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ON INTERACTIVITY IN FUZZY CONTROLLERS

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Abstract

We sketch the usual construction of a fuzzy logic controller out of a family of control rules and consider the problem if such a controller then works in accordance with these rules. This is not always the case. And we solve the problem of getting a necessary and sufficient condition for the controller rules to generate a controller which works in accordance with these rules, and we find additionally that every controller can be generated by such a special set of control rules.

1. Introduction

We use here fuzzy sets theory for the description of fuzzy controllers.

First some remarks on notation. The generalized membership value for point x and fuzzy set A will be $A(x) \in [0,1]$, i.e. we use $A(x)$ instead of $\mu_A(x)$. The cartesian product $A \times B$ of fuzzy sets A, B is given by $A \times B(\langle x, y \rangle) = A(x) \wedge B(y)$ with \wedge for min. The height of fuzzy set A is $\text{hgt}(A) = \sup_x A(x)$, the union $\bigcup_i A_i$ of a family of fuzzy sets $A_i, i \in I$ is given by $(\bigcup_i A_i)(x) = \sup_{i \in I} A_i(x)$, and for a fuzzy set A and a fuzzy relation R we consider sup-min composition $A \circ R$ with $A \circ R(x) = \sup_y (A(y) \wedge R(y, x))$.

2. Construction of fuzzy controller

For the description and execution of control policies in case of complex processes or unsharp data by using fuzzysets the construction of fuzzy logic controller was introduced by Mamdani /3,6/ and considered since in a lot of papers cf. e.g. /4,5, 7,2/ .

Usually, the main way to construct such a fuzzy controller is by first considering a set of control rules, written down using linguistic values of some suitable variables and, secondly, combining them into a fuzzy algorithm that constitutes the fuzzy controller. The control rules connect in the simplest case values of an input variable α with values of an output variable β like

if value(α) = X_1 , then value(β) = U_1 ,
 if value(α) = X_2 , then value(β) = U_2 ,

shorter written in most cases as

if X_1 then U_1 ,
 if X_2 then U_2 ,

or symbolically

$$X_1 \Rightarrow U_1 , \quad X_2 \Rightarrow U_2 , \quad \dots \dots \dots \quad (1)$$

Here, the X_i, U_i are fuzzy sets on respective universes of discourse $\underline{X}, \underline{U}$, indicated through the use of linguistic terms.

The whole fuzzy controller R is given by a family of such rules $X_i \Rightarrow U_i$, also often called "fuzzy implications". Mathematically, each one of these rules $X_i \Rightarrow U_i$ is modelled by a fuzzy binary relation, i.e. a fuzzy mapping, as

$$X_i \Rightarrow U_i = X_i \times U_i \quad (2)$$

on the cartesian product $\underline{X} \times \underline{U}$ of corresponding universes of discourse. And the whole controller R is modelled by the set theoretic sum of all these rules as

$$R = \bigcup_i (X_i \times U_i) . \quad (3)$$

Finally, the connection between the values X, U of input and output variables is described by the relation equation

$$U = X \circ R, \quad (4)$$

i.e. with - in set theoretic terms - U as the full picture of X under the fuzzy mapping R.

3. Interactivity of rules for fuzzy controllers

Of course, having constructed the fuzzy controller R by (3) out of the rules (1) one is interested that it works in accordance with these rules, i.e. one likes to have

$$U_i = X_i \circ R \quad \text{for all } i. \quad (5)$$

Unfortunately, this is not always the case /1/. Moreover, often fuzzy controllers are part of decision algorithms and hence from $U_j \neq X_j \circ R$ it may happen that the decision coming from U_j is different of that from $X_j \circ R$. This means, in sense of control theory, that the decision-making rules which gave rise to (1) are interacting.

To have a closer look at this problem we first will give two definitions.

Definition 1. A family $(X_t \Rightarrow U_t)_{t \in I}$, I any index set, of rules for a fuzzy controller R is a generating family of R or $X_t \Rightarrow U_t, t \in I$ are generating rules for R iff the equality $R = \bigcup_{t \in I} (X_t \Rightarrow U_t)$ holds true.

Definition 2. A generating family $(X_t \Rightarrow U_t)_{t \in I}$ of a fuzzy controller R is called interactive iff there exist an index $j \in I$ such that $U_j \neq X_j \circ R$. Otherwise the rules $X_t \Rightarrow U_t, t \in I$ are called non-interactive.

4. Results on non-interactivity

Because for all fuzzy sets A, B it holds true

$$A = B \quad \text{iff} \quad A \subseteq B \text{ and } B \subseteq A, \quad (6)$$

the problem of non-interactivity, i.e. the problem if (5) holds true, splits into the two conditions

$$U_t \subseteq X_t \circ R \quad \text{for all } t \in I, \quad (7)$$

$$U_t \supseteq X_t \circ R \quad \text{for all } t \in I, \quad (8)$$

where we assume for the rest of this note that $X_t \Rightarrow U_t, t \in I$ are generating rules of the fuzzy controller R .

Both conditions (7), (8) will be treated separately. And always we assume $t, k \in I$.

The results will be given without proofs here.

Theorem 1.

$$U_t \subseteq X_t \circ R \quad \text{iff} \quad \text{hgt}(U_t) \leq \text{hgt}(X_t) .$$

In the applicational papers always the generating families have been finite. This result, yet, holds true for any generating family and any universes of discourse $\underline{X}, \underline{U}$.

Theorem 2.

Put $(U_t < U_k) =_{\text{def}} \{u \in \underline{U} \mid U_t(u) < U_k(u)\}$. Then we have

$$U_t \supseteq X_t \circ R \quad \text{iff} \quad \text{for all } k \in I:$$

$$\text{hgt}(X_t \cap X_k) \leq \inf_{u \in \underline{U}} \{U_t(u) \mid u \in (U_t < U_k)\}.$$

Hence, for both cases (7) and (8) we have a necessary and sufficient condition, i.e., we have a necessary and sufficient condition for the non-interactivity of any given generating family of a fuzzy controller.

Corollary. A sufficient condition for the non-interactivity of the generating rules $X_t \Rightarrow U_t, t \in I$ of a fuzzy controller R is that the fuzzy sets $X_t, t \in I$ are pairwise disjoint normal fuzzy sets.

Finally, we ask for the existence of non-interactive generating families.

Theorem 3.

Every fuzzy controller has a non-interactive generating family of control rules.

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