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Detecting car-following and platoons within traffic using the dynamic time warping algorithm

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SHORT SUMMARY

The increase in adaptive cruise control (ACC) penetration rate might increase energy consumption, reduce road capacity, and decrease safety levels. Those externalities can be linked with the string instability they create while platooning. To assess ACC's future impact, it is necessary to know the percentage of platoons within traffic and their length. However, no method exists to quantify platoons. This article proposes a new approach to detect car-following based on dynamic time warping. This methodology is then extended to platooning. The method is then validated on a trajectory dataset both in free-flow and congested conditions. The framework appears promising as it highlights efficiently if a vehicle is car-following or platooning, and it determines the size of the platoon.

Keywords: traffic-flow theory, car-following, platooning, dynamic time-warping

1 INTRODUCTION

With the increase of adaptive cruise control(ACC) vehicle penetration rate, observed vehicle behavior, and especially the car-following behavior, will be drastically modified. Vehicles equipped with adaptive cruise control (ACC), the first level of automation according to (SAE, 2018), will increase the traffic safety level by reducing the front-rear collision risk. It is expected that the penetration rate of ACC vehicles on European roads will reach 25% by 2030 (Calvert et al., 2017). This estimation might even be conservative as systems such as Openpilot (Comma, 2022) are slowly gaining popularity in the US. Openpilot allows equipping more than 150 compatible models with a full-range ACC, lane keeping, and automatic lane changing. To predict future traffic, we must estimate what a mixed traffic composed of AV and human-driven vehicles (HDV) would look like and compare it to HDV traffic.

Experiments that compare ACC platoons with HDV platoons have taken place in the last years (Makridis et al., 2021; Gunter et al., 2019). Studies based on these experimental data have shown that ACC platoons lead to a capacity drop (Li, 2021; Gomez Patino, 2022). Based on these data, other studies have shown that ACC platoons lead to string instability and that the instability is higher when the target time gap is short (Ciuffo et al., 2021). (Ciuffo et al., 2021) also showed that, with ACC vehicles, the time to collision (TTC) is shorter if the platoon size is equal to or larger than 6. Another study showed that ACC platoons led to higher fuel consumption in the case of a short time-gap setting (Charlottin et al., 2024).

In summary, previous studies have shown that ACC platoons lead to higher fuel consumption and reduced TTC value compared with HDV platoons. However, those studies have been executed in a controlled environment. To have a global vision of ACC impact in traffic, we need to analyze the size of platoons in traffic. Most authors conceptualize a platoon as a string of vehicles in car-following. However, they do not agree on any formal mathematical definition. Previous studies have developed methods to detect whether a vehicle is in car-following (Duret, 2011) or in free-flow (Hoogendoorn, 2005). To our knowledge, however, there is no direct method to detect a platoon other than the visual analysis of the space-time diagram.

This study develops a method to detect if two vehicles are in car-following and extends the method to platoon identification. This identification is executed in both free-flow and congested traffic. The method is based on the analysis of trajectories using the dynamic time warping algorithm (DTW) developed by (Senin, 2008). The remainder of this article is organized as follows. In section 2, we define a platoon mathematically. Section 3 proposes a method to detect if two vehicles are in car-following or platooning. Section 4 describes the results of this method on several motorway trajectories. Section 5 presents the conclusions and recommendations for future research.

2 PLATOON DEFINITION

The generic definition of car-following is that the follower reacts to any action of its leader based on the speed (or the acceleration) and the position of the leader (see (Gerlough & Huber, 1975)). Car-following is possible in any traffic state, as a vehicle can follow a leader that cruises at the free-flow speed. In this paper, we assume that the reaction time is specific for each vehicle and constant over time for each trajectory. Following this generic definition, we can define that the speed of vehicle 1 is a function of vehicle 0 speed and vehicle 1 position:

$$v_1(t + \tau_1) = F(x_1(t), v_0(t)) \quad (1)$$

Where v_n is the speed of the n-th vehicle, x_n is its position. τ_n is the n-th vehicle reaction time and t is the time.

Considering a third vehicle 2 following vehicle 1, we can write:

$$v_2(t + \tau_2) = F(x_2(t), v_1(t)) \quad (2)$$

We then have vehicle 2 speed as a function of vehicle 1 speed and vehicle 1 speed as a function of vehicle 0 speed. We then have a string of three vehicles, all car-following each other. In this situation, every vehicle's speed is locally constrained by the speed of its leader.

Let's now assume that the speed of the first vehicle of the string constrains the speed of both vehicles 1 and 2. Then, if we assume that we can sum the reaction time and that it exists a universal car-following rule, we can write the following relation between the position of 2 and the speed of vehicle 0:

$$v_2(t + \tau_1 + \tau_2) = F(x_2(t + \tau_1), F(x_1(t), v_0(t))) \quad (3)$$

In this case we have a relation to describe the case a three vehicles platoon.

Let us assume that we have n vehicles that are car-following. The position of the n-th vehicle as a function of the first vehicle in the platoon can be written as follows:

$$x_n(t + \sum_{i=1}^n \tau_i) = \int F(x_n(t + \sum_{i=1}^{n-1} \tau_i), F(x_1(t), v_0(t))) \quad (4)$$

In such a situation, the n-th vehicle speed is directly constrained by the vehicle's 0 speed. Then, we can describe the platooning and the string of car-following using those two equations:

Definition 1. *A group of n vehicles forms a string of cars following if each vehicle is car following the vehicle in front of itself.*

Definition 2. *A group of n vehicles forms a platoon if for each vehicle that is not the first in line; its speed is constrained by the first vehicle of the platoon.*

Based on those definitions, to detect if a vehicle belongs to a platoon, we follow the following methodology:

- First, for each leader-follower pair, we compute the reaction time.
- Then, we compare each leader-follower pair trajectories.
- Finally, we identify the string of vehicles that are all car-following and then compare the string's first vehicle trajectory to all the other vehicles' trajectories.

3 METHODOLOGY

Selecting the car-following model

In this paper, we will use the Newell's car-following model (Newell, 2002). A study has shown that the model can detect macroscopic patterns in congestion (Chiabaut et al., 2010), a requirement for our study. Furthermore, the model has been used to estimate the reaction time distribution within traffic by (Duret et al., 2008), another requirement as we assumed each vehicle had its own reaction time.

According to Newell's car-following model, if vehicle 1 follows vehicle 0 during a time interval $[t_{init}, t_{max}]$, we can write:

$$\frac{dx_1(t + \tau)}{dt} = \frac{dx_0(t)}{dt}, \forall t \in [t_{init}, t_{max}] \quad (5)$$

This means that if vehicle 1 is following vehicle 0, the space-time trajectories of vehicle 0 at time t and vehicle 1 at time $t + \tau$ will have a similar shape.

Consequently, detecting whether two vehicles are in car-following involves studying the difference in the shape of their space-time trajectories at times t for the leader and $t + \tau$ for the follower. If this difference tends toward zero, the two vehicles are in car-following.

Let us now assume that n vehicles are following each other. According to Newell's car-following model, we can link the speed of the n -th vehicle to the speed of the first vehicle.

$$\frac{dx_n(t + \sum_{i=1}^{n-1} \tau_i)}{dt} = \frac{dx_0(t)}{dt}, \forall t \in [t_{init}, t_{max}] \quad (6)$$

This shows that in Newell's car-following model, a straightforward linear relation connects the positions of non-adjacent vehicles within a platoon, indicating a simple extension for detecting car-following at the platoon level. Based on this, we can detect if two vehicles belong to the same platoon just by comparing their space-time trajectories.

Selecting the metric to compare the trajectories

A first method to compare the trajectories would be to study the evolution of the distance over time using a simple metric such as the Euclidean distance or any other algebraic distance. This would not take into account the difference between the predicted position and the actual position (named ϵ). If we add this error term, equation 5 becomes:

$$\frac{dx_0(t)}{dt} = \frac{dx_1(t + \tau)}{dt} + \epsilon(t), \forall t \in [t_{init}, t_{max}] \quad (7)$$

Based on that, the distance metric we select has to follow those two requirements:

- First, to be a metric that computes the distance continuously (in opposition to the distance that just highlights the local min or max distance that exists between two curves);
- Second, be robust to any local variations that do not affect the global shape of the curve.

A metric that satisfies those two requirements is given by the dynamic time-warping (DTW). DTW computes an optimal alignment between two time series, minimizing the cost of matching corresponding points. The alignment is achieved through a dynamic programming approach, where a cost matrix is constructed to represent the dissimilarity between each pair of time points. The algorithm then efficiently identifies the optimal path through this matrix, reflecting the optimal alignment of the two-time series. It is widely used in other fields, especially in signal comparison where its efficiency has been proven (Müller, 2007).

Algorithmic applied method

We apply the presented methods to the HighD dataset (Krajewski et al., 2018). This dataset was collected in German Autobahns using drones by a team from Ika and the University of Aachen. It captures highway scenes in two directions on 400-meter sections. The trajectories were extracted from the video recordings using a neural network. The dataset includes records of both free-flow and congested traffic conditions. The data contains each vehicle's timestep, position, speed, acceleration, current lane, time and distance gap, and time to collision. The data is collected at a 24Hz rate. Trajectories were recorded on straight motorway sections near Köln and Düsseldorf.

We selected a few samples of free-flow traffic and congested scenes from the dataset to test our method. Table 1 presents the selected recordings.

For each vehicle, we compute the individual value of τ using Duret's method (Duret, 2011; Duret et al., 2008). The method uses as input a list of w values between leader and follower trajectories. For each w value, we calculate τ^1 . The selected reaction time is the one that minimizes the objective function $\frac{\sigma(\tau)}{\mu(\tau)}$.

Algorithm 1 presents the used method to compute the DTW between two trajectories.

¹in Newell CF $w = \delta/\tau$ where δ is the minimal time headway

Table 1: Selected HighD Trajectories. The recording number and direction correspond to the definitions in the HighD dataset documentation.

Recording	Congestion Type	Direction	number of trajectories
23	Free Flow	To the North bound	1191
25	Heavy Congestion	To the North bound	1232
26	Heavy Congestion	To the North bound	1123
36	Limited Congestion	To the North bound	1237

Algorithm 1 Test car-following between vehicle 0 and vehicle 1

```

1: function TESTCARFOLLOWING(data, 0, 1, wlist)
2:   x1  $\leftarrow$  Subset of data where 'id' is 0
3:   x2  $\leftarrow$  Subset of data where 'id' is 1
4:   Try tau,  $\leftarrow$  compute Tau(data, 0, 1, wlist)
5:   Catch Exception tau  $\leftarrow$  1.2            $\triangleright$  in case the trajectories are too far away
6:   T  $\leftarrow$  Frames where 'leader' is 0 in x2
7:   X1  $\leftarrow$  Differencing of x1 within frames [min(T), max(T)] with smoothing 0.1  $\triangleright$  we
      apply this smoothing at the rate of the data to reduce the effect of potential acquisition
      errors.
8:   X2  $\leftarrow$  Differencing of x2 within frames [min(T) +  $\tau$ , max(T) +  $\tau$ ] with smoothing 0.1
9:   distance  $\leftarrow$  DTW(X1, X2)
10:  return distance
11: end function

```

To run the platoon detection algorithm, we first divide the trajectory dataset into several strings of leader-followers. A vehicle belongs to a string of leader-followers if the value of DTW between local leader and local follower is under the car-following detection threshold ².

For each vehicle of the string of car-following, we run the DTW algorithm between the global leader and each vehicle. Then, we define the local leader as the second vehicle in the string and repeat the process. The algorithm continues as long as the string length is superior or equal to two vehicles. A description of this method is presented in figure 1.

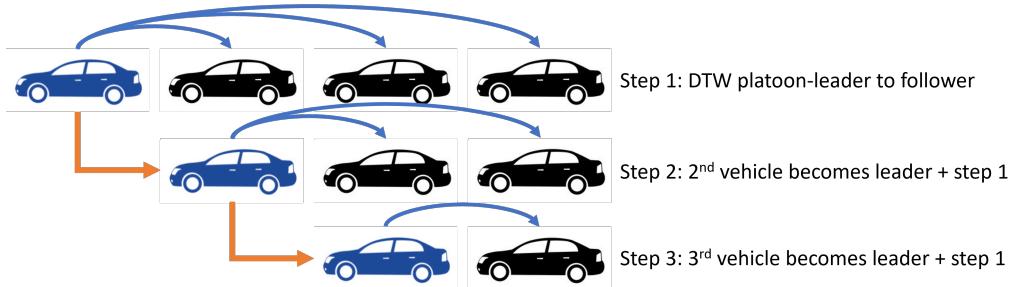


Figure 1: Scheme of the algorithm used to detect if a vehicle belongs to a platoon. The blue arrows describe the DTW calculation between the platoon (or sub-platoon) leader and follower. The orange arrows present a step in the algorithm where the leader considered is the vehicle placed one position behind.

Then, we define that a platoon exists while the DTW value between vehicle n and vehicle $n+i$ is under a threshold. The threshold is computed by comparing manual labeling of trajectories with the DTW value. If this condition is verified we have a platoon that extends from the n -th to the $n+i$ -th vehicle. We then start back the process of comparing DTW using the $n+i+1$ vehicle trajectory as the reference. To do so, we plot the speed time diagrams, slicing them by time to visually compare the shape of the trajectories and the obtained DTW value.

²the value of the DTW threshold will be determined in the results section

4 RESULTS

Car-following analysis

At first, we analyze the capacity of the proposed framework to effectively detect if a vehicle is car-following. To do so, we plotted the speed time diagrams, slicing them by time to visually compare the shape of the trajectories and the obtained DTW value. We manually annotated if a vehicle is in car-following if its trajectory is visually identical to the one of its preceding vehicle. We do that on three shockwaves and 60 seconds of free-flow traffic, leading to 107 trajectories. An example is given in Figure 2 and Table 2.

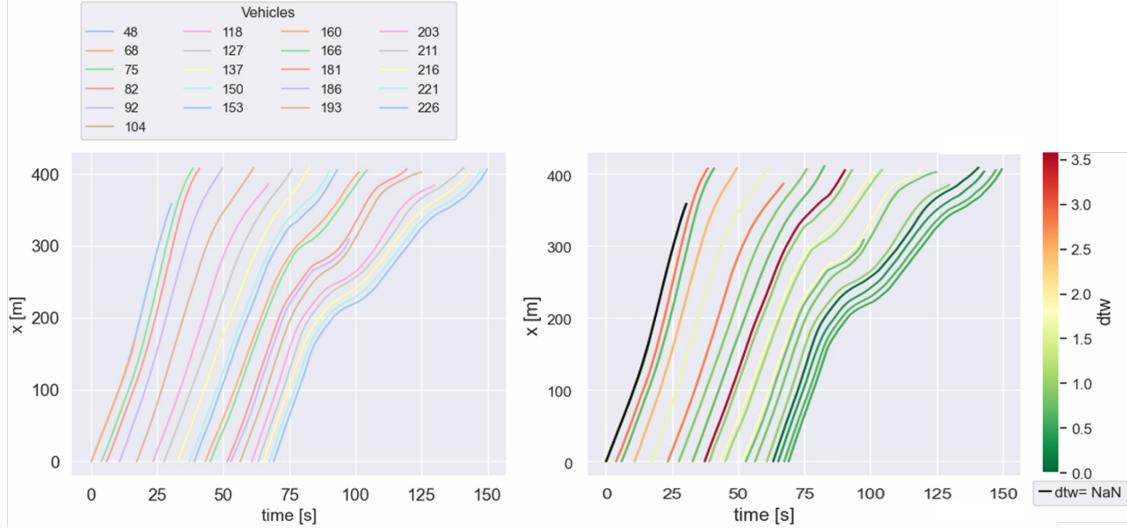


Figure 2: Trajectories used to label a potential car following (left) and values of the DTW algorithm (right).

Table 2: Summary of the manual annotations versus the DTW results of the 21 first trajectories in lane 2 of recording 25 (heavy congestion).

Vehicle	Leader	DTW	Annotation	Explanation
48.0	NaN	NaN	No car-following	Global leader
68.0	NaN	NaN	No car-following	Global leader 2
75.0	68.0	0.97	No car-following	Follower is faster
82.0	75.0	0.36	Car-following	
92.0	82.0	0.87	No car-following	Follower is faster
104.0	92.0	0.63	No car-following	Headway is larger than 4 seconds
118.0	104.0	0.96	No car-following	Headway is larger than 4 seconds
127.0	118.0	0.41	Car-following	
137.0	127.0	0.36	Car-following	
150.0	137.0	1.20	No car-following	Follower is slower
153.0	150.0	0.43	Car-following	
160.0	153.0	0.65	Car-following	
166.0	160.0	0.47	No Car-following	Follower is slower
181.0	166.0	0.66	Car-following	
186.0	181.0	0.38	Car-following	
193.0	186.0	0.39	Car-following	
203.0	193.0	0.43	No car-following	Follower is slower
211.0	203.0	0.15	Car-following	
216.0	211.0	0.27	Car-following	
221.0	216.0	0.33	Car-following	
226.0	221.0	0.31	Car-following	

Table 2 shows that DTW values lower than 0.4 consistently indicate car-following. DTW values between 0.4 and 0.8 show a potential car-following. DTW values higher than 0.8 suggest that there is no car-following. This observation is consistent all over the set of observations we did. We apply this method to other scenes in both free-flow and congested traffic conditions. Figure 3 compares the results of the manual annotation with the DTW values returned by the algorithm.

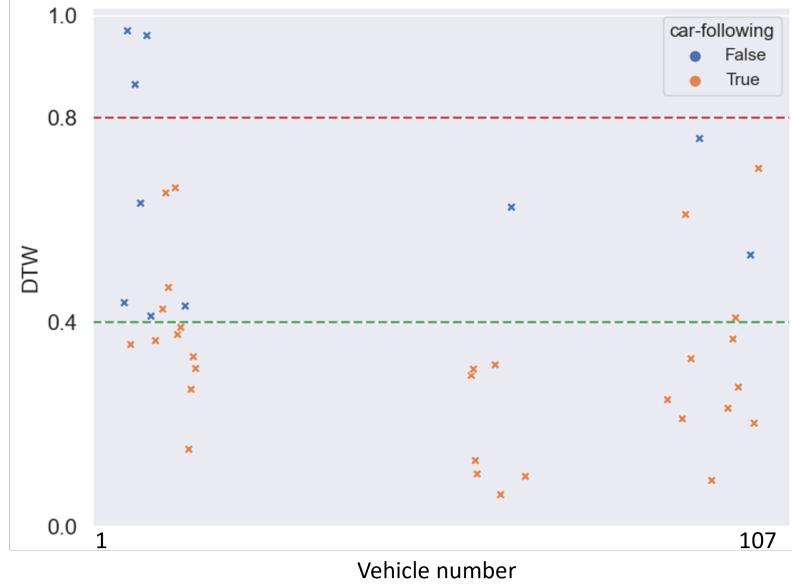


Figure 3: Scatter plot of the obtained DTW values. The color indicates if the manual labeling identified a vehicle in car-following situation or not. The plot is truncated to show DTW values lower than 1.

In this case study of 107 trajectories, every trajectory with a DTW value under 0.4 was labeled manually 'car-following', and every trajectory over 0.8 was labeled 'no car following'. Based on these findings, we will assume that a vehicle is in car-following if the DTW value is under 0.8. The comparison of manual labeling versus DTW value shows that in the case of free-flow scenarios, the distribution of DTW values is more spread. The uniformity of trajectories in free-flowing traffic can obscure subtle differences that might otherwise be detectable. This means that manual labeling might be insufficient in this case. Then, to validate the method in free-flow cases, we compare the value of DTW to the distance headway in the free-flow scenario. Figure 4 shows the violin plot of the DTW distribution versus the distance headway range.

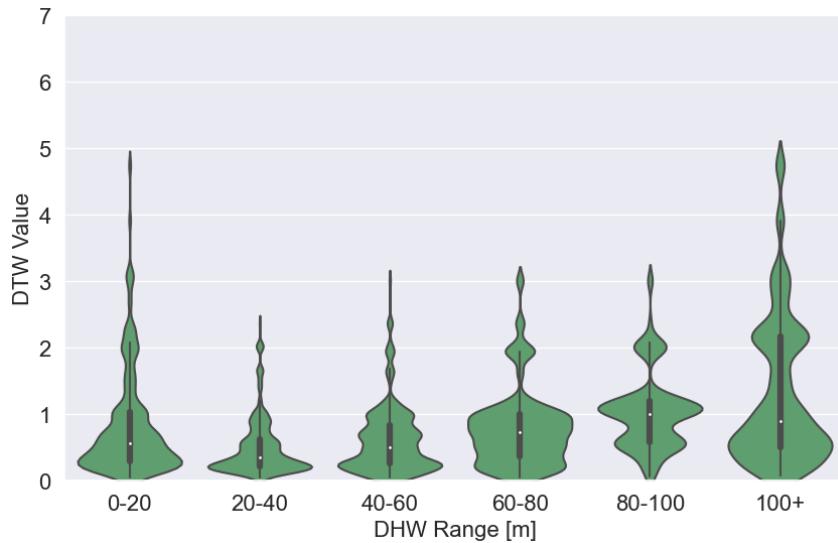


Figure 4: Distribution of DTW values for different distance headway range for the free flow scenario.

In free-flowing, the value of DTW is smaller if the follower is close to its leader. One notable exception is for distance headways that are under 20m. In this case, the percentage of vehicles that do not have precisely the same trajectory decreases. In this distance range, there might be an issue for the driver's safety that would lead him to brake to keep his safety distance. This leads to an end of car-following as described by car-following models that translate to a higher DTW value.

Those elements confirm that DTW is a good indicator of car-following. Indeed, it highlights small values in the case of a follower that translates perfectly to its leader speed in the case of congestion. In free-flow traffic, the fact that DTW values are strongly higher in the case of vehicles that are not close to each other highlights the fact that our method works both in free-flowing and congested traffic.

We then examine the impact of traffic state on the values of DTW. We hypothesize that the more congested a situation is, the more vehicles will be in car-following.

First, we study the raw distribution of DTW across speed ranges of 10m/s as shown in Figure 5.

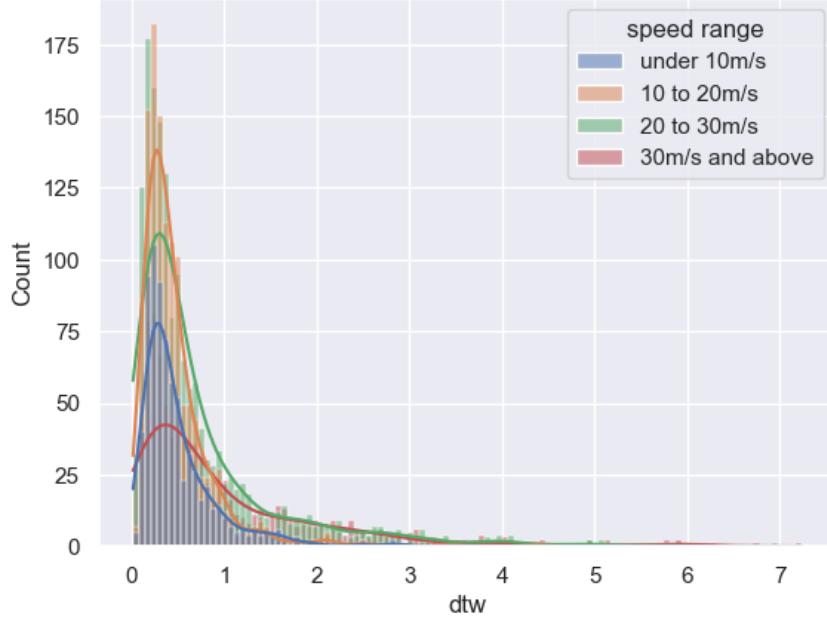


Figure 5: Distribution of DTW Values across Speed ranges, the lines are the KDE plots.

Maximal values of DTW by speed range show that the smaller the speed range, the smaller the maximal DTW values. Under 10 m/s, the maximal value is 3.63; from 10 to 20 m/s, it is 4.41; from 20 to 30 m/s, it is 6.96; for 30 m/s and above speed range, it is 7.23. Figure 5 shows that the distributions are centered on smaller values of DTW if the speed range is smaller. It shows that the more congested a situation is, the more vehicles are car-following, which is something to be expected.

Using the Spearman correlation test, we test the hypothesis that the distribution of the DTW is correlated with the speed range. We obtain a Spearman correlation value of 0.165 and a p-value of 1.1×10^{-28} . The correlation value shows that the higher the speed, the higher the DTW value will be.

Platooning analysis

For the analysis of the platooning, we use the DTW results of lane 2 of recording 25. First, we label all the continuous strings of car-following, defined as being a continuity of leader-follower pair with a DTW value under 1. We increase the value to one to avoid excluding false negatives. This selection criteria allowed us to include 209 out of 278 vehicles in 24 strings.

We apply the platoon detection method for each string as presented in Figure 1. Two vehicles belong to the same platoon when the DTW value is lower than 0.8. We get a mean size of a platoon of 5.8 and a median size of 3. The longest platoon size is 32 vehicles. Figure 6 presents the comparative size of car-following strings versus the size of the platoons. To do so, we sort the sizes of both the strings and the platoons and plot them.

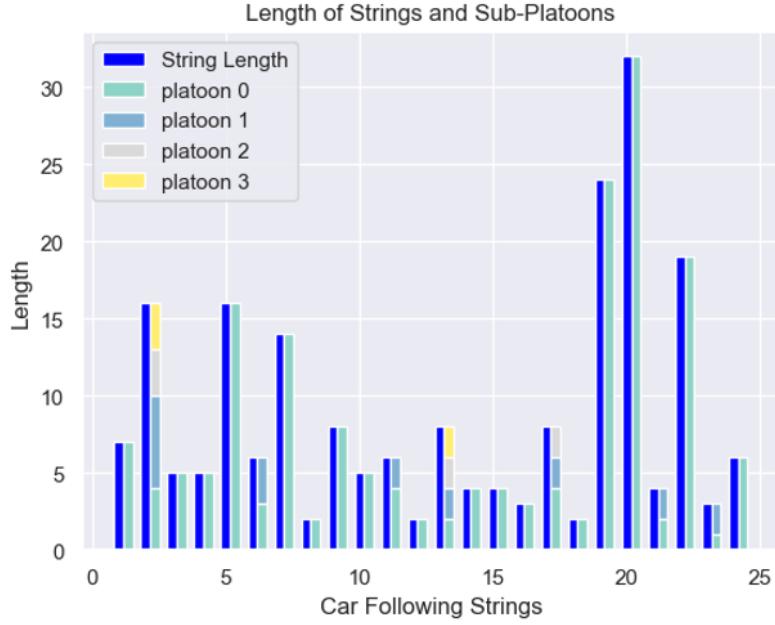


Figure 6: Barplot of the size of the strings of car-following and of the platoons

This result highlights that we can effectively cut strings of car-following into platoons. To examine the validity of the obtained platoons we compare trajectories with the identified platoons in Figure 7.

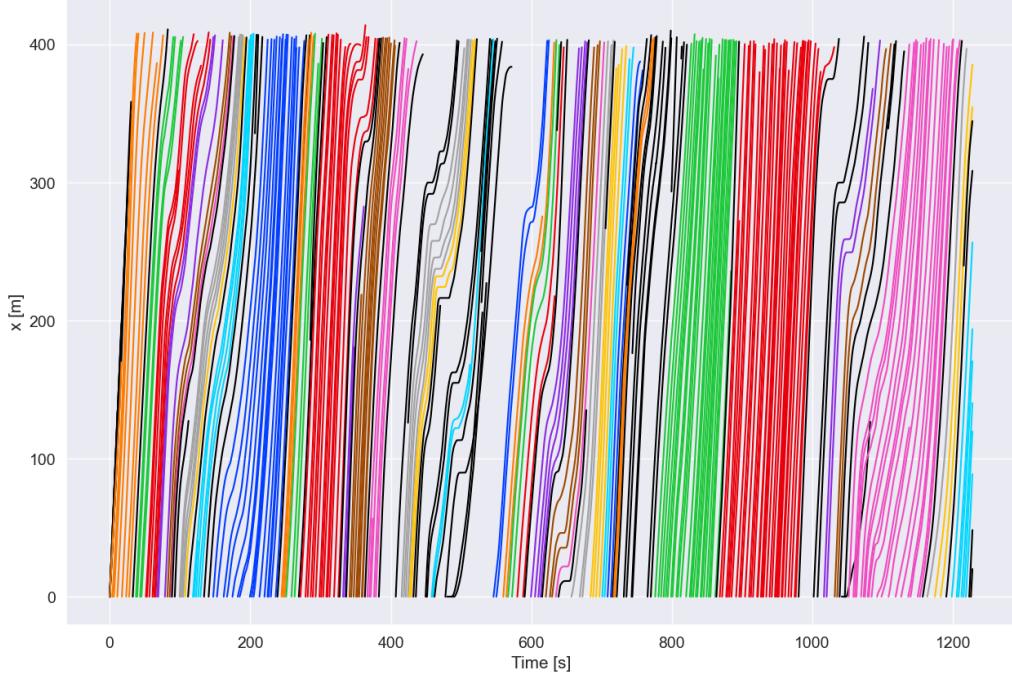


Figure 7: Space time diagram of all the detected platoons within lane two of record 25. Black trajectories indicate that there is no platoon, and trajectories with the same color indicate that all vehicles belong to the same platoon.

Figure 6 shows that the algorithm can, in the majority of cases, detect if two vehicles belong to the same platoon as the different groups of vehicles seem to have the same trajectory from the recognized leader to themselves. From the 36 detected platoons, we detected two outliers in which the size was too large and two platoons that were too short. This highlights that a unique threshold value all over the platoon is perhaps too restrictive.

Within the set of trajectories labeled as being part of the same platoon, we observe in some

situations small periods of time where the platoon does not exist according to our definition. However, in the rest of the trajectory, the vehicles are platooning. This highlights that platooning is not a constant state, even with trajectories as short as the ones in HighD.

We then examine the impact of the position of the vehicle in the platoon on its DTW value. To do so, we plot the difference between the platoon-based DTW and direct DTW values. Figure 8 presents the vehicle's position in a platoon versus the difference of DTW (local versus platoon).

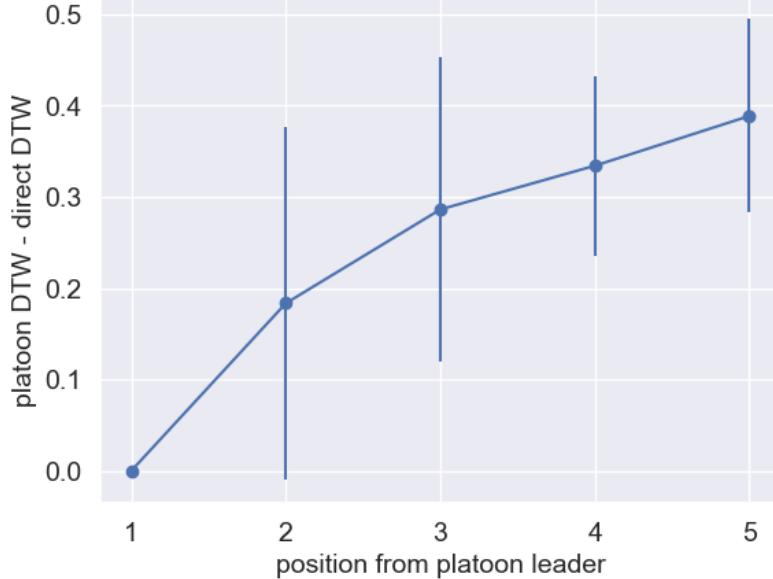


Figure 8: DTW difference between direct leader DTW and platoon leader to n-th vehicle DTW versus position in the different platoons. The point is the mean observed difference, horizontal bar represents the standard deviation.

Figure 8 shows that the longer the distance from the platoon leader, the higher the DTW difference. This highlights that choosing a unique DTW threshold value is too restrictive.

5 CONCLUSIONS

This study proposes a method to detect if vehicles are in car-following using DTW. We extend this method to detect if vehicles belong to a platoon.

The method allows us to detect if vehicles are in car-following based on the DTW value. In the case of platooning, the algorithm is able to detect if a vehicle trajectory is similar to a defined platoon leader. In some cases, the algorithm classifies two sets of trajectories as being in two different platoons. This often happens when a vehicle executes a lane change. However, before this event, the two sets of trajectories belong to the same platoon. To determine the impact of lane change, the algorithm needs to be used in datasets with longer trajectory lengths.

In the example used in this paper, we have been applying the platoon detection algorithm to a congested motorway scenario with several shockwaves. However, the vehicle's trajectories are the one of HDV vehicles for the vast majority. Before applying this method to trajectories databases that include ACC trajectories (Gloudemans et al., 2023), we have to determine if the ACC behavior impacts the performance of our framework. To do so, the method should be applied first to an ACC platoon dataset such as the Open ACC dataset. By doing so we would be able to detect if the threshold for ACC is somewhat different to the framework for HDV.

This methodology could be used to assess the impact of ACC. If the length of the platoons in highly automated traffic is greater, we may notice an increase in overconsumption and hazardous traffic events, as these two variables are dependent on the size of the platoons. Conversely, if the lengths are smaller, this implies that those two elements observed in platoon formations are insignificant in actual traffic. Regardless of the scenario, this technique can be a powerful tool for evaluating ACC traffic.

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The authors confirm their contribution to the paper as follows:

- Study conception and design: Thibault Charlottin, Christine Buisson, Silvia Varotto;
- Analysis and interpretation of results: Thibault Charlottin, Christine Buisson, Silvia Varotto;
- Data processing and figure creation: Thibault Charlottin;
- Draft manuscript preparation: Thibault Charlottin, Christine Buisson, Silvia Varotto.

All authors reviewed the results and approved the final version of the manuscript.

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