a first example, a cubic restoring forcing function is employed here to  
 284 demonstrate the functionality of the nonlinear programming. The specific  
 285 form of the nonlinear restoring force is given by:

$$F_{\text{NL}}(t, u_m) = K_d(u_m + \beta_{\text{NL}}u_m^3) \quad (22)$$

286 Where  $\beta_{\text{NL}}$  represents an adjustable nonlinear parameter. The value of the  
 287 parameter  $\beta_{\text{NL}}$  plays a crucial role in determining whether the stiffness force  
 288 composed of the linear part  $K_d u_m$  and of the nonlinear part  $K_d \beta_{\text{NL}} u_m^3$  ex-  
 289 hibits nonlinear phenomena across the considered range of amplitudes. The  
 290 order of magnitude of these two parts weighs in the activation of nonlinear  
 291 behavior. The sign of  $\beta_{\text{NL}}$  gives the sign of the force that determines whether  
 292 the programmed behavior results in either a hardening or softening behavior.  
 293 For considered amplitudes, the parameter is set to  $\beta_{\text{NL}} = 2 \times 10^{11} \text{ m}^{-2}$  for

294 designing hardening behavior, and  $\beta_{\text{NL}} = -3 \times 10^{11} \text{ m}^{-2}$  for designing soft-  
 295 ening behavior. The restoring forces for both cases are plotted in Fig. 3. In  
 296 the case of the softening behavior, the overall potential can become positive  
 297 at high oscillating amplitudes, leading to possible instability due to the in-  
 298 jection of energy into the system. This case is used for illustrative purposes.  
 299 Nevertheless, the logarithmic restoring force, later considered in this paper,  
 300 solves the instability issue.

301

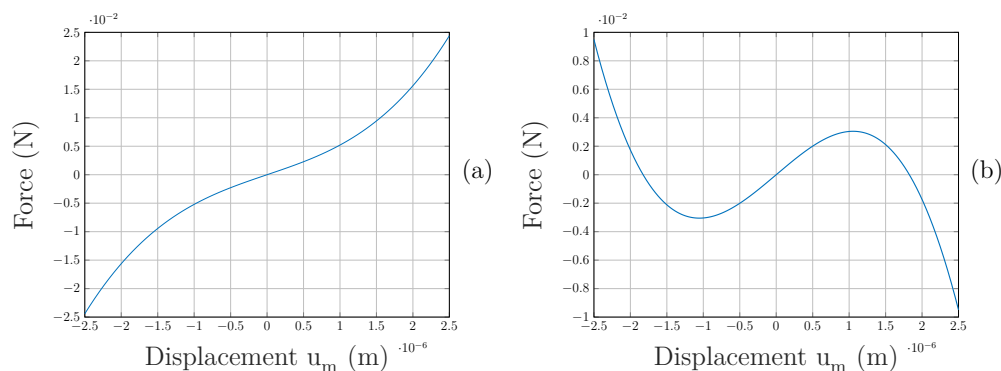


Figure 3: Nonlinear restoring force  $F_{\text{NL}} = K_d u_m + K_d \beta_{\text{NL}} u_m^3$  for (a) hardening behavior  $\beta_{\text{NL}} = 2 \times 10^{11} \text{ m}^{-2}$  and for (b) softening behavior  $\beta_{\text{NL}} = -3 \times 10^{11} \text{ m}^{-2}$ .

302 The choice of the damping parameter  $\mu_R$  is motivated by the need to  
 303 create bifurcations. Indeed, a condition on the existence of the bifurcations  
 304 emerges from the calculations of the centre manifold [51]. The given condi-  
 305 tion is expressed as a function of the linear stiffness and of the damping of  
 306 the resonator. Throughout the paper, the choice of  $\mu_R = 1/8$  ensures the  
 307 existence of the bifurcation.

308

309 The choice of the linear stiffness parameter  $\mu_K$  is motivated by the ne-  
 310 cessity to align the nonlinear resonance frequencies of the ER with the fre-  
 311 quencies of the acoustic mode of the tube. Indeed, the nonlinear softening  
 312 behavior involves large amplitude oscillations of the ER at lower frequencies,  
 313 and as a result, the ER is more efficient within the considered frequency  
 314 bandwidth. The choice of  $\mu_K$  determines the frequency of the linear reso-  
 315 nance of the ER. It allows the selection of the targeted linear mode frequency  
 316 of the resonator. Choosing an appropriate  $\mu_K$  is essential to tune the ER  
 317 nonlinear resonance with the acoustic tube resonance for improved sound

318 absorption. Let  $f_t$  be the targeted frequency of the linear resonance of the  
 319 ER, and  $f_0$  be the frequency of the intrinsic mode of the ER. For simplicity,  
 320 in this paper,  $\mu_M$  is set to 1. The parameter  $\mu_K$  can be found using the  
 321 following equation:

$$\frac{\mu_K}{\mu_M} = \left(\frac{f_t}{f_0}\right)^2 \quad (23)$$

322 The choice of this parameter depends on the intensity of the considered  
 323 nonlinearity. The parameter  $\mu_K$  has been chosen by empirical and numerical  
 324 means. The chosen parameters for the creation of hardening behavior are  
 325  $\mu_M = \mu_K = 1$ , and  $\mu_R = 1/8$ .

326 Knowing that  $f_0 = 531$  Hz, the frequency gap between the linear first mode  
 327 of the ER and the acoustic mode of the tube (596 Hz) is 65 Hz. In order to  
 328 produce the nonlinear softening behavior, the parameter  $\mu_K$  is chosen such  
 329 as the frequency gap between the mode of the ER and the acoustic mode  
 330 of the tube should be close to 65 Hz. Consequently,  $f_t = 660$  Hz, which  
 331 gives the parameters  $\mu_M = 1$ ,  $\mu_K = 1.5496$ , and  $\mu_R = 1/8$  for producing the  
 332 softening behavior.

#### 333 4.2.2. Results

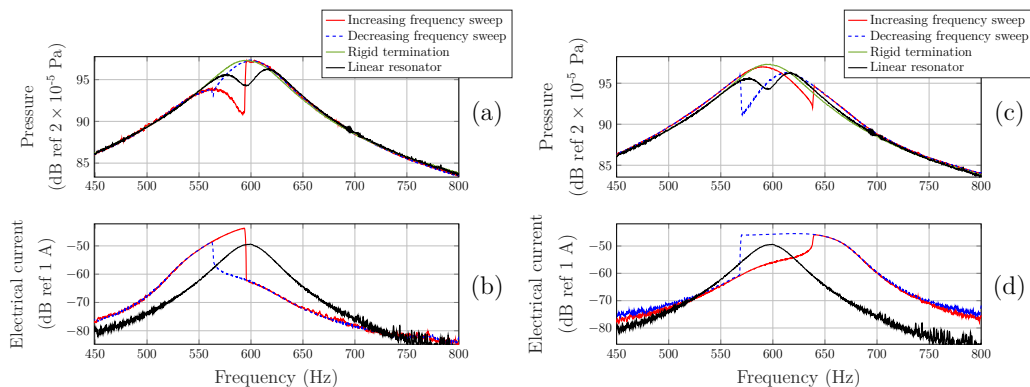


Figure 4: Experimental results for the cubic restoring force: variation of the pressure amplitude for the system with (a) hardening behavior, (c) softening behavior. Variation of the electrical current amplitude for the system with (b) hardening behavior, (d) softening behavior.

334 The variation of the pressure and electrical current amplitudes are plot-  
 335 ted in Fig. 4 for both increasing and decreasing frequency sweep, and for the  
 336 programmed hardening and softening behaviors. Variations of the electrical

337 current depicted in Fig. 4b clearly indicate the programming of a hard-  
 338 ening behavior. Indeed, in the case of increasing frequency excitation, the  
 339 measured pressure jumps (at 596 Hz) to a lower energy stable equilibrium  
 340 point, whereas, in the case of decreasing frequency excitation, the measured  
 341 pressure jumps (at 563 Hz) to a higher energy equilibrium point. For the  
 342 softening behavior, shown in Fig. 4d, the opposite phenomenon occurs. The  
 343 decreasing frequency measurement experiences a jump to lower energy equi-  
 344 librium points at lower frequencies (569 Hz), while the increasing frequency  
 345 measurement jumps to a higher energy equilibrium point (638 Hz). The pres-  
 346 sure plotted in Fig. 4a and 4c presents the coupled behavior of the acoustic  
 347 mode of the tube. The same phenomena occurs in both cases characterized  
 348 by a significant reduction of the sound level in the tube at the frequencies  
 349 corresponding to the nonlinear resonance. It is due to the activation of the  
 350 ER nonlinear behavior that starts to act with large response amplitudes, and  
 351 as a result it pumps the energy of the tube mode. However, from Fig. 4c one  
 352 can observe a slight increase in the sound level in the reduced section tube  
 353 from 600 Hz to 700 Hz. This issue can be solved by optimization means,  
 354 which is out of the scope of the present study.

The results obtained from the numerical integration of the model outlined in

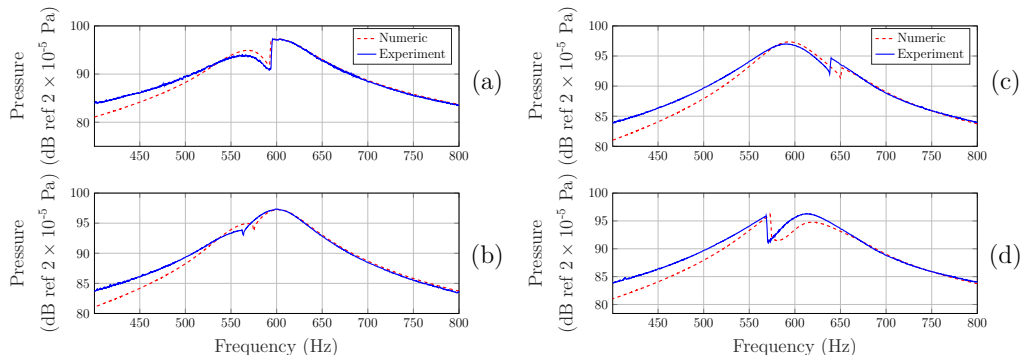


Figure 5: Experimental and corresponding numerical results obtained from direct numerical integration of Eq. (19): variation of the pressure amplitude for the hardening behavior with (a) increasing frequency sweep, (b) decreasing frequency sweep and for the softening behavior with (c) increasing frequency sweep, (d) decreasing frequency sweep.

355 Section 3 are compared to the experimental ones, and the outcomes are pre-  
 356 sented in Fig. 5. One can see that the model describes correctly the behavior  
 357 of the coupled experimental system despite a noticeable frequency shift of the  
 358 bifurcations illustrated in Fig 5. This shift can mainly be explained by model  
 359



360 uncertainties. Among model uncertainties, an important role is played by the  
 361  $S_d$  parameter, which can differ significantly from the membrane surface area.  
 362 In terms of modeling, it is worth highlighting that the parameter  $S_d$  imposes  
 363 significant challenges when it comes to its estimation and measurement [46].  
 364 This parameter affects the coupling between the acoustic mode and the ER,  
 365 and hence the bifurcation frequencies. The inaccurate approximation at low  
 366 frequencies is due to the presence of a coupling of an anti-resonance at lower  
 367 frequencies that is not modeled. It might also be responsible for the frequency  
 368 shift of the bifurcations. The influence of the antiresonance is discussed in  
 Appendix A.1. Nevertheless, despite the strong approximations employed

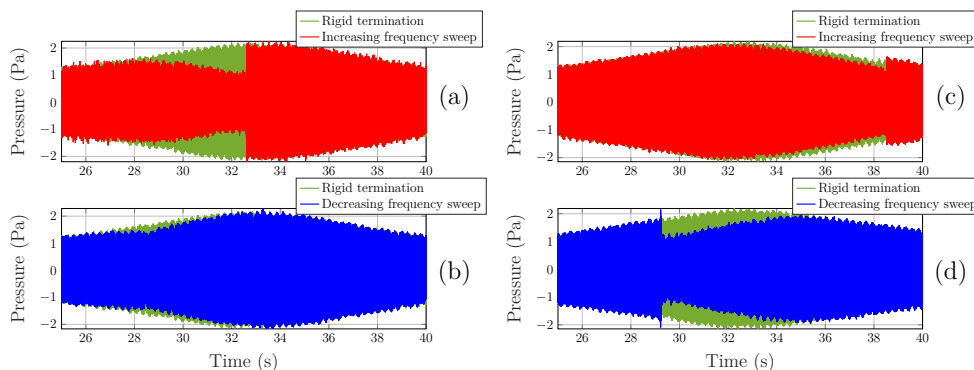


Figure 6: Experimental time histories for the cubic restoring force: pressure inside the reduced-section tube for the hardening behavior with (a) increasing frequency sweep (b) decreasing frequency sweep and for the softening behavior with (c) increasing frequency sweep (d) decreasing frequency sweep.

369 in the reduced numerical model, it well captures the trends of the measured  
 370 pressure. It confirms that the actual behavior of the ER is extremely close  
 371 to the desired behavior. As a result, the sound level starting point of the  
 372 model is different from the experiment, modifying the amplitudes and so the  
 373 frequencies of the bifurcations. Moreover, either larger frequency bandwidth  
 374 or better sound absorption are realized using these cubic restoring forces.  
 375 The tuning parameters should be optimized in future studies. Experimental  
 376 time histories of the pressure inside the reduced-section tube are plotted in  
 377 Fig. 6 for both hardening and softening behaviors. One can see that the  
 378 incident pressure level is divided by two for the increasing frequency sweep,  
 379 which is coherent with the reduction of more than 3 dB seen in the frequency  
 380 plots. The bifurcation and its frequency are clearly recognizable.  
 381

382 4.3. The piece-wise linear restoring forcing function

383 4.3.1. Design

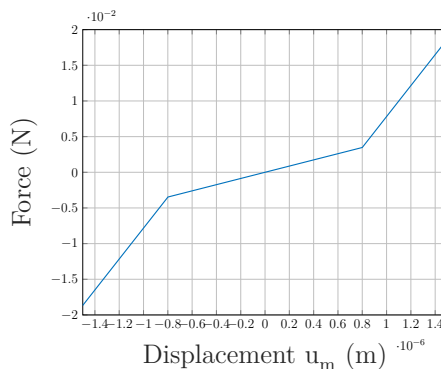


Figure 7: Piece-wise linear restoring forcing function.

384 The linear piece-wise potential behavior, which has not been previously  
 385 explored in the context of acoustics, is introduced in this study. This behavior  
 386 shows that the method presented here is not limited to smooth restoring  
 387 forcing functions and can be applied to synthesize non-smooth functions as  
 388 well. Here, the goal is to create a system with a hardening behavior similar  
 389 to a duffing-type behavior with 2 different slopes depending on a clearance  
 390 of  $2\delta$ . The associated restoring forcing function force plotted in Fig. 7 reads:

$$F_{NL}(t, u_m) = \begin{cases} K_2 u_m - (K_1 - K_2)\delta & \text{if } u_m < -\delta \\ K_1 u_m & \text{if } -\delta < u_m < \delta \\ K_2 u_m + (K_1 - K_2)\delta & \text{if } \delta < u_m \end{cases} \quad (24)$$

391 This force is not differentiable on its whole definition domain, as in  $u_m = \pm\delta$ ,  
 392 where an infinite number of derivatives exist. The force is plotted in Fig. 7  
 393 for the values presented in Table 3. The clearance parameter  $2\delta$  defines the  
 394 displacement of the membrane threshold at which nonlinear phenomena are  
 395 initiated.

396

397 The selected parameters for the experiment are  $\mu_M = 1$ ,  $\mu_K = 1.2598$ ,  
 398 and  $\mu_R = 1/8$ . The selection of  $\mu_K$  has been driven by the aim of aligning  
 399 the linear resonance frequency of the ER with the acoustic mode of the tube.

400

$\delta$	$8 \times 10^{-7}$ m
$K_1$	$K_d$
$K_2$	$5K_1$

Table 3: Values of the piece-wise linear experiment. (see Eq. (21))

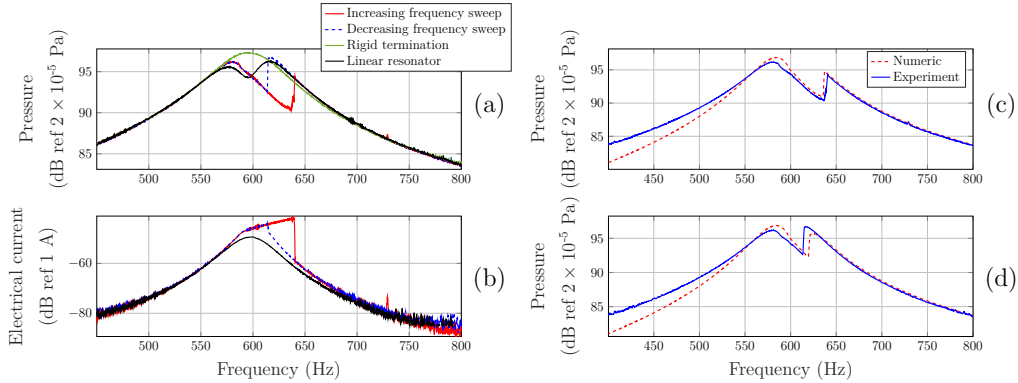


Figure 8: Experimental results for the piece-wise linear restoring force: variation of (a) the pressure amplitude (b) the electrical current. Experimental and corresponding numerical results obtained from direct numerical integration of Eq. (19): variation of the pressure amplitude with (a) Increasing frequency sweep, (b) Decreasing frequency sweep.

#### 4.3.2. Results

401 The variations of the pressure and electrical current amplitudes are plot-  
402 ted in Fig. 8a and Fig. 8b. One can observe in Fig. 8b that the created  
403 behavior looks like a hardening duffing-type oscillator which is depicted in  
404 Fig. 4b. Moreover, a significant reduction of the sound level can be seen and  
405 is not limited to the frequencies of the nonlinear resonance. It is explained  
406 by the linear nature of the oscillator which is tuned to the frequency of the  
407 first mode of the primary system. The linear oscillator pumps the energy  
408 before activating its nonlinear behavior. This leads to a shift of the nonlin-  
409 ear behavior to higher frequencies, and as a result to energy pumping shifted  
410 from the maximum peak amplitude of the primary system. The experimen-  
411 tal outcome is compared to results obtained from numerical integration of  
412 Eq. (19) for increasing frequency sweep in Fig. 8c and decreasing frequency  
413 sweep in Fig. 8d. Both results fit closely the measurements, confirming  
414 that the actual behavior follows the programmed piece-wise linear behavior.  
415 These results show that via accurate choice of parameters, the programming  
416 of the piece-wise linear restoring forcing function can lead to promising re-  
417

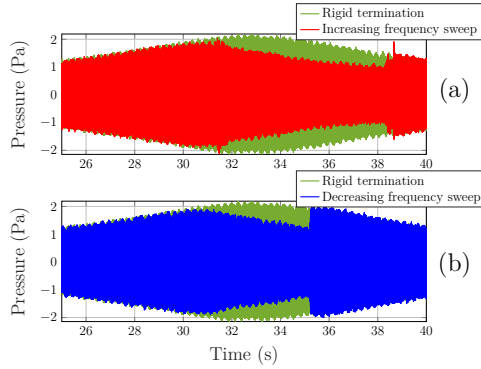


Figure 9: Experimental time histories for the piecewise linear restoring force: pressure inside the reduced-section tube for the hardening behavior with (a) increasing frequency sweep (b) decreasing frequency sweep.

418 sults. Indeed, this behavior shows the potential of such nonlinear ER, as  
 419 the performed sound absorption is greater than the linear ER for a larger  
 420 frequency bandwidth. Experimental time histories of the pressure inside the  
 421 reduced-section tube are plotted in Fig. 9.

#### 422 4.4. The logarithmic potential

##### 423 4.4.1. Design

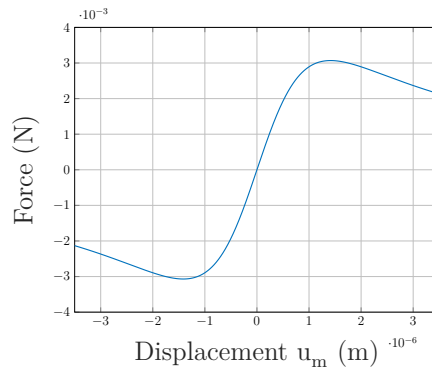


Figure 10: Logarithmic potential force  $F_{NL} = \frac{K_d u_m}{1 + \beta_{NL} u_m^2}$  with  $\beta_{NL} = 5 \times 10^{11} \text{ m}^{-2}$

424 To the best of our knowledge, the logarithmic potential force has not  
 425 been experimentally implemented yet. It would create softening behavior  
 426 that cannot become unstable [50]. Let us introduce the following potential

427 function:

$$V(u_m) = -\frac{1}{2}K_d \ln(1 + u_m^2) \quad (25)$$

428 A key point is that this force cannot cause unstable behaviors as the potential  
429 cannot become positive. It gives the equivalent nonlinear restoring force:

$$F_{\text{NL}}(u_m) = \frac{K_d u_m}{1 + u_m^2} \quad (26)$$

430 Nevertheless, this nonlinear force in its current state activates a nonlinear  
431 behavior at high excitation amplitudes. In order to enable the activation of a  
432 nonlinear behavior at lower excitation amplitudes, we introduce a coefficient  
433  $\beta_{\text{NL}}$  as follows:

$$F_{\text{NL}}(u_m) = \frac{K_d u_m}{1 + \beta_{\text{NL}} u_m^2} \quad (27)$$

434 The force is plotted Fig. 10 with  $\beta_{\text{NL}} = 5 \times 10^{11} \text{ m}^{-2}$ . The logarithmic  
435 potential restoring force can be briefly studied by developing the Taylor series  
436 of the force:

$$F_{\text{NL}}(u_m) = \frac{K_d u_m}{1 + \beta_{\text{NL}} u_m^2} = \sum_{n=0}^{\infty} (-1)^n K_d \beta_{\text{NL}}^n u_m^{2n+1} \quad (28)$$

437 The Taylor series of the force shows that logarithmic potential restoring  
438 force is equivalent to a cubic softening restoring force if the Taylor series is  
439 truncated to its two first terms. Therefore, the choice of a similar  $\mu_K$  than  
440 chosen for the cubic behavior should give similar results. Let us choose  $\mu_K =$   
441  $1.5496$  which is the same as the cubic softening restoring force. Nevertheless,  
442 the Taylor series highlights that the logarithmic potential restoring force is  
443 weaker than the cubic softening restoring force [50]. Both behaviors are  
444 compared in Fig. 11a for equal parameters  $|\beta_{\text{NL}}| = 3 \times 10^{11} \text{ m}^{-2}$  and  $\mu_K = 1$ .  
445 One can observe that the softening behavior is activated at higher amplitudes  
446 for equal parameters. Therefore, the logarithmic potential softening restoring  
447 force is weaker than the cubic softening behavior, as expected. To solve  
448 this issue, the value of  $\beta_{\text{NL}}$  should be raised, as depicted in Fig. 11b for  
449  $\mu_K = 1$ , and  $\beta_{\text{NL}} = 5 \times 10^{11}$  for the logarithmic potential restoring force and  
450  $\beta_{\text{NL}} = -3 \times 10^{11} \text{ m}^{-2}$  for the cubic softening restoring force. The chosen  
451 parameters for the experiment are  $\mu_M = 1$ ,  $\mu_K = 1.5496$ ,  $\mu_R = 1/8$  and  
452  $\beta_{\text{NL}} = 5 \times 10^{-11} \text{ m}^{-2}$ .

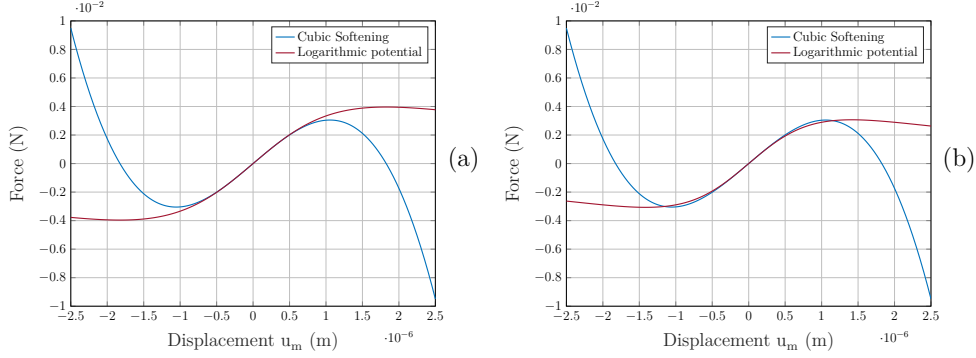


Figure 11: Restoring forces of the Logarithmic potential and Cubic Softening nonlinearities with (a)  $|\beta_{\text{NL}}| = 3 \times 10^{-11} \text{ m}^{-2}$  (b)  $\beta_{\text{NL}} = 5 \times 10^{11} \text{ m}^{-2}$  for the logarithmic potential restoring force and  $\beta_{\text{NL}} = -3 \times 10^{11} \text{ m}^{-2}$  for the cubic softening restoring force.

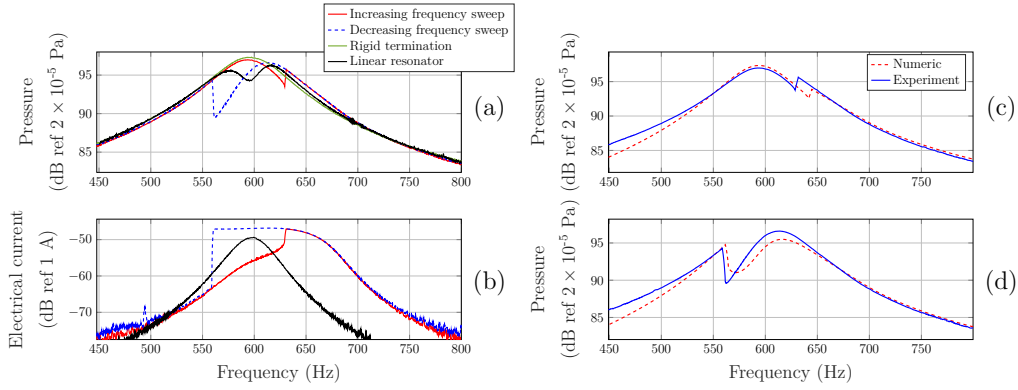


Figure 12: Experimental results for the logarithmic potential restoring force: variation of (a) the pressure amplitude, (b) the electrical current. Experimental and corresponding numerical results obtained from direct numerical integration of Eq. (19): variation of the pressure amplitude: (a) increasing frequency sweep (b) decreasing frequency sweep.

#### 4.4.2. Results

453 The variations of the pressure and electrical current amplitudes are plot-  
 454 ted in Figs. 12a and 12b for both increasing and decreasing frequency sweeps  
 455 of the excitation. One can observe that the ER presents a softening behav-  
 456 ior. Moreover, energy transfer from the acoustic mode of the tube to the  
 457 nonlinear ER occurs at the frequencies of the nonlinear resonance, following  
 458 the same phenomenon as the previous cases.  
 459

460

461 The numerical results obtained from the integration of Eq. (19) are com-

462 pared to experimental ones in Fig. 12c for the increasing frequency sweep,  
 463 and in Fig. 12d for the decreasing frequency sweep. The numerical resolution  
 464 of the model predicts well the behavior of the ER, despite the frequency  
 465 shift of the jump, as for the other target behaviors. Despite the fractional  
 466 nature of the restoring forcing function, it shows that the digital ER control  
 467 is efficient in implementing the programmed nonlinear behaviors. The loga-  
 468 rithmic potential restoring force shows great potential for sound absorption  
 469 as the frequency bandwidth of efficiency is broadened. It also features sound  
 470 absorption being equal to or greater than the linear oscillator.

As previously mentioned, the nonlinear behavior can be attained at higher

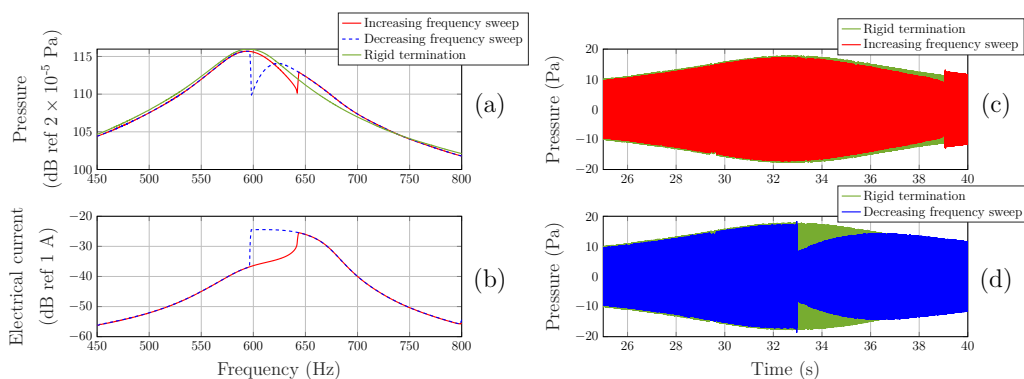


Figure 13: Experimental results for the logarithmic potential restoring force at high excitation amplitude with  $\mu_M = 1$ ,  $\mu_K = 1.5496$ ,  $\mu_R = 1/8$  and  $\beta_{NL} = 3 \times 10^9$ : variation of (a) the pressure amplitude, (b) the electrical current. Experimental time histories of the pressure inside the reduced-section tube with (c) increasing frequency sweep (d) decreasing frequency sweep.

471 or lower excitation amplitudes. It is due to the nonlinear parameter  $\beta_{NL}$   
 472 which determines the magnitude of the nonlinear term. The nonlinear term  
 473 activates the nonlinear response when its order of magnitude becomes similar  
 474 to the one of the linear term. Based on this analysis, let us choose the pa-  
 475 rameters  $\mu_M = 1$ ,  $\mu_K = 1.5496$ ,  $\mu_R = 1/8$  and  $\beta_{NL} = 3 \times 10^9$ . The variations  
 476 of the pressure and electrical current amplitudes are plotted in Figs. 13a  
 477 and 13b. One can observe that the nonlinear phenomena are reproduced at  
 478 higher excitation amplitudes. It highlights a softening behavior. The time  
 479 histories of the pressure inside the reduced-section tube are plotted for both  
 480 increasing and decreasing frequency sweeps in Figs 13c and 13d.  
 481

## 482 5. Conclusion

483 The article features the programming of diverse nonlinear behaviors at  
484 low excitation amplitudes and their potential application for noise reduction  
485 purposes. A Real-Time-Based method and algorithm are employed to syn-  
486 thesize both polynomial and non-polynomial nonlinear behaviors. In this  
487 contribution, we test such control algorithm to achieve cubic, piece-wise lin-  
488 ear, and logarithmic restoring forces, with comparison to numerical results.  
489 Piece-wise linear and logarithmic resonator dynamics are behaviors that are  
490 hardly achievable by passive means, even at high excitation levels, neverthe-  
491 less, they are highly interesting applications. Observe that the logarithmic  
492 dynamics, while being fractional, can be implemented using this method.  
493 Moreover, the nonlinear ER is here for the first time tested against the at-  
494 tenuation of the primary acoustic mode of a coupled tube. This paper proves  
495 that our nonlinear ER correctly performs the expected trends for various tar-  
496 get dynamics, in a coupled environment. Despite not being optimized, the  
497 nonlinear oscillators created in this study led to a significant noise reduction  
498 within the tube. To design the oscillators for efficient noise reduction pur-  
499 poses, an analytical study can be performed using the method of multiple  
500 scales [28, 52]. This perspective of study enables the programming of addi-  
501 tional nonlinear phenomena. For example, the programming of compound  
502 piece-wise nonlinearities for multi-energy levels of activations, as discussed  
503 in [34], can be performed to create fast energy triggering. Moreover, non-  
504 periodic responses such as modulate [53] should be investigated. This paper  
505 is the first step toward the conception of a distributed set of nonlinear sys-  
506 tems to create programmable nonlinear meta-materials for vibro-acoustical  
507 applications.

## 508 Acknowledgement

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## 514 Appendix A. Model of the experiment



515 *Appendix A.1. The influence of the non-modeled antiresonance*

516 This appendix aims at explaining the differences between the model and  
 517 the experiment. Taking into account the inevitable model uncertainties (in  
 518 both ER and tube cavity models) of the simplified numerical model, the shift  
 519 of the experimental bifurcation with respect to the numerical one is under-  
 520 standable. However, the low-frequency inaccuracy of the numerical results  
 521 is due to an antiresonance of the tube cavity at lower frequencies than the  
 522 acoustic mode of the tube, as depicted in Fig A.14. The model does not  
 523 take into account the antiresonance at 115 Hz, which could impact the cou-  
 524 pling with the ER, and consequently, it could contribute to the discrepancy  
 525 between measurements and simulations. The influence of the antiresonance  
 526 can not be ignored. Nevertheless, notice that slight pressure variations in  
 527 the experimental set-up may trigger the bifurcation earlier or later than sim-  
 528 ulated with numerical means. Moreover, considering both the complexity of  
 529 the actual system and the simplicity of the numerical model (reduced to one  
 530 mode), the agreement between measurements and simulations is reasonable.

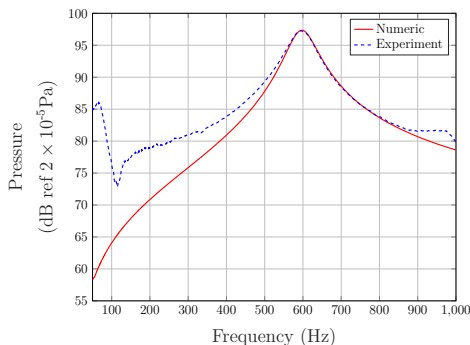


Figure A.14: Variation of the pressure amplitude inside the reduced-section tube with a rigid termination

531

532 *Appendix A.2. The influence of the damping parameter*

533 The form of the damping parameter of the primary system  $c_a$  has been  
 534 chosen arbitrarily to model multiple physical damping phenomena. As a re-  
 535 sult, a preliminary study regarding its influence on the results is needed. The  
 536 damping parameter is obtained using the rigid termination measurement.

537 The determination of its value has been done using three measurements at  
 538 different excitation amplitudes. The value of the damping parameter is:

$$c_a = 0.0386 \pm 1.1 \times 10^{-4} \text{ kg.s}^{-1} \quad (\text{A.1})$$

539 Let us consider the parameter  $c_{a,p}$  such as  $c_{a,p} \in [0.8c_a; 1.2c_a]$ . Its defi-  
 540 nition takes into account a variation of 20% of  $c_a$ , which is larger than the  
 541 calculated uncertainty. Let us plot the experimental data along with simu-  
 542 lations realized with different values of  $c_{a,p}$  in the case of a cubic hardening  
 543 restoring force with  $\mu_K = \mu_M = 1$ ,  $\mu_R = 1/8$  and  $\beta_{textNL} = 2 \times 10^{11} \text{ m}^{-2}$ .  
 544 The variation of the pressure amplitude inside the reduced-section tube is  
 545 presented for increasing frequency sweep in Fig. A.15a, and decreasing fre-  
 546 quency sweep in Fig. A.15b. One can observe that the bifurcation frequency  
 547 changes depending on the damping parameter of the acoustic mode of the  
 548 tube. The lower value of the parameter gives the lower frequency of bifur-  
 549 cation, and the higher value of the parameter gives the higher frequency of  
 bifurcation. Moreover, let us plot the quality of the prediction of the

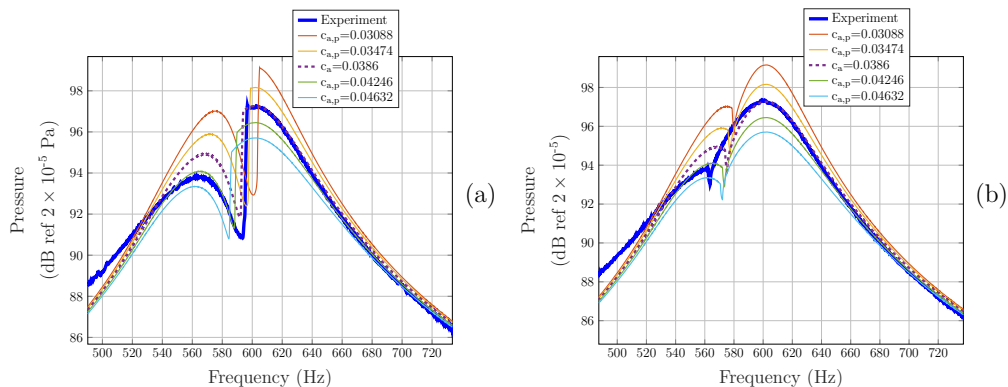


Figure A.15: Variation of the pressure amplitude inside the reduced-section tube as a function of frequency for multiple values of damping  $c_a$  with (a) increasing frequency sweep (b) decreasing frequency sweep.

550  
 551 simulations with each  $c_a$  using the classical indicator:

$$\text{Indicator} = \sum_i (x_{c_a}^i - y^i)^2 \quad (\text{A.2})$$

552 where  $x_{c_a}^i$  stands for the values of the pressure spectra predicted by simu-  
 553 lations for each value of the damping parameter  $c_a$ . The variable  $y^i$  denotes

554 the values of the pressure spectra from experimental data. One can see in  
 555 Fig. A.16 that the best value given by this indicator is the measured value  
 $c_a = 0.0386 \text{ kg}\cdot\text{s}^{-1}$ .

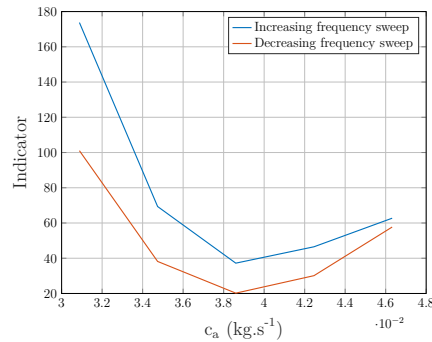


Figure A.16: Least mean square indicator of the difference of simulations done with multiple values of  $c_a$

556

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