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1 Feasibility of ultrasonic non-destructive testing of
2 ancient funerary urns : experimental and analytical
3 parametric model

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6 **Abstract**

Non-invasive modalities are being developed as the computer tools and industrial non-destructive testing capabilities become more widespread. Using the same investigation methods as those applied to modern objects, we seek to study ancient funerary urns containing human bones. The exploration of ancient urns is key to increasing our understanding of the practices of burial and cremation in archaeo-anthropology. But those urns present at archaeological sites are exposed to diversified environments (wet, temperate or dry environment), and can be subjected to chemical or mechanical destructive actions over time. As ancient funeral urns containers are sometimes made of lead, stone or ceramic, the use of X-ray scanners is difficult or even impossible for *in situ* control. In this work, we propose an engineering technique based on ultrasonic non-destructive testing and wave propagation in urns. An analytical parametric model is developed using mathematical signal processing tools and validated by means of laboratory experiments under controlled conditions of propagation through artificial urns for two ultrasonic frequencies (500 kHz and 1 MHz). The aim is to characterise an ancient urn by creating an analytical model, and to use a parametric identification algorithm to determine the presence or absence of bone fragments. The parametric identification algorithm based on the Levenberg-Marquardt method makes it possible to determine the geometrical (thickness) and physical (wave velocity and

attenuation, mass density) parameters when the urn is filled with water, with water and bones, and with water, bones and sand. We show that the model makes it readily (processing time ≈ 15 sec) possible to find the different times of flight between the transducer and the different walls of the urn and the parameters with an accuracy less than 10%.

- ⁷ *Keywords:* Non-destructive testing, Ultrasonic waves, Parametric model,
⁸ Ancient funerary urn
-

9 1. Introduction

10 The exploration of ancient funerary urns is key to increasing our understand-
11 ing of the practices of inhumation and cremation in archaeo-anthropology, and
12 to understanding past human societies, [1], [2], [3] and [4]. For a long time,
13 tomb excavation only aimed to reveal ancient funerary objects. But after years of
14 archaeological research, the study of bone fragments has become the most impor-
15 tant element of funerary archaeo-anthropology. The characterisation of ancient
16 funerary urns and fragments provides information not only about the lives of indi-
17 viduals, but also about the evolution of an entire society. Anthropological excava-
18 tion greatly expanded in the 1980s, focusing on bone fragments and skeletons, and
19 more particularly on skeletal (sarcophagi, for example) and bone (funerary urns)
20 receptacles. While the main aim of funerary archaeology is to reconstruct funer-
21 ary gestures, field anthropology focuses on bone remains and aims to reconstruct
22 skeletons and their relationship with funerary artefacts. Based on the contents and
23 natures of the bones, it attempts to reconstruct the various funerary rites and the
24 different layouts of a grave or burial site. But tombs, and/or the artefacts found at
25 archaeological sites such as urns, may be exposed to humid, temperate or dry en-
26 vironments, and possibly to destructive chemical or mechanical action over time.

27 Depending on these archaeological facts, the issues developed and the time
28 available, excavation and laboratory investigation procedures have to evolve and
29 adapt. Archaeoscience borrows principles and methods from life and earth sci-
30 ences, from biological and chemical sciences, and from medical and medico-legal
31 sciences, [5]. Archaeoscience takes into account a large number of macroscopic,
32 microscopic and molecular indicators, both organic (plants, animals, humans) and
33 inorganic (minerals, metals). Most often, because of their complex protocols,
34 these methods have to be carried out in the laboratory using specific instruments,
35 which can be heavy and expensive, such as medical scanners. There are very
36 few *in situ* methods other than photography, photogrammetry and the simple but
37 highly effective method of drawing.

38 Non-invasive methods are being developed as computer and digital tools, and
39 non-destructive testing (NDT) capabilities become more advanced and widespread.
40 In the same way as industrial NDT has been used on manufactured objects, X-ray
41 tomodensitometry is being used in the laboratory on ancient funerary urns con-
42 taining human bones, [6], [7], [8], and [9]. Because ancient funerary urns are
43 sometimes bowls made of lead, stone, or ceramic, the use of X-ray modalities
44 to determine the presence or absence of bone fragments of anatomical elements
45 is difficult, and nearly impossible for *in situ* exploitation. Anthropologists who

46 practice and manage sections of human archaeology are therefore interested in
47 alternative, non-destructive approaches that would allow them to analyse remains
48 and specimens in the laboratory and/or *in situ* without damage or destruction.

49 Ultrasound techniques are particularly adapted for bone characterisation. There
50 is a substantial literature on the characterisation of bones by propagative methods,
51 [10], but in this case the bones are often fresh or the post-mortem delays are short
52 (< 1 year on average). There is no suitable method for the study of older, antique
53 or even prehistoric bone fragments, even less so when they have been conditioned
54 or transformed.

55 In this work, ultrasonic wave propagation in funerary urns was studied. First,
56 an analytical model was developed using mathematical tools for signal processing,
57 and validated by means of laboratory experiments under controlled conditions
58 of propagation through artificial urns. Second, an optimisation algorithm of the
59 difference between model and measures, enabled us to evaluate the geometrical
60 (thickness) and physical (mass density, ultrasonic wave velocity and attenuation)
61 parameters of the urn, and to detect the presence or absence of bones inside it.

62 The optimisation algorithm, based on the Levenberg-Marquardt procedure for
63 minimising the difference between the ultrasonic wave recorded at a position
64 around the urn, and the corresponding numerical modelled signal, enabled us to
65 calculate the 4 unknown geometrical and physical parameters of the urn, and to
66 detect the presence of bones inside it. This parametric identification algorithm
67 converged rapidly in less than fifteen seconds, which made it possible to identify
68 an urn and its contents with an acceptable degree of accuracy (deviation lower
69 than 20%) for *in situ* exploitation. Three artificial clay urns, one empty, one con-
70 taining bones, and one containing bones and sand, were used in this study. The
71 study comprised 4 steps:

- 72 • proposition of a representative **analytical model** of the interaction of an
73 ultrasonic wave with an urn.
- 74 • development of an **optimisation algorithm** for **parametric identification**
75 of the urn.
- 76 • study of the **convergence** of the method for 3 configurations (empty urn,
77 urn containing bones, and urn containing bones and sand).
- 78 • study of the **relevance, reproducibility** and the **resistance** of the model and
79 the identification algorithm for several measurement points around the urn.

80 2. Materials & Methods

81 2.1. Description of the urns

82 Three artificial urns reproducing ancient funerary ceramics were used, made
83 of (*terracotta*) clay by a potter specialized in archaeology, (Figures 1 and 2). The
84 urns were built in successive homogeneous clay layers, whose thicknesses and
85 heights differed by a few millimeters. The dimensions of the urns, measured with
86 a caliper at several points, were approximately identical. The outer diameter, D_{out} ,
87 was 130 ± 0.2 mm. The height of the urns was 135 ± 0.03 mm. The thickness, E_{urn} ,
88 of the urn walls was measured for each urn separately using a caliper. The first
89 urn was empty, the second contained only bones and the third contained bones and
90 coarse sand (average particle size more than 0.5 mm). The bones were fragments
91 of real human bones from Hammamet in Tunisia (1st century AD), composed of
92 different pieces, such as femoral heads, lower jaws, diaphysis of tibias or fibulae,
93 flat bones such as scapulae, and vertebrae.



Figure 1: Photos of the artificial funerary urns, empty (left), containing bones (centre), containing bones and sand (right).

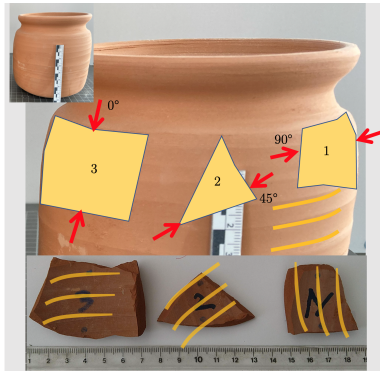


Figure 2: Orientation of the clay layers in an artificial funerary urn manufactured according to the concepts of the 1st century AD. The yellow lines represent the orientation of the layers in the clay samples. The red arrows represent the chosen orientation of the axis, so that it is perpendicular to the acoustical beam when measuring the acoustical parameters of the clay.

94 *2.2. Ultrasonic non-destructive testing devices*

95 *2.2.1. 1MHz configuration: Ultrasonic electro-mechanical 2D-scanner*

96 Ultrasonic experiments were performed in echo mode using an ultrasonic
 97 electro-mechanical 2D-scanner with a horizontal main arm carrying a vertical sup-
 98 port holder for a 1MHz-transducer, (Figure 3). The transducer and holder were
 99 immersed in a circular water tank of 2m diameter. The temperature of the water
 100 ranged from 21°5 to 21°7, and the reference velocity, V_{water} , of the ultrasonic wave
 101 in water was 1474 ± 1 m/s.

102

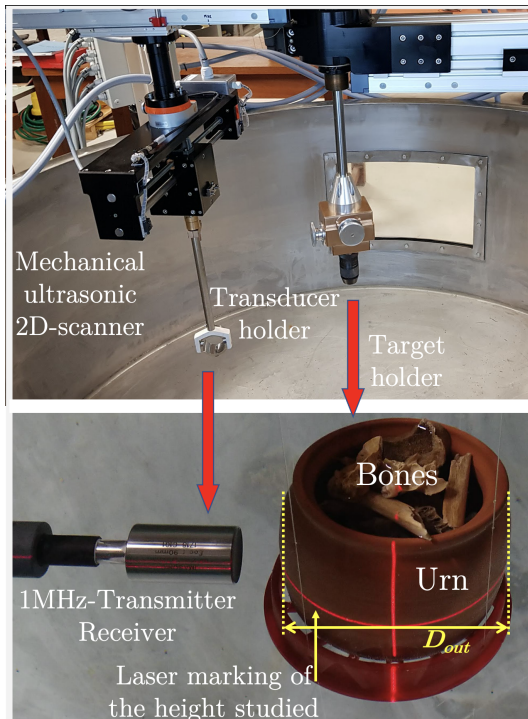


Figure 3: Photos of the ultrasonic electro-mechanical 2D-scanner and the bone-containing urn placed at the centre.

103 By means of stepping motors and an electronic motorization rack (Rhonax,
 104 Thyez, France) to manage the mechanical displacements, the transducer could be
 105 adjusted and moved linearly relative to the tested urn positioned at the geometrical
 106 centre of the scanner. The urns, whether they were empty or containing bones and
 107 no sand or bones and sand, were immersed in the water tank, so that their inner
 108 cavity was always filled with water, whatever else they contained.

109

110 2.2.2. 500kHz configuration: Portable inspection unit

111 A second ultrasonic experiment was carried out, again in echo mode, using
 112 a portable inspection unit (diameter of 170 mm, height of 150 mm) consisting
 113 of a 3D-printed support to position a 500kHz-transducer in contact with the urn.
 114 To ensure the wave transmission with the minimum loss of energy, a 2cm-thick
 115 gel pad (Aquaflex™, Parker laboratories INC, Fairfield, USA) used for medical
 116 applications, was placed between the front of the transducer and the urn.

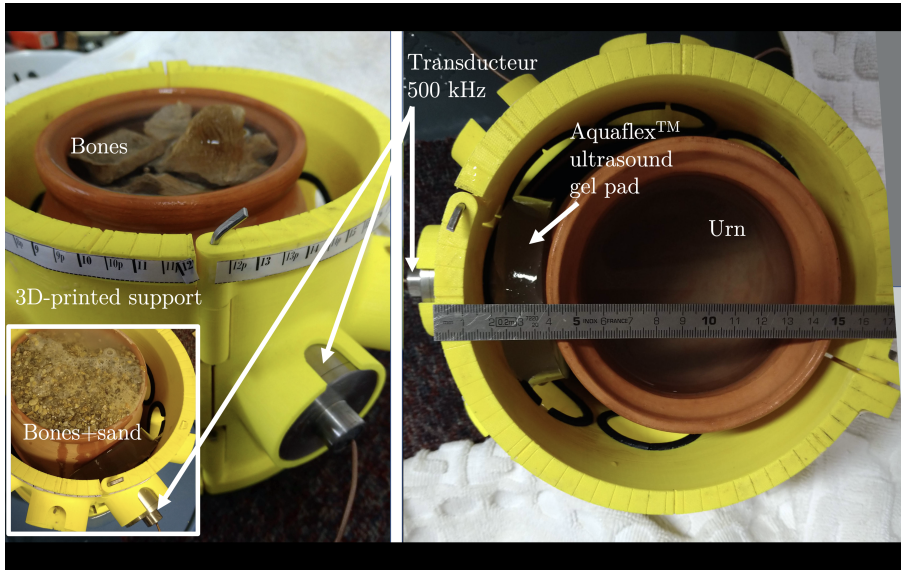


Figure 4: Photos of the ultrasonic portable inspection unit and the funerary urn inside, when the inner cavity was filled with water (right), and water and bones (left).

117 *2.2.3. Transducer and electro-acoustical devices*

118 The transducers (Imasonic, Voray-sur-l'Ognon, France) were piezo-composite
 119 transducers with a nominal frequency, f_0 , of 1 MHz, and 500 kHz. The -6 dB-
 120 bandwidths were of 1.5 MHz (from 0.5 MHz to 2 MHz), and 530 kHz (from
 121 230 kHz to 760 kHz). The 1MHz-transducer had a housing height of 60 mm and
 122 a diameter of 40 mm (active diameter of 30 mm) (Figure 5-a), a focal length of
 123 90 mm in the Fresnel zone, an aperture (axial (0x) and lateral (0y)) of 4.6 mm
 124 and a slice thickness (0z) of 3 mm. The 500kHz-transducer was 55 mm high
 125 and 45 mm in diameter (active diameter 42 mm) (Figure 5-b). The shape and
 126 dimensions of the acoustical field of this contact transducer are not relevant. Each
 127 transducer was used as a transmitter to send a incident wave, then as a receiver to
 128 record the waves reflected by the urn.

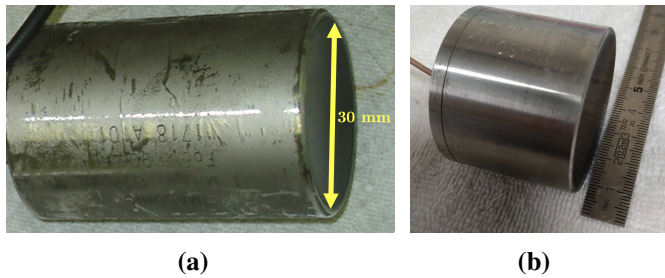


Figure 5: Photos of the 1 MHz (left) and 500 kHz (right) ultrasonic transducers.

129 Each transducer was driven using a pulse-receiver generator (Sofranel 5077
 130 PR, Olympus, USA) including the voltage amplifier. The radio-frequency sig-
 131 nals (RF-signals) were conveyed from the 12-bit oscilloscope (Lecroy HDO 6104,
 132 Teledyne Inc., Thousand Oaks, CA, USA), including the data digitizer, to a per-
 133 sonal computer using a USB interface file transfer, and stored. Each RF-signal
 134 contained $N=4096$ samples, and the sampling frequency, F_e , was 20 MHz. The
 135 voltage amplifier delivered an output voltage from 40 mV_{pp} . The envelope was
 136 defined by the modulus of the Hilbert transform of the RF-signal. All process-
 137 ing algorithms were implemented using Matlab[®] (The MathWorks, Inc., Natick,
 138 Massachusetts, USA).

139 2.3. Analytical ultrasonic 1D-model

140 2.3.1. General synoptic

141 Figure 6 shows the synoptic diagramme of the non-destructive echo-mode test-
 142 ing system of a funerary urn.

143 The electro-mechanical and electro-acoustical devices and the transducer there-
 144 fore serve as a continuous, linear, stationary causal filter of the acoustical signa-
 145 ture of the urn.

146

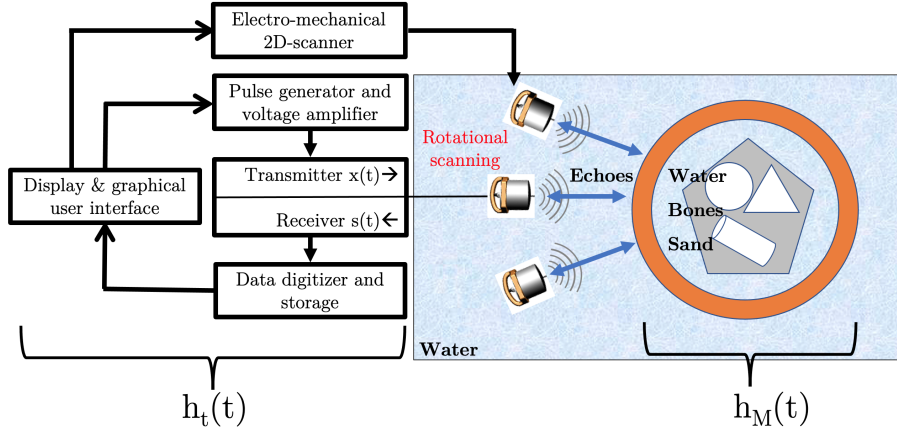


Figure 6: Synoptic diagramme of the non-destructive testing system.

147 The transmitted, $x(t)$, and the recorded, $s(t)$, signals are connected by convo-
 148 lution:

$$s(t) = x(t) \otimes h_M(t) \quad (1)$$

$$x(t) = h_t(t) \otimes e(t) \quad (2)$$

149 where \otimes denotes the convolution operation.

150 h_M is the ultrasonic response of the urn studied. This variable is unknown, and its
 151 model and comparison with experimental measurements are the objectives of the
 152 proposed parametric ultrasonic non-destructive testing.

153 h_t is the electro-acoustical impulse response of the electro-acoustical acquisition
 154 system. It is assumed to be linear and known.

155 e , also known, is the electrical input conveyed to the transducer via the pulse
 156 generator. In pulse mode, this signal is comparable to a Dirac delta function,
 157 $\delta(t)$ (in terms of distribution), which is the neutral element of the convolution
 158 operation:

$$\delta(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases} \quad (3)$$

159 The transmitted signal, $x(t)$ (Eq. 2), can therefore be written as a copy of the
 160 electro-acoustical impulse response:

$$x(t) = h_t(t) \quad (4)$$

161 And the recorded signal, $s(t)$ (Eq. 1) can therefore be written as:

$$s(t) = h_t(t) \otimes h_M(t) \quad (5)$$

162 In this work, only the pure compression waves (which from now on will be
163 called ultrasonic waves) were taken into account, and the shear waves propagat-
164 ing in the walls of the urn or in the bones fragments were considered negligible.
165 The water, urns, bones and sand were considered to be weakly dispersive me-
166 dia/materials. As a result, attenuation was considered to be slightly different at
167 500 kHz and 1 MHz, while the wave velocities were taken to be constant and in-
168 dependent of the frequency. Only the propagation time (which from now on will
169 be called the time of flight, TOF) and the amplitudes of the sound waves were
170 modelled and measured.

171 2.3.2. Modelling of the electro-acoustical impulse responses of the devices

172 Lasaygues et al., [11], have shown that the electro-acoustical impulse re-
173 sponse can be obtained by echo-mode measurements on objects that are infinite-
174 dimensional (with respect to the wavelength) and perfectly reflecting, such as a
175 large flat aluminum plate. In *in situ* NDT, the signal corresponding to the first
176 back-and-forth travel between the transducer and the object under investigation
177 is generally used as the electro-acoustical impulse response. This is a common
178 means of calibrating ultrasonic NDT devices. But here, the object under investi-
179 gation was an urn made of clay, and this material is not perfectly reflective. The
180 first waves observed cannot be used to calibrate the device, and cannot be used as
181 electro-acoustical impulse response.

182 In this work, the electro-acoustical impulse response of the devices was modelled
183 using a pseudo-Ricker wavelet (damped sinusoid):

$$h_t(t) = \sin(2\pi \frac{f_0}{F_e} t) \exp(\frac{-t^2}{\sigma^2}) t \frac{1}{\sigma} \quad (6)$$

184 f_0 is the centre frequency of the transducer (1 MHz and 500 kHz, in this work),
185 F_e is the sampling frequency (20 MHz in this work), and σ is a factor of the
186 sampling frequency (F_e). Figures 7 and 8 show examples of a pseudo-Ricker
187 wavelet with the parameters used in this work.

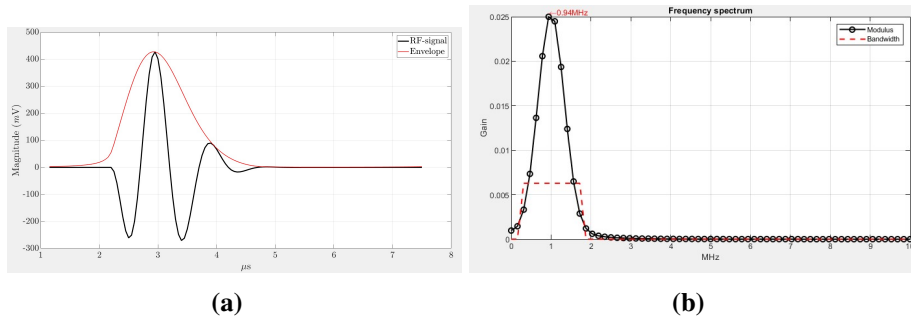


Figure 7: Temporal and spectral representation of the pseudo-Ricker wavelet h_t (4096 samples, $f_0=1$ MHz, $F_e=20$ MHz, $\sigma=20$). The dotted red curve on the temporal representation represents the envelope of the signal (i.e., the modulus of the Hilbert transform). The dotted red line on the spectral representation marks the limits of the -6dB-bandwidth.

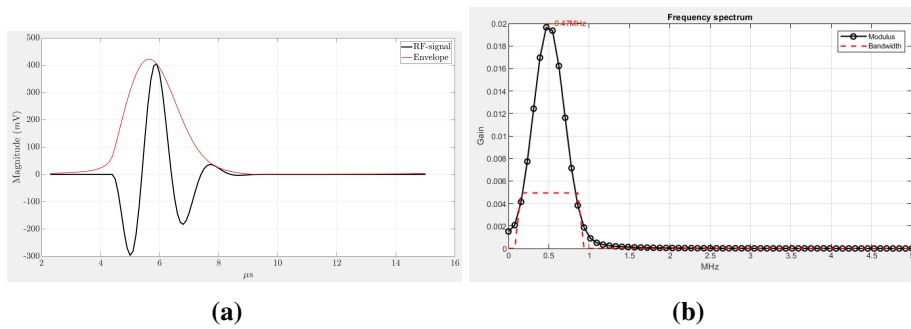


Figure 8: Temporal and spectral representation of the pseudo-Ricker wavelet h_t (4096 samples, $f_0=500$ kHz, $F_e=20$ MHz, $\sigma=20$). The dotted red curve on the temporal representation represents the envelope of the signal (i.e., the modulus of the Hilbert transform). The dotted red line on the spectral representation marks the limits of the -6dB-bandwidth.

188 2.3.3. Modelling of the response of the urn

189 The funerary urns were placed in the centre of each device at a distance of
 190 $H = 86.5 \pm 0.4$ mm from the active surface of the 1MHz-transducer, in their Fresnel
 191 zone (90 mm) (electro-mechanical 2D-scanner), or at the distance of 2 cm of
 192 the 500kHz-transducer front face (portable inspection unit). In this work, because
 193 of the narrow ultrasonic beam aperture, the effect of the side lobes was neglected.
 194 The waves reach the urn walls, which are assumed to be plane and perpendicular,
 195 with an almost plane wave front (i.e., the incidence angle was null). The walls of
 196 the urn were modelled by a succession of two infinite perfect rectangular paral-
 197 lelepipedes, perpendicular to the acoustical beams, of thickness, E_{urn} , equal to that

198 of the urn walls, and separated by a distance equal to that of their inner cavity,
199 ($D_{\text{out}} - 2E_{\text{urn}}$). The parallelepipeds were supposed to be made of clay (Figure 9).
200 The geometry of the bones contained in the urn was not known. The objective of
201 the work was to determine the presence or absence of bones inside the urn, not
202 their shape or the nature (whether burnt or not, for example) of the fragments.
203 As a first approximation, the bones were therefore modelled by a medium whose
204 properties were different from those of the empty urn containing water only. This
205 inner medium was defined by its physical and acoustical parameters assumed to
206 be known (ρ_{cavity} , V_{cavity} and α_{cavity}). These parameters would be prior parameters
207 of the problem. The cavity was considered successively as being water, water and
208 bones, and water, bones and sand.
209

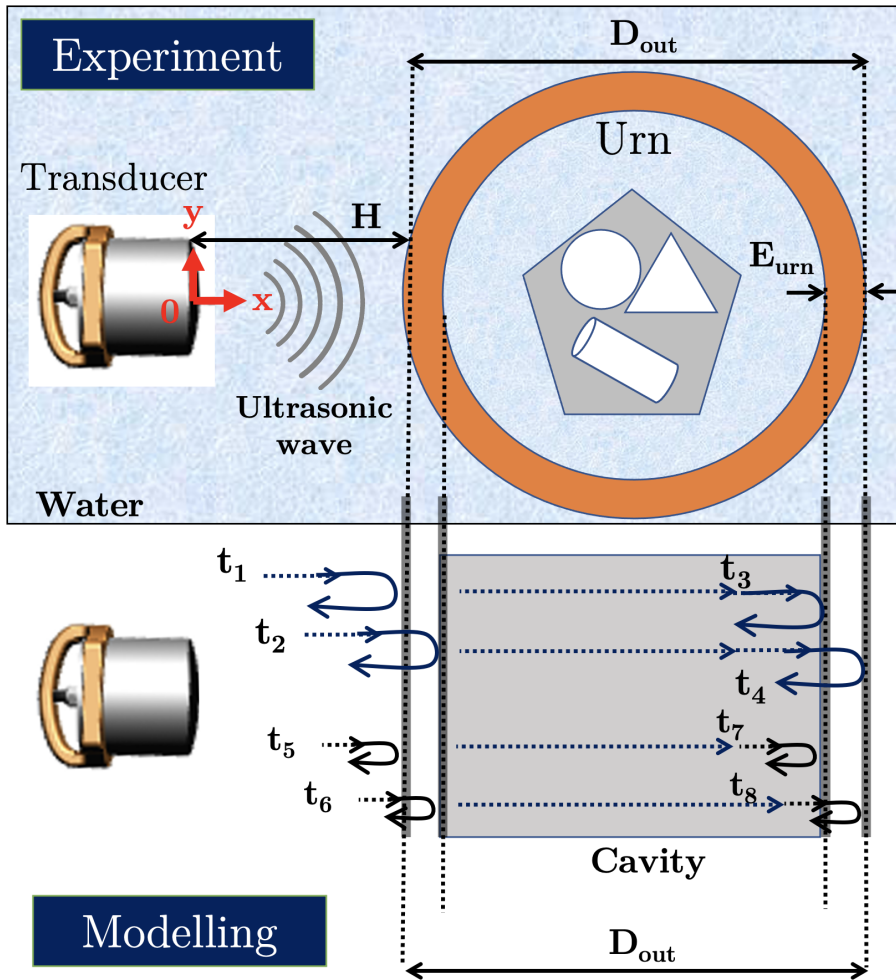


Figure 9: Schematic model of the ultrasonic non-destructive testing of a funerary urn using ultrasounds in echo mode.

210 The response $h_M(t)$ of the urn was modelled as the sum of pulse signals, which
 211 were comparable to Dirac delta functions (in terms of the distribution) :

$$h_M(t) = \sum_n A_n \delta(t - t_n) \quad (7)$$

212 where A_n were the amplitudes of the wave packets corresponding to the time-
 213 delays of the reflected waves, located at the TOF, t_n . The indices n were between
 214 1 and 8. For indices n equal to 1 to 4, the values of the amplitudes, A_n , and

215 TOF, t_n , corresponded to the first four specular waves (direct waves) successively
 216 reflected on the four interfaces. In this work, the approximation for multiple re-
 217 flected waves was limited to the order 2, which means that the back-and-forth
 218 travels corresponding to the reflection of waves on the interfaces beyond two times
 219 (multiple waves) were no longer recordable. Four multiple waves were then mod-
 220 elled for indices n equal to 5 to 8.

221

222 The TOF t_n depended on the velocity of the ultrasonic wave in the crossed
 223 medium (the water, V_{water} , the urn, V_{urn} , and the cavity V_{cavity}), the urn wall
 224 thickness, E_{urn} , the urn inner diameter, $D_{out} - 2E_{urn}$, and the distance between
 225 the transducer and the urn, H . For example, the TOF t_1 was corresponding to the
 226 time delay related to the first back-and-forth travel between the transducer and the
 227 first urn interface:

$$t_1 = 2 \frac{H}{V_{water}} \quad (8)$$

228 The TOF t_2 was corresponding to the propagation through the water to the first
 229 urn interface in water, the propagation through the thickness of the urn at velocity
 230 V_{urn} , and the back travel to the transducer:

$$t_2 = 2 \left(\frac{H}{V_{water}} + \frac{E_{urn}}{V_{urn}} \right) = t_1 + \frac{2E_{urn}}{V_{urn}} \quad (9)$$

231 Similarly, t_3 was computed as a function of t_2 by adding the propagation in
 232 the inner cavity inside the urn:

$$t_3 = t_2 + \frac{2D_{in}}{V_{cavity}} \quad (10)$$

233 The other TOF t_n , for $n = 4$ to 8, were calculated with the same formula, by
 234 adapting the number of back-and-forth travels.

235

236 The amplitudes A_n depended on the transmission and reflection coefficients on
 237 the considered interfaces (water/urn and cavity/urn), and on the wave attenuation
 238 coefficients in the media.

239 The coefficient of reflection of a wave on a medium 1/medium 2 interface was
 240 defined under normal incident conditions as:

$$R_{1,2} = \frac{[Z_2 - Z_1]^2}{[Z_2 + Z_1]^2} \quad (11)$$

241 and the transmission coefficient was defined as:

$$T_{1,2} = \frac{4Z_1Z_2}{[Z_2 + Z_1]^2} \quad (12)$$

242 where $Z_1 = \rho_1 V_1$ and $Z_2 = \rho_2 V_2$ were the acoustical impedances [$MRayls = kg/m^2/s$],
 243 of medium 1 and of medium 2. ρ was the mass density [kg/m^3], of the medium,
 244 and V was the velocity [m/s], of the ultrasonic wave propagating in the medium.
 245 Equation 12 was admitted by supposing that all the energies of the waves were
 246 reflected or transmitted, and that all the higher order phenomena of energy loss
 247 (heat, diffusion or dispersion) were neglected. The reflection and transmission
 248 coefficients were calculated at the water/urn and cavity/urn interfaces, at the wa-
 249 ter/transducer interface, and inside the wall thickness. These coefficients represent
 250 the energy distribution between the waves that cross the interface and those that
 251 are reflected. They are written as the ratio between the amplitudes of the reflected
 252 waves, A_n^R , (respectively transmitted waves, A_n^T), and the incident waves, A_n^I :

$$A_n^R = -A_n^I R \quad (13)$$

$$A_n^T = A_n^I T \quad (14)$$

253 The minus sign models the phase shift inversion of the acoustical signals dur-
 254 ing a reflection on an interface.

255 The amplitude $A_n(x_2)$ of a wave at the distance x_2 is a function of the initial am-
 256 plitude $A_n(x_1)$ and the wave attenuation coefficient α [$dB/m/f_0$] of the wave
 257 over the distance $|x_2 - x_1|$:

$$A_n(x_2) = A_n(x_1) e^{-\alpha|x_2-x_1|} \quad (15)$$

258 The attenuation of the waves in the water and in the thickness of the urn wall
 259 was taken into account. The first amplitude, A_1 , and the first TOF, t_1 , corre-
 260 sponded to the first wave packet of the recorded signals. Indeed, the variables, H
 261 and V_{water} in Equation 8, were known, so that the first wave packet characterised
 262 precisely the first interface of the wall of the urn.

263
 264 Identifying the variable h_M (Eq.7) consisted therefore in determining the geo-
 265 metrical parameter, E_{urn} , the physical parameter, ρ_{urn} , and the acoustical param-
 266 eters, V_{urn} and α_{urn} . D_{out} was the known diameter of the urn, measured using
 267 a caliper. The physical parameters (ρ_{cavity} , V_{cavity} and α_{cavity}) of the inner cavity

268 containing either water, water and bones, or water, bones and sand were also a
 269 *prior* parameters.

270 2.4. Parametric identification algorithm

271 The parametric identification algorithm proposed in this work to identify the
 272 set of geometrical and physical parameters of the urn, was based on an adaptive it-
 273 erative optimisation procedure. This procedure consisted in iteratively minimising
 274 a cost function, C , which was the difference between the signal recorded, s_{mea} ,
 275 and the corresponding signal modelled, s_{mod} (Figure 10), and adapting the model
 276 at each iteration:

$$C = |s_{mea}(t) - s_{mod}(t)| \quad (16)$$

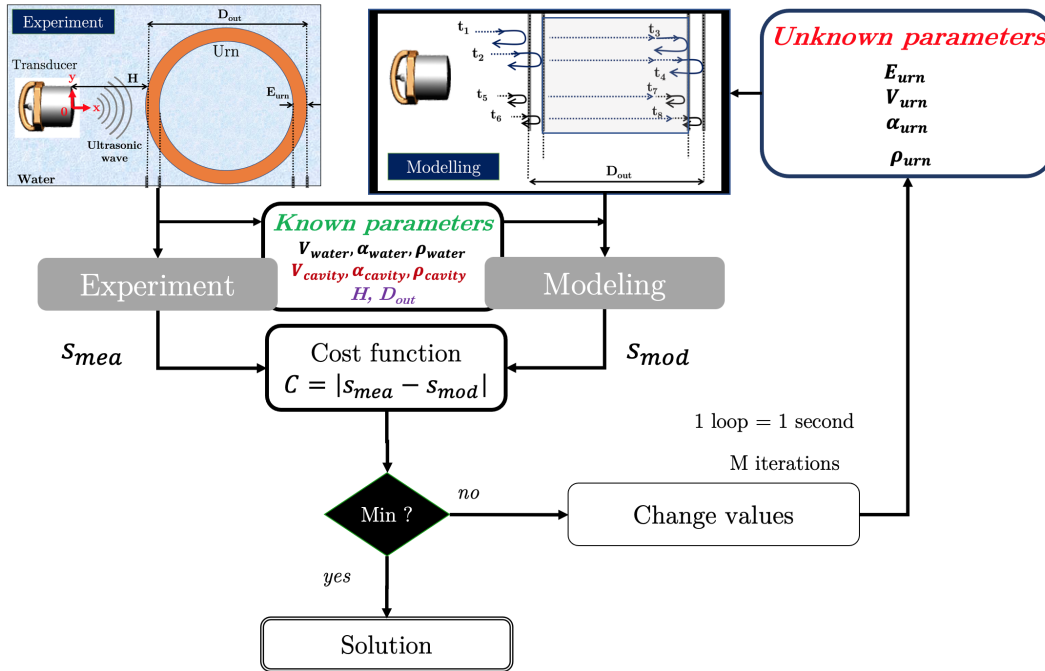


Figure 10: Adaptive iterative optimisation algorithm for the identification of the geometrical and physical parameters of a funerary urn.

277 The solution was calculated using an iterative algorithm for solving nonlinear
 278 least squares problems, based on the Levenberg-Marquardt method. The algo-
 279 rithm started with lower bound parameters given by the user, and found a mini-
 280 mum to the cost function, C . If this minimum was lower than the defined threshold

281 (10^{-5} in this study) then the algorithm stopped, and the parameters for that itera-
282 tion were the estimated ones. Otherwise, the algorithm was continued iteratively
283 by modifying the model parameters, until it reached the upper bound. If it reached
284 the upper bound, there was no convergence and no solution found.

285 The parametric identification algorithm was studied in order to know the num-
286 ber of iterations needed, the speed of convergence to reach the threshold, and also
287 the behaviour and the resilience of the solutions to transducer positioning varia-
288 tions. To test the convergence, the parameters obtained by the parametric identifi-
289 cation algorithm had to be compared with experimental mean parameters for the
290 urns and clay. These initial values were also used to define the lower and upper
291 bounds of the algorithm. To study the behaviour of the algorithm as a function of
292 the position on the urn, parametric identifications were carried out all around the
293 urns at several positions in the same orthogonal plane.

294 2.5. Initial geometrical and physical parameters

295 In the study, two categories of geometrical and physical parameters were re-
296 tained.

- 297 • The **known** or **prior** parameters were the distance, H , between transducer
298 and urn, the ultrasonic wave velocity, V_{water} , and attenuation, α_{water} , in water,
299 the mass density, ρ_{water} , of the water, the outer diameter of each urn, D_{out} ,
300 and the parameters of the inner cavity, V_{cavity} , α_{cavity} , and ρ_{cavity} , which could
301 be water, bones, or bones and sand.
- 302 • The **unknown** parameters to be found were the thickness of each urn, E_{urn} ,
303 the ultrasonic wave velocity, V_{urn} , and attenuation, α_{urn} , in the clay parts,
304 and the mass density, ρ_{urn} , of the clay.

305 The parameters to be found had previously been measured by independent
306 methods on three clay pieces (figure 2) from an urn constructed identically to
307 those studied, and the values were first compared with the literature, [12]. For
308 each parameter, 10 measurements had been performed, and the mean values were
309 used for the study.

- 310 • D_{out} and E_{urn} were measured with a caliper (Absolute Digimatik Solar®,
311 Mitutoyo, Kanagawa, Japan).
- 312 • The ultrasonic wave velocities, V_{urn} , and attenuations, α_{urn} were measured
313 by ultrasonic interferometry for both 500kHz- and 1MHz-frequency, based
314 on the work of Lasaygues et al, [13].

315 • The mass densities, ρ_{urn} , were measured based on Archimedes' principle
316 with a micrometric balance and a density kit (Voyager® 610 GX, Ohaus
317 Corporation, Florham Park, NJ, USA).

318 To estimate the mean acoustical parameters, V_{cavity} , α_{cavity} , of bone fragments
319 without or with sand, the same methods were used on characteristic mixed sam-
320 ples, placed in the field of the transducers without separating interfaces. For the
321 mass densities, ρ_{cavity} , several measurements were carried out on several different
322 fragments, and the averages were calculated.

323 *2.6. Rotational scanning*

324 To study the behaviour of the algorithms in the face of variations in trans-
325 ducer positioning, each transducer was moved at different angles in its orthog-
326 onal plane around the urn, either with the electro-mechanical 2D-scanner using
327 the 1MHz-transducer, or with the portable inspection unit using the 500kHz-
328 transducer. Eight positions were tested with an angular step of 45° , and eight RF-
329 signals were recorded with $N = 4096$ samples, for both configurations, 500 kHz
330 and 1 MHz. For each RF-signal, the parametric identification algorithm was used
331 to determine the geometrical and physical parameters of the urns.

332 *2.7. Human-machine interface*

333 A human-machine interface (Figure 11) has been finally developed to allow a
334 non-specialist user to perform modelling process from a recorded RF-signal. This
335 interface allows the user to choose different configurations (intermediate display,
336 recording of the modelled signals) and to modify the processing parameters (vari-
337 able depending on the material used), and provides the values of the geometrical
338 and physical parameters of the urn.

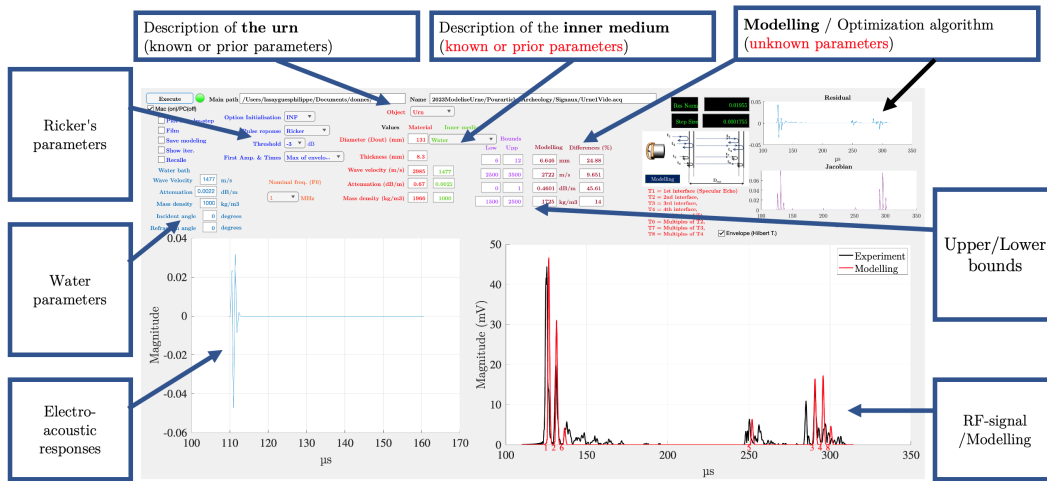


Figure 11: Human-machine interface for non-destructive testing of a funerary urn.

339 3. Results

340 3.1. Experimental RF-signals

341 Figures 12, 13, and 14 show the experimental recorded RF-signal at $y = 0^\circ$
 342 for the urn no1, no2 and no3 (inner cavity filled with water, water and bones,
 343 and water, bones and sand, respectively) when using the electro-mechanical 2D-
 344 scanner.

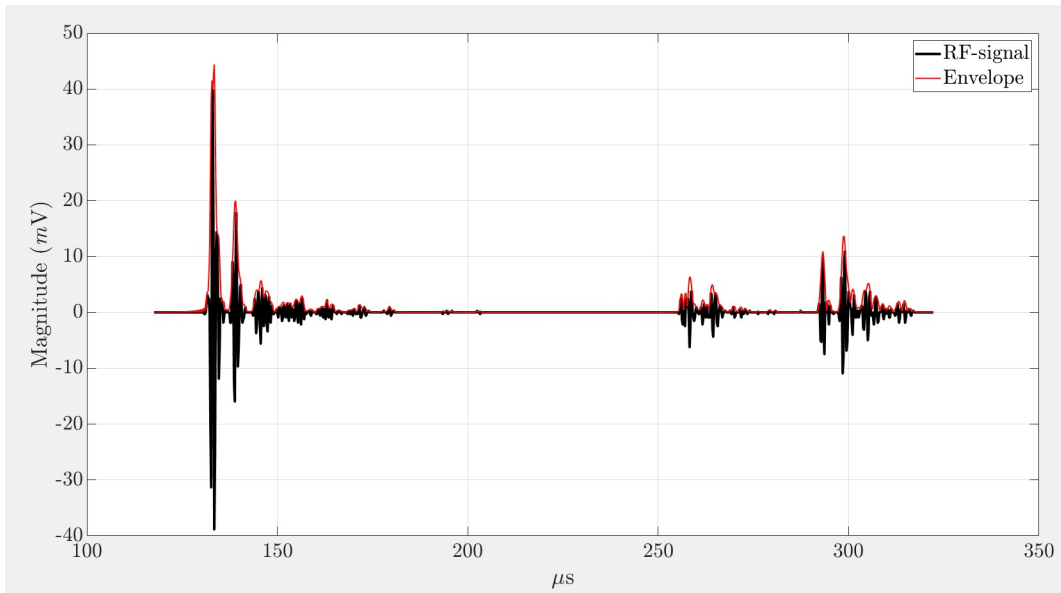


Figure 12: Temporal representation of the recorded RF-signal (black) and the envelope (red) for the artificial funerary urn no1 at $y = 0^\circ$ (inner cavity filled with water). 1MHz configuration, scanner.

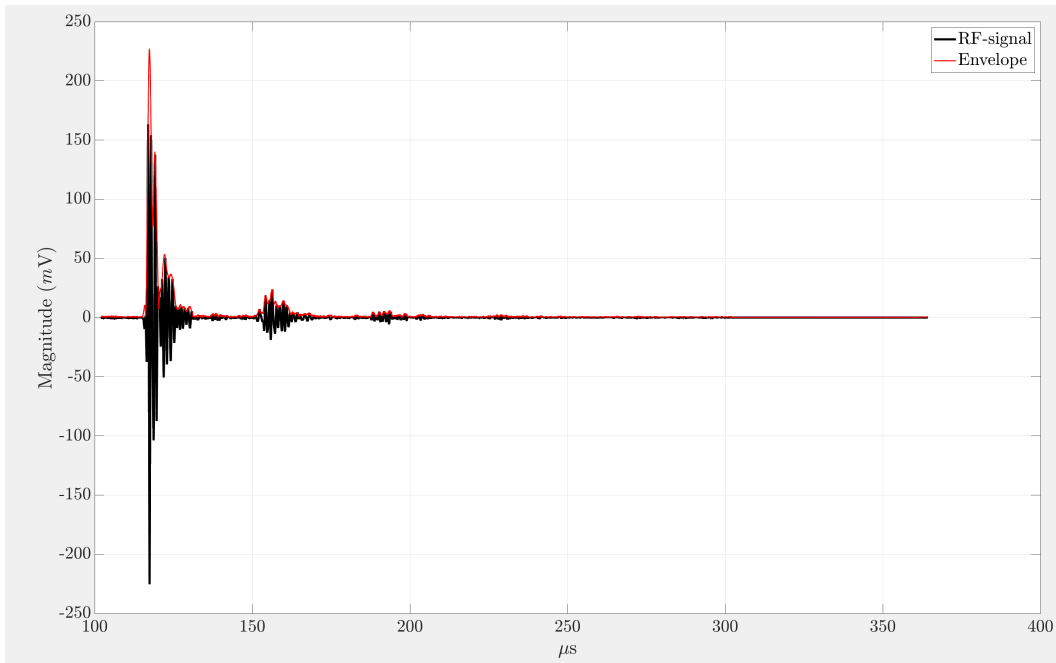


Figure 13: Temporal representation of the recorded RF-signal (black) and the envelope (red) for the artificial funerary urn no2 at $y = 0^\circ$ (inner cavity filled with water and bones). 1MHz configuration, scanner.

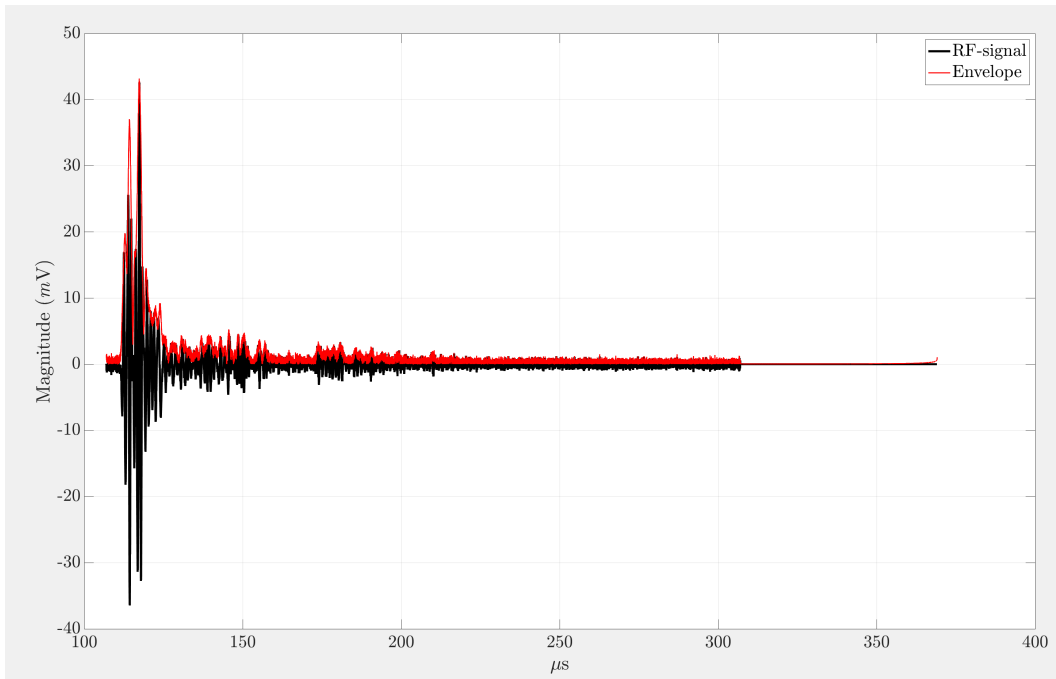


Figure 14: Temporal representation of the recorded RF-signal (black) and the envelope (red) for the artificial funerary urn no3 at $y = 0^\circ$ (inner cavity filled with water, bones and sand). 1MHz configuration, scanner.

345 Figures 15, 16, and 17 show the experimental recorded RF-signal for the urn
 346 no1, no2 and no3 (inner cavity filled with water, water and bones, and water,
 347 bones and sand, respectively) when using the portable inspection unit (500kHz
 348 configuration).

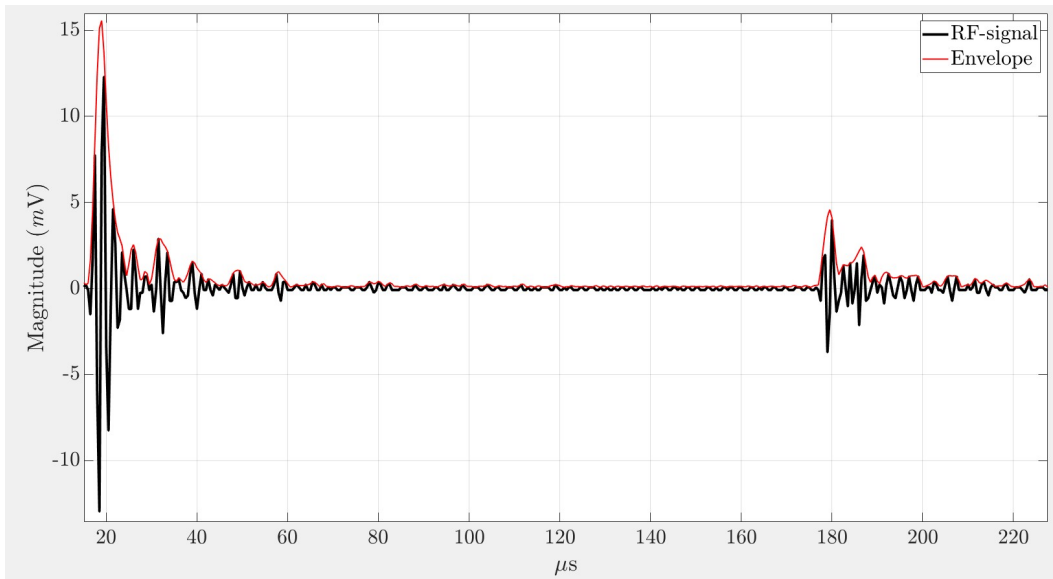


Figure 15: Temporal representation of the recorded RF-signal (black) and the envelope (red) for the artificial funerary urn no1 at $\gamma = 0^\circ$ (inner cavity filled with water). 500kHz configuration, portable inspection unit.

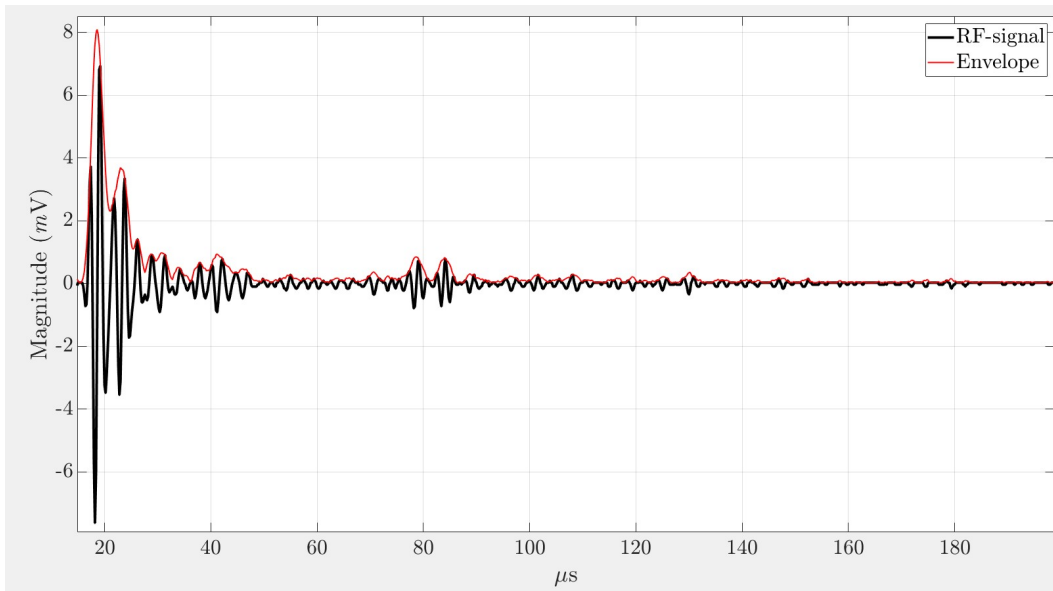


Figure 16: Temporal representation of the recorded RF-signal (black) and the envelope (red) for the artificial funerary urn no2 at $y = 0^\circ$ (inner cavity filled with water and bones). 500kHz configuration, portable inspection unit.

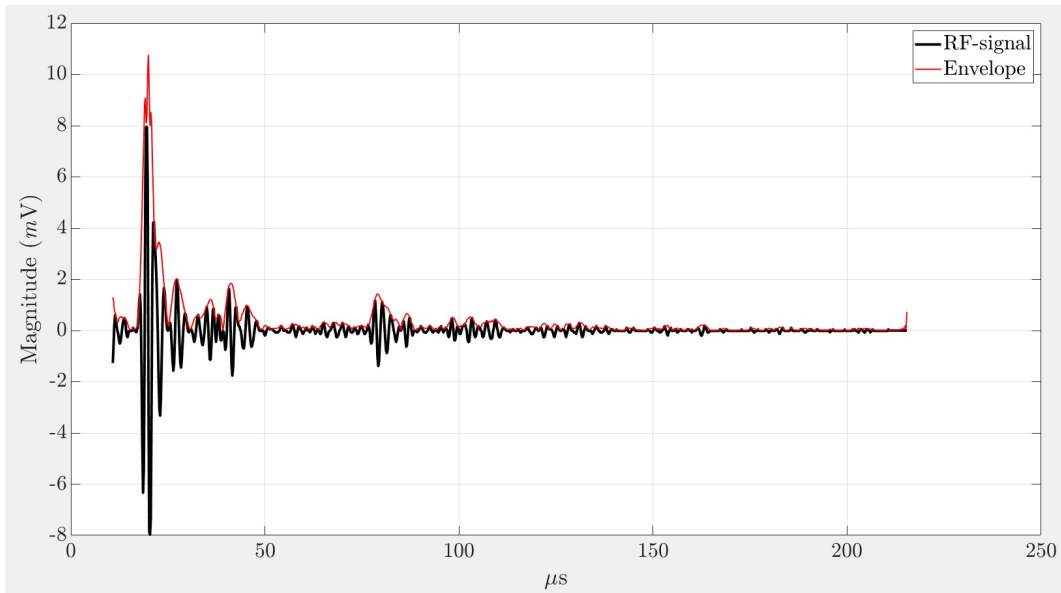


Figure 17: Temporal representation of the recorded RF-signal (black) and the envelope (red) for the artificial funerary urn no3 at $y = 0^\circ$ (inner cavity filled with water, bones and sand). 500kHz configuration, portable inspection unit.

349 *3.2. Prior parameters*

350 Table 1 summarises the geometrical parameters of the three artificial funerary
 351 urns (measured with caliper).

352

Description	Units	Parameters	Values [Average±SD]
<u>Urn no1</u>			
Diameter	mm	D_{out}	129±0.2
Thickness	mm	E_{urn}	8.72±0.2
<u>Urn no2</u>			
Diameter	mm	D_{out}	131±0.2
Thickness	mm	E_{urn}	8.35±0.3
<u>Urn no3</u>			
Diameter	mm	D_{out}	131±0.3
Thickness	mm	E_{urn}	8.67±0.2

Table 1: Mean diameters and thicknesses measured with a caliper of the three artificial funerary urns for 10 positions around the urn.

353 Table 2 summarises the experimental physical parameters of the three clay
354 pieces (Figure 2) from an urn constructed identically to those studied. For each
355 parameter, 10 measurements were taken and the average values were used for the
356 study.

Description	Units	Parameters	Values [Average±SD]
<u>Sample no1</u>			
US velocity ¹	m/s		3171±1.74
US attenuation (1 MHz)	dB/m		0.52±0.02
US attenuation (500 kHz)	dB/m		0.60±0.02
Mass density	kg/m ³		1993.1
<u>Sample no2</u>			
US velocity ¹	m/s		2717±1.3
US attenuation (1 MHz)	dB/m		0.70±0.01
US attenuation (500 kHz)	dB/m		0.59±0.02
Mass density	kg/m ³		1956.4
<u>Sample no3</u>			
US velocity ¹	m/s		3067±1.96
US attenuation (1 MHz)	dB/m		0.80±0.02
US attenuation (500 kHz)	dB/m		0.62±0.01
Mass density	kg/m ³		1949.9
<u>Mean values²</u>			
US velocity	m/s	V_{urn}	2985±238
US attenuation (1 MHz)	dB/m	α_{urn}	0.67±0.016
US attenuation (500 kHz)	dB/m	α_{urn}	0.60±0.011
Mass density	kg/m ³	ρ_{urn}	1966±23.29

¹ Mean value measured for 1 MHz and 500 kHz. ²Mean values used to compare experiments and models for both 1MHz and 500kHz configurations.

Table 2: Experimental physical parameters measured for the three clay pieces by ultrasonic interferometry and Archimedes' principle.

357 Table 3 summarises the experimental *prior* parameters of the surrounding wa-
358 ter, and of the media considered in the inner cavity. When the inner cavity was
359 filled with water, the values considered were those of the water in the tank.

360

Description	Units	Parameters	Values [Average \pm SD]
<u>Water</u>			
Distance ¹	mm	H	86.5 \pm 0.4
Associated TOF	μ s	t_1	117.36 \pm 0.5
US velocity	m/s	V_{water}	1480 \pm 5
US attenuation ²	dB/m	α_{water}	0.0022
Mass density	kg/m ³	ρ_{water}	1000
<u>Bones</u>			
US velocity	m/s	V_{cavity}	3500
US attenuation (1 MHz)	dB/m	α_{cavity}	3
US attenuation (500 kHz)	dB/m	α_{cavity}	2
Mass density	kg/m ³	ρ_{cavity}	2000
<u>Bones with sand</u>			
US velocity	m/s	V_{cavity}	4200
US attenuation (1 MHz)	dB/m	α_{cavity}	3
US attenuation (500 kHz)	dB/m	α_{cavity}	2
Mass density	kg/m ³	ρ_{cavity}	1900

¹ Distances between the transducer and the urn (1MHz configuration, scanner). ² Wave attenuation in water was assumed to be the same at 1 MHz and at 500 kHz.

Table 3: Experimental geometrical and physical parameters of the surrounding water, and of the media contained by the inner cavity.

361 3.3. Modelling of the wave propagation in a funerary urn

362 Figures 18 and 19 show the modelled impulse response, h_M , calculated with
363 the experimental parameters for the urn no1 (using the mean parameters reported
364 in Tables 1 and 2) at $y = 0^\circ$, when the inner cavity was filled with water, and the
365 convolution with the pseudo-Ricker wavelet (ie., the modelled signal $s(t)$, Eq. 5).

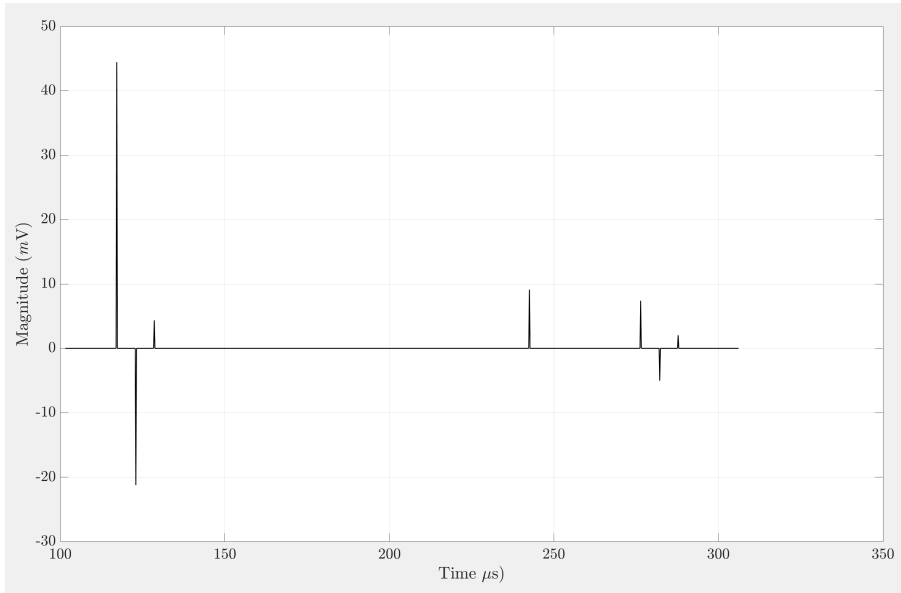


Figure 18: Temporal representation of the modelled impulse response, $h_M(t)$, calculated with the experimental parameters for the urn no1 at $y = 0^\circ$, when the inner cavity was filled with water.

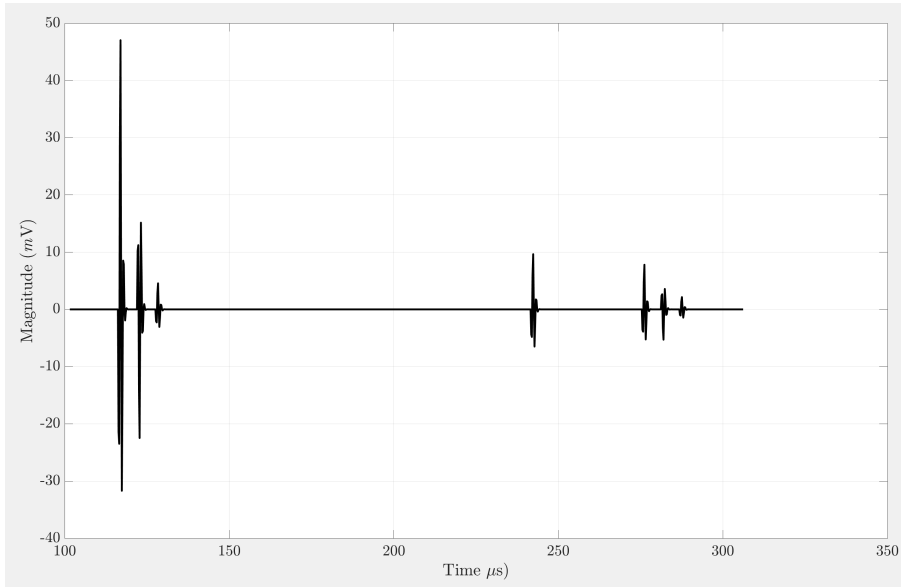


Figure 19: Temporal representation of the modelled signal, $s(t)$, calculated by the convolution of the pseudo-Ricker wavelet (1 MHz) and the modelled impulse response, $h_M(t)$.

366 *3.4. Characterisation of the funerary urn*

367 Tables 4 and 5 show the estimated geometrical and physical parameters of
 368 the three artificial funerary urns, and the lower and upper bounds chosen. The
 369 relative deviation expressed as a percentage (%) between the experimental and
 370 model parameters was defined by:

$$100 \times \frac{s_{mod}}{s_{mea}} - 1 \quad (17)$$

Parameters	Units	Bounds [Lower-Upper]	Experimental [Average]	Estimated [Modelled]	Deviations (Eq.17)
<u>Urn no1</u>	Empty				
E_{urn}	mm	[6-12]	8.72	8.73	0.11%
V_{urn}	m/s	[2000-3500]	2985	2983	0.07%
α_{urn} (1MHz)	dB/m	[0-1]	0.67	0.69	2.90%
ρ_{urn}	kg/m ³	[1500-2500]	1966	1966	0.01%
<u>Urn no2</u>	Bones				
E_{urn}	mm	[6-12]	8.35	8.55	2.34%
V_{urn}	m/s	[2000-3500]	2985	3037	1.71%
α_{urn} (1MHz)	dB/m	[0-1]	0.67	0.71	5.63%
ρ_{urn}	kg/m ³	[1500-2500]	1966	2019	2.63%
<u>Urn no3</u>	+sand				
E_{urn}	mm	[6-12]	8.67	9.01	3.77%
V_{urn}	m/s	[2000-3500]	2985	3224	7.41%
α_{urn} (1MHz)	dB/m	[0-1]	0.67	0.72	6.94%
ρ_{urn}	kg/m ³	[1500-2500]	1966	2110	6.82%

Table 4: Estimated geometrical and physical parameters of the three artificial funerary urns at $y = 0^\circ$. 1MHz configuration, scanner

Parameters	Units	Bounds [Lower-Upper]	Experimental [Average]	Estimated [Modelled]	Deviations (Eq.17)
Urn no1 Empty					
E_{urn}	mm	[6-12]	8.72	8.70	0.23%
V_{urn}	m/s	[2000-3500]	2985	2990	0.17%
α_{urn} (500kHz)	dB/m	[0-1]	0.60	0.60	0.01%
ρ_{urn}	kg/m ³	[1500-2500]	1966	1965	0.05%
Urn no2 Bones					
E_{urn}	mm	[6-12]	8.35	8.45	1.18%
V_{urn}	m/s	[2000-3500]	2985	3025	1.32%
α_{urn} (500kHz)	dB/m	[0-1]	0.60	0.66	9.37%
ρ_{urn}	kg/m ³	[1500-2500]	1966	1987	1.06%
Urn no3 +sand					
E_{urn}	mm	[6-12]	8.67	8.98	3.45%
V_{urn}	m/s	[2000-3500]	2985	3035	1.65%
α_{urn} (500kHz)	dB/m	[0-1]	0.60	0.71	15.49%
ρ_{urn}	kg/m ³	[1500-2500]	1966	2200	10.64%

Table 5: Estimated geometrical and physical parameters of the three artificial funerary urns. 500kHz configuration, portable inspection unit.

371 The acceptability criterion was defined so that the difference between the ex-
372 perimental and model parameters was less than 10^{-5} . Under these conditions, the
373 total number of iterations was 14 for the parameters estimated from the modelled
374 signals for urns 1 and 2, and 12 for urn 3. The computation time was 1 sec per
375 iteration. The convergence of the algorithm in all the trials of this work did not
376 exceed 15 seconds.

377

378 Figure 20 shows steps 1, 2, 3, 4, 7 and 14 (the final one) for urn no1 at $y = 0^\circ$
379 (inner cavity filled with water), for geometrical parameters given in Table 1, and
380 the mean physical parameters given in Tables 2 and 3.

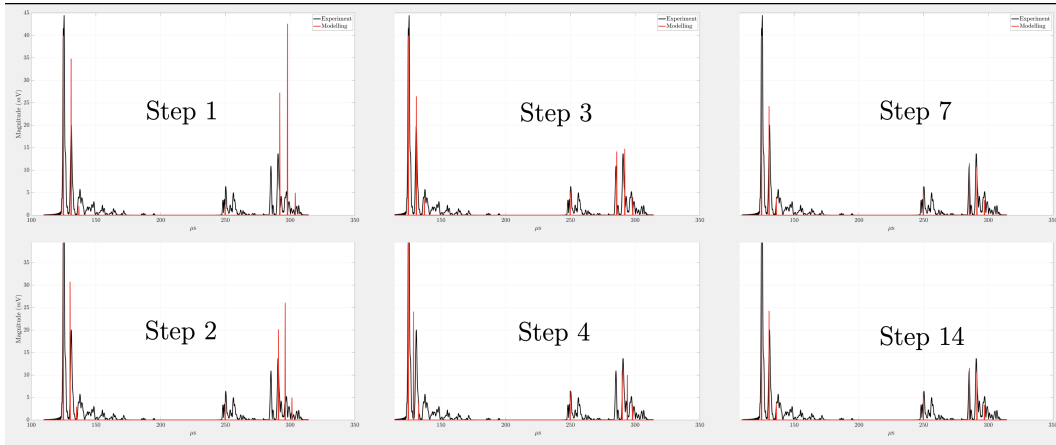


Figure 20: Temporal representation of the envelopes of the experimental RF-signal (black) obtained for the urn no1 and of the modelled response, h_M , obtained by the algorithm at iterations no1, 2, 3, 4, 7, and 14 (red). 1MHz configuration, scanner

381 The final results, (i.e., the modelled signal $s(t)$) are presented in Figure 21 for
 382 the 1MHz-configuration (scanner), and in Figure 22 for the 500kHz-configuration
 383 (portable inspection unit).

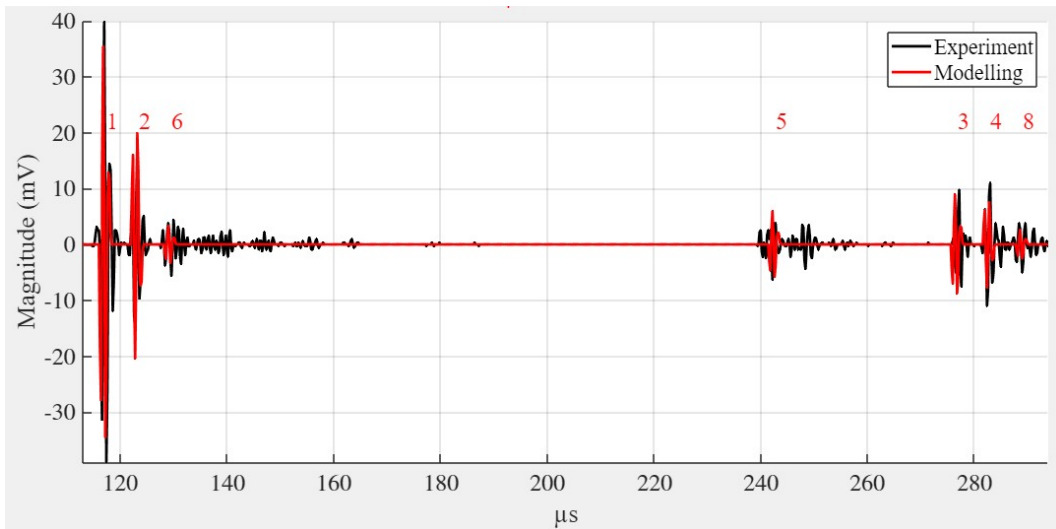


Figure 21: Temporal representation of the experimental recorded RF-signal (black) and the modelled signal, $s(t)$, (red) for the artificial funerary urn no1 at $y = 0^\circ$ (inner cavity filled with water). 1MHz configuration, scanner.

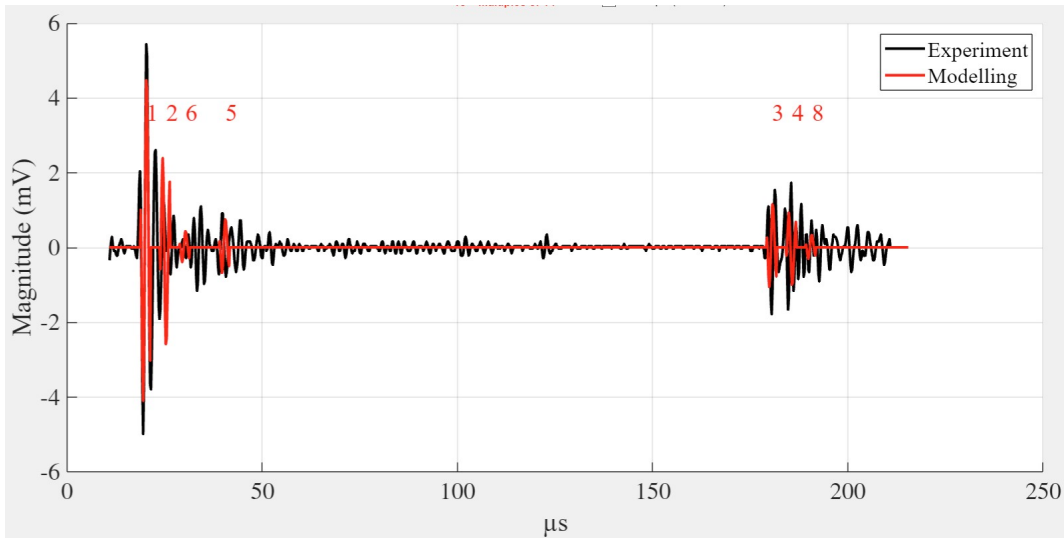


Figure 22: Temporal representation of the experimental recorded RF-signal (black) and the modelled signal, $s(t)$, (red) for the artificial funerary urn no1, (inner cavity filled with water). 500kHz configuration, portable inspection unit

384 The numbers (in red) in the figures correspond to the different TOF (Figure 9).
 385 Figures 23, 24, 25 and 26 show the temporal representations of the experimental
 386 RF-signals obtained for the artificial funerary urns when the inner cavity was filled
 387 respectively with water and bones (urn no2), and water, bones and sand (urn no3),
 388 and the corresponding modelled signals, for 1MHz-configuration (scanner), and
 389 the 500kHz-configuration (portable inspection unit). In these cases, the physical
 390 parameters for the inner cavity were those of the bones, without and with sand
 391 (Table 3).

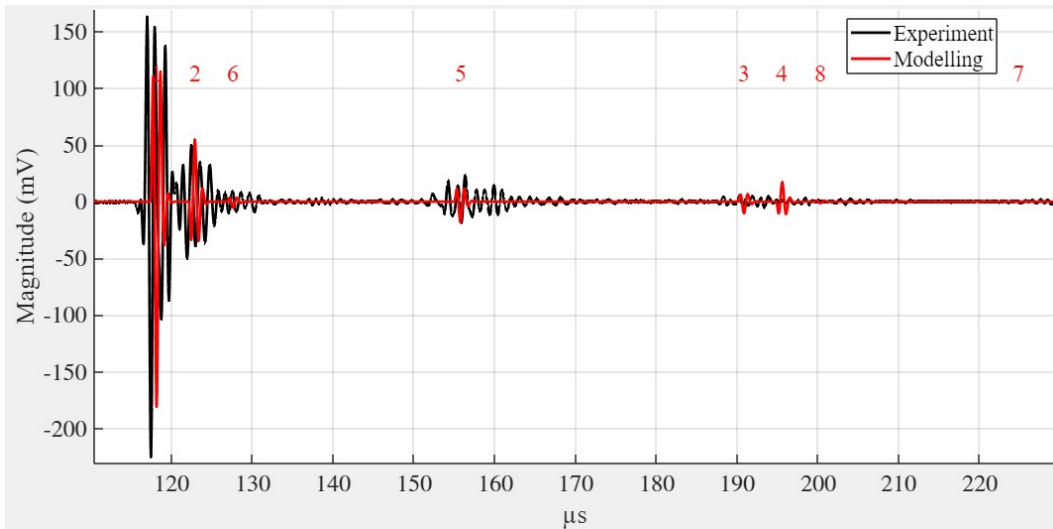


Figure 23: Temporal representation of the experimental recorded RF-signal (black) and the modelled signal, $s(t)$, (red) for the artificial funerary urn no2 at $y = 0^\circ$ (inner cavity filled with water and bones). 1MHz configuration, scanner.

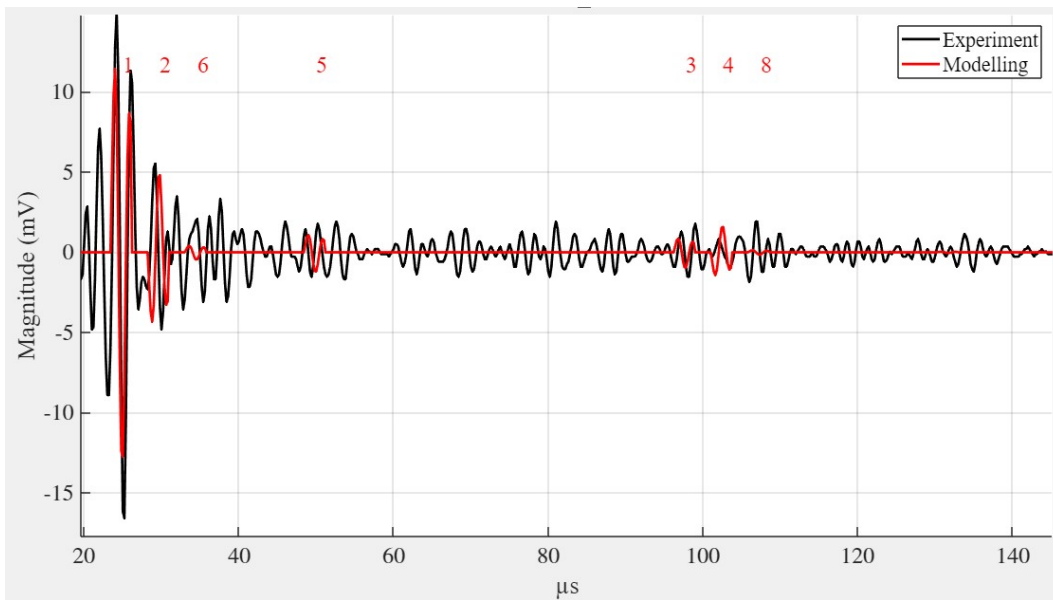


Figure 24: Temporal representation of the experimental recorded RF-signal (black) and the modelled signal, $s(t)$, (red) for the artificial funerary urn no2 (inner cavity filled with water and bones). 500kHz configuration, portable inspection unit.

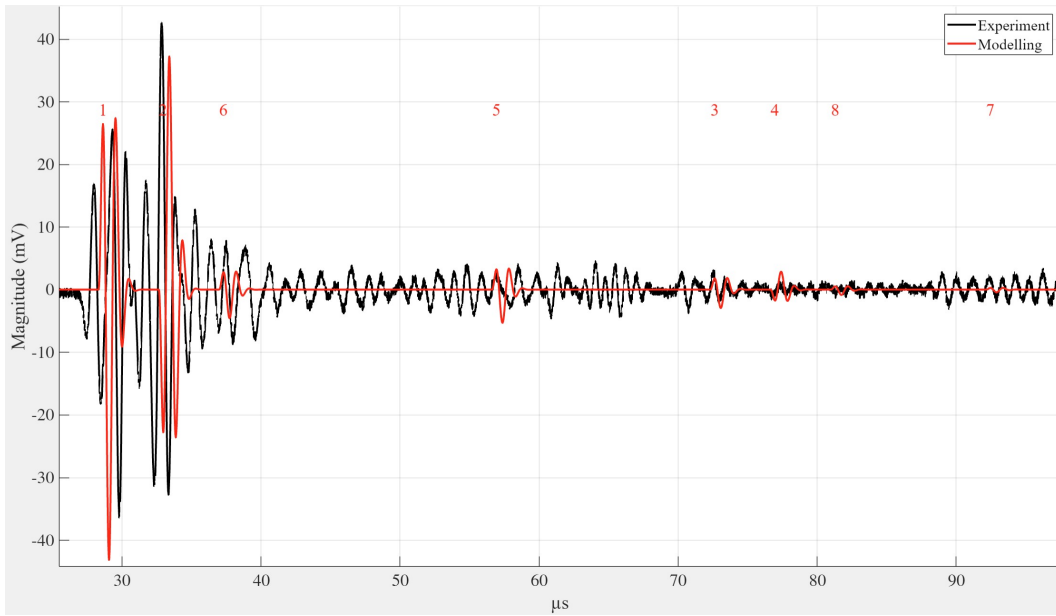


Figure 25: Temporal representation of the experimental recorded RF-signal (black) and the modelled signal, $s(t)$, (red) for the artificial funerary urn no3 at $y = 0^\circ$ (inner cavity filled with water, bones and sand). 1MHz configuration, scanner.

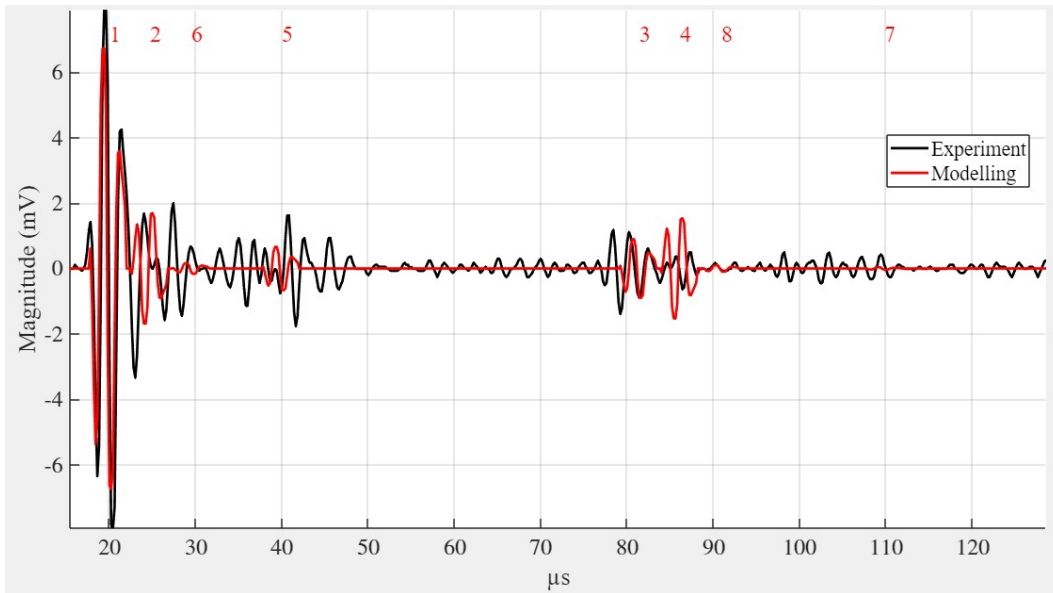


Figure 26: Temporal representation of the experimental recorded RF-signal (black) and the modelled signal, $s(t)$, (red) for the artificial funerary urn no3 (inner cavity filled with water, bones and sand). 500kHz configuration, portable inspection unit.

392 *3.5. Rotational scanning*

393 Figures 27 and 28 show the model parameters of the artificial funerary urns,
 394 when the inner cavity was filled with water (no1), with water and bones (no2), and
 395 with water, bones and sand (no3), for 8 positions over 360° and both experimental
 396 configurations.

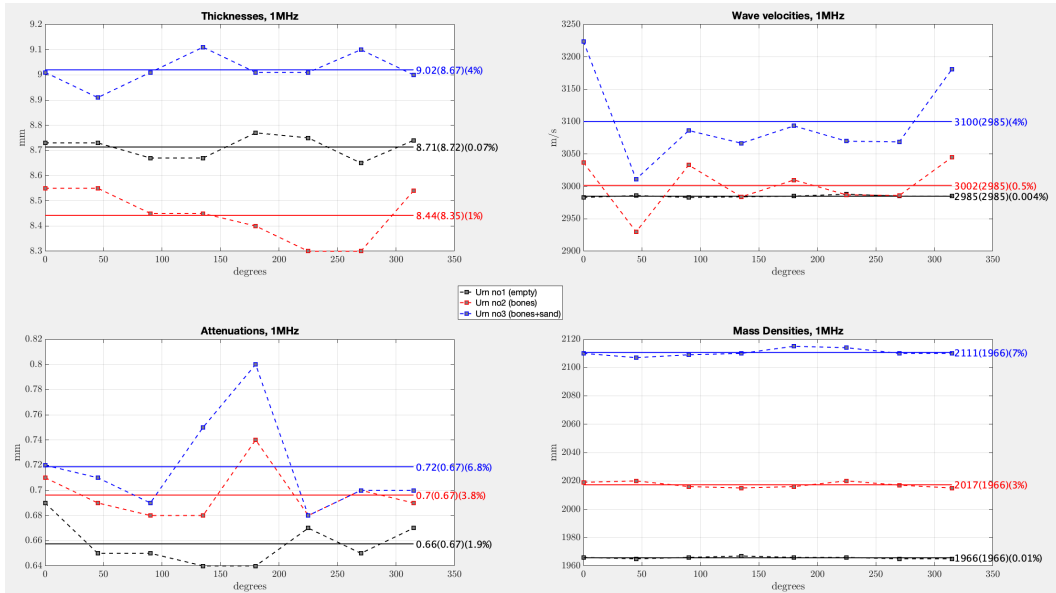


Figure 27: Angular parametric study of the urns. The symbols represent the values per angle for the urns, when the inner cavity was filled with water (black), water and bones (red), and water, bones and sand (blue). The solid line corresponds to the average of the parameters. The mean parameter is shown on the right, along with the reference and deviation (percentage) values. 1MHz configuration, scanner.

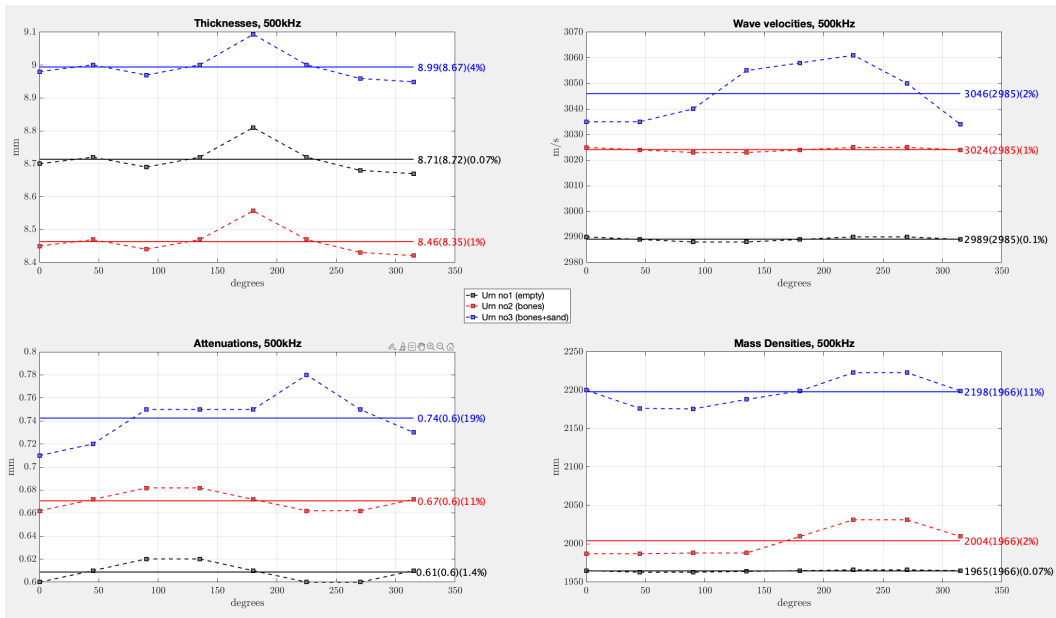


Figure 28: Angular parametric study of the urns. The symbols represent the values per angle for the urns, when the inner cavity was filled with water (black), water and bones (red), and water, bones and sand (blue). The solid line corresponds to the average of the parameters. The mean parameter is shown on the right, along with the reference and deviation (percentage) values. 500kHz configuration, portable inspection unit.

397 Tables 6 and 7 show the mean parameters of the artificial funerary urns for 8
 398 angles of incidence over 360°, for both experimental configurations.
 399

Parameters	Units	Experimental [Average]	Estimated [Average \pm SD]	Deviations (Eq.17)
Urn no1	Empty			
E_{urn}	mm	8.72	8.71 \pm 0.04	0.07%
V_{urn}	m/s	2985	2985 \pm 1.5	0.02%
α_{urn} (1MHz)	dB/m	0.67	0.66 \pm 0.02	1.9%
ρ_{urn}	kg/m ³	1966	1966 \pm 0.7	0.01%
Urn no2	Bones			
E_{urn}	mm	8.35	8.44 \pm 0.1	1.1%
V_{urn}	m/s	2985	3002 \pm 35.5	0.55%
α_{urn} (1MHz)	dB/m	0.67	0.7 \pm 0.02	3.77%
ρ_{urn}	kg/m ³	1966	2017 \pm 2	2.54%
Urn no3	+sand			
E_{urn}	mm	8.67	9.02 \pm 0.06	3.88%
V_{urn}	m/s	2985	3100 \pm 64.2	3.71%
α_{urn} (1MHz)	dB/m	0.67	0.72 \pm 0.04	6.78%
ρ_{urn}	kg/m ³	1966	2111 \pm 2.4	6.85%

Table 6: Estimation of the parameters of an artificial funerary urn for 8 angles of incidence over 360°. 1MHz configuration, scanner.

Parameters	Units	Experimental [Average]	Estimated [Average±SD]	Deviations (Eq.17)
Urn no1	Empty			
E_{urn}	mm	8.72	8.71 ± 0	0.07%
V_{urn}	m/s	2985	2989 ± 0.8	0.14%
α_{urn} (500kHz)	dB/m	0.60	0.61 ± 0.01	1.44%
ρ_{urn}	kg/m ³	1966	1965 ± 1.1	0.07%
Urn no2	Bones			
E_{urn}	mm	8.35	8.46 ± 0	1.34%
V_{urn}	m/s	2985	3024 ± 0.8	1.29%
α_{urn} (500kHz)	dB/m	0.60	0.67 ± 0.01	10.53%
ρ_{urn}	kg/m ³	1966	2004 ± 18.2	1.89%
Urn no3	+sand			
E_{urn}	mm	8.67	8.99 ± 0	3.6%
V_{urn}	m/s	2985	3046 ± 10.5	2.0%
α_{urn} (500kHz)	dB/m	0.60	0.74 ± 0.02	19.19%
ρ_{urn}	kg/m ³	1966	2198 ± 17	10.56%

Table 7: Estimation of the parameters of an artificial funerary urn for 8 angles of incidence over 360°. 500kHz configuration, portable inspection unit.

400 4. Discussion

401 In this paragraph, we will discuss the results and their interpretation from the
402 point of view of the modelling assumptions, as well as from the point of view
403 of the *in situ* non-destructive testing of ancient funerary urns. Future research
404 directions will also be highlighted.

405 4.1. Presence of water in the urn

406 What needs to be observed in our study is the presence of water in the urns
407 in our study. Whatever the experimental device, the urns are always filled with
408 water, to allow the ultrasonic waves (at the frequencies considered, 500 kHz and
409 1 MHz) to propagate with low variations in attenuation (<1 dB/m). This is obvi-
410 ously a very important limitation of this study, as the urns found at archaeological
411 excavation sites are not (necessarily) filled with water. However, it is important
412 to know that ultrasounds can also be propagated through the air. This is a spe-
413 cial technology that will have to be tested in future experiments. There have been
414 major technical advances in this field, [14].

415 *4.2. Study of RF- and modelled signals*

416 The signals for a cavity containing water, water and bones, and water, bones
417 and sand, are different and more or less noisy; signal-to-noise ratio between 3 dB
418 (water, bones and sand) to 15 dB (water only). The first echo (t_1) on the first
419 water/urn interface (whatever the urn) has a higher dynamic range than the other
420 wave packets that follow (t_2 to t_8). This is due to the acoustical impedance con-
421 trast between the water and the clay. The acoustical impedance of the clay (≈ 6
422 MRays) is greater than that of the water (≈ 1.5 MRays), and the reflection coef-
423 ficient (Eq. 11) between the two media is high (≈ 0.7). The acoustical intensity
424 of the waves transmitted into the urn is therefore greatly reduced. Conversely, the
425 waves reflected in the inner cavity do not lose too much energy.

426 Despite all the variations in acoustical intensity, the wave packets are fairly
427 well identifiable, although in the case of urn no3 (water, bone and sand), the
428 signal-to-noise ratio is low, and the wave packets more difficult to identify. With
429 regard to this last point, the benefits of the model are significant. The wave packets
430 are identified on the modelled signals, enabling the corresponding wave packets
431 to be identified on the RF-signals by superposition whatever the signal-to-noise
432 ratio.

433 In the case of an empty urn (i.e. filled with water only), the TOF t_7 does not
434 appear. This TOF corresponded to the second reflection inside the inner cavity.
435 The wave velocity being that of the waves in water, it was slower than in the wall
436 of the urn, and the TOF was longer, and higher than the recording time of the
437 RF-signals. For urns containing bones and/or sand, the waves were accelerated
438 (and attenuated) in the inner cavity because the acoustical parameters were higher
439 than those of water alone, and then the multiple echoes arrived early enough to be
440 recorded.

441 *4.3. Relevance of the analytical parametric model*

442 To define the analytical parametric model composed of four interfaces, a num-
443 ber of hypotheses were introduced, not all of which had the same impact on the
444 model of the signals. The chosen number of back-and-forth travels (equal to 2)
445 was well suited. For the empty urn, it would be possible to increase the number of
446 back-and-forth travels (and therefore the number of TOF), but some of these TOF
447 would be outside the time limits of the recorded signals, and therefore unusable.
448 The empty urn was not the main objective of the study and there is no inter-
449 est in modifying the recording times or increasing the number of points, which
450 would result in longer processing times. With bones and sand (a more realistic

451 archaeological condition), all the TOF would be closer together (the wave veloc-
 452 ities increase), but as attenuation would be greater, the wave packets would not
 453 be distinguished from the noise. Adding back-and-forth travels and TOF to the
 454 model would also introduce a bias which would add processing time. In fact, the
 455 parametric identification algorithm would then have more difficulty in converging
 456 quickly. Tests were carried out which confirmed these remarks.

457 Ray theory was adopted for this modelling process. This means that the in-
 458 cident wave is symbolised by a straight ray perpendicular to the interface. This
 459 assumption is valid at high frequencies, i.e. short wavelengths compared with
 460 the thickness and diameter of the urn (Table 8). The higher the frequency, the
 461 greater the number per wavelength, and the more valid this high-frequency as-
 462 sumption. Wave packets on 1MHz signals are better resolved (i.e. visible), but
 463 noisier, whereas 500kHz signals are less noisy, but the wave packets are harder
 464 to identify. For non-destructive testing to be extended to other archaeological
 465 objects, such as urns from different periods, pottery or vases, and for this high-
 466 frequency hypothesis to be plausible, the frequency (and the wavelength) of the
 467 transducers used will have to be adapted to the dimensions/thickness of the ob-
 468 jects.

469 The calculated deviations (Tables 6 and 7) between the estimated and experi-
 470 mental parameters, at several points around the urns, are greater at 500 kHz than
 471 1 MHz, particularly for the urns containing bones, especially for the attenuation
 472 measurements. This is also due to the lower resolution of the signals at low fre-
 473 quencies. This can be seen in Figures 22, 24 and 26. The superposition of the RF
 474 and modelled signals is correct for TOF, but less good for amplitudes, particularly
 475 for t_6 , or t_7 .

	Units	Water	Water	Urn	Urn
Wave velocity	m/s	1480	1480	2985	2985
Frequency	MHz	0.5	1	0.5	1
Wavelength (λ)	mm	≈ 3	≈ 1.5	≈ 6	≈ 3
Thickness E_{urn}	mm	≈ 8.6	≈ 8.6	≈ 8.6	≈ 8.6
Number of λ		≈ 3	≈ 6	≈ 1.5	≈ 3
Diameter D_{out}	mm	≈ 130	≈ 130	≈ 130	≈ 130
Number of λ		≈ 43.3	≈ 86.6	≈ 21	≈ 43.3

Table 8: Values used for high-frequency approximation.

476 Shear waves were neglected in this work, considering that their attenuation

477 was very strong in the clay the urns were made of, but also that the incidence of
478 the transmitted wave was very precisely normal to the water/urn interface. This
479 hypothesis is discussed. At a distance of $H = 86.5$ mm, the angular aperture of
480 the ultrasonic beam from the 1MHz-transducer is less than 3° . As the radius
481 of curvature of the urn was much larger, the wavefront and the water/urn inter-
482 faces were considered to be parallel, so that any mode conversion (compression to
483 shear waves, and vice versa) was highly negligible compared with the refraction
484 of compression waves. For the 500kHz-transducer, the mode conversion prob-
485 lem did not arise because the transducer was placed in contact with the urn, with
486 an angular aperture of 2° . There was therefore no propagation of shear waves
487 in the urn. It should also be noted that the critical angle for compression waves,
488 $\sin^{-1}(V_{\text{water}}/V_{\text{urn}})$, is 30° . Beyond this angle of incidence, no more compression
489 waves propagate. This angle could be exceeded in the experiments carried out in
490 this study.

491 Also, a layer of clay is about 20 mm high, so the focal area (16 mm^2) can
492 cover one or two layers. From one layer to the next, the acoustical properties of
493 the clay may vary slightly. This explains why the estimated and the experimental
494 values differ with deviations lower than 10%. This also explains the additional
495 wave packets that appear in the RF-signals (most pronounced at 1 MHz) after the
496 TOF (t_5) and (t_6), and that do not appear in the analytical parametric model, as they
497 are not taken into account deliberately. Numerical modelling process, using finite-
498 element modelling for example, makes it possible to add these physical consider-
499 ations to the model, and to highlight multiple propagation phenomena difficult to
500 understand. But they have first nothing to add to the objective of the identification
501 of urns containing or not containing bones. And, second, they would add diffi-
502 culties and constraints (the equations for calculating the times (Eqs. 8, 9 and 10)
503 would be different), and, consequently, the processing time would increase which,
504 for *in situ* use, would not be relevant.

505 The strongest assumptions made on the analytical parametric model concern
506 the physical parameters (wave velocity and attenuation, mass density) used as
507 *known (prior)* information, and considered to be homogeneous and identical for
508 all the urns. Even if the urns were made by the same specialist, they would contain
509 intra-urn differences. Those intra-urn differences surely also existed in ancient
510 urns. The acoustical parameters measured in the clay (used to define the bounds
511 of the optimisation algorithm) were of the same order of magnitude but there were
512 differences between the 3 small samples. The standard deviation is 238 m/s for the
513 velocities, which is significant and shows that the assumption of homogeneity of
514 the clay and the urn can be questioned. There are therefore inter-urn differences,

515 which was expected, but also intra-urn differences, which can be significant. In
516 the context of an *in situ* NDT procedure, the parametric information on the objects
517 will not be known. It will be necessary to have information about the material
518 (type of clay or ceramic, or object material) or to increase the values of the lower
519 and upper bounds of the algorithm, thus increasing the processing time.

520 The parametric values, as well as the deviations between the experimental and
521 estimated parameters, are larger when the urn was filled with water, bone and
522 sand, and become smaller when only water was present. Similarly, the standard
523 deviations are greater for full urns than for empty urns, and for all parameters ex-
524 cept mass density. This can be explained by the fact that with bones and sand/or
525 no sand, the waves were more strongly attenuated (2-3 dB/m) in the inner cavity
526 because of multiple diffraction phenomena and/or intrinsic absorption. The bones
527 diffracted the waves in the inner cavity in all spatial directions, and the sand ab-
528 sorbed a large proportion of the acoustical intensity. The reflected waves (those
529 returning to the transducer) had less energy. The wave packets were less distin-
530 guishable from the surrounding noise, and the analytical parametric model had
531 difficulty finding the 2nd order TOF (t_5 to t_8).

532 The values of the physical parameters of bone and sand also need to be dis-
533 cussed. Laboratory measurements of velocity, attenuation and mass density car-
534 ried out on "characteristic mixtures" of bones and of sand cannot be generalised,
535 although these mean values (*in situ*) should always be of the same order of mag-
536 nitude. The values measured do not take into account the type of bone - there
537 are very large differences between cortical and porous bone - or the density (com-
538 pactness) of the mixture of fragmentary bones, which increases still further in the
539 presence of sand. If the bones are all porous and the sand not very compact, the
540 wave velocities and attenuations will be lower (close to what they would be if they
541 were propagated in water) than for a mixture of cortical bone and very compact
542 sand.

543 4.4. Reproducibility and resistance of the parametric identification algorithm

544 The optimisation algorithm based on the Levenberg-Marquardt method is well
545 suited to the task of identifying the geometrical and physical parameters of a fu-
546 nerary urn, and detecting the presence or absence of bone inside it. It is fast (a
547 few seconds), and accurate (deviations $< 10\%$). However, when the urn contains
548 bones and sand, the algorithm converges with greater difficulty (greater error) on
549 the TOF of the third and fourth interfaces, and remains focused on the first two,
550 with greater signal-to-noise ratio (lower noise). For bone-containing urn, the er-
551 rors were higher, since the minimisation algorithm sought the same number of

552 variables, for the same threshold (10^{-5}), but could converge on several minima,
553 reach the upper bounds, and ultimately find no solution, or a false solution. The
554 Levenberg-Marquardt algorithm is less well-suited to very noisy signals, but is
555 still suitable for our study.

556 With the rotational study, the algorithm was tested for reproducibility and re-
557 sistance. Values at 500 kHz are more homogeneous and standard deviations lower
558 for all parameters. Low-frequency signals are less well resolved than those ob-
559 tained at 1 MHz. Errors are greater in relation to the same reference, but con-
560 versely, the signals are very similar and less sensitive to position, and therefore
561 to variations in the urn, than those at 1 MHz. The algorithm is highly resistant to
562 variations, and the lower the frequency, the greater the reproducibility.

563 There are also problems related only to numerical algorithms. For example is
564 the strong non-linearity of the cost function C (Eq. 16) makes the gradient-based
565 minimisation algorithms only converge to the solution if they are started close
566 to the global minimum, i.e., provided that the initial model is already of good
567 “quality” (low parameter bias). We know that, in this analytical and numerical
568 form, the proposed algorithm has application limits. We have to admit that it
569 will make mistakes, unless two possible ways are forwarded. Firstly, the linear
570 propagation assumption could be revised, and more complex and time-consuming
571 non-linear algorithms introduced, such as the full waveform inversion method,
572 [15]. Secondly, it would be possible to use commercial (or home-made) software,
573 which can be expensive, and would increase processing time. This is not desirable.

574 **5. Conclusions**

575 The aim of this work was to study the feasibility of non-destructive ultrasonic
576 testing of ancient funerary urns. More specifically, the aim was to propose a
577 method for identifying the geometrical and physical parameters of an urn, and
578 detecting the presence or absence of bone inside it.

579 Firstly, an analytical model based on assumptions and approximations relating
580 to ultrasonic wave propagation in urns was proposed. This mathematical model
581 is based on the convolution of a series of Dirac distributions, which models the
582 impulse response of the propagating medium, by a pseudo-Ricker wavelet, which
583 models the impulse response of the ultrasonic device. The Dirac distribution se-
584 ries is representative of homogeneous propagation of compression waves only,
585 in an isotropic, weakly dispersive medium. The pseudo-Ricker wavelet is repre-
586 sentative of linear and causal electro-acoustics. This model represents wave/urn
587 interaction phenomena very well when the urn is filled with water. It remains

588 valid when the urn contains bones, or bones and sand. Although slightly biased,
589 it correctly detects the presence or absence of bone fragments. However, it does
590 not allow us to discriminate between buried and burnt bones, the nature of the
591 fragments (trabecular or cortical), or the quantity of bone contained in the urn.
592 Several improvements need to be considered.

593 The model is limited to four parameters that only concern the urn: urn thick-
594 ness, wave velocity and attenuation in the clay, and clay mass density. Propa-
595 gation in the inner cavity, which is assumed to be known in this study, should
596 be introduced into the model, wave/bone interaction phenomena should be taken
597 into account, and the number of variables should be increased to seven, with the
598 addition of bone and sand velocities, attenuations and mass densities.

599 The parametric identification algorithm, based on the minimisation of a func-
600 tional between a measure and a model, is effective for four variables, and en-
601 ables us to correctly identify a funerary urn and detect the presence or absence
602 of bone inside it. Deviations between experimental and estimated parameters do
603 not exceed 10%, which is acceptable. Convergence, computation time, resilience
604 and, more generally, the benefit/performance ratio are very favourable to *in situ*
605 use. Adapted to the linear modelling process, limited to the wave/urn interac-
606 tion phenomena alone, the processing time is less than 15 seconds on average (a
607 few minutes for a complete examination) with a result that enables a specialist to
608 compare two urns with each other. But we would have to test other configurations,
609 other materials, without any *known or prior* information, and adapt it to a seven-
610 variable modelling process. This would require a paradigm shift from a linear
611 to a non-linear approach. Modelling process would be more complex, requiring
612 more in-depth numerical notions. Processing times would increase sharply, and
613 the benefit/performance ratio would have to be discussed, particularly in relation
614 to the precision expected by archaeologists and ceramists.

615 From our point of view, it would be preferable to keep on with linear modelling
616 process, which is simpler and faster, and work on AI algorithms (deep learning,
617 for example) that could take into account the model's imperfections and devia-
618 tions, and enable us to establish a classification of the materials used for ancient
619 funerary urns; different clays or materials, different types of manufacturing pro-
620 cess, different shapes; and discriminate the nature of the inner cavity.

621 **Declaration of Competing Interest**

622 The authors declare no conflict of interest. The funders had no role in the
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