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STOCHASTIC CLOSURE FOR PARTIALLY-RESOLVED TURBULENCE

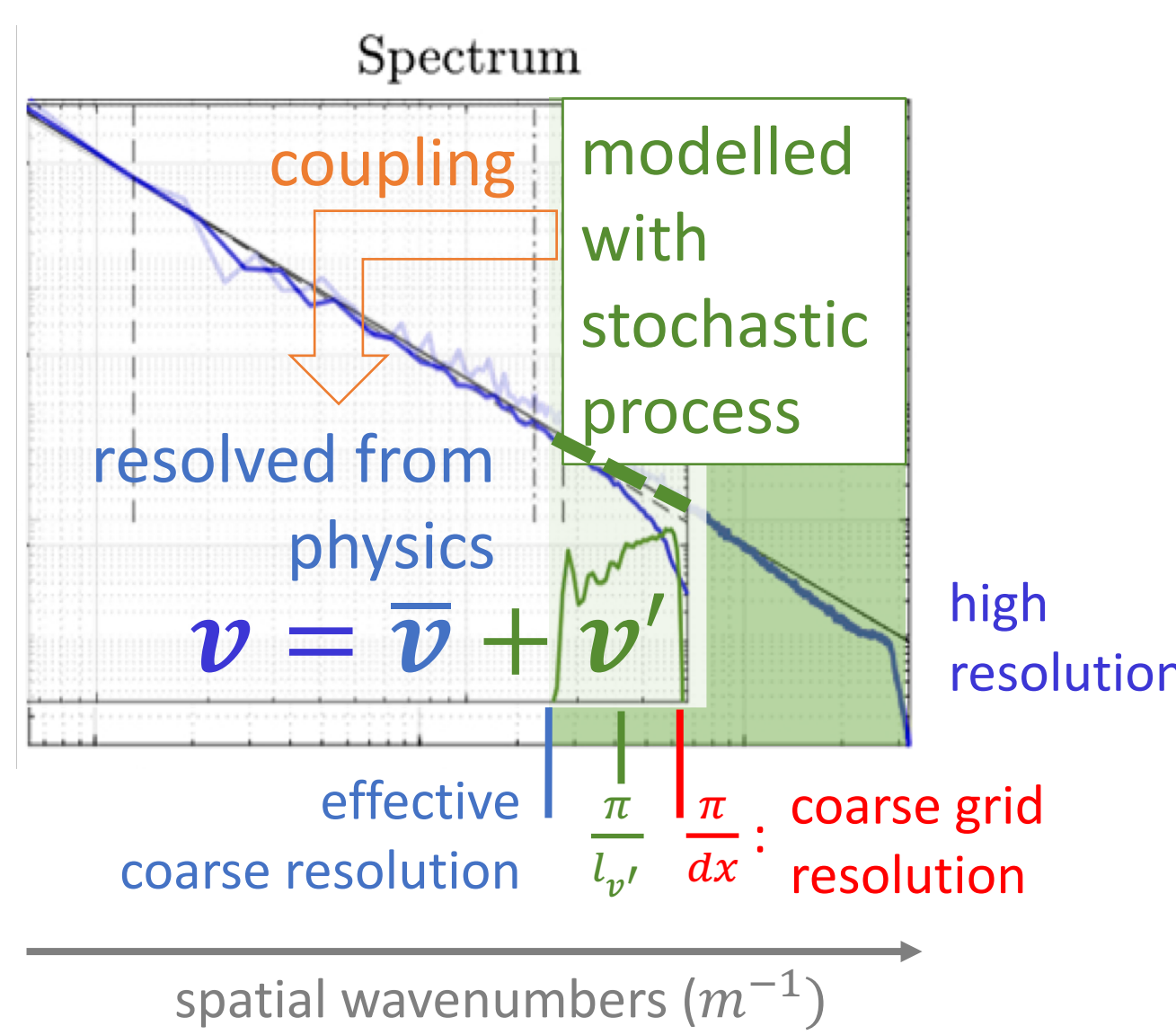
Resseguier V., Caville C., Talla M., Moneyron A., Lepape G., Picard A., Ladvig M., Hascoët E., Wallian L., Jacquet P., Stabile G., Mémin E., Heitz D., Chapron B.



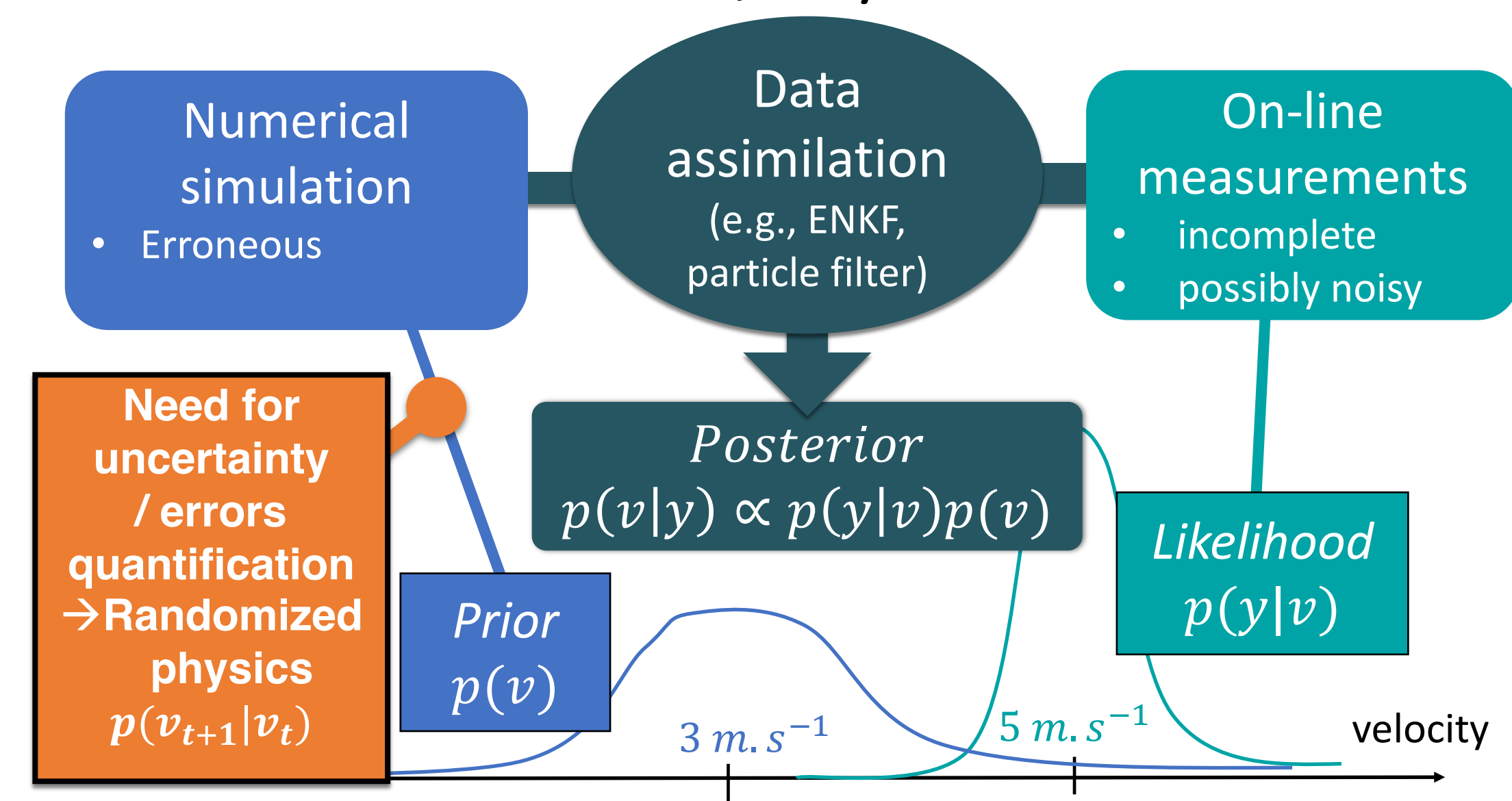
Abstract

To better constraint and explain data-based algorithms (e.g., data assimilation, Bayesian inference, machine learning), we tie *priors* to physics. Although statistical physics propose stochastic processes to be used as *priors*, many real systems hardly accommodate with some usual statistical simplifications (e.g., stationarity, delta-correlation). Therefore, our framework stands halfway between analytic stochastic descriptions and heavy numerical simulations. We rely on turbulence multiscale decomposition and stochastic transport. This framework is applied to wave-turbulence interactions and model errors quantification for data assimilation in geophysical fluid mechanics and reduced order models.

Multi-scale velocity decomposition



Data assimilation / Bayesian inference



Stochastic subgrid parametrization

Stochastic transport

We consider $v' = \sigma \dot{B}_t$, a divergence-free (conditionally-)Gaussian process, δ -correlated in time & correlated in space to be injected in physics equation, typically through stochastic transport of any Q along the flow $\frac{dx}{dt} = \bar{v} + \sigma \dot{B}_t$

$$(1) \frac{DQ}{Dt} = \partial_t Q + \bar{v}^* \cdot \nabla Q + \sigma \dot{B}_t \cdot \nabla Q - \nabla \cdot \left(\frac{a}{2} \nabla Q \right) = S_Q \text{ (sources, e.g. forces) and } \begin{cases} a = \mathbb{E}\{\sigma \dot{B}_t (\sigma \dot{B}_t)^T\} / dt \\ \bar{v}^* = \bar{v} - \nabla \cdot a / 2 \end{cases}$$

Some references:

- Kunita (1990); Piterbarg & Ostrovskii (1997); Mikulevicius & Rozovskii (2004). *SIAM J Math Anal* 35.5: 1250-1310.
- Location Uncertainty (LU) : Mémin (2014). *Geophys. Astro. Fluid* 108.2: 119-146; Resseguier et al. (2017). *Geophys. Astro. Fluid*. hal-01391420
- Stochastic Advection by Lie transport (SALT) : Holm (2015) *Proc. Royal Society A*. 471.2176 (2015): 20140963.

Several possible calibrations:

- Brute-force data-driven : PCA from residues v'
 $v'(x, t) = \sum_{i=1}^n \xi_i(x) \dot{W}_t^{(i)}$
- Physics-driven : e.g., using scale symmetry
 $v'(x, t) = \nabla \times \psi_\sigma * \dot{B}$ with $|\hat{\psi}_\sigma(\kappa)|^2 \sim A \kappa^{-\mu}$
 $(v' \cdot \nabla) g(x, t) = \sum_{i=1}^n F_i[g](x, t) \dot{W}_t^{(i)}$
- More in the review: Resseguier et al. (2021). *Arch Comp Eng*. hal-02558016.

Notations
Velocity wave-number
 $\kappa = |\kappa|$

Geophysical Fluid Dynamics application for forecastings & reanalyses

The example of randomized SQG model for $\frac{l_v'}{l_v} \ll 1$

(SQG : Surface Quasi-Geostrophic)

Stochastic transport of buoyancy b

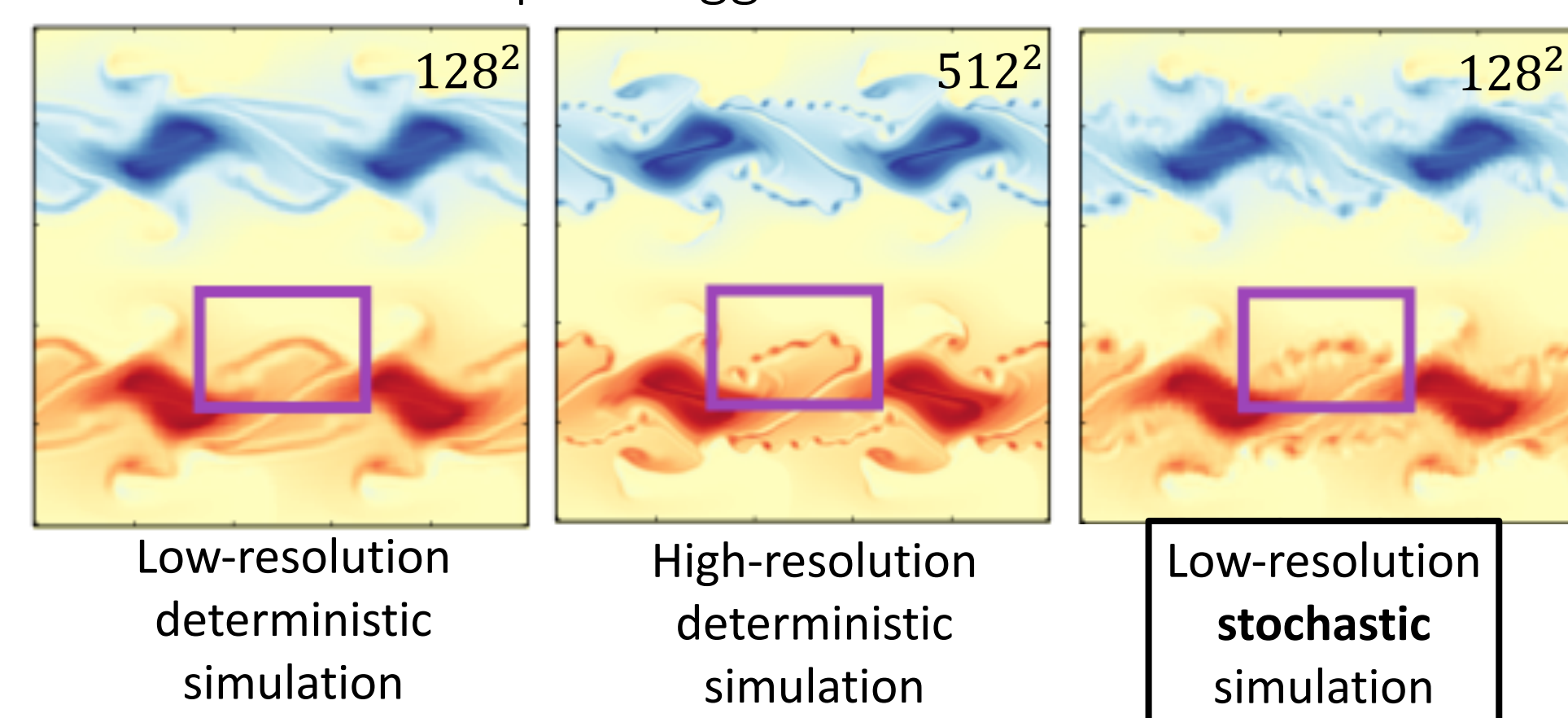
$$\frac{Db}{Dt} = S_b \text{ (forcing \& dissipation)}$$

with $\bar{v} = \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} b$

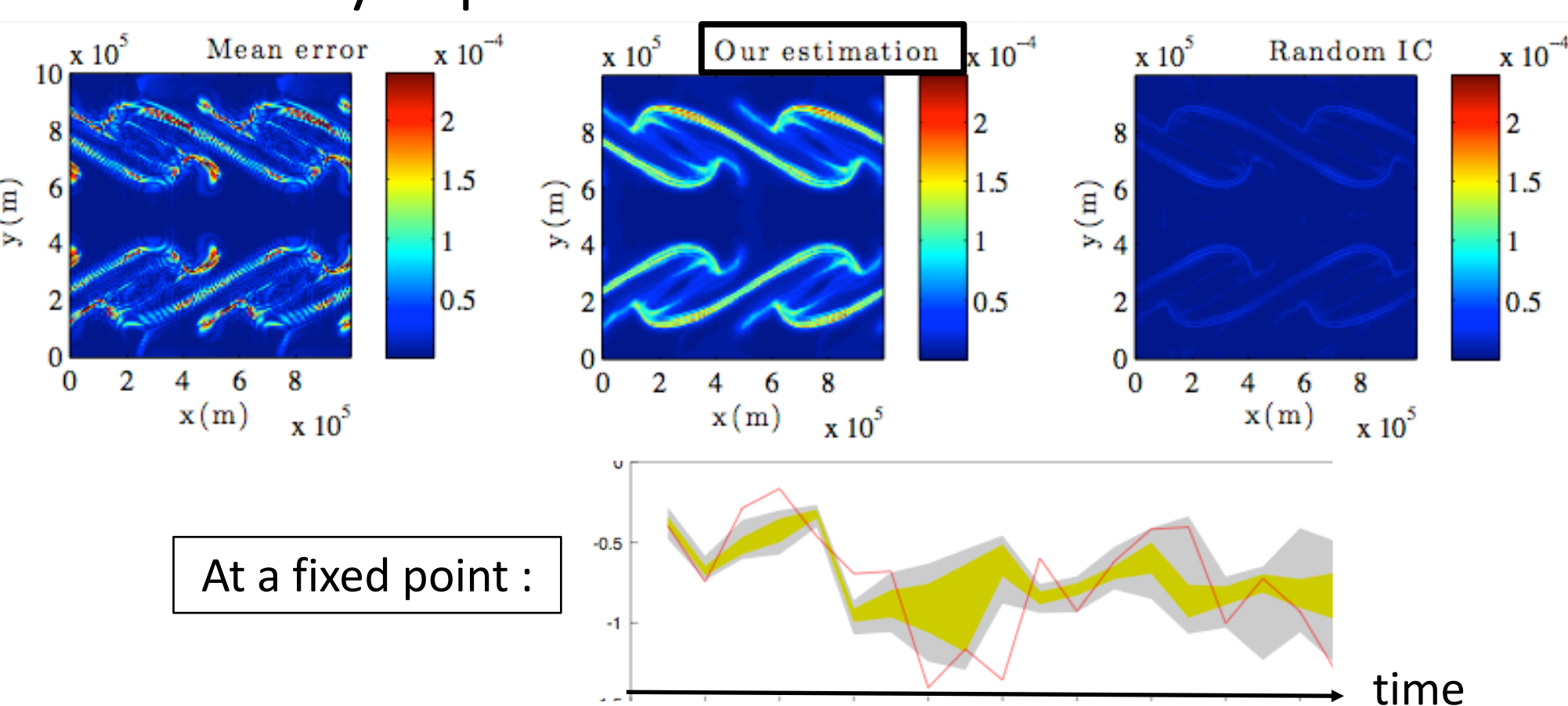


From one realization, better small-scale representation.

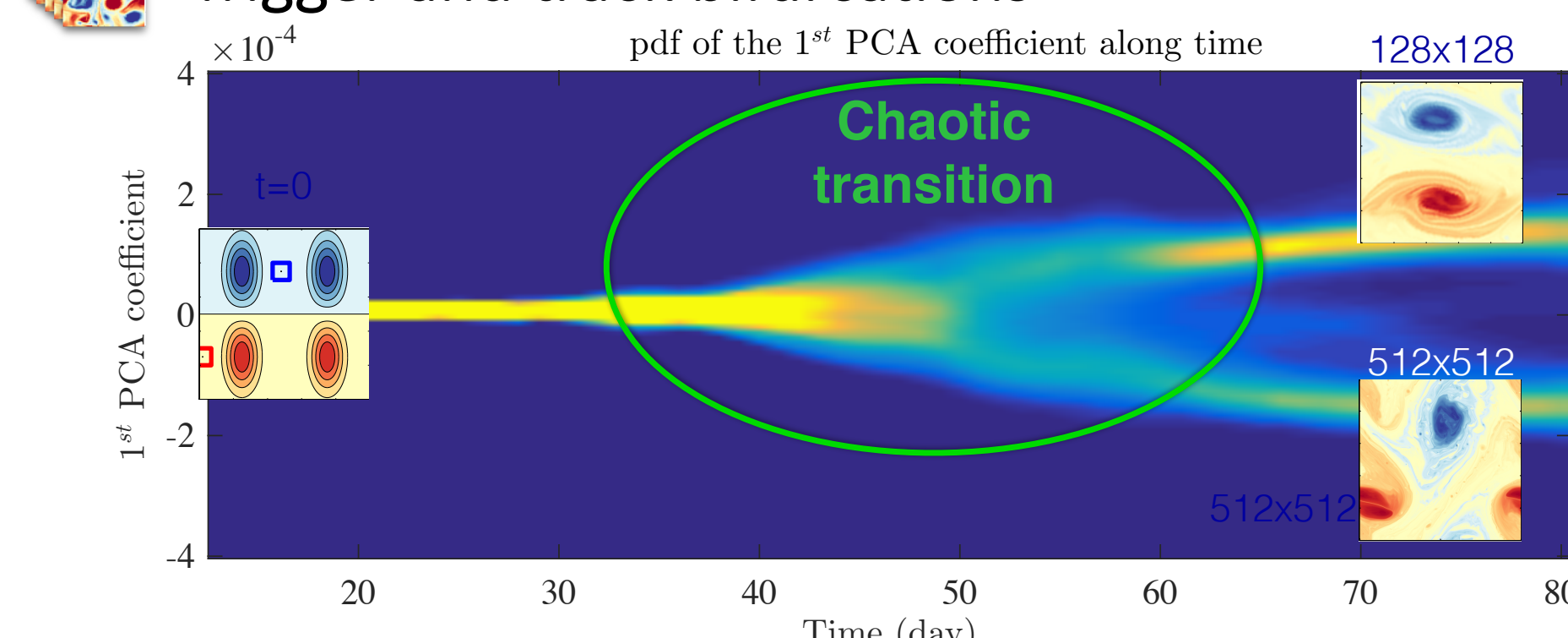
The stochastic transport triggers instabilities.



From an ensemble of simulations, accurate uncertainty quantification ("covariance inflation") to eventually improve data assimilation.

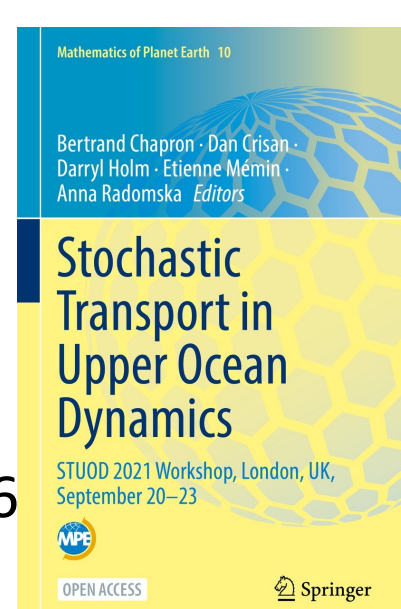


Trigger and track bifurcations



References

- Resseguier et al. (2017). *Geophys. Astro. Fluid*. hal-01391476v3
 - Resseguier et al. (2020). *Nonlin. Proc. Geo*. hal-02371789v3
 - Resseguier et al. (2021). *Arch Comp Eng*. hal-02558016
- Further reading with LU/SALT data assimilation :
- Cotter et al. *SIAM-ASA J Uncertain*. 8.4 (2020) 1446-1492.

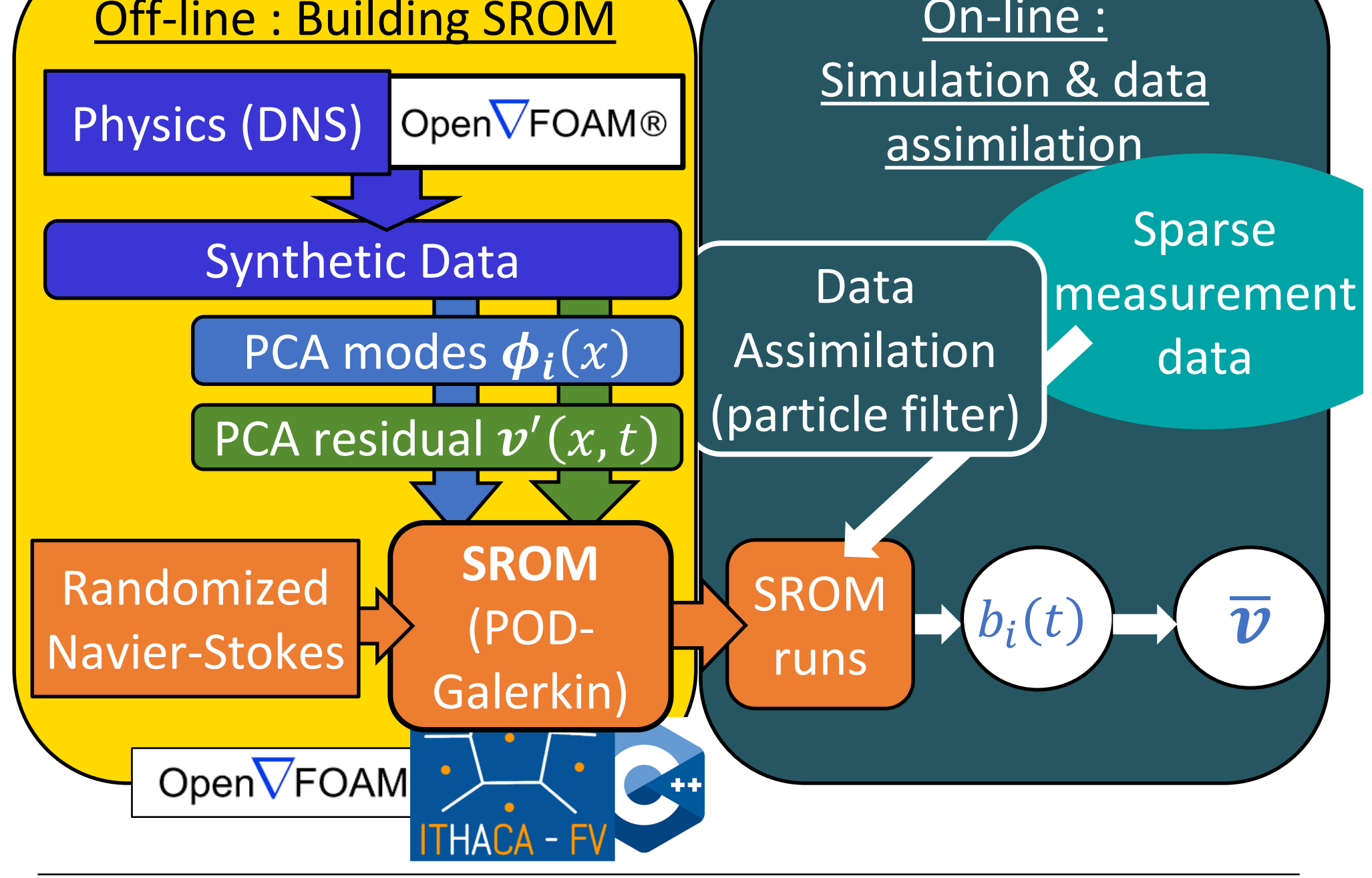


Surrogate application for system monitoring & control

Stochastic reduced order model (SRM) from randomized Navier-Stokes & subsampled synthetic data for real-time data assimilation

Stochastic transport of $\bar{v}(x, t) = \sum_{i=0}^n b_i(t) \phi_i(x)$

$$\frac{D\bar{v}}{Dt} = S_{\bar{v}} = \text{forces, constrained in a small space Span}(\{\phi_i\})$$



POD-Galerkin and statistical estimations with projected additive & multiplicative noises

$\int_{\Omega} \phi_i \cdot (\text{Randomized Navier-Stokes})$ leads to :

$$M \frac{d}{dt} b(t) = H(\bar{b}(t)) + K(\sigma \dot{B}) \bar{b}(t) \quad \begin{matrix} \text{Full order : } M \sim 10^7 \\ \text{Reduced order : } n \sim 10 \end{matrix}$$

2nd order polynomial with coefficients given by :

$$\mathbb{E}\{K_{jq}(\dots) K_{ip}(\dots)\} / dt \approx \Delta t K_{jq} \left[\frac{\partial v_p}{\partial b_j} \frac{\partial b_i}{\partial b_j} \frac{\partial v'}{\partial t} \right]$$

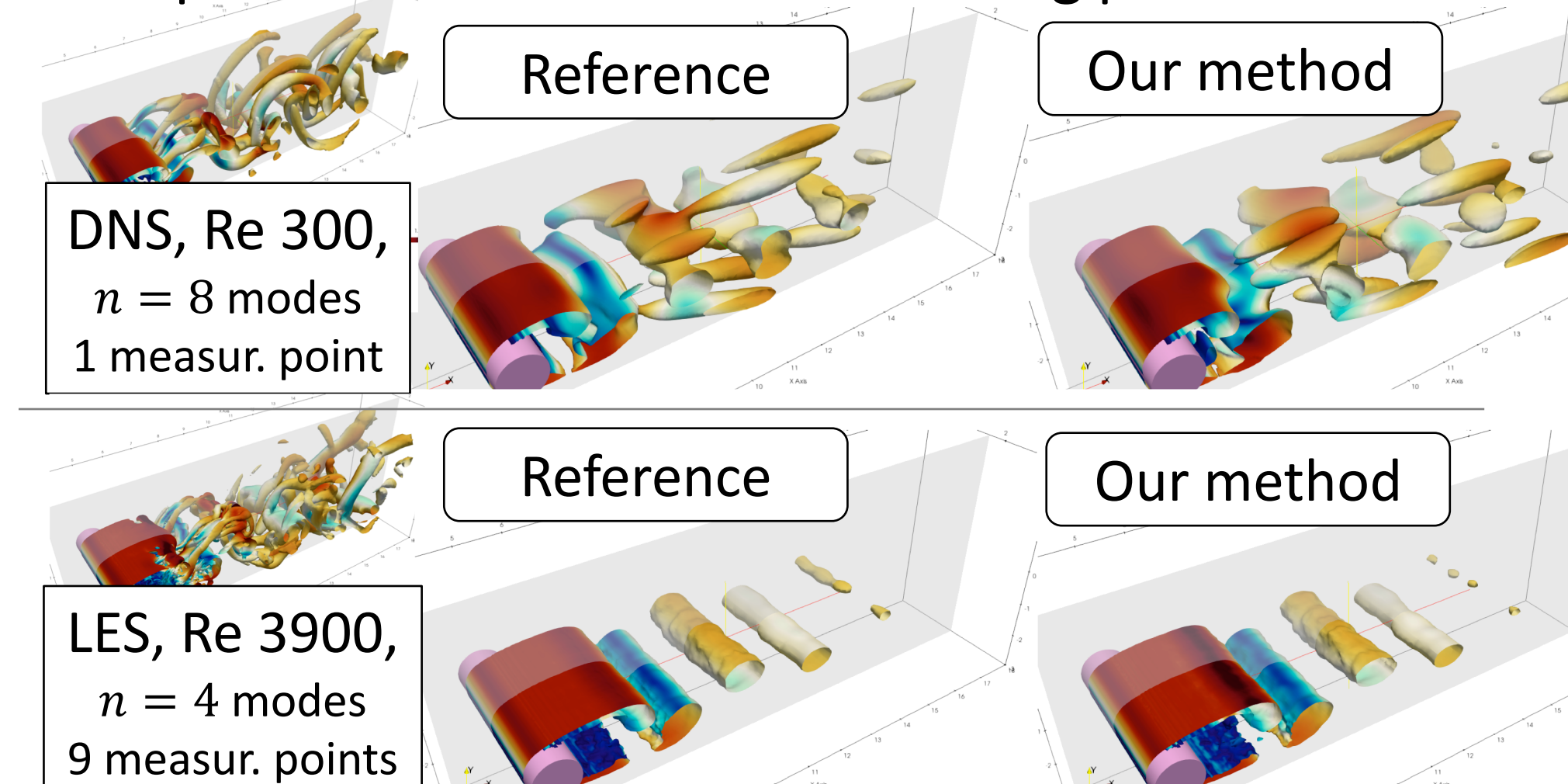
Estimator for the noise covariance

- $\frac{D\bar{v}}{Dt} = S_{\bar{v}}$
- $(\phi_j)_j$
- $a \approx \Delta t v'(v')^T$

Randomized Navier-Stokes, PCA modes, PCA residual v-prime from synthetic data

$$K_{jq}[\xi] = \int_{\Omega} \phi_j \cdot \mathcal{P}[-(\xi \cdot \nabla) \phi_q]$$

Flow prediction far outside the learning period



References

- ITHACA-FV: Stabile & Rozza (2018). *Comp. Fluids* arXiv:1710.11580
- Resseguier et al. (2021). *SIAM-ASA J Uncertain*. hal-03169957
- Resseguier et al. (2022). *J Comp. Phys*. hal-03445455

Wave-turbulence interaction application for better observation analyses

Ray tracing for fast waves : $c_g^0 / v \gg l_v' / l_v$

For a wave $h = H e^{i\phi(\epsilon t, \epsilon x)}$

the (Stratonovich) dispersion ratio reads: $\omega - v \cdot k = \omega_0(k)$

with $\omega = -\frac{\partial \phi}{\partial t}$, $k = \nabla \phi$, and $\omega_0 \propto k^\alpha$.

Along the 4-dimensional stochastic flow,

$$\begin{cases} \frac{dx_r}{dt} = c_g^0 + \bar{v} + \sigma \dot{B}_t \\ \frac{dk}{dt} = -\nabla(\bar{v} + \sigma \dot{B}_t)^T k \end{cases} \quad \begin{matrix} \text{(eikonal)} \\ \text{(equations)} \end{matrix}$$

the wave action spectrum N is transported

$$\frac{DN}{Dt} = 0$$

and energy adds up: $H^2(x, t) = \int \omega_0(k) N(x, k, t) dk$

Single-ray statistics

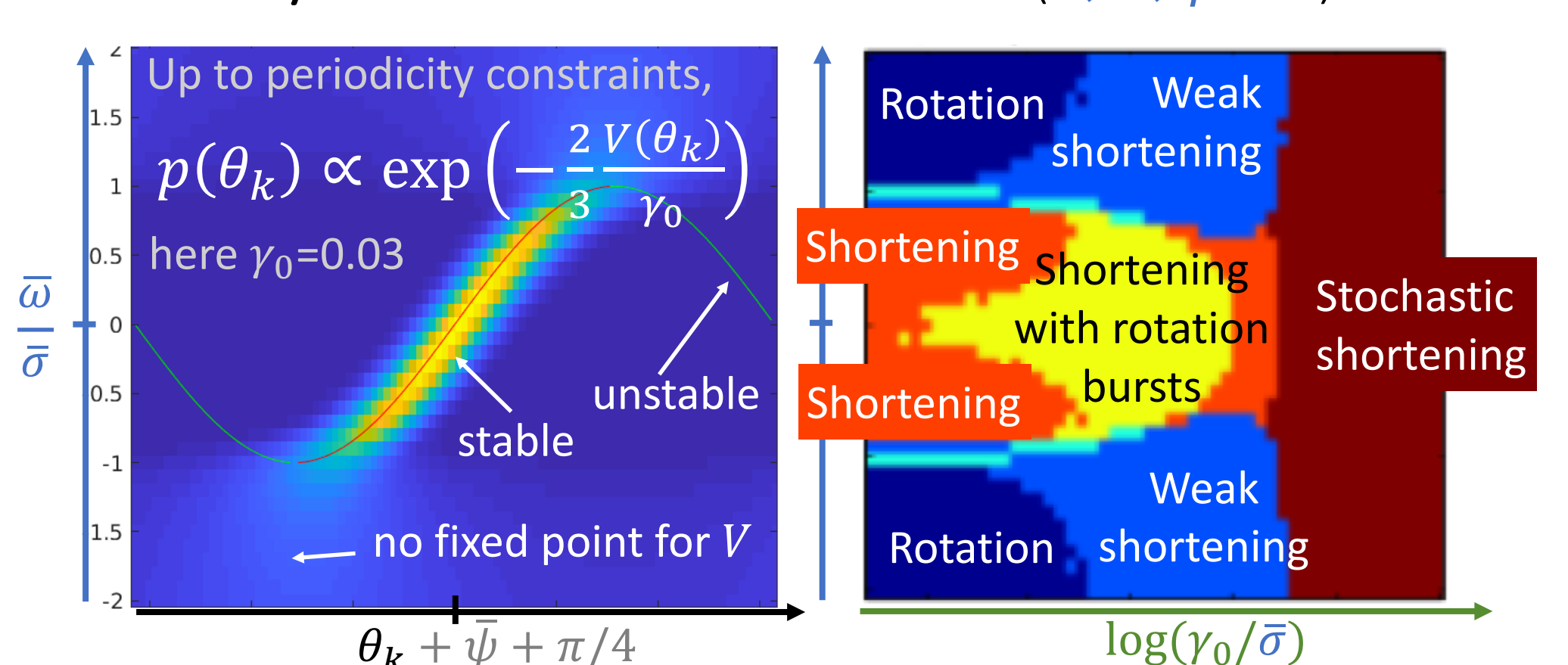
For isotropic v' stretching, stochastic shortening, wave-number log-normality

$$\begin{cases} \frac{d}{dt} \log k = -\bar{\sigma} \sin \zeta(\theta_k) + \gamma_0 + \sqrt{\gamma_0} \dot{B}_t^{(3)} \\ \frac{d}{dt} \theta_k = -\frac{\partial v}{\partial \theta_k}(\zeta(\theta_k)) + \sqrt{3\gamma_0} \dot{B}_t^{(4)} \end{cases}$$

with $V = \frac{\bar{\sigma}}{4}(\sin \zeta - \frac{\bar{\sigma}}{\sigma} \zeta)$, $\zeta = 2(\theta_k + \bar{\psi})$, and $\gamma_0 = \frac{1}{8} \mathbb{E} \|\nabla(\sigma \dot{B}_t)^T\|^2 / dt$.

Then, $\mathbb{E}N(x, k, t) = \iint dx_r^0 dk^0 N^0(x_r^0, k^0) p(x, k | x_r^0, k^0, t)$

Stationary solution for linearized flow ($\bar{\sigma}, \bar{\omega}, \bar{\psi}$ cst.)



Wave groups trapped in an adversarial turbulent jet

Our fast wave approximation still valid for strong current if l_v' / l_v small enough (steep v spectrum or fine resolution).

For $\bar{u} \approx \bar{U}_0 - \frac{1}{2} \bar{\beta} y^2$ & $\bar{v} \approx 0$, $\bar{U}_0, \bar{\beta} < 0$,

$$\frac{d^2}{dt^2} y_r + |c_g^0 \bar{\beta}| y_r = c_g^0 \sqrt{3\gamma_0} \dot{B}_t^{(4)}$$

with analytic oscillatory \mathcal{GP} solution slowly diffusing along y (jet trap)

($\sigma_{y_r}^2 \propto t$ whereas the usual branching regime ($\bar{v} = 0$) predicts $\sigma_{y_r}^2 \propto t^3$)

Reference

- Resseguier, Hascoët & Chapron (2024). *Submitted to J Fluid Dyn*.

Conclusion

Stochastic transport models were successfully applied to several fluid dynamics problems. Possible model improvements could consider time-correlated and/or non-Gaussian subgrid velocities, through e.g. rough path advection and Gaussian multiplicative chaos.