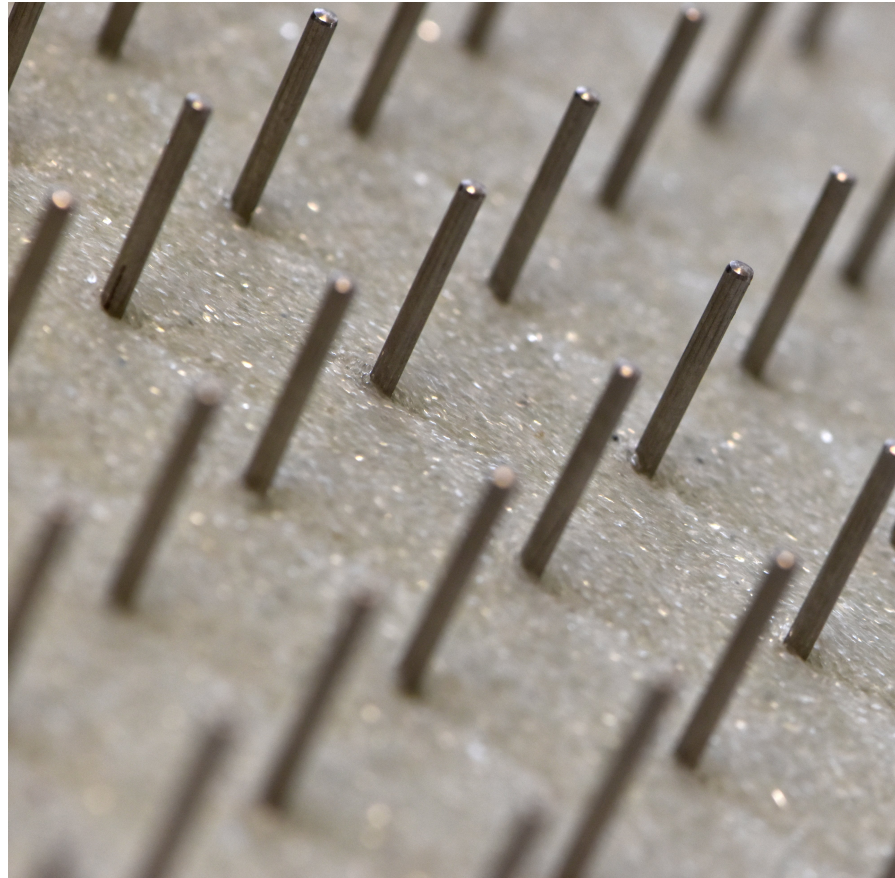


# Flow of a granular material through a forest of obstacles



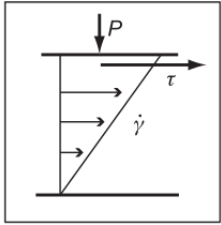
Baptiste Darbois Texier, Yann Bertho, Philippe Gondret

Laboratoire FAST, CNRS, Université Paris Saclay

AERC 9-12th April 2024 Leeds

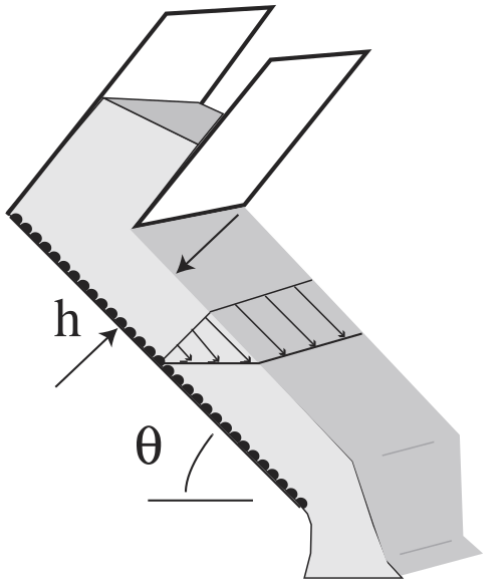


# Rheology of dry granular materials

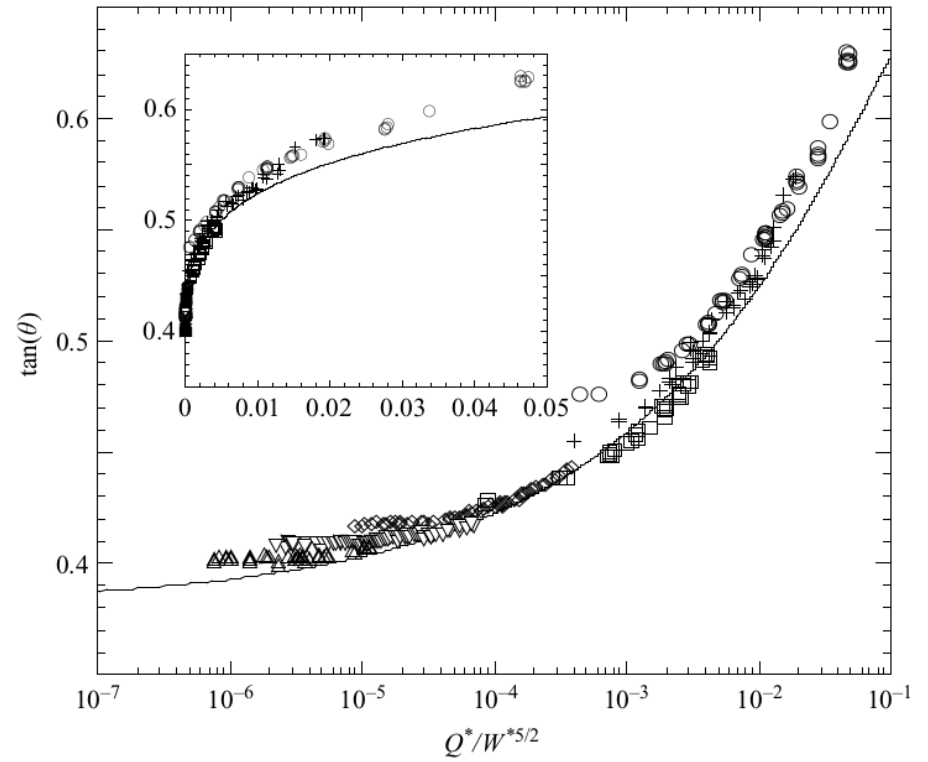


$$\tau = \mu(I)P$$

$$I = \frac{d\dot{\gamma}}{\sqrt{P/\rho_g}}$$



$$\frac{\tau}{P} = \tan \theta$$



$$\mu(I) = \tan \theta_1 + \frac{\tan \theta_2 - \tan \theta_1}{1 + I_0/I}$$

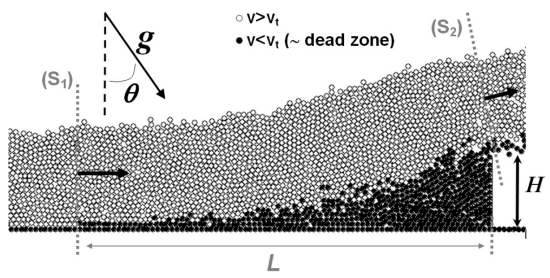
Pouliquen, O. (1999) Phys. Of Fluids.

GDR MiDi (2004) EPJ E.

Jop, P., Forterre, Y., & Pouliquen, O. (2005) JFM.

Jop, P., Forterre, Y., & Pouliquen, O. (2006) Nature.

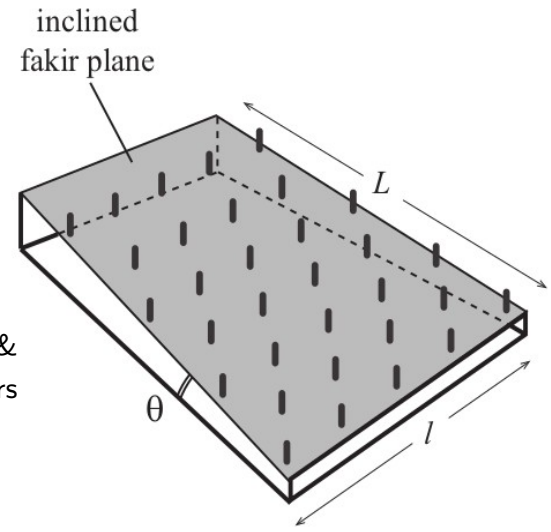
# Obstacles on inclined planes



Faug, T., Beguin, R., & Chanut, B. (2009) PRE

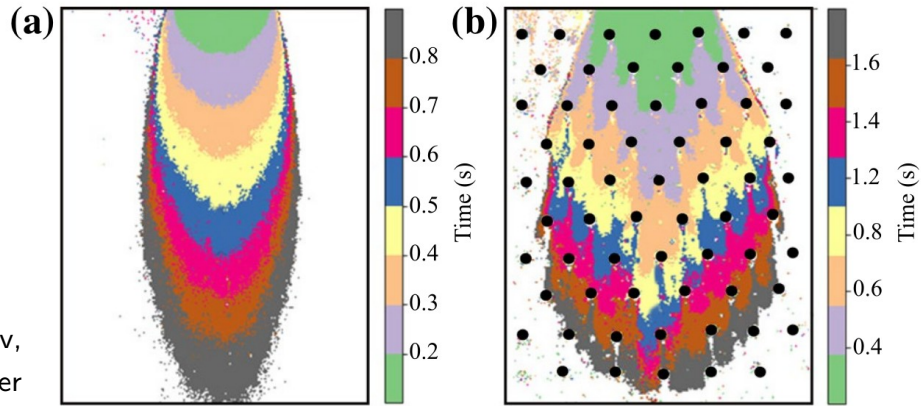


Benito, J., Bertho, Y., Ippolito, I., & Gondret, P. (2012) Europhysics Letters



Cui, X., & Gray, J. (2013) JFM

Luong, T.H., Baker, J.L. & Einav, I. (2020) Granular Matter



Effect of a forest of pillar on the stationary granular flow?

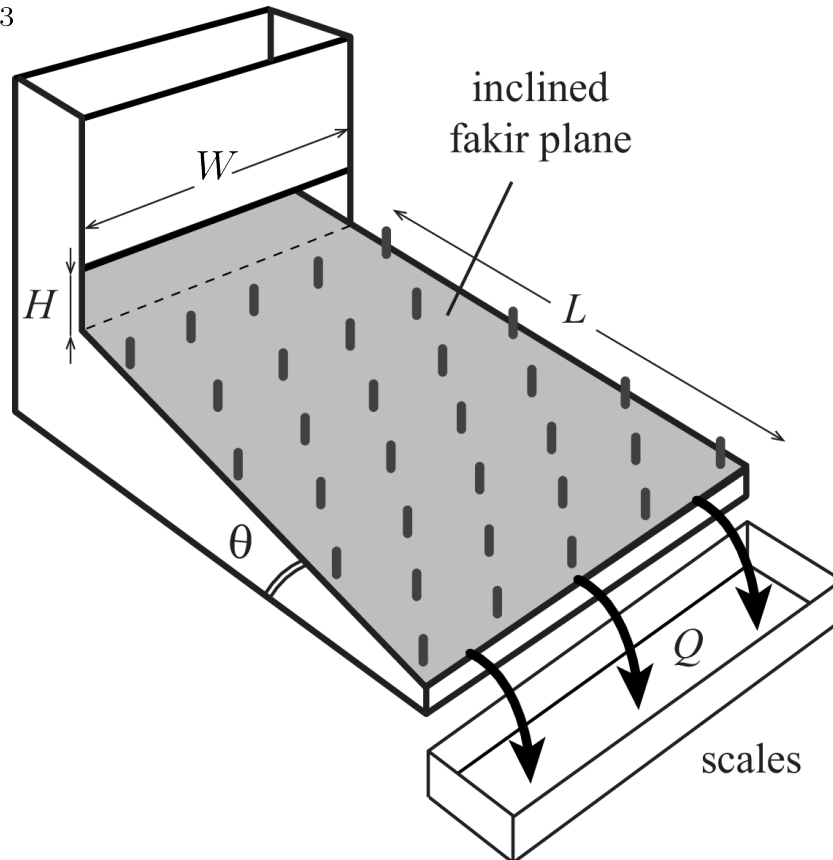
# Setup

Glass beads

$$d = 450 \pm 40 \mu\text{m}$$

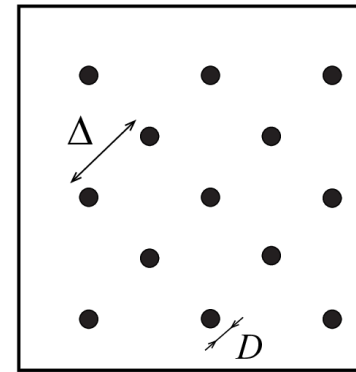
$$\rho_g = 2520 \text{ kg}\cdot\text{m}^{-3}$$

$$\phi = 0.60 \pm 0.01$$



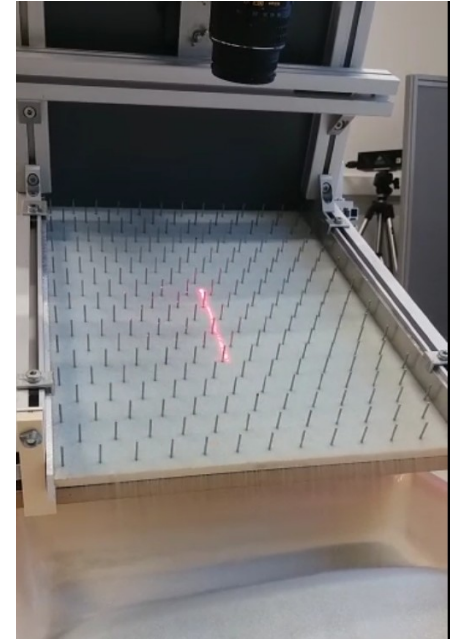
$$L = 50 \text{ cm}$$

$$W = 37 \text{ cm}$$



$$D = 2 \text{ mm}$$

$$\chi = \frac{1}{\Delta^2}$$

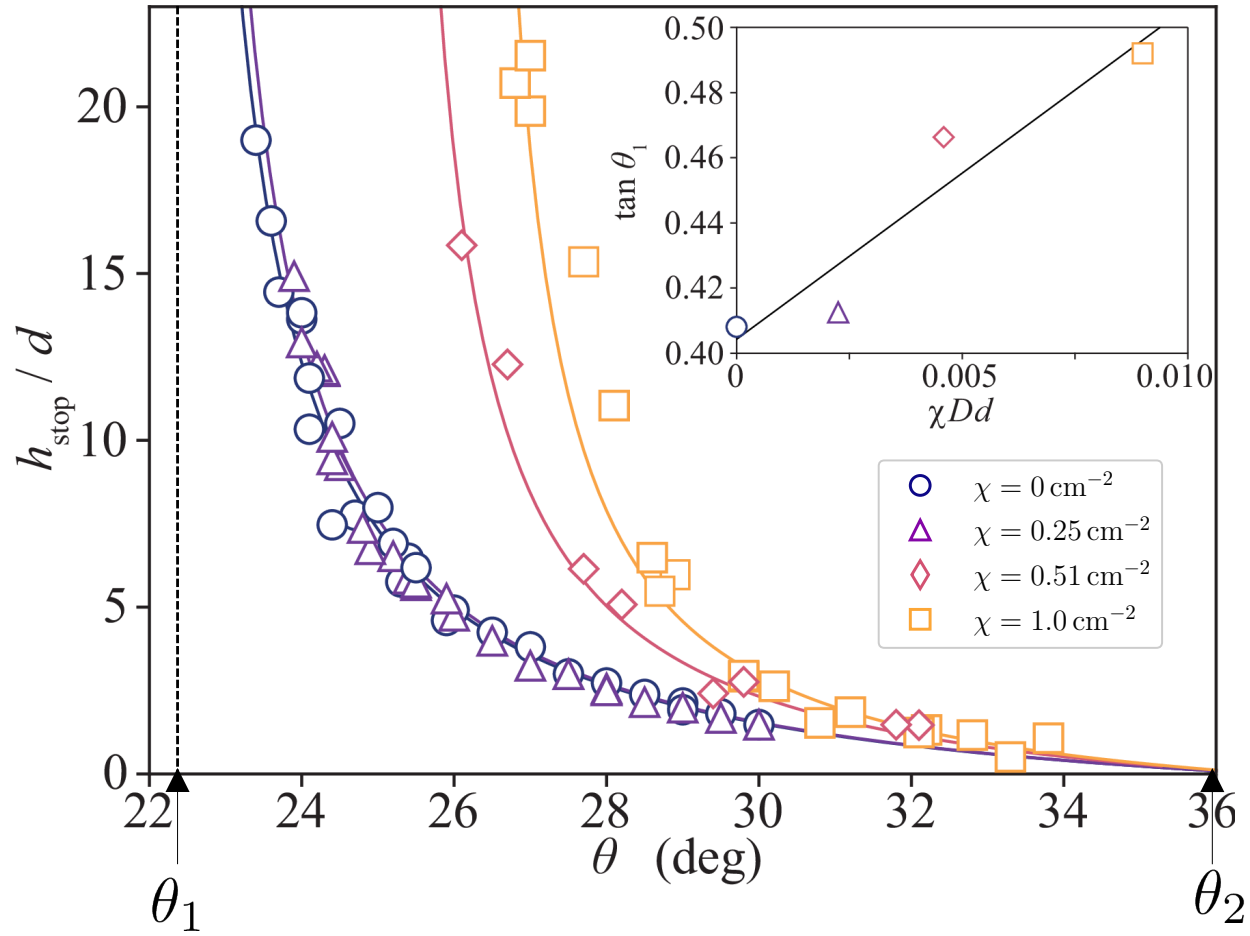
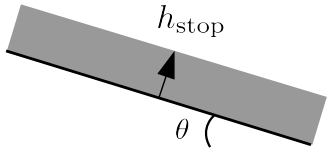


$$\tilde{Q} = \frac{Q}{\rho W g^{1/2} d^{3/2}}$$

$$\tilde{Q}(h, \theta, \chi) ?$$

**Downslope granular flow  
through a forest of obstacles**

# Stopping conditions



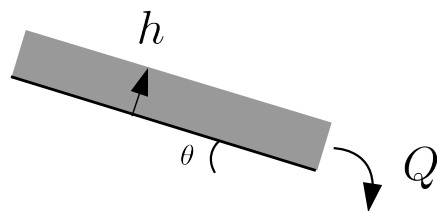
$$\frac{h_{\text{stop}}(\theta)}{d} = \lambda \frac{\tan \theta_2 - \tan \theta}{\tan \theta - \tan \theta_1}$$

$\chi = 0 \rightarrow$

$$\begin{aligned} \theta_1 &= 22.2^\circ \\ \theta_2 &= 36.6^\circ \\ \lambda &= 1.55 \end{aligned}$$

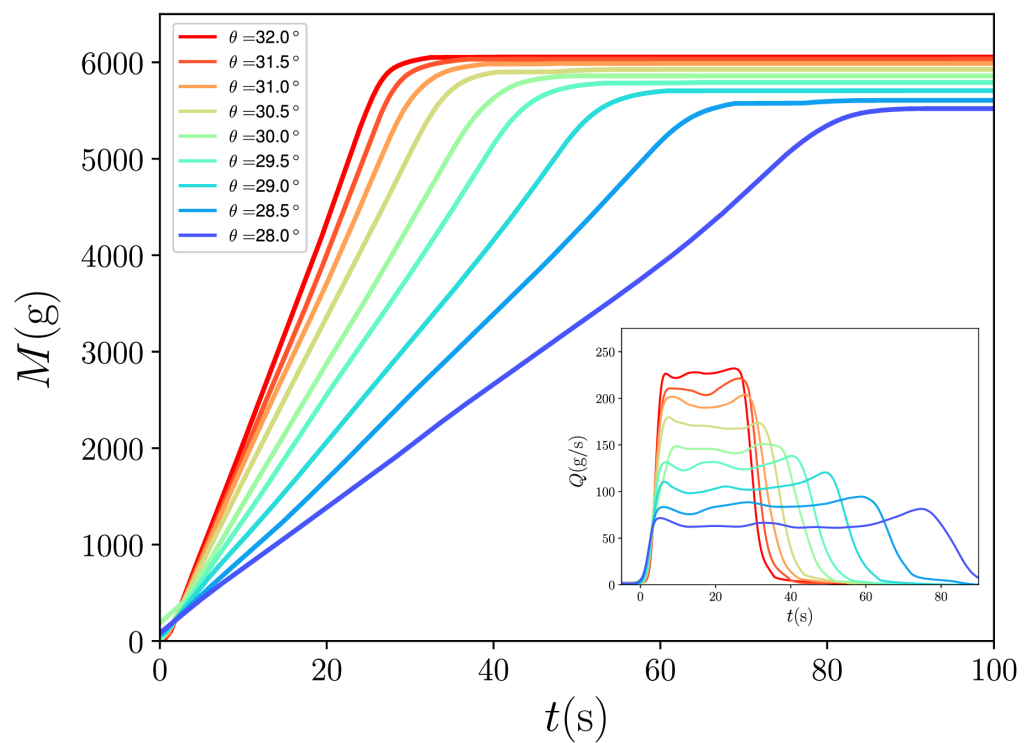


Steady flow

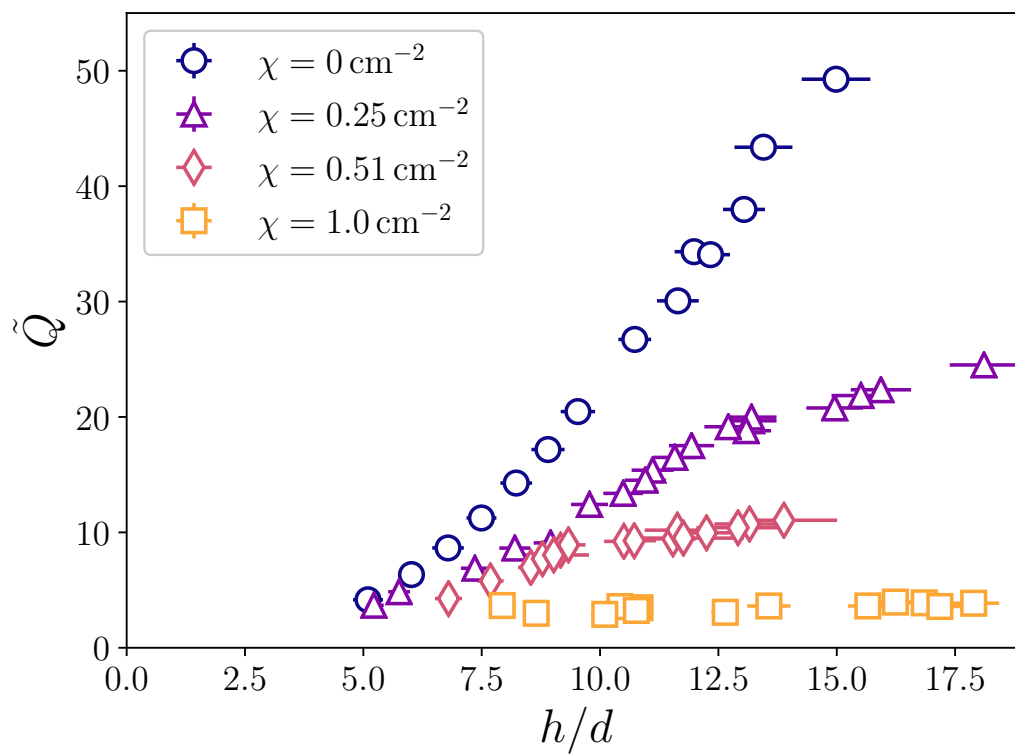


$$\tilde{Q} = \frac{Q}{\rho W g^{1/2} d^{3/2}}$$

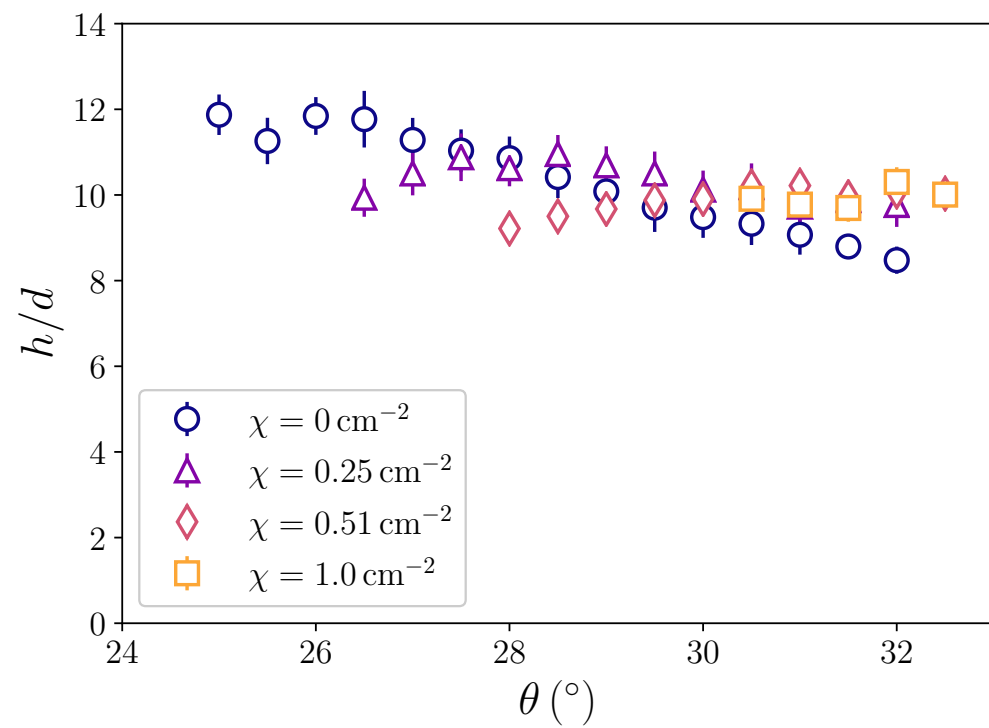
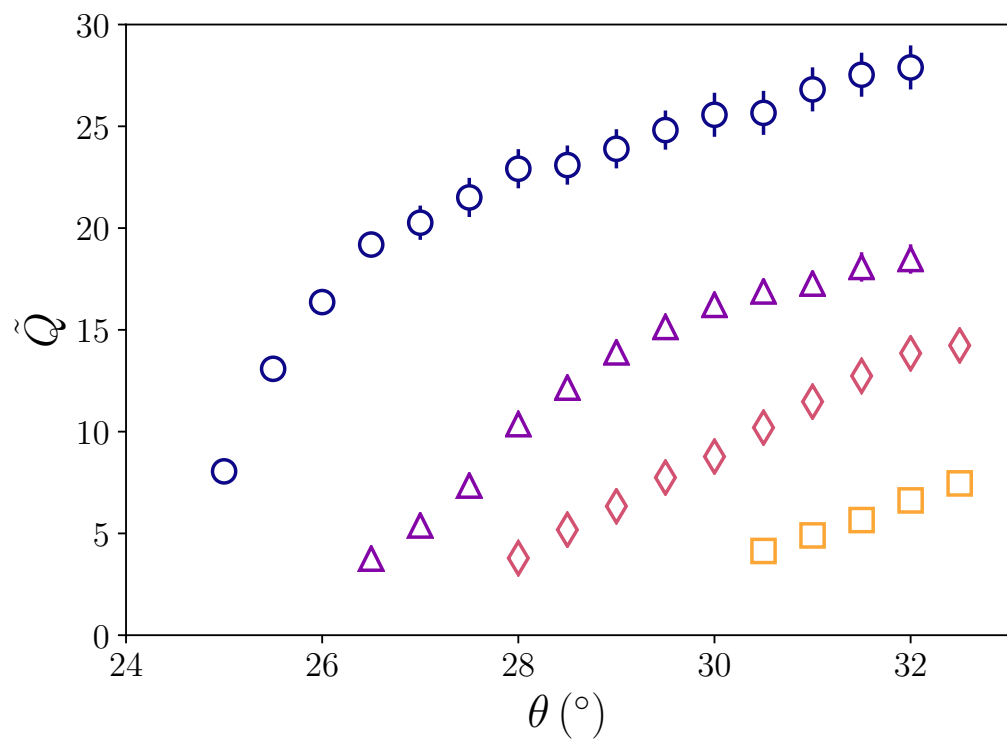
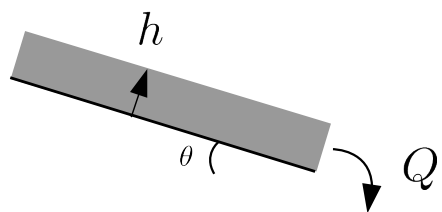
$h/d = 12$   $\chi = 0.51 \text{ cm}^{-2}$



$\theta = 29^\circ$

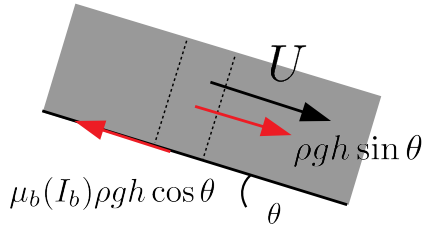


Steady flow





## Model without pillars



- Depth average approach
- $\mu(I)$  rheology

Jop, P., Forterre, Y., & Pouliquen, O (2006) Nature

$$\rho g h \sin \theta - \mu_b(I_b) \rho g h \cos \theta = 0$$

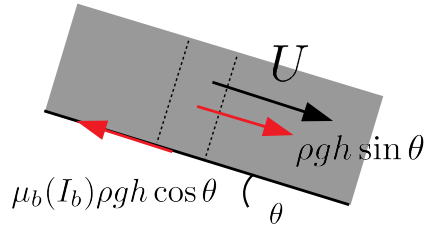
$$\text{with } \mu(I_b) = \tan \theta_1 + \frac{\tan \theta_2 - \tan \theta_1}{1 + I_0/I_b} \quad I_b = \frac{\dot{\gamma}_b d}{\sqrt{P_b/\rho_g}} \quad \dot{\gamma}_b = \frac{5U}{2h} \quad U = \frac{Q}{\rho W h}$$

$$\tilde{Q}(h, \theta) = \frac{2I_0}{5} \frac{\tan \theta - \tan \theta_1}{\tan \theta_2 - \tan \theta} \sqrt{\phi \cos \theta} \left( \frac{h}{d} \right)^{5/2}$$

# Model without pillars

- Depth average approach
- $\mu(I)$  rheology

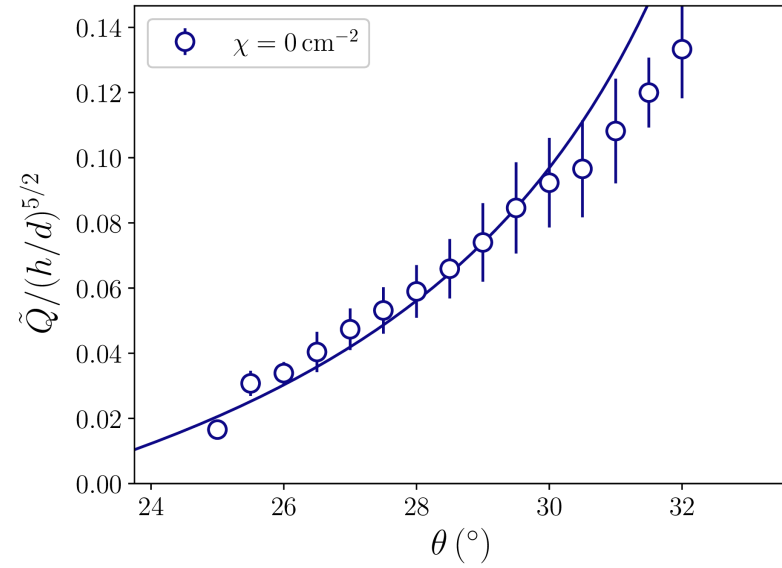
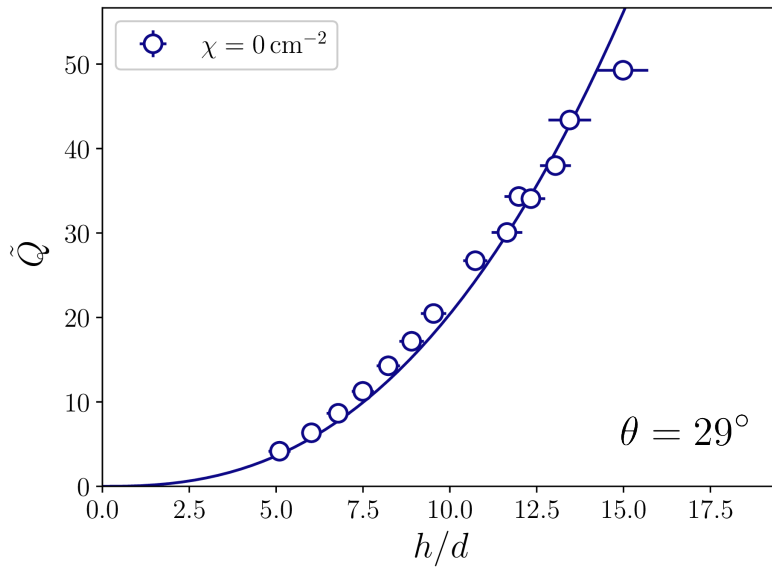
Jop, P., Forterre, Y., & Pouliquen, O (2004) Nature



$$\rho g h \sin \theta - \mu_b(I_b) \rho g h \cos \theta = 0$$

$$\text{with } \mu(I_b) = \tan \theta_1 + \frac{\tan \theta_2 - \tan \theta_1}{1 + I_0/I_b} \quad I_b = \frac{\dot{\gamma}_b d}{\sqrt{P_b/\rho_g}} \quad \dot{\gamma}_b = \frac{5U}{2h} \quad U = \frac{Q}{\rho W h}$$

$$\tilde{Q}(h, \theta) = \frac{2I_0 \tan \theta - \tan \theta_1}{5 \tan \theta_2 - \tan \theta} \sqrt{\phi \cos \theta} \left( \frac{h}{d} \right)^{5/2}$$



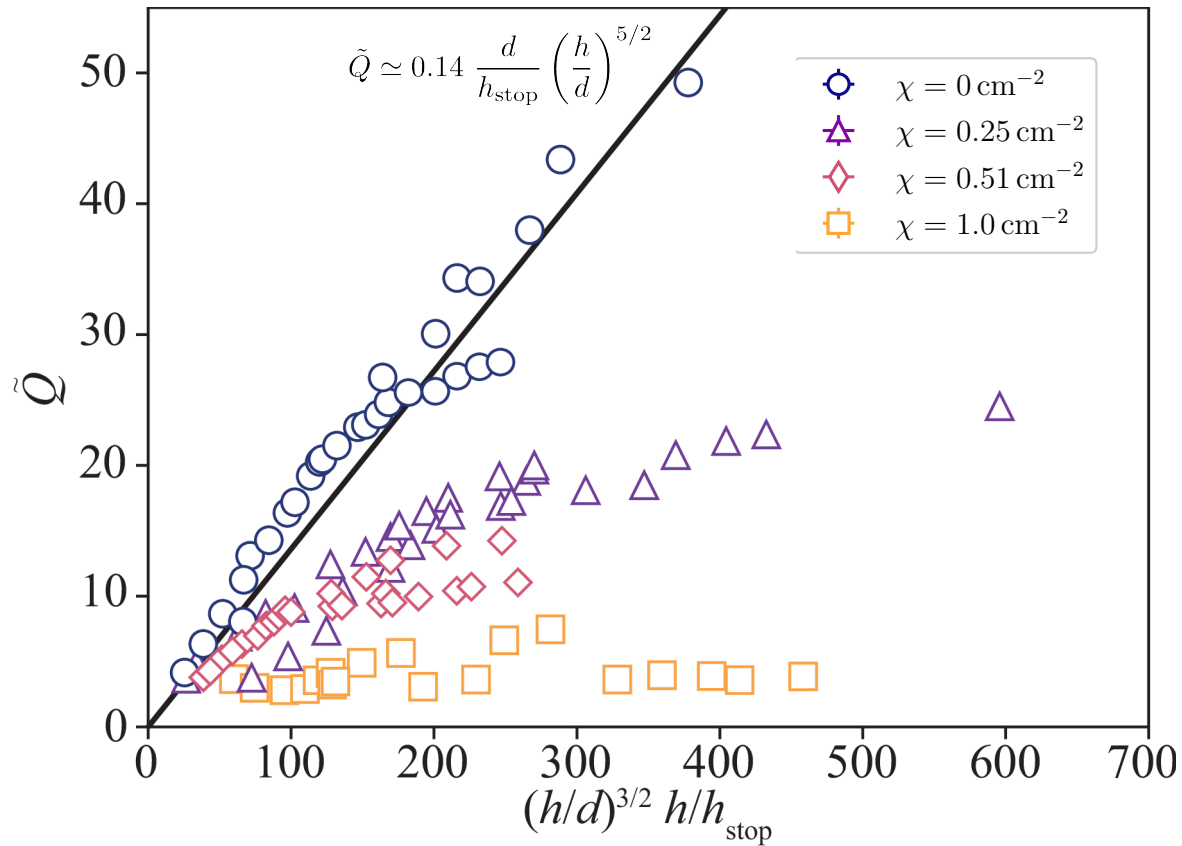
$\theta_1 = 22.2^\circ$   
 $\theta_2 = 36.6^\circ$   
 $I_0 = 0.35$

Model

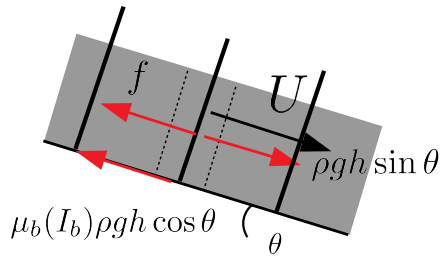
$$\tilde{Q}(h, \theta) = \frac{2I_0 \tan \theta - \tan \theta_1}{5 \tan \theta_2 - \tan \theta} \sqrt{\phi \cos \theta} \left(\frac{h}{d}\right)^{5/2}$$

$$\frac{h_{\text{stop}}(\theta)}{d} = \lambda \frac{\tan \theta_2 - \tan \theta}{\tan \theta - \tan \theta_1}$$

$$\tilde{Q} \simeq 0.14 \frac{d}{h_{\text{stop}}} \left(\frac{h}{d}\right)^{5/2}$$



## Model with pillars

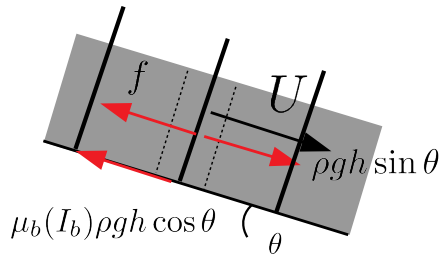


Force on a single pillar  $f_h = \beta \rho g h^2 D \cos \theta$   $f_i = C_D \rho h D U^2$  Faug, T. (2015) EPJE

Average stress for a forest of pillars  $f = \chi(f_h + f_i)$

Force balance  $\rho g h \sin \theta - \mu_b(U, h) \rho g h \cos \theta - \beta \rho g \chi h^2 D \cos \theta - C_d \rho \chi h D U^2 = 0$

## Model with pillars



Force on a single pillar  $f_h = \beta \rho g h^2 D \cos \theta$   $f_i = C_D \rho h D U^2$  Faug, T. (2015) EPJE

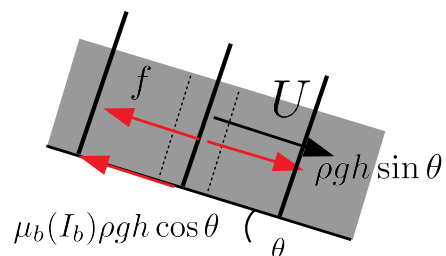
Average stress for a forest of pillars  $f = \chi(f_h + f_i)$

Force balance  $\rho g h \sin \theta - \mu_b(U, h) \rho g h \cos \theta - \beta \rho g \chi h^2 D \cos \theta - C_d \rho \chi h D U^2 = 0$

$$C_d = 0 \longrightarrow \tilde{Q} = \frac{2I_0}{5} \frac{\tan \theta - \tan \theta_1 - \beta \chi h D}{\tan \theta_2 + \beta \chi h D - \tan \theta} \sqrt{\phi \cos \theta} \left( \frac{h}{d} \right)^{5/2}$$

$C_d \neq 0 \longrightarrow$  Numerical solution

# Model with pillars



Force on a single pillar  $f_h = \beta \rho g h^2 D \cos \theta$       $f_i = C_D \rho h D U^2$

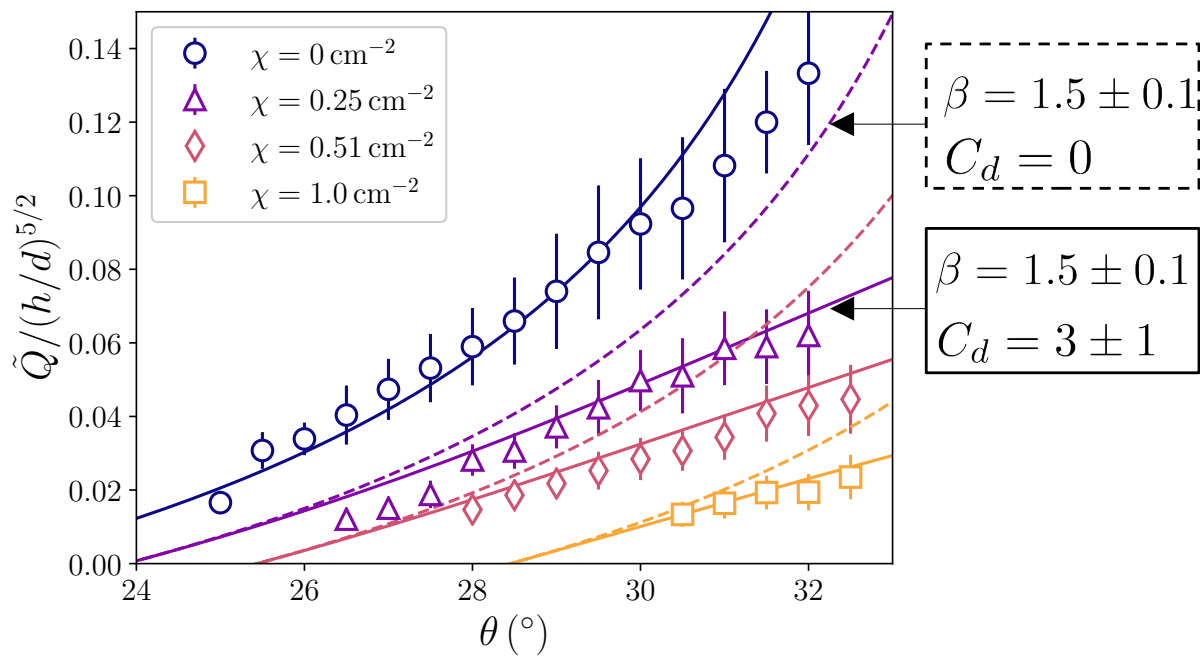
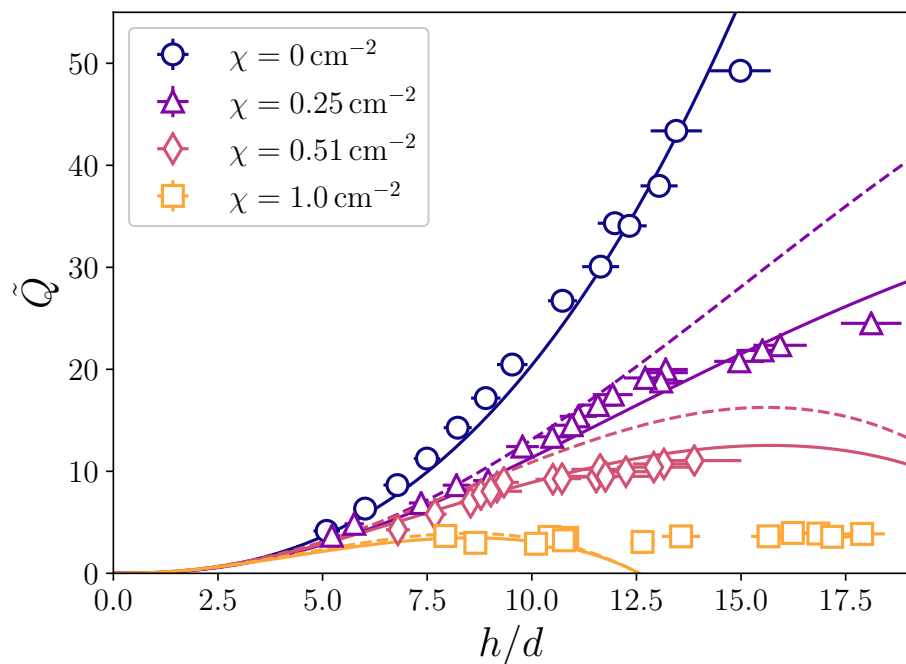
Faug, T. (2015) EPJE

Average stress for a forest of pillars  $f = \chi (f_h + f_i)$

Force balance  $\rho g h \sin \theta - \mu_b(U, h) \rho g h \cos \theta - \beta \rho g \chi h^2 D \cos \theta - C_d \rho \chi h D U^2 = 0$

$C_d = 0 \longrightarrow \tilde{Q} = \frac{2I_0}{5} \frac{\tan \theta - \tan \theta_1 - \beta \chi h D}{\tan \theta_2 + \beta \chi h D - \tan \theta} \sqrt{\phi \cos \theta} \left( \frac{h}{d} \right)^{5/2}$

$C_d \neq 0 \longrightarrow$  Numerical solution



## Conclusion

- Measurements of the stationary flow of grains through a forest of pillars
- Model based on the  $\mu(I)$ -rheology that accounts for the presence of pillars

$$\tilde{Q} = \frac{2I_0}{5} \frac{\tan \theta - \tan \theta_1 - \beta \chi h D}{\tan \theta_2 + \beta \chi h D - \tan \theta} \sqrt{\phi \cos \theta} \left( \frac{h}{d} \right)^{5/2}$$

- Effective rheology for dry grains and rigid pillars

$$\mu_{\text{eff}}(I, \chi D h) = \tan \theta_1 + \beta \chi D h + \frac{\tan \theta_2 - \tan \theta_1}{1 + I_0/I}$$



## Conclusion

- Measurements of the stationary flow of grains through a forest of pillars
- Model based on the  $\mu(I)$ -rheology that accounts for the presence of pillars

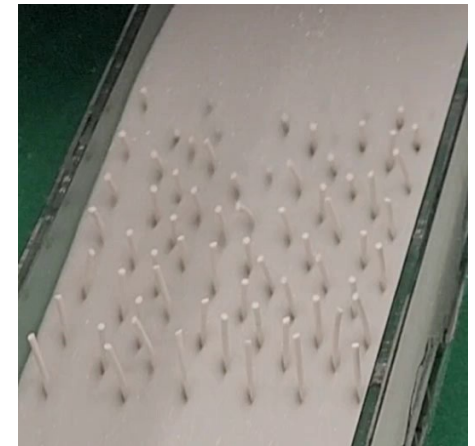
$$\tilde{Q} = \frac{2I_0}{5} \frac{\tan \theta - \tan \theta_1 - \beta \chi h D}{\tan \theta_2 + \beta \chi h D - \tan \theta} \sqrt{\phi \cos \theta} \left( \frac{h}{d} \right)^{5/2}$$

- Effective rheology for dry grains and rigid pillars

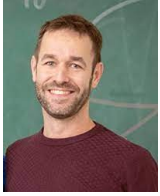
$$\mu_{\text{eff}}(I, \chi Dh) = \tan \theta_1 + \beta \chi Dh + \frac{\tan \theta_2 - \tan \theta_1}{1 + I_0/I}$$

## Perspectives

- Interaction between pillars
- Disordered networks
- Flexible pillars



Thank to Simon Herbez

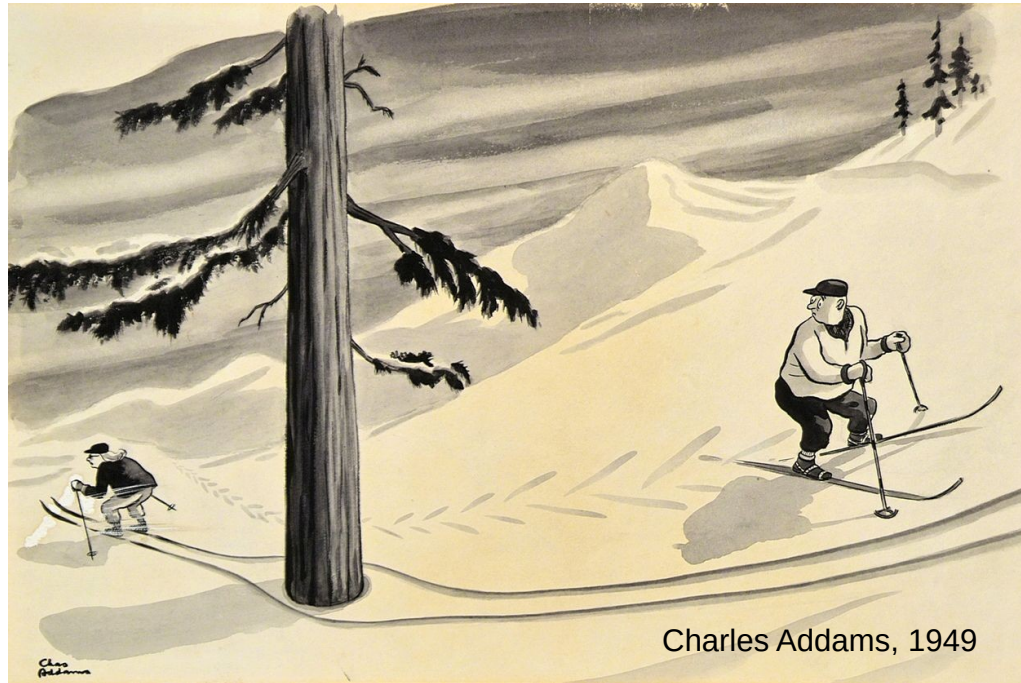


Yann Bertho



Philippe Gondret

Thanks for your attention



Charles Addams, 1949