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# Unsteady aerodynamic modeling of whirl flutter on a bending wing

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**Abstract.** With the new generation of propeller engines, aircrafts become more prone to whirl flutter, an aeroelastic instability possibly causing irreversible structural failures. The prediction of the motion induced aerodynamic force on the propeller is essential to catch such kind of instability. Classical theory uses a quasi-steady aerodynamic model to obtain analytically the dependence of the forces and moments on the movement. This paper proposes to use an unsteady aerodynamic theory and to take into account the inflow disturbance, to obtain a more realistic model. This leads to a complex dependency of the aerodynamic loads on the vibratory frequency, which is bypassed by the use of a rational matrix approximation (RMA) of the aerodynamic transfer function to obtain a linearized stability problem. Aeroelastic stability studies are performed on a classical two degrees of freedom structural model, and on another one including one more degree of freedom to represent the wing bending. Results demonstrate a strong dependence of the stability boundaries on the aerodynamic model used, and highlight the precision and convenience of the method involving a rational matrix approximation of the aerodynamic transfer function. Wing flexibility is of importance, as some unconventional stability boundaries are obtained in comparison to the classical 2-dof model.

## 1. Introduction

Propeller aircraft engines, and more generally engines with a large rotating part (turboprops, high bypass ratio turbojets, etc.) are widely used in the industry and are subject to numerous developments in order to reduce their fuel consumption. In this context, unconventional architectures such as open rotors appear, and it becomes necessary to consider the influence of these systems on the aircraft stability in flight. Indeed, the tendency to lengthen the blades but also the wings on which these propulsion devices are fixed increases their flexibility and promotes the whirl flutter risk [1]. This phenomenon of aeroelastic instability is characterized by a divergent spiral movement of the axis of rotation which can lead to the destruction of the engine. During pre-project design, it is highly convenient to assess the stability of various propulsive configuration without requiring heavy numerical calculations. Therefore, it is of first interest to develop analytic aeroelastic models that can simulate with a reasonable level of precision the whirl flutter instability.

A reference model of whirl flutter is based on the work of Houbolt & Reed [2] who proposed an analytic expression of the motion induced aerodynamic loads on a rigid blade propeller whose

axis orientation is subject to small perturbations. In their work, the aerodynamic forces and moments are obtained using the thin airfoil theory with a strip method. In the present study, this model is combined with the Blade Element Momentum Theory [3] to model the acceleration and rotation of the flow by the propeller. Another more complex aerodynamic model is used based on Theodorsen's theory [4] combined here again with the Blade Element Momentum Theory. Whereas the thin airfoil theory supposes that the flow instantly adapts to the movement of the blades (quasi-steady theory), Theodorsen's theory allows to take into account a phase lag by modeling the wake unsteady vorticity and added mass effects. Due to the complex dependency of the aerodynamic loads on the vibratory frequency, a rational matrix approximation of the aerodynamic transfer function is constructed following the methodology in [5] in order to perform linear stability analysis. This step, which was not necessary in the work of Houbolt & Reed, leads to the apparition of new state space variables modeling the dynamics of the flow. These two models of the aerodynamic loads on the propeller are then coupled with firstly, a classical 2-degrees of freedom structural model and secondly, with a more realistic one with 3 degrees of freedom to take into account a bending wing.

## 2. Structural models

Two structural models are used in this study to perform aeroelastic stability analysis. The first one, used by Houbolt & Reed [2], is presented Figure 1. It models a rotor with rigid blades rotating at speed  $\Omega$  around a rigid axis connected to the frame by a non-conservative elastic link with rotational stiffnesses ( $K_\theta$ ,  $K_\psi$ ) and their associated damping coefficients ( $C_\theta$ ,  $C_\psi$ ). The system has therefore two degrees of freedom, the pitch and yaw angle ( $\theta$ ,  $\psi$ ), modeling the movement of a nacelle on a rigid wing. The second one displayed Figure 2 is an improvement of the previous model, which has been thoroughly studied in the literature, with a vertical spring/damper system ( $K_z$ ,  $C_z$ ) allowing vertical motion. This system models a bending wing (vertical translation) with a flexibly mounted nacelle (pitch and yaw angle). The structure has three degrees of freedom which are the vertical translation of the pivot point ( $u_o^z$ ), the pitch and yaw angle ( $\theta$ ,  $\psi$ ).

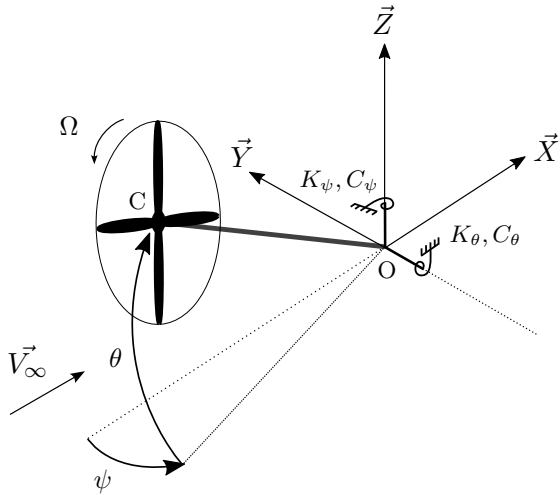


Figure 1: Classical structural model: rigid wing

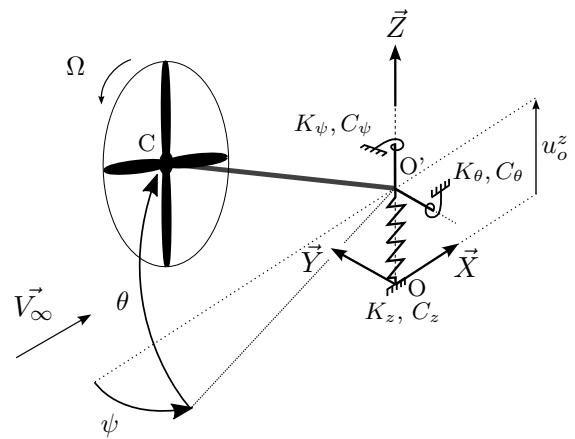


Figure 2: Improved structural model: bending wing

When there are three blades or more (to avoid periodicity of the rotor's inertia in the reference frame), the mass, damping and stiffness matrices are independent of time and are written for

the improved structural model as:

$$\mathbf{M}_{\text{struct}} = \begin{pmatrix} m & 0 & 0 \\ 0 & I_{yz} + mL_a^2 & 0 \\ 0 & 0 & I_{yz} + mL_a^2 \end{pmatrix}, \quad \mathbf{C}_{\text{struct}} = \begin{pmatrix} C_z & 0 & 0 \\ 0 & C_\theta & -I_x \Omega \\ 0 & I_x \Omega & C_\psi \end{pmatrix},$$

$$\mathbf{K}_{\text{struct}} = \begin{pmatrix} K_z & 0 & 0 \\ 0 & K_\theta & 0 \\ 0 & 0 & K_\psi \end{pmatrix}$$

where  $m$  is the propeller mass,  $I_x$  its inertia along its rotation axis,  $I_{yz}$  its transverse inertia and  $L_a$  the mast length. For the classical structural model, the matrices are obtained by deleting the first column and first line of the ones listed above.

Taking into account the aerodynamic forces and moments, the motion equation can be written for both structural models as:

$$\mathbf{M}_{\text{struct}} \ddot{\mathbf{u}} + \mathbf{C}_{\text{struct}} \dot{\mathbf{u}} + \mathbf{K}_{\text{struct}} \mathbf{u} = \mathbf{l}_{\text{aero}}(\ddot{\mathbf{u}}, \dot{\mathbf{u}}, \mathbf{u}) \quad (1)$$

where  $\mathbf{u}$  is the vector containing the degrees of freedom ( $\mathbf{u} = (\theta, \psi)^T$  for the classical structural model and  $\mathbf{u} = (u_o^z, \theta, \psi)^T$  for the improved one) and  $\mathbf{l}_{\text{aero}}$  is the external aerodynamic load applied on the propeller which depends on the dynamics of the system. The next two sections will focus on the process used to obtain a linearized expression of  $\mathbf{l}_{\text{aero}}$  regarding the degrees of freedom of the system.

### 3. Quasi-steady aerodynamic model

In order to perform linear stability analysis, a linearized expression of the aerodynamic loads  $\mathbf{l}_{\text{aero}}$  of (1) has to be obtained. Considering a propeller with rigid blades, it can be expressed in a generic way as a function of the propeller center displacement  $\mathbf{u}_c$  and orientation  $\theta_c$  (Figure 3).

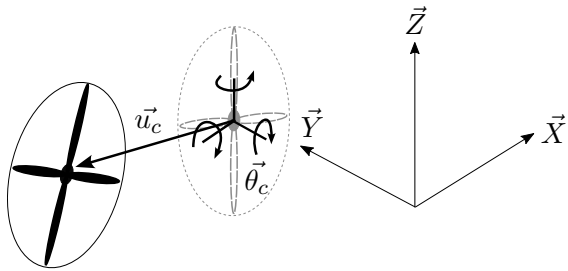


Figure 3: Motion of the propeller center

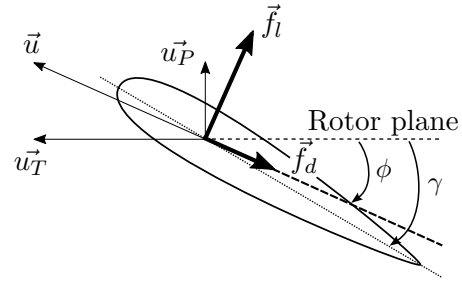


Figure 4: Lift & drag on blade section profile and associated velocities at the  $\frac{1}{4}$  chord point

Figure 4 presents a blade profile with the associated in-plane  $u_T$  and out-of-plane  $u_P$  relative velocity of the blade section to the airflow. With a thin airfoil approximation, the aerodynamic pitching moment about the quarter chord is equal to zero since the profile is considered symmetric. Therefore, the aerodynamic lift, drag and pitching moment are:

$$f_l = \frac{1}{2} \rho a c (u_T^2 + u_P^2) (\gamma - \phi) \quad (2)$$

$$f_d = \frac{1}{2} \rho c C_d (u_T^2 + u_P^2) \quad (3)$$

$$m_{1/4} = 0 \quad (4)$$

where  $\rho$  is the air density,  $a$  the static lift coefficient curve slope at the origin,  $c$  the chord of the profile,  $\gamma$  the pitch angle,  $\phi$  the inflow angle and  $C_d$  the drag coefficient. The coefficient  $a$  used here is equal to the ideal lift curve slope ( $2\pi$ ) multiplied by a correction factor taken from [2] to include flow compressibility and finite blade length effects. The thin airfoil theory is a quasi-steady theory since it supposes that the lift and drag force get instantly their stationary values, but it will leads here to unsteady aerodynamic forces and moments as the velocities  $u_T$  and  $u_P$  evolve as a function of the propeller center motion. In order to obtain a linear expression of the loads, the blade section apparent velocities are perturbed around their steady state values -  $u_T = u_T^0 + u'_T$  and  $u_P = u_P^0 + u'_P$  - and the expression of lift and drag are linearized for small disturbed velocities. Then, these disturbed velocities  $u'_T$  and  $u'_P$  are expressed as a function of  $\mathbf{u}_c$  and  $\boldsymbol{\theta}_c$  which gives the expression of the force on a blade profile in the rotating frame depending on the propeller center motion. By integrating along blade span and summing for all blades, the resulting aerodynamic force and moment on the propeller center in the reference frame can be written:

$$\mathbf{f}_c = \mathbf{M}_{f\theta} \boldsymbol{\theta}_c + \mathbf{M}_{fdu} \dot{\mathbf{u}}_c + \mathbf{M}_{fd\theta} \dot{\boldsymbol{\theta}}_c + \mathbf{f}_{stat} \quad (5)$$

$$\mathbf{m}_c = \mathbf{M}_{m\theta} \boldsymbol{\theta}_c + \mathbf{M}_{mdu} \dot{\mathbf{u}}_c + \mathbf{M}_{md\theta} \dot{\boldsymbol{\theta}}_c + \mathbf{m}_{stat} \quad (6)$$

There is no dependency on  $\mathbf{u}_c$  because the flow is supposed translationally invariant. The vectors  $\mathbf{f}_{stat}$  and  $\mathbf{m}_{stat}$  represent the thrust and the aerodynamic moment when the propeller is fixed in its initial position. The other matrices characterize the dependency of the aerodynamic force and moment on the propeller motion. All the geometric characteristics of the propeller (number of blades, evolution of chord and pitch angle with the radius etc.) are taken into account in these matrices. It is important to note that the steady state values  $u_T^0$  and  $u_P^0$  include modeling of the disturbed inflow which is created when the propeller generates thrust. Indeed, in this situation the flow is accelerated and put in rotation to satisfy the conservation of momentum. This phenomenon is modeled with the Blade Element Momentum Theory [3]: the current tube passing through the propeller is discretized into rings of different radius on which balance of linear and angular momentum are performed in order to obtain the induced velocities "seen" by the blade profiles. It leads to the apparition of two corrective function  $V_a(r)$  and  $V_t(r)$  in the expression of the steady state velocities  $u_P^0 = \Omega r - V_t(r)$  and  $u_T^0 = V_\infty + V_a(r)$ . The benefits of expressing the aerodynamic loads in terms of the propeller center degree of freedom is that it can be used with any structural model, as long as  $\mathbf{u}_c$  and  $\boldsymbol{\theta}_c$  can be linked to the degrees of freedom of the structure.

The reference modeling of Houbolt & Reed [2] considers a "windmilling" propeller with a blade pitch angle  $\gamma$  chosen so that no thrust is generated when the propeller is in its reference position (meaning  $\mathbf{f}_{stat} = \mathbf{0}$  and  $\mathbf{m}_{stat} = \mathbf{0}$ ). They develop a method for obtaining the aerodynamic loads on the propeller when no forward translation  $u_c^x$  and roll  $\theta_c^x$  of the propeller center are considered, as well as no modeling of the perturbation of the inflow since the propeller generates no steady thrust. When these assumptions are made, Houbolt & Reed's expressions of the aerodynamic loads on the propeller are recovered with the aerodynamic model presented herein.

To obtain the expression of  $\mathbf{l}_{aero}$  in (1), the aerodynamic loads must be expressed at point  $O$  and as a function of the vector containing the degrees of freedom:  $\mathbf{u} = (\theta, \psi)^T$  for the classical structural model or  $\mathbf{u} = (u_o^z, \theta, \psi)^T$  for the improved one. For propeller with three blades or more, time dependent terms compensate each other and the following constant coefficient equation is obtained:

$$\mathbf{M}_{struct} \ddot{\mathbf{u}} + \mathbf{C}_{struct} \dot{\mathbf{u}} + \mathbf{K}_{struct} \mathbf{u} = -\mathbf{C}_{aero} \dot{\mathbf{u}} - \mathbf{K}_{aero} \mathbf{u} + \mathbf{l}_{stat} \quad (7)$$

The matrices  $\mathbf{C}_{aero}$  and  $\mathbf{K}_{aero}$  highlight added damping and added stiffness effects induced by the aerodynamic forces and moments. We suppose that prestress effects due to the initial

aerodynamic loads are negligible ( $\mathbf{l}_{stat} = \mathbf{0}$ ) and the system is transformed in state-space format:

$$\mathbf{B}\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} \quad (8)$$

$$\text{with } \mathbf{y} = \begin{pmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \mathbf{K}_{\text{struct}} + \mathbf{K}_{\text{aero}} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{\text{struct}} + \mathbf{M}_{\text{aero}} \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{K}_{\text{struct}} + \mathbf{K}_{\text{aero}} \\ -\mathbf{K}_{\text{struct}} - \mathbf{K}_{\text{aero}} & -\mathbf{C}_{\text{struct}} - \mathbf{C}_{\text{aero}} \end{pmatrix}$$

The analysis of stability comes down to the study of the generalized eigenvalue system  $\mathbf{A}\hat{\mathbf{y}} = \lambda\mathbf{B}\hat{\mathbf{y}}$  with  $\mathbf{y} = \hat{\mathbf{y}}e^{\lambda t}$ . The system is stable if all the eigenvalues have a negative real part and becomes unstable if one of the eigenvalues has a strictly positive real part.

#### 4. Unsteady aerodynamic model

The previous quasi-steady aerodynamic model supposes that the flow, and thus the loads, instantly adapt to the movement of the blade profile. In reality, a phase shift can appear between the aerodynamic loads and the profile displacement due to wake memory effects. Theodorsen developed a theory for harmonic motion to tackle this phenomenon by introducing the lift deficiency function defined in the complex plane, as well as added mass effects [4]. It leads to the following expression of lift and pitching moment about the  $\frac{1}{4}$  chord point in the complex domain (physical lift and moment are obtained by taking the real part of their complex expressions):

$$\tilde{f}_l = \pi \rho c \frac{c}{4} \left( -\dot{u}_P \cos(\gamma) + \dot{u}_T \sin(\gamma) + \frac{c}{4} \dot{\varepsilon} \right) + \frac{1}{2} \rho a c C(k) \left[ (u_T^2 + u_P^2)(\gamma - \phi) + \sqrt{u_T^2 + u_P^2} \frac{c}{2} \varepsilon \right] \quad (9)$$

$$\tilde{m}_{1/4} = -\pi \rho c \left( \frac{c}{4} \right)^2 \left[ \cos(\gamma)(u_T \varepsilon - \dot{u}_P) + \sin(\gamma)(u_P \varepsilon + \dot{u}_T) + \frac{3c}{8} \dot{\varepsilon} \right] \quad (10)$$

where  $\varepsilon$  is the angular velocity of the blade section in the rotating frame and  $C(k)$  the lift deficiency function which depends on the reduced pulsation  $k = \frac{\omega c}{2\sqrt{u_P^0{}^2 + u_T^0{}^2}}$ ,  $\omega$  being the pulsation of  $u_T$ ,  $u_P$  and  $\varepsilon$ . Drag is still given by (3).

These expressions of the aerodynamic loads are much more complex than the one given by the quasi-steady model. The thin airfoil theory used in the previous section can be seen as an approximation of the Theodorsen theory when the non-circulatory terms are neglected (terms independent of  $C(k)$ ), when the angular velocity  $\varepsilon$  is taken equal to zero and when the lift deficiency function is equal to one. It is important to note that Theodorsen's expression of the lift deficiency function is used meaning effects of the returning wake (considered in Loewy's expression of the lift deficiency function [6]) are supposed negligible. As mentioned in [7], this hypothesis seems appropriate for the high advance ratio considered in the present study (defined by the expression  $\frac{V_\infty}{\Omega R}$  where  $R$  is the radius of the blade). The lift deficiency function causes an intricate dependency of the loads on the pulsation  $\omega$  of the blade profile apparent motion in the rotating frame. Due to this cumbersome dependence two methods are used to solve the stability problem.

#### 4.1. Non-linear solution of the stability problem: pk-method

The first way consists in solving non-linearly the stability problem with the classical pk-method [8]. By conducting the same approach as in the previous section (linearization for small disturbed velocities, integration along blade span and summation over blades), it is possible to obtain the state space equation. However, as the pulsation  $\omega$  in the lift deficiency function is linked to the vibratory pulsation, the state space matrices depend on the imaginary part of the sought eigenvalue (noted  $\lambda^I$ ) and a non-linear stability problem of the following form appears:

$$A(\lambda^I)\hat{\mathbf{y}} = \lambda B(\lambda^I)\hat{\mathbf{y}} \quad (11)$$

The pk-method solves this problem iteratively. An initial value of  $\lambda^I$  is chosen, usually the windless pulsation of the structural mode in consideration, which allows to find the eigenvalue  $\lambda$  of (11) via a linear analysis. The imaginary part of this newly found  $\lambda$  is then reinjected in (11) as  $\lambda^I$  and a new eigenvalue analysis is performed. This iterative procedure stops when no significant change is observed between the two successive eigenvalues imaginary parts. Although this method is well-known, it can be expensive in computing time to obtain a full stability graph (see section 5), especially when many degrees of freedom are considered, and requires a mode-tracking algorithm during the iteration procedure.

#### 4.2. Linearized solution of the stability problem: Rational Matrix Approximation

To avoid these drawbacks and deal with a linear stability analysis, another method consists in constructing a rational matrix approximation (RMA) of the transfer function between the displacement of the propeller center and the aerodynamic loads following the methodology presented by Morino et al. [5]. This approach is further detailed by Gennaretti & Greco for rotary-wings [9] and an application to whirl flutter is conducted in [7]. In the aforementioned paper, multiple whirl flutter stability analysis are conducted on the classical structural model (Figure 1) using Theodorsen's theory to obtain the aerodynamic loads on the propeller. The unsteady aerodynamic model developed herein is similar to the one developed in [7], but adds a modeling of the inflow with the Blade Element Momentum Theory. First, the velocities  $u_T$  and  $u_P$  are perturbed around their steady state values and the aerodynamic loads are linearized for small disturbed velocities. The perturbed velocity values are then expressed regarding the disturbed apparent velocity of the propeller center in the rotating frame  $\boldsymbol{\nu} = (\nu_1, \nu_2, \nu_3, \omega_1, \omega_2, \omega_3)^T$ . The expression of force and moment on a blade profile as a function of  $\boldsymbol{\nu}$  is obtained which leads, after integration along blade span, to the apparition of a 6x6 transfer matrix  $\mathbf{E}(s)$  relating the force and moment in the rotating frame exerted on a blade to the apparent velocity  $\boldsymbol{\nu}$ :

$$\tilde{\mathbf{l}}_{aero}^n = \begin{pmatrix} \tilde{\mathbf{f}}_n \\ \tilde{\mathbf{m}}_n \end{pmatrix} = \mathbf{E}(s)\tilde{\boldsymbol{\nu}} + \tilde{\mathbf{l}}_{stat}^n \quad (12)$$

where  $s$  is the complex pulsation. In the quasi-steady model the matrix  $\mathbf{E}(s)$  is constant, whereas it has here a complicated dependence on  $s$  because of the lift deficiency function. To obtain a linear stability problem, the matrix  $\mathbf{E}(s)$  is approximated under the following form:

$$\mathbf{E}(s) \approx \hat{\mathbf{E}}(s) = s\mathbf{A}_1 + \mathbf{A}_0 + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \quad (13)$$

where  $\mathbf{A}_1$  and  $\mathbf{A}_0$  are 6x6 matrices,  $\mathbf{C}$  is a 6x $N_a$  matrix,  $\mathbf{A}$  a  $N_a \times N_a$  matrix and  $\mathbf{B}$  a  $N_a \times 6$  matrix,  $N_a$  being the number of poles introduced in the approximation. Details of this process are given in Appendix. Casting (12) in time domain leads to the following finite state

formulation:

$$\mathbf{l}_{aero}^n = \mathbf{A}_1 \dot{\boldsymbol{\nu}} + \mathbf{A}_0 \boldsymbol{\nu} + \mathbf{C} \mathbf{r} + \mathbf{l}_{stat}^n \quad (14)$$

$$\dot{\mathbf{r}} = \mathbf{A} \mathbf{r} + \mathbf{B} \boldsymbol{\nu} \quad (15)$$

The vector  $\mathbf{r}$  contains the additional "aerodynamic" state variables included in the approximation. They can be seen as additional degrees of freedom characterizing the flow's own dynamics, expressed through the lift deficiency function. The rotating frame disturbed apparent velocity of the propeller center  $\boldsymbol{\nu}$  can be linked to the displacement of the propeller center in the reference frame through the following relationship:

$$\boldsymbol{\nu} = \mathbf{K}_1 \begin{pmatrix} \dot{\mathbf{u}}_c \\ \dot{\boldsymbol{\theta}}_c \end{pmatrix} + \mathbf{K}_0 \begin{pmatrix} \mathbf{u}_c \\ \boldsymbol{\theta}_c \end{pmatrix} \quad (16)$$

By means of equations (14)(15)(16) and after summation for all blades, the motion equation in the reference frame can be obtained:

$$\mathbf{M}_{struct} \ddot{\mathbf{u}} + \mathbf{C}_{struct} \dot{\mathbf{u}} + \mathbf{K}_{struct} \mathbf{u} = -\mathbf{M}_{aero} \ddot{\mathbf{u}} - \mathbf{C}_{aero} \dot{\mathbf{u}} - \mathbf{K}_{aero} \mathbf{u} + \mathbf{C}_t \mathbf{r}_t + \mathbf{l}_{stat} \quad (17)$$

$$\dot{\mathbf{r}}_t = \mathbf{A}_t \mathbf{r}_t + \mathbf{B}_t^1 \mathbf{u} + \mathbf{B}_t^2 \dot{\mathbf{u}} \quad (18)$$

In opposition to (7), the motion equation is now dependent on the aerodynamic state vector  $\mathbf{r}_t$  that gathers the fluid state variables of all the blades. Flow added mass effects are taken into account through the matrix  $\mathbf{M}_{aero}$ . It is important to note that the Theodorsen theory is designed only for harmonic motion. Therefore, the motion equation is not valid for damped or amplified motion but it is not a problem for the calculation of stability boundaries as vibrations are purely harmonic at the flutter point (neither amplified nor damped). The static aerodynamic loads are supposed negligible ( $\mathbf{l}_{stat} = \mathbf{0}$ ) and the system is transformed into state space format:

$$\begin{pmatrix} \ddot{\mathbf{u}} \\ \dot{\mathbf{u}} \\ \dot{\mathbf{r}}_t \end{pmatrix} = \begin{pmatrix} -\mathbf{M}^{-1} \mathbf{C} & -\mathbf{M}^{-1} \mathbf{K} & \mathbf{M}^{-1} \mathbf{C}_t \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_t^2 & \mathbf{B}_t^1 & \mathbf{A}_t \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}} \\ \mathbf{u} \\ \mathbf{r}_t \end{pmatrix} \quad (19)$$

with  $\mathbf{M} = \mathbf{M}_{struct} + \mathbf{M}_{aero}$ ,  $\mathbf{C} = \mathbf{C}_{struct} + \mathbf{C}_{aero}$  and  $\mathbf{K} = \mathbf{K}_{struct} + \mathbf{K}_{aero}$ .

The terms  $\mathbf{B}_t^2$ ,  $\mathbf{B}_t^1$  and  $\mathbf{C}_t$  introduce periodic terms in (19). Instead of the direct eigenvalue analysis of section 3, a Floquet analysis is performed here to estimate the stability of the aeroelastic system [10].

## 5. NUMERICAL RESULTS

Various numerical studies are conducted in order to assess the influence of the different structural and aerodynamic models. At first, the classical 2-dof structural model of Figure 1 is examined. In a second part, the behavior of the 3-dof structure of Figure 2 is analyzed.

### 5.1. 2-dof classical structural model: rigid wing

In this part, the model parameters are taken from [11]. The stability analysis with regards to the mounting stiffnesses is very common in the study of whirl flutter. Indeed, the flexibility of the connection between the engine pylon and the wing is a key parameter for the stability of the system. Figure 5 represents the stability boundary as a function of  $K_\theta$  and  $K_\psi$ , obtained with the quasi-steady aerodynamic model. The blue area corresponds to the whirl flutter instability (eigenvalue with a positive real part and a non-zero imaginary part), with a backward divergent precessional motion of the propeller center. The red area is associated to a static divergence instability (eigenvalue with a positive real part and an imaginary part equal to zero) i.e. a



divergent motion without any oscillations. It can be seen that decreasing the mounting stiffnesses favors the instability. The case where the two stiffnesses are equal is the most unfavorable since the stiffnesses required to obtain a stable system are the largest in this configuration. Due to the flow compressibility and finite blade length correction factors, the graph Figure 5 is slightly different than the one obtained by Mair et al [11] but if these coefficients are not taken into account, results become equivalent.

The model developed in this paper allows to take into account propellers that are not in windmilling configuration, i.e. that generate thrust. Figure 6 presents this effect by considering propellers with two different pitch angles. The first pitch angle, corresponding to the windmilling configuration (the one of Figure 5), is chosen so that for each blade section, the incident flow velocity is aligned with the chord of the profile (when the propeller is in its reference configuration meaning  $\theta_c = 0$ ,  $\dot{\theta}_c = 0$ ,  $\dot{\mathbf{i}}_c = 0$ ). Therefore the incidence angle  $\gamma - \phi$  is null and no static thrust is generated. The second pitch angle generates an incidence angle of  $4^\circ$  for all the blade profiles. Looking at the stability boundaries, it can be seen that thrust has a stabilizing effect since the unstable area is reduced when the incidence angle of the blades is increased. The differences between the two configurations concern mainly the whirl flutter instability as the static divergence boundaries coincides between the two cases.

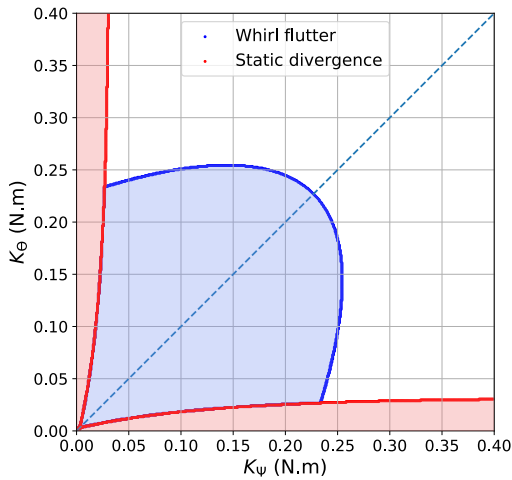


Figure 5: Stability boundaries (quasi-steady aerodynamic model)

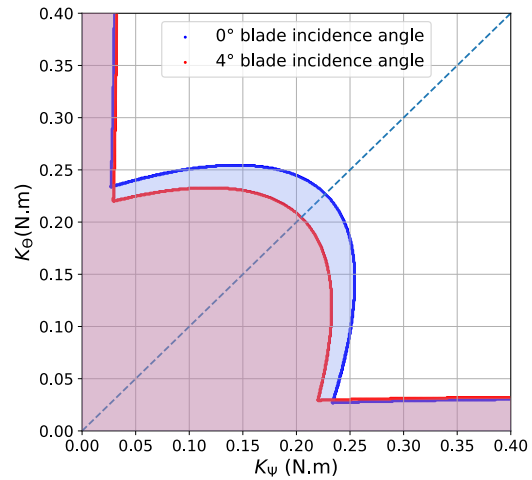


Figure 6: Influence of thrust on the stability boundaries (quasi-steady aerodynamic model)

The previous results were obtained with the quasi-steady aerodynamic model. As explained in Section 4, this theory does not model the wake unsteady vorticity as well as added mass effects. Theodorsen's theory, used here in the so called unsteady aerodynamic model, takes into account these effects and will now be employed to obtain the stability graphs. The main novelty is that it uses the lift deficiency function which provokes a phase lag between the movement of the system and the aerodynamic loads. To catch this non-trivial behavior, the rational matrices approximation of Section 4.2 is used. Before performing any aeroelastic analysis, the precision of this approximation must be assessed. Figure 7 presents the evolution of the imaginary part of the coefficient  $E_{33}$  of the exact and approximated transfer matrices (results are obtained with 14 poles in (13)). This coefficient is chosen because it is the one with the most error with respect to the exact transfer function. Nevertheless, the precision is largely satisfactory which validates the effectiveness of the RMA. The precision of the approximation on the other ones is better than it is here, and is not shown for the sake of conciseness.

The quality of the rational matrix approximation being validated, it is now possible to obtain the stability boundaries of the classical structural model with the unsteady aerodynamic model using the rational matrix approximation (RMA). Figure 8 compares this stability boundary with the one obtained with the quasi-steady aerodynamic model. The boundary obtained with the pk-method is also shown for validation. The stability frontier obtained with the RMA and the pk-method are really close which validates the methodology using the RMA. The advantage of this approach is that once the approximation of (13) is performed, the entire stability graph can be computed with linear eigenvalue analysis. With the pk-method, an iterative algorithm must be used for each operating point (pair of  $K_\theta$  and  $K_\psi$ ) and for each mode of the structure in order to obtain the modal damping. This can be costly in computational time, especially when convergence problems are encountered or for structures with more degrees of freedom (and therefore more modes). In comparison to the quasi-steady theory, the unsteady model leads to a reduced unstable area, and thus to more stable results. The static divergence boundaries coincides in the two models. The results given by the simplest model (quasi-steady aerodynamic model) proves here to be conservative. However, the sensibility of the results to the aerodynamic theory used is significant and clearly justify the need of precise aerodynamic model to properly study whirl flutter.

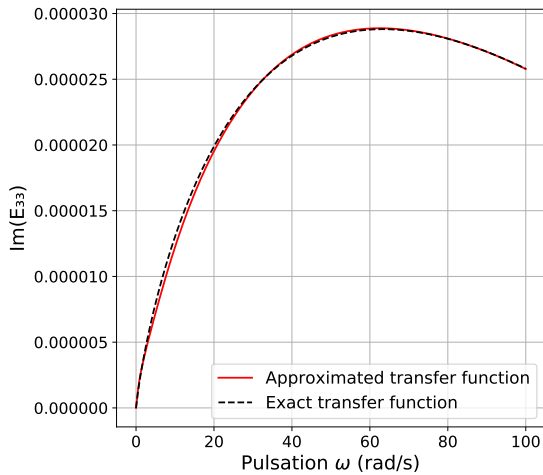


Figure 7: Imaginary part of the (3,3) element of the aerodynamic transfer matrix

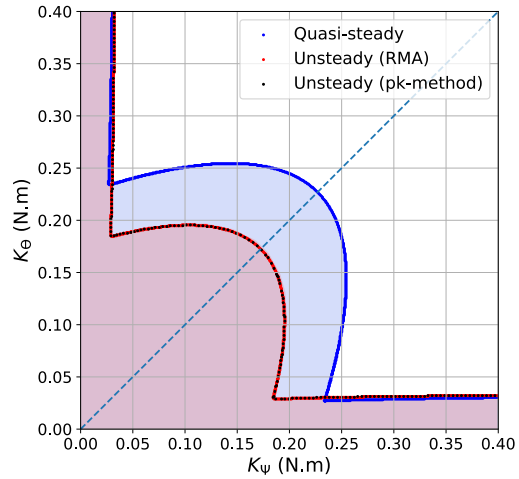


Figure 8: Stability boundaries: quasi-steady vs unsteady aerodynamic model

### 5.2. 3-dof structural model: pure bending wing

The improved structural model introduces a vertical spring/damper system representing the bending of the wing on which the propeller is fixed (Figure 2). As in the previous section, stability analysis regarding the mounting stiffnesses  $K_\psi$  and  $K_\theta$  are carried out. All the model parameters are still taken from [11], except for  $K_z$  and  $C_z$  which are chosen here arbitrarily since the authors used the model of Figure 1. Four values of vertical stiffnesses are retained -  $K_z = \{1, 2, 5, 1000\} \text{ N.m}^{-1}$  - as they allow to have an overview of the system behavior, and no vertical structural damping is considered ( $C_z = 0 \text{ N.m}^{-1}.\text{s}$ ). The resulting stability analysis obtained with the quasi-steady and unsteady aerodynamic model are presented in Figure 9.

Starting from a low  $K_z$  value (Figure 9a), it can be seen that unlike the classical structural model, the system presents here a whirl flutter instability area that has an asymmetric shape. It is due to the vertical spring that breaks the symmetry of the aerodynamic matrices in (7), (17)

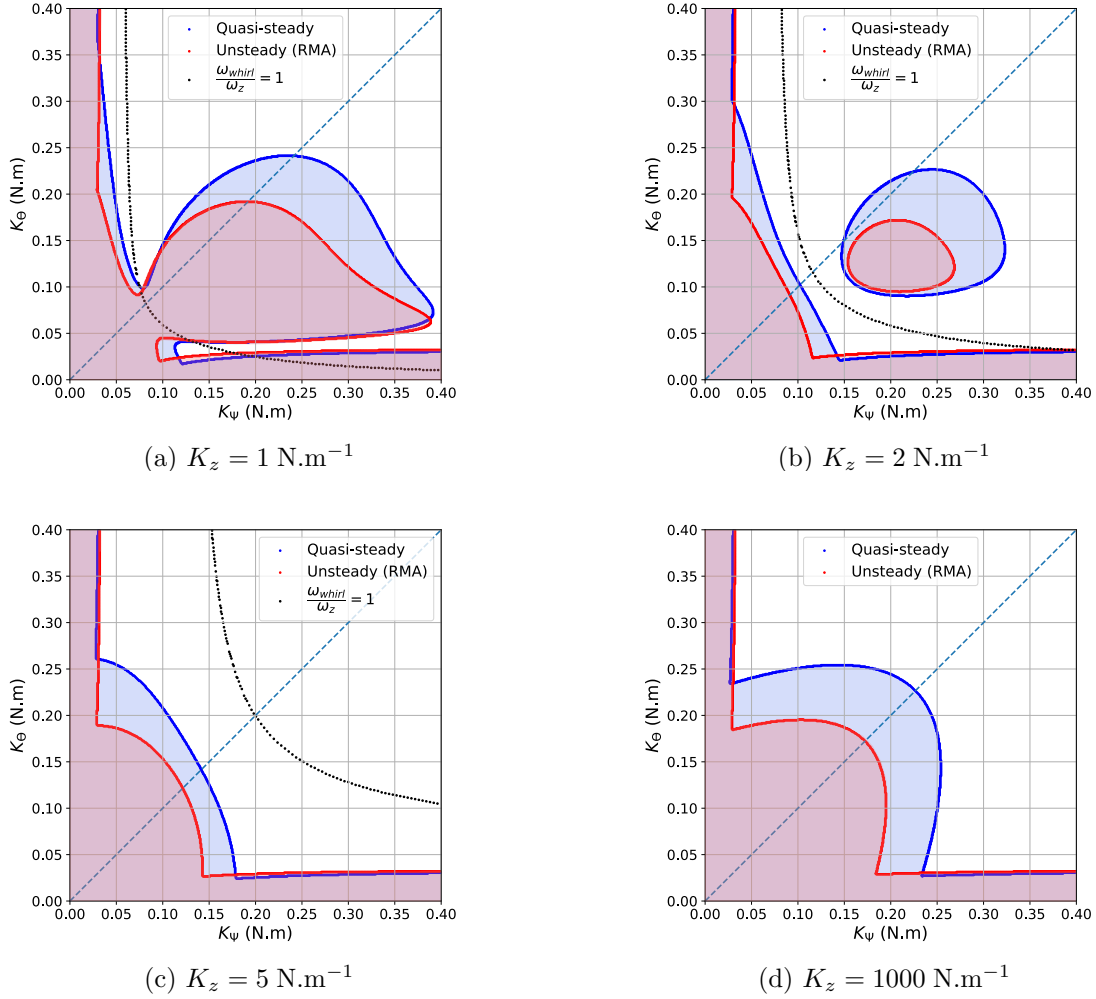


Figure 9: Stability frontiers for the improved structural model

and (18). If the same spring was added along the  $\vec{Y}$  axis, a symmetric graph would be obtained. As in Figure 5, two unstable areas correspond to a static divergence of the system when one of the mounting stiffnesses is too low. The effect of the spring along the vertical axis appears to be double. At first, it seems to be destabilizing for certain low values of  $K_\theta$  as some areas that were stable for the rigid wing structural model (Figure 8) are here unstable. However, some peninsular zones are characterized by a renewed stability. For the quasi-steady aerodynamic model, the dotted curve represents coalescence between the frequency of the aeroelastic whirl mode prone to instability and the frequency of the vertical spring structural mode (characterized by its pulsation  $\omega_z = \sqrt{\frac{K_z}{m}}$ ). It can be seen that this curve matches these stable peninsular areas. Therefore, the authors of [12] consider that the vertical spring acts locally as a tuned mass damper, concentrating and dissipating energy from the whirl mode leading to instability. When the vertical stiffness is increased to  $2 \text{ N.m}^{-1}$  (Figure 9b), the virtual tuned mass damper extracts enough energy to create a strip of stability isolating an "island" of instability. This unfamiliar compartment attests of the complex behavior that can be introduced when considering more flexible structures than the classical two-degrees-of-freedom model. Eventually, this island disappears when the stiffness is increased to  $5 \text{ N.m}^{-1}$  (Figure 9c), but the whirl flutter instability

area is still flattened by the tuned mass damper effect. When  $K_z$  is taken high enough (Figure 9d), the system has very few vertical displacement and acts as the classical structural model with a stability graph similar to the one of Figure 8. As for the classical structural model, results obtained with the unsteady aerodynamic model are found to be more stable than those given by the quasi-steady model for every choice of  $K_z$  values.

## 6. CONCLUSION

This paper proposed to study whirl flutter with two aerodynamic models. The first one, widely used in the literature, is based on the thin airfoil theory to model the motion induced aerodynamic loads on the propeller (quasi-steady model). The second one, more complex, use the Theodorsen theory which leads to an intricate dependency of the aerodynamic loads on the vibratory frequency (unsteady model). To avoid dealing with a computationally costly non-linear stability problem (solved with the pk-method), a rational matrix approximation (RMA) of the aerodynamic transfer matrix is constructed which introduces novel aerodynamic state variables that models the flow's own dynamics. Both models are coupled with the Blade Element Momentum Theory to take into account disturbed inflow effects. Stability analysis are performed on two structural models, with respectively two and three degrees of freedom. In comparison to the quasi-steady model, the unsteady aerodynamic model gives more stable results. For the unsteady aerodynamic model, results obtained with the RMA show consistency with the pk-method, and testify of the advantages of this technique which leads to a linear cost-effective stability problem. The 3-dof structural model presents a complex stability behavior. Even though only one degree of freedom is added, stability frontiers are much more sophisticated than the classical structural model and "islands" of instability are observed for certain stiffnesses combinations. It highlights the need to take into account the whole structure when whirl flutter stability analysis are performed.

Results presented in this paper dealt with a rigid blade propeller on structural models with few degrees of freedom. The use of full finite element models of the nacelle/wing system is already functional and will be investigated in future works. Blade flexibility will also be taken into account to see its influence on the stability boundaries. From an aerodynamic perspective, other unsteady models will be considered since the methodology used in this paper to build a rational matrix approximation of the aerodynamic transfer function is generic and can be applied with almost no limitation to any aerodynamic theory.

## Acknowledgments

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## Appendix

The approximation process of (13), presented in [5], is carried out in three steps.

Step 1: The exact transfer function  $\mathbf{E}(s)$  is evaluated numerically on a finite number of complex pulsations  $[i\omega_1, \dots, i\omega_{N_f}]$  and an order  $M$  is chosen to solve the linear least-square problem:

$$\mathbf{E}(s) \approx \hat{\mathbf{E}}(s) = s\mathbf{A}_1 + \mathbf{A}_0 + (\mathbf{I}s^M + \sum_{i=0}^{M-1} \mathbf{D}_i s^i)^{-1} (\sum_{i=0}^{M-1} \mathbf{R}_i s^i) \quad (\text{A.1})$$

This allow to find the matrices  $\mathbf{A}_1$ ,  $\mathbf{A}_0$ ,  $\mathbf{D}_i$  and  $\mathbf{R}_i$  that minimizes the error on the approximation of  $\mathbf{E}(s)$  with  $\hat{\mathbf{E}}(s)$ . (A.1) is then re-written under the following form:

$$\mathbf{E}(s) \approx \hat{\mathbf{E}}(s) = s\mathbf{A}_1 + \mathbf{A}_0 + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \quad (\text{A.2})$$

with  $\mathbf{C} = (\mathbf{I} \ 0 \dots 0)$ ,  $\mathbf{A} = \begin{pmatrix} -\mathbf{D}_{M-1} & \mathbf{I} & & \mathbf{0} \\ \vdots & & \ddots & \\ -\mathbf{D}_1 & \mathbf{0} & & \mathbf{I} \\ -\mathbf{D}_0 & \mathbf{0} & & \mathbf{0} \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} \mathbf{R}_{M-1} \\ \vdots \\ \mathbf{R}_0 \end{pmatrix}$ .

Step 2: The previous step leads to a good fit of the approximated transfer matrix  $\hat{\mathbf{E}}$  with the exact transfer matrix  $\mathbf{E}$ . However, the least-square process can lead to the apparition of spurious unstable poles in the transfer function. These poles are purely numerical and do not have physical meaning since the fluid-only system is always stable and the aerodynamic loads on the propeller cannot diverge for small velocities of the propeller hub. If taken into account, it would lead to fallacious stability analysis on the complete aeroelastic system. To avoid this problem, the unstable poles are truncated (eigenvalues of  $\mathbf{A}$  with a positive real part) and the matrices  $\mathbf{C}$ ,  $\mathbf{A}$  and  $\mathbf{B}$  are reduced according to this truncation.

Step 3: With the truncation of the unstable poles, the precision of the approximation is inevitably degraded. The fit can be further improved by solving successive linear least-square problems to find the new matrices  $\mathbf{A}_1$ ,  $\mathbf{A}_0$ ,  $\mathbf{C}$  and  $\mathbf{B}$  while keeping the matrix  $\mathbf{A}$  constant to guaranty a stable system. In the end, the approximation of (13) is obtained, which contains only stable poles and has an appropriate level of fidelity with the exact transfer function.

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