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PROBABILISTIC vs. FUZZY PRODUCTION RULES
IN EXPERT SYSTEMS

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0. Introduction

The use of grades or degrees of strength of implication, leading to grades or degrees of certainty attached to the conclusions, constitute what we will call systems of graded production rules.

(MYCIN and PROSPECTOR are among the Expert Systems which use such rules). Graded rules represent a major advance, for two reasons:

- 1) immediate improvement of the ES using them,
- 2) the opening up of a new (to ES) area of theory, which will (with degree of certainty 1) lead to valuable new improvements in Expert Systems.

This area of theory is Multiple-Valued Logic or Fuzzy and Probabilistic Logic, and is not so new that it does not already have very many results of use to Expert Systems Designers. We touch on just a few of these.

The best known (to the traditionally educated, the only known) multiple-valued logic is the probabilistic one, with what may be called "Bayesian implication." So designers of graded systems have tried to base themselves on this, twisting it to try to fit cases for which it is not suited.

But Bayesian implication is just one among many implications, and the others often fit particular situations better.

1. Classical and "Plausible" Production Rules.

Citing Friedman [1], who builds on Polyá [2], we may distinguish the four modes of inference shown in Figure 1. The first and the last

Figure 1

CLASSICAL AND PLAUSIBLE PRODUCTION RULES

Ponens	$\frac{a \rightarrow b \quad a}{b}$	Confirmation	$\frac{a \rightarrow b \quad b}{\text{"a"}}$
Denial	$\frac{a \rightarrow b \quad \neg a}{\neg b}$	Tollens	$\frac{a \rightarrow b \quad \neg b}{\neg a}$

are classical: Modus ponens derives from "if a then b" and "a", the conclusion "b"; modus tollens derives from "if a then b" and "not-b", the conclusion "not-a". There is no doubt about the validity of these rules. On the other hand, the second and third rules are illegitimate classically, although we all use them all the time on an informal basis. Denial derives from "if a then b" and "not-a", the conclusion "not-b", while confirmation derives from "if a then b" and "b", the conclusion "a".

In defence, for example, of confirmation, consider (like Karl Popper) the case in which a is a scientific theory and b a set of consequences of it. The verification of b surely strengthens belief in a. The use of multiple-valued logics (MVL) can help to put such arguments on a firm basis.

Now in MVL, Modus ponens becomes

$$\frac{a \rightarrow b \quad a}{b} \quad \Rightarrow \quad \frac{a \rightarrow b = \rho \quad a = \alpha}{b := \beta}$$

E.g. Bayesian

$$\begin{array}{rcl}
 a \rightarrow b & = & \rho \\
 a & = & 1 \\
 \hline
 b & := & \rho \cdot 1
 \end{array}
 \quad \Rightarrow \quad
 \begin{array}{rcl}
 & = & \text{pr}(B/A) \\
 & = & \text{pr}(A) \\
 \hline
 & = & \text{pr}(B) \text{ "posterior"} \\
 & = & \text{pr}(A \cap B).
 \end{array}$$

This is really "retentive modus ponens"

$$\begin{array}{r}
 a \rightarrow b \\
 \\
 a \\
 \hline
 a \wedge b
 \end{array}$$

For more general values of a,

$$\begin{array}{rcl}
 \text{Bayesian} \\
 a \rightarrow b & = & \rho \\
 a & = & \alpha \\
 \hline
 b & := & \beta \geq \rho \cdot \alpha
 \end{array}
 \quad \Rightarrow \quad
 \begin{array}{rcl}
 & = & \text{pr}(B/A) \\
 & = & \text{pr}(A) \\
 \hline
 & = & \text{pr}(B) \text{ "posterior"} \\
 & \geq & \text{pr}(B/A) \cdot \text{pr}(A)
 \end{array}$$

And in general, in MVL, the four modes become as shown in Figure 2.

And now, for certain MVL "ply" operators, \rightarrow , the plausible modes of denial and confirmation can be legitimated.

Figure 2

CLASSICAL AND PLAUSIBLE PRODUCTION

RULES IN MULTIPLE-VALUED

LOGIC

Ponens

$$a \rightarrow b = \rho$$

$$a = \alpha$$

$$b := \beta$$

Denial

$$a \rightarrow b = \rho$$

$$\neg a = \bar{\alpha}$$

$$\neg b := \bar{\beta}$$

Confirmation

$$a \rightarrow b = \rho$$

$$b = \beta$$

$$a := \alpha$$

Tollens

$$a \rightarrow b = \rho$$

$$\neg b = \bar{\beta}$$

$$\neg a := \bar{\alpha}$$

These can now be
legitimated

Thus it is important to take a look at some of the other MVL or "Fuzzy" ply operators (implication operators), beside the Bayesian one. So wide is the variety of them [3], [4], that semantics for them are badly needed. Fortunately, the checklist Paradigm [5] provides such semantics for many of the operators. We will illustrate this for a central group of the most important ones.

2. The Checklist Paradigm.

Imagine a list of n items, to be checked "yes" or "no", and two situations in which the list is used. The paradigm covers many kinds of possible situations; for present purposes we may take the two situations to be deciding on the degree of assent to two propositions, A and B respectively. Concretely, the list might be a list of symptoms, the situations this attribution to two different patients, and the propositions "The first (second) patient suffers from the ailment characterized by these symptoms." Use of a checklist leads to the configuration shown in Figure 3. We are interested in the degree to

Figure 3

CHECKLIST PARADIGM

OF THE ASSIGNMENT OF FUZZY VALUES

	No for B	Yes for B	Row Total	
No for A	α_{00}	α_{01}	r_0	Define $a = \frac{r_1}{n}$
Yes for A	α_{10}	α_{11}	r_1	
Column total	c_0	c_1	n	

Define

$$b = \frac{c_1}{n}$$

which saying yes to A implies saying yes to B. If, as in the figure, we call a the proportion of yes-to-A answers and b the proportion of yes-to-B answers, we seek a measure of the degree to which a implies

Look now at classical material implication, as shown in Figure 4,

Figure 4

CLASSICAL MATERIAL IMPLICATION

 $a \rightarrow b$

		b	
		0	1
a			
0		1	1
1		0	1

Top line "by default"

Bottom line "by performance".

which shows how the composite statement $a \rightarrow b$ is judged false (0) or true (1) in terms of the falsity or truth of the components a and b . On the top line, where a is false, "if a then b " is classically judged always true "by default". On the bottom line, where a is true, "if a then b " is judged false or true according to the value of b , that is, "by performance".

Comparison of Figures 3 and 4 immediately suggests two implication measures $m(a \rightarrow b)$. The Classical measure is

$$m_1 = \frac{\alpha_{00} + \alpha_{01} + \alpha_{11}}{n} = 1 - \frac{\alpha_{10}}{n}$$

The performance measure is

$$m_2 = \frac{\alpha_{11}}{r_1} = 1 - \frac{\alpha_{10}}{r_1}$$

Other suitable measures also exist (as discussed in [5]), but these two will suffice here.

Depending on how the α shift around in the table of Figure 3 without altering the row and column totals, the classical measure turns out to have the following attainable upper and lower bounds:

$$\max(1-a, b) \leq m_1 \leq \min(1, 1-a+b),$$

that is,

$$a \rightarrow_6 b \leq m_1 \leq a \rightarrow_5 b,$$

where \rightarrow_6 is Kleene-Dienes implication and \rightarrow_5 is Lukasiewicz implication.

In the expected configuration of the table,

$$E(m_1) = 1-a + ab,$$

a new implication operator which has been called Kleene-Dienes-Lukasiewicz and numbered 5.5.

For the performance measure, on the other hand, in the expected configuration

$$E(m_2) = b,$$

while the attainable upper and lower bounds are given by

$$\max(0, \frac{a+b-1}{a}) \leq m_2 \leq \min(1, \frac{b}{a});$$

that upper bound is $a \rightarrow_4 b$, where \rightarrow_4 is the Gaines 43 operator. Other suitable measures connect with other implication operators in a similar fashion.

3. Legitimization of the "Plausible" Modes.

For most of the fuzzy "ply" operators, justification is offered for the previously "illegitimate" modes. This is because, away from the all-or-nothing corners where a and b are confined to the values zero and one, it is factually true of these operators that the plausible schemata lead to solutions.

For example

Confirmation:

$$a \rightarrow b = \rho$$

$$b = \beta$$

actually leads to

$$a := \alpha$$

sometimes sharply, or anyway

$$a \in [\alpha_1, \alpha_2] .$$

a range or interval of values.

Most exciting, with many of the operators, certain combinations of values of $a \rightarrow b$ and a are out of bounds, that is, could not have been produced by an a with that value and any b whatsoever. This offers the valuable opportunity to modify the production rules during operation according to a scheme like that in Figure 5.

Figure 5

Modus Ponens

MODIFICATION OF THE PRODUCTION RULES
DURING OPERATION

$$\begin{array}{l}
 a \rightarrow b = \rho \\
 a = \alpha
 \end{array}
 \left. \vphantom{\begin{array}{l} a \rightarrow b = \rho \\ a = \alpha \end{array}} \right\} \times$$

$$\begin{array}{l}
 a \rightarrow b := \rho' \\
 a := \alpha'
 \end{array}$$

4. Conclusions.

Having worked for some time with a panoply of fuzzy implication operators, we have become ever more convinced that there is no unique a priori best choice among them. One or more should be chosen according to the kind of problem actually in hand. In particular, the Bayesian operator has no special claims beyond familiarity. Graded systems are here to stay, and they will benefit from the use of a much more extensive kit of tools than they have employed hitherto.

5. References

- [1] Friedman, Leonard. Reasoning by plausible inference. In Bibel, W and Kowalski, R. (eds). 5th Conference on Automated Deduction. Berlin: Springer (1980) 126-142.
- [2] Polyá, M. Patterns of Plausible Inference. Princeton: Princeton University Press (1954).
- [3] Bandler, W and Kohout, L.J. Fuzzy power sets and fuzzy implication operators. Fuzzy Sets and Symptoms, 4 (1980), 13-30
- [4] Willmott, R. Two fuzzier implication operators in the theory of fuzzy power sets. Fuzzy Sets and Systems, 4 (1980), 31-36.
- [5] Bandler, W and Kohout, L.J. Semantics of implication operators and fuzzy relational products. Int. J. Man-Machine Systems, 12 (1980), 89-116. Reprinted in Mamdani, E.H. and Gaines, B.R. (eds). Fuzzy Reasoning and Its Application. London: Academic Press (1981) 219-246.