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Fractals and fracture

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Abstract

The relation between quantities resulting from fracture energetics and those describing the geometrical structure of a surface is of fundamental importance in investigating the physical nature of a fracture. Since the Mandelbrot's works, fractal geometry has been extensively applied to characterize the roughness of fracture surfaces and to correlate it with mechanical properties. We will first make a survey of several experimental methods to determine the fractal dimension, D_f , according to the roughness of fracture surfaces. Then we will discuss the experimental results reported in the bibliography and the relations inferred to correlate the fractal dimension of fracture surfaces with mechanical properties. These results show that a general conclusion cannot easily be drawn. Some works report a positive variation of fracture toughness along with D_f and others a negative one. To other researchers, there is no correlation and the fractal dimension of the fracture surface is a universal constant. The discrepancy between these results is related to the methods used to calculate the fractal dimension. Theoretical and experimental problems exist and have to be solved before correlating the fractal dimension to mechanical behaviour.

Keywords: Fractal; Fracture

1. Introduction

As early as 1947, Kostron [1] reported that the measured values for the size and area fractions of microstructural features depend on the magnification used to examine the materials. It is also now well known that the length of a coastline is dependent on the magnification [2]. But extensive studies on these phenomena in the field of materials sciences are more recent and were developed after the works of Mandelbrot [3] who introduced the

fractal concept. Many topics in materials sciences, computers or mechanics like percolation [4], diffusion [5], roughness [6] or fracture, have been studied through fractal analyses. These developments are generally associated with quantitative fractography using image analysis which is a powerful tool to study fractographic images [7] associated with mathematical morphology such as dilation for example. Numerous researchers have reported on the fact that fracture surfaces are self-similar and can be analysed by fractal geometry [8,9]. We will first present and discuss different methods to evaluate the fractal dimension. We will then present results found from our bibliography concerning correlations between mechanical properties and fractal dimension. These results are discussed and two possible conclusions are drawn: firstly, the fractal dimension is correlated with mechanical properties and the question consists in explaining the relation we obtained from a physical point of view [10,11]; secondly, the fractal dimension is universal for all fracture paths [12]. These two possibilities have been reported by many papers published over the last 10 years. However we wonder about the adequacy of the methods used to calculate the fractal dimension.

2. Determination of the fractal dimension

2.1. Definitions

Generally, all the objects studied in mathematics or physics are continuous, linear and smooth, but Nature shows us that such is not always the case. Natural objects are generally rough and discontinuous. As shown in Fig. 1, the Von Koch flake is a typical fractal object which cannot be described with Euclidean geometry. Firstly, when the number of iterations tends to be infinite, the total length increases indefinitely, whereas the surface is finite. Secondly, the Von Koch flake is self-similar: each part of this object is the same as the whole for all ranges of scale investigated. In this case, we can determine simply the fractal dimension, D_f , which characterizes in fact the irregularity of the object. D_f is a decimal number which exceeds the Euclidean dimension E . For the Von Koch flake shown in Fig. 1, in which the initiator generates four patterns ($N = 4$) with a ratio r for the new fractal generator ($r = 1/3$) the fractal dimension is found to be:

$$D_f = \frac{\ln N}{\ln 1/r} = \frac{\ln 4}{\ln 3} = 1.2619. \quad (1)$$

Through this example, we can see that the fractal dimension D_f exceeds the Euclidean dimension, E , which is unity. D_f will take place for a curve between $D_f = 1$ (smooth line) and $D_f = 2$ (Euclidean dimension of surfaces). The dimension increases as tortuosity increases. In the same way, the fractal dimension of a surface varies between 2 and 3. To avoid problems resulting from the comparison of different values, we prefer to use the fractal dimensional increment $\Delta(\Delta = D_f - E)$ which corresponds to the decimal part of the fractal dimension and which varies between 0 and 1.

It is very interesting to see that the theory about non-integer dimension is based on mathematical objects and analyses developed 80 or 100 years ago (Hausdorff, Minkowski, etc.), but the applications in the field of materials sciences are only recent (15 years). The fact that

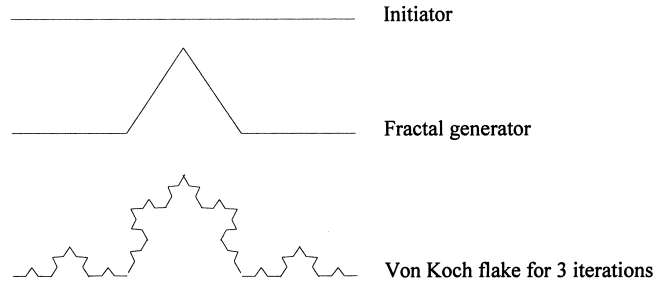


Fig. 1. Example of the Von Koch flake with its generator, initiator and after three iterations.

when a material is fractured by impact loading, the fracture surface that is formed is irregular, was found to be statistically self-similar over two orders of magnification by different authors [8, 9] and it can be considered fractal.

The mathematical aspects will not be introduced here but the reader should consult Tricot [13] who explained the different approaches carried out by Hausdorff, Besicovitch, Bouligand, Minkowski, Borel and Cantor at the outset of the century. The methods used to determine the fractal dimension are based on the concept of recovery. To resume, it is interesting to note that the fractal increment of a G set is generally given by the equation:

$$D(G) = \lim_{\epsilon \rightarrow 0} \left(1 - \frac{\ln L(G_\epsilon)}{\ln \epsilon} \right) \quad (2)$$

where L is a measure of the G set and ϵ the measure length.

Three main methods are generally applied to analyse the fractal dimension of the fractured surface of a material. Firstly, in the 'Vertical Section Method', the fractured specimens are mounted in epoxy resin and cut to reveal the vertical sections through the fractured surfaces. They were then polished carefully and the fractal dimension of the profile was calculated by means of the compass or the Minkowski method. Secondly, the roughness of the fractured surface is measured by profilometer analyses (profilometer, confocal microscopy, etc.) and then, the fractal dimension of the profile can be determined for example with the box counting or the Variation method. Thirdly, the sample can be coated and polished parallel to the crack plane and then, the Slit Island Method (S.I.M.) can be used. We will then explain all these methods.

2.2. The compass method

Let C be a curve, N the number of sub-parts in which the initial curve is divided and r the length of each sub-part (Fig. 2). The measured length $L(r)$ of the curve will be $L(r) = N(r) \cdot r$ and the fractal dimension is obtained from the slope of the $\ln(N(r))$ versus $\ln(1/r)$ curve using a least-squares fitting procedure.

$$D = \lim_{r \rightarrow 0} \left[1 - \frac{\ln(N(r))}{\ln(1/r)} \right] \quad (3)$$

as $N(r) = L(r)/r$, and if we assume through a heuristic argument that a linear relation exists

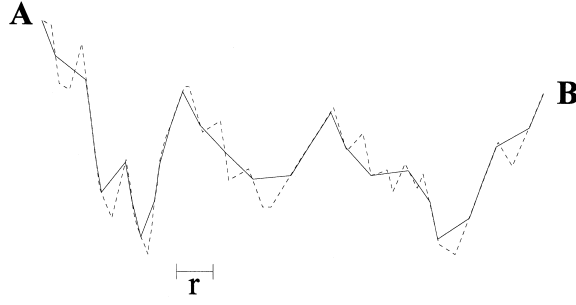


Fig. 2. Compass method. The polygonal line associated with the AB curve is composed by segments of the same length r .

between $\ln(N(r))$ and $\ln(1/r)$, the following relation is obtained:

$$\ln(L(r)) \sim \Delta \ln(r). \quad (4)$$

Underwood and Banerji [11] proposed to normalise length L :

$$L(r) = L_0 r^{-\Delta} \quad (5)$$

or

$$\ln(L(r)) = -\Delta \ln(r) + \ln(L_0). \quad (6)$$

Obviously the dimensional equation is not respected: the left shows a *Length* while the other side shows a *Length*^{- Δ} . To overcome this difficulty, Huang et al. [14] proposed the following correction:

$$L = L_0^{1+\Delta} r^{-\Delta} \quad (7)$$

where $r = L_0$, $L = L_0$, but since Δ is variable, $L_0^{1+\Delta}$ has no physical meaning. This theoretical point of view must be improved.

This method must also be avoided because of a Mandelbrot paper [15] in which he explains the importance of the yardstick value. If the value is too small or too great, the fractal dimension can often be wrong and meaningless. The Minkowski method will then be preferred.

2.3. The Minkowski method

When a surface profile Γ is covered by discs of radius r [7], we obtain a ‘sausage’ named the ‘Minkowski sausage’ (Fig. 3). This sausage has an area $A(\Gamma_r)$ and a fractal increment Δ :

$$\Delta = \lim_{r \rightarrow 0} \left[1 - \frac{\ln A(\Gamma_r)}{\ln(r)} \right]. \quad (8)$$

If the profile is fractal, the log–log plot of the sausage area versus the disc radius is generally reported to be linear and independent of the scale (Fig. 4). Keustermans [16] gives three pieces of advice for a fractal analysis. Firstly, the profile can have locally a morphology which is not the same as for the average profile. Secondly, r_{\max} must be small vis-à-vis the length of the

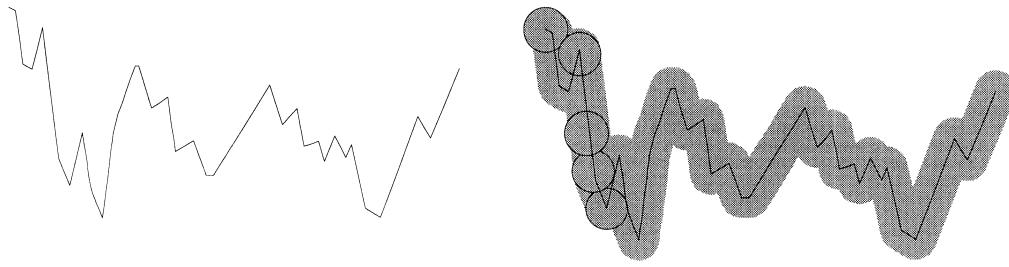


Fig. 3. Example of Minkowski sausage.

profile (for example $r_{\max} < L/6$). Finally, two or three decades of magnification are necessary to verify the fractal hypothesis. Many authors [8, 11, 17] obtained a log–log plot with a sigmoidal appearance instead of a linear curve, and a variable Δ . The deviation from the linearity could be explained by a difference between the characteristic size for the micro and macrostructure in relation with the fracture size. Furthermore, according to these authors, the fractal dimension varies. However, these results can be explained by the non-respect of Keusterman’s precautions. This method is often used because it is simple to perform through image analysis.

However, some problems can appear in image analysis, and we will now describe physical problems met when calculating the fractal dimension using this technique.

- (i) The material investigated and the resin used to embed it do not have the same hardness. As a consequence polishing leads to a difference between the two levels and microscopic observations give a bad resolution and a crack profile less tortuous than in reality. It has been proved that the lack of resolution is equivalent to filtering the profile by a low pass filter, and decreases the calculated fractal dimension.
- (ii) When the Minkowski sausage method or another technique based on image analysis are applied, we have to transform digitalized images defined in grey level into binary ones. Value ‘true’ defines the material and value ‘false’ defines resin. We obtain all the grey level at the interface and the threshold used to scatter grey level image to binary one introduces

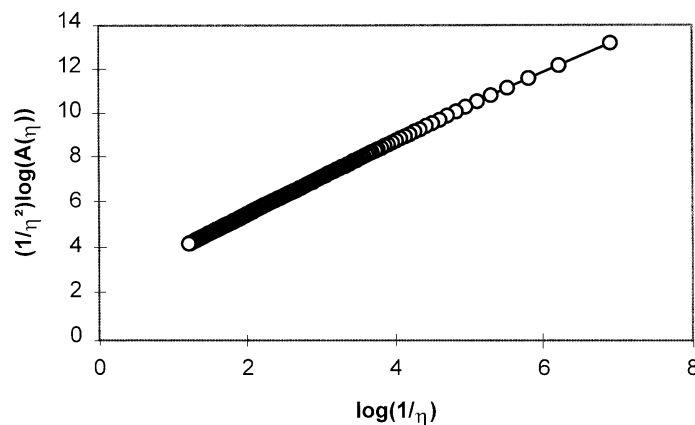


Fig. 4. Example of log–log plot between log (area of the sausage) and log (disk radius).

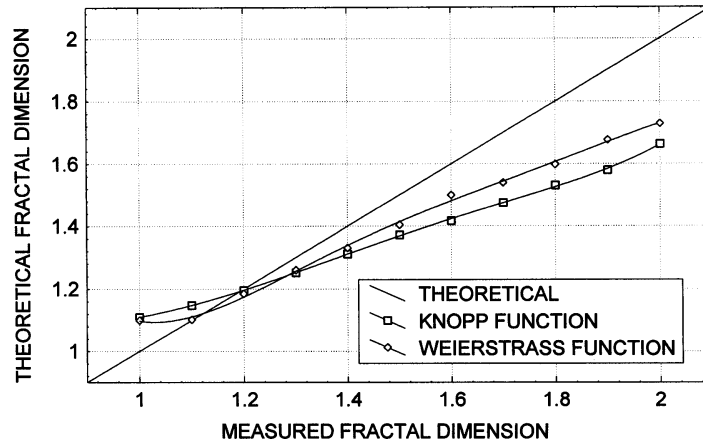


Fig. 5. Plot of the theoretical fractal dimension of the Knopp and Weierstrass function calculated by the Minkowski method with image analyser on a 512*512 pixels image. The straight line defines nil error.

an artefact in the interface: the threshold used to get a binary image defines a crack profile that is not the right one.

- (iii) The Minkowski fractal dimension is obtained by covering profiles with balls of radii r and the fractal dimension is obtained by studying the convergence of a limit function when r decreases. With digitalized images, when r decreases, the discretization effect increases because the balls could not be considered as circular. So an artefact is introduced. We studied fractal curves (Knopp and Weierstrass function) [13] with an image analyser (512×512 pixels) and applied the Minkowski algorithm. It could be observed (Fig. 5) that the theoretical fractal dimension is never obtained. For the highest fractal dimension, the maximal error is over 30% of the range of the fractal dimension, without microscopy artefacts.

2.4. The box counting method

This method applies simply when data files of profile roughness are obtained by profilometering techniques, confocal microscopy [18] or back-scattered electron [8]. Fig. 6 shows schematically the method used to obtained the fractal dimension. If N_η is the number of

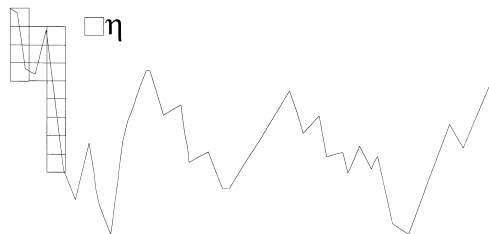


Fig. 6. Box counting method. A grid covered the curve and the log (number of squares) is plot versus log (η).

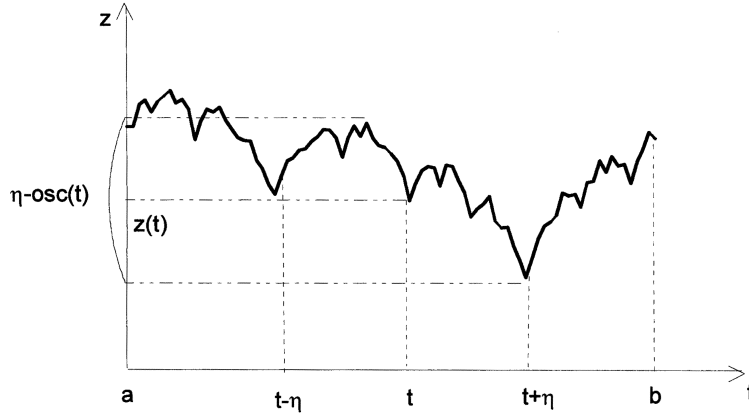


Fig. 7. η -oscillation of $z(t)$ used in the variation method.

square boxes of size η required to cover the length of the line, the area obtained is $N_\eta \eta^2$ and the fractal increment is:

$$\Delta = \lim_{\eta \rightarrow 0} \left[1 - \frac{\ln N_\eta \eta^2}{\ln \eta} \right]. \quad (9)$$

With the data file, this method can simply give the fractal dimension but it is not very precise.

2.5. The Variation method

Introduced by Tricot et al. [13, 19–21], the Variation method is based on the η -oscillation (Fig. 7). For a Γ -curve parametrized by $z(t)$ in $[a, b]$, one can define the oscillation as:

$$osc_\eta(t) = \sup_{|t-t'| \leq \eta} (z(t') - z(t)) - \inf_{|t-t'| \leq \eta} (z(t') - z(t)). \quad (10)$$

An η -oscillation is associated with a given t . The box area is replaced by the variation:

$$\text{var}_\eta(z) = \int_a^b osc_\eta(t) dt \quad (11)$$

and the fractal dimension increment Δ is given by:

$$\Delta = \lim_{\eta \rightarrow 0} \left[1 - \frac{\ln \text{var}_\eta(z)}{\ln \eta} \right]. \quad (12)$$

We have tested this method on mathematical curves used to simulate a fractal surface profile as Weierstrass–Mandelbrot, Knopp or Brownian motion with well known fractal dimension. A complete study of those curves will soon be published but we will only present here three remarks. At first, there is a problem at the borders. When the η -oscillation is calculated, a 2η large window is considered. Or, when $t < a + \eta$ or $t > b - \eta$, oscillations are not defined. Dubuc et al. proposed [20] to take the calculated value for $a + \eta$ if points are located in $[a,$

$a + \eta]$ and the calculated value for $b - \eta$ if points are located in $[b - \eta, b]$. It seems however more appropriate to calculate the fractal dimension with the points which are in $[a + \eta, b - \eta]$ only. The second problem is related to the choice of the η_{\max} value as arises in the Minkowski method. If η_{\max} is too high, the local oscillations can be neglected. However, for a curve with a high fractal dimension which presents many irregularities, we cannot neglect them. The choice of η_{\max} will therefore depend on the irregularity of the profile. Finally, the number of points used to calculate the η -oscillation is a very important parameter. 1000 points are generally needed in a 2η interval to obtain a precision in the η -oscillation calculus of 1%. So, if we use a file of 1024 points (for example obtained by confocal microscopy) and if we suppose that profile looks like a Weierstrass graph, 1024×1000 points are needed to limit the calculated error to less than 1%. It is very difficult with an experimental profile on a metallic fracture sample to record such a high number of points. Therefore, the method has to be improved and we proposed a method to calculate the fractal dimension avoiding these artefacts: this method, called the ANAM method [22], is based on the minimization of the autocorrelation by taking the average of the local difference of the profile on the whole interval η . We introduced the K_η^α function defined as:

$$K_\eta^\alpha = \frac{1}{b-a} \int_{x=a}^{x=b} \left[\frac{1}{\eta^2} \int_{t_1=0}^{\eta} \int_{t_2=0}^{\eta} |z(t+t_1) - f(t-t_2)|^\alpha dt_1 dt_2 \right]^{1/\alpha} dt \quad (13)$$

which gives the fractal increment:

$$\Delta = \lim_{\eta \rightarrow 0} \left[1 - \frac{\ln K_\eta^\alpha}{\ln \eta} \right] \quad (14)$$

by linear regression of $\log K_\eta^\alpha$ vs $\log \eta$ for different η values.

However, as fractal curves are not derivable over all the definition interval, it is then difficult to estimate the error. As discretization implies errors which rise exponentially with the fractal dimension, the number of points we need to calculate $A(\Gamma_r)$ with a given approximation rise exponentially with the fractal dimension. It is then necessary to modelize the error so the following relation is stated [23]:

$$VAR_\eta = d\eta^d + \frac{e}{\eta}, \quad (15)$$

where d and e are two constants. Applied to the ANAM method, we obtain:

$$K_\eta^\alpha = d'\eta^d + \frac{e'}{\eta}. \quad (16)$$

We have to find Δ , d' , e' , which minimize this function by non-linear regression. In fact, there is no mathematical reason, except for the discretization phenomenon, so that VAR_η or K_η^α is a function with a $1/\eta$ term. By statistical analysis, we take coefficient e or e' only if its value is significantly different from 0 (Student's test). Applied on the Weierstrass Mandelbrot curves with 1000 points, the ANAM method gives better results than the Variation one and the correction of the discretization error improves both (Fig. 8).

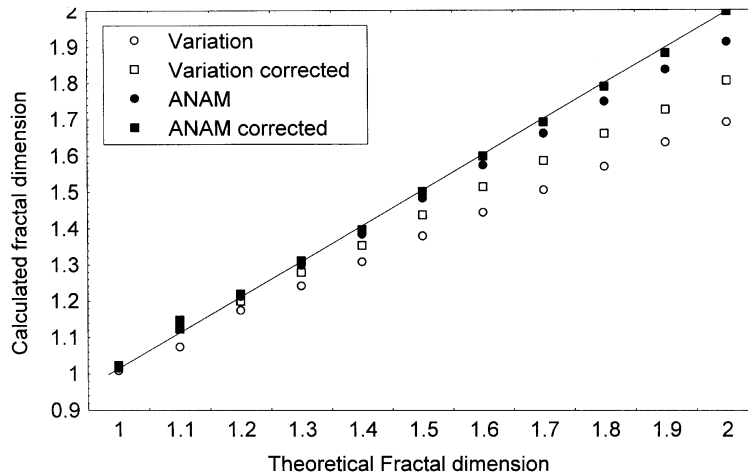


Fig. 8. Plot of the theoretical fractal dimension of the Weierstrass function versus the computed fractal dimension. Fractal dimensions are computed by Variation and ANAM methods with or without error correction. The straight line defines nil error.

On the other hand, when a profile is recorded by confocal microscopy, a noise is superimposed on the material signal (Fig. 9). For example, if a white noise is added to the true signal, the fractal dimension tends to approach the white noise fractal dimension (i.e. 2). We have just shown that the convergence criteria used to calculate the fractal dimension were computed at different scales. The results we obtained depend on the scale used:

- small scale—high frequency artefact (e.g. noise);
- large scale—low frequency artefact (e.g. perpendicularly of the sample to the measure instrument).

The calculation of the fractal dimension of the crack profile is calculated by a least square regression applied to a function that depends on the scale versus the scale. So we do not measure the intrinsic crack fractal dimension but the fractal dimension on a set of different phenomena and the result depends on the different scale where the fractal dimensions are

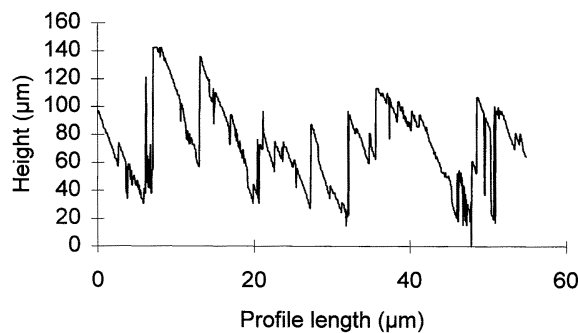


Fig. 9. Example of profile obtained by confocal microscopy and analysed with a computer.

calculated. We objectively believe that the fractal dimensions commonly reported in the bibliography are not the right ones for crack profiles and correspond to a number that depends on the experimental method, on the low and high frequency noise and to some extent on the crack profile itself. As a consequence, before computing a fractal dimension, all noises have to be studied. A correction was proposed by Lange et al. [18] who introduced a light smoothing to diminish the noise, but the roughness of the profile is affected by the degree of smoothing. It then seems difficult to correlate fractal dimension with mechanical characteristic through this method.

2.6. The slit island method (SIM)

This method, proposed by Mandelbrot et al. [10] to measure the fractal dimension of fracture surfaces, consists of a first attempt to correlate the fractal dimension with the mechanical properties of fractured materials. Since this first work, there has been considerable research and this method has been widely used. The SIM starting point is based on the assumption that the fractal dimension of a coastline of geometrically similar islands, resulting from the intersection of the fractured surface by a plane is $D_f = D'_f - 1$, if the fractal dimension of the surface is D'_f .

Experimentally, the surfaces of the fractured specimens are electrolytically plated with nickel or chromium in order to preserve the details of the fractured surface, mounted in a resin and then polished parallel to the fracture plane. After etching, the etched material appears like grey 'islands' surrounded by bright areas of nickel called 'lakes'. The islands contain 'lakes within islands' and 'islands within lakes' but generally only the former are taken into account [10]. The distribution of islands and lake areas and that of islands and lake perimeters of random sections exhibit hyperbolic distributions analogous to those found in geographic islands and lakes [24].

In *The Fractal Geometry of Nature* [25], Mandelbrot extends the relationship between the perimeter and area for a regular geometric figure to irregular ones. The theoretical base of the method considers that the ratio:

$$C(r) = \frac{(\sum P_i(r))^{1/D'_f}}{(\sum A_i(r))^{1/2}} \quad (17)$$

is a constant, independent of the shapes and the sizes of the islands and independent of the length of the yardstick r used to measure the perimeter $P_i(r)$ and the area $A_i(r)$ of the i th island (in spite of the fact that the area is finite but the perimeter length should be infinite because the perimeter is fractal).

Usually, the perimeter and area of the islands (generally 100 islands or more) are measured by using an electronic planimeter, by digitizing the island perimeter on a computer, or by quantitative image-analysis algorithms. The length of the coastlines of the slit islands is generally measured with a fixed scale for each successive section.

The fractal dimension increment Δ is calculated by plotting the variation of the logarithm of the area (in ordinate) versus the logarithm of the perimeter, for all the islands of each layer examined. Δ is obtained from the slope $2/D_f$ of the straight line obtained by linear regression

analysis. Different magnifications are used to obtain length measurements over a wide range of scales (typically 2–4 order of magnitude for the perimeter range) and imply the fractal character of the fractured surface. Mandelbrot et al. [10] claimed that ‘such linear relationships among all variable data while covering such a broad range of sizes are almost unique in metallurgy’. For Charpy specimens of 300 grade maraging steel, heat treated between 300 and 430°C and tested at room temperature, Mandelbrot et al. [10] obtained a negative relation

Table 1
Main usual results of relations between the fractal increment and the mechanical properties

Material investigated	Mechanical properties	Testing variable	Mechanical properties range	Δ range	Method Δ	Variation M.P. (Δ)	Ref.
300 Maraging steel	Charpy	Tempering	35–160 (J)	0.1–0.27	SIM	–	10
AISI 4340	Fracture X	Tempering	X	0.072–0.091	VSM + R	X	11
HSLA steel	Charpy	T, test	5–35 (J)	0.01–0.53	SIM	+	28
HSLA steel	Charpy	T, test	5–35 (J)	0.012–0.036	P	O	28
30CrMnSiNi2	K_{ISCC}	Tempering	19–57 (MPa \sqrt{m})	0.08–0.12	VSM + R	+	77
4340 steel	Bend. Cor.	T. Q., Method	X	0.594–0.753	SIM	X	26
4340 steel	Bend. Cor.	T. Q., Method	X	0.063–0.55	VSM	X	26
24SiMnCrNi2A	K_{IC}	Tempering	90–150 (MPa \sqrt{m})	0.12–0.22	SIM	–	37
30CrMnSiNi2A	K_{IC}	T, Test	60–105 (MPa \sqrt{m})	0.11–0.18	SIM	–	37
Dual phase	CVN	Tempering	4.9–8.2 (J)	0.2–0.8	SIM	–	38
Dual phase	Fatigue (ΔK_{th})	Martensite %	8–17 (MPa \sqrt{m})	0.09–0.22	SIM	+	41
Al-SiC	K_{IC}	Material	8.7–36 (MPa \sqrt{m})	0.084–0.248	R		55
Resin/C. comp.	Bending	Carbon fibre %	1.2–1.6 (MPa \sqrt{m})	0.1–0.38	SIM	–	39
Al alloy	K_Q	Heat treatment	28.5–45 (MPa \sqrt{m})	0.18–0.25	Cor. function	=	78
7475 Al alloy	Tensile	Grain diameter	X	0.143–0.362	VSM	X	79
Ti Alloy 6211	Tensile	Method	X	0.087–0.126	VSM	X	8
Ti Alloy 6211	Tensile	Method	X	0.171	SEM profile	X	8
Ti Alloy 6211	Tensile	Method	X	0.32	SIM	X	8
TA6V2XZr	DTE	Zr %	180–380 (ft.lb)	0.41–0.46	SIM	O	17
TA6V2XZr	DTE	Zr %	180–380 (ft.lb)	0.03–0.11	VSM	O	17
Ti Alloy	Tensile	Tempering	900–950 MPa	0.78–0.91	SIM	O	44
Cement	K_{IC}	Material	0.38–0.59 (MPa \sqrt{m})	0.085–0.117	Confocal + R	+	18
Concrete	K_{IC}	Material	800–1300 (Psi \sqrt{inch})	0.065–0.12	P + BC	–	40
Concrete	K_{IC}	Material	1.6–5.16 (MPa \sqrt{m})	0.033–0.06	VSM	O	64
Ocala chart	K_{IC}	Tempering	1.05–1.55 (MPa \sqrt{m})	0.15–0.32	SIM/R	+	43
Alumina + glass ceramic	K_{IC}	Material	1.6–3.9 (MPa \sqrt{m})	0.05–0.31	SIM	+	42
Dental porcelains	K_{IC}	Material	0.74–0.8 (MPa \sqrt{m})	0.16–0.33	P + V	–	80
AlN ceramic, glass	K_{IC}	Material	0.6–3.6 (MPa \sqrt{m})	0.024–0.145	R	+	47

Method used to calculate the fractal dimension—SIM: Slit Island; VSM: Vertical Section; R: Richardson plot; P: Profilometry; SEM: Secondary Electron Intensity Brightness Profile; V: Variation; B.C.: Box Counting. Relation between mechanical properties and fractal increment—(X): not defined; (O) no correlation; (–) mechanical properties decrease as fractal dimension increases; (+): mechanical properties increase as fractal dimension increases; (=) no variation

between the impact energy and the fractal dimension increment (the energy decreases as Δ rises). These results and others provided by other work are shown in Table 1.

These results are surprising. Normally one expects exactly the opposite trend as it is generally agreed that during fracture a rougher surface (a more tortuous path) is associated with an increase in the energy absorbed and then leads to a higher toughness of the material. Numerous objections were made and a number of publications questioned the applicability of the SIM.

Firstly, the results obtained by the SIM differ from those obtained via other techniques. For example, Pande et al. [17] found a Δ variation between 0.41 and 0.46 with the SIM and between 0.03 and 0.1 with the vertical section method although the variations between Δ and the dynamic tear energy are similar. Krupin [26] obtained different values for SIM (between 0.594 and 0.753) and the vertical section method (0.063 and 0.55) and the results are not correlated. One reason advanced by Huang et al. [27] is that perhaps the different methods describe the roughness from different orientations. Another argument used by Ray et al. [28] concerned the errors associated with the measurement of Δ . By statistical analyses of the linear regression, two fractal dimensions can be distinguished only if their difference is at least 0.05. Regarding this situation, the values reported by Pande et al. [8] are not significant. Secondly, Ray et al. [28], using SIM, found some relationship between impact energy and fractal dimension but no correlation using the Richardson method. This result is physically meaningless and difficult to interpret. Thirdly, the fractal dimension is often found to be yardstick dependent [14, 29–31], so the Δ determined by the SIM is not the intrinsic one. Particularly Lung and Zhang [32] showed that the fractal dimension defined by the perimeter–area method on the Von Koch flake depends on the generator. All those authors concluded that non linear variation between the logarithm of the perimeter and the logarithm of the area exists and therefore the fractal dimension can be evaluated by SIM only with short yardsticks. Meisel [33] answered these objections and showed, on fractal constructions, that when excluding too long (where there was a tendency to dispersion) or too short yardsticks, the fractal dimension found by the SIM has an intrinsic value.

Finally, some propositions have recently been made to improve the method proposed by Mandelbrot et al. [10]. Mu et al. [30] assumed that the perimeter of the island is directly proportional to its maximum diameter and that fractal dimension can be obtained with the slope of the straight line representing the variation of the logarithm of the perimeter versus the logarithm of the maximum diameter for a constant yardstick length. The results obtained by this method are close to those obtained by the perimeter–area method when r is small enough. Another interesting feature is put into relief by Mu et al. [30] using the square grid method to obtain various yardsticks. The fractal dimension measured by the perimeter–maximum diameter method is yardstick independent over a wide range, but two values of Δ are found: one for long yardsticks and another for short ones. These results could be explained in terms of multifractals and need further explanation.

The choice of lakes within islands is questioned by Huang et al. [27]. The results obtained for the variation of the impact toughness of CC 45 steel versus the fractal dimension showed that different conclusions may be drawn. For ‘lakes within islands’, a negative correlation was found as in the usual works on steels, but if ‘islands within lakes’ are considered, the correlation becomes positive. The authors explained those surprising results by the fact that in

ductile fracture, the plastic deformation creates dimples on the surface so that each island of the fracture surface does not correspond to a lake of the matching surface.

The main criticism of the SIM concerns the self-similarity of the islands in fracture section which is a prerequisite condition. If the structure is self-affine rather than self-similar then Δ can take two or more values [34]. Mandelbrot [25] defined a fractal as ‘a shape made of parts similar to the whole in some way’ and in other words, $C(r)$ in Eq. (17) and D_f' must be constant. In a recent work, Imre [35] shows that if the islands have a distribution of the form factor, the SIM can not be applied. For example, if the term $C(r)$ and the perimeters of the i th islands are correlated by a power function $C(r) = P_i^\delta$, then:

$$\log P_i(r) = (D_f + \delta)\log A_i. \quad (18)$$

The apparent fractal dimension is $D_f + \delta$, and then the perimeter–area relation does not hold. It can also be seen that if the area of the islands is constant and the perimeter varies, then an infinite fractal dimension can be found. This restriction applied also to the perimeter–maximum diameter method developed by Mu et al. [30]. Preliminary observations of the distribution or constancy of form factors and Δ are required in a certain scaling range (probably two orders of magnitude) before applying the SIM method [36]. However, the determination of the self-similarity is difficult to resolve. Pande et al. [17] claimed that the condition is not verified because the ratio of the length to the breadth of the islands is not size invariant, but Meisel [33] did not agree with these analyses. According to this author, the ratio length versus breadth is characteristic only for similar Euclidean figures but is not required for self-similar fractal curves. All those criticisms and notes show that the SIM is not a method which gives simple and indisputable results.

3. Discussion of the results and correlation between fractals and mechanical properties

As previously stated for the Slit Island Method, many correlations are made in the scientific review between the toughness characterized by K_{IC} or the impact energy (CVN) and the fractal dimension. Does a correlation exist between toughness and Δ ? Are these correlations relevant? This paragraph aims to review the latest works.

We must first study the experimental correlations. All investigators found that the materials exhibit self-similar fracture behaviour and that fracture is fractal. The methods used to determine the fractal dimension are SIM, confocal microscopy, vertical section, etc. and are reported in Table 1 with the results obtained and the materials investigated. Three conclusions can be drawn.

- (i) Mandelbrot et al. [10] were the first to correlate the impact energy obtained by the Charpy impact test and Δ . It was shown that with increasing toughness, the fractal dimension exhibits a decreasing trend. As always with the slit island analyses Mu and Lung [37], Su et al. [38], Lin and Lai [39], Saouma et al. [40], Wang et al. [41] find the same correlation as Mandelbrot between, G_{IC} , K_{IC} , ΔK_{th} or the impact energy and Δ .

The origin of the negative correlation between the toughness of the material and the fractal dimension of the fractured surfaces can be associated with the choice of the yardstick used to measure the perimeter and the area [28] in the Slit Island Method. For CrMnSiNi₂A and $\eta = 1.85 \mu\text{m}$ they find a negative correlation between K_{IC} and Δ , and the correlation becomes positive for $\eta = 0.08 \mu\text{m}$ and $\eta = 0.15 \mu\text{m}$.

(ii) The above results are all the more questionable as the contrary can be expected. Unlike the previous works, Ray and Mandal [28] found a positive correlation between the fractal dimension of a microalloyed steel specimen broken at subambient temperatures and CVN. Mecholsky et al. [42,43] found such a positive correlation for ceramic materials. A general relation was obtained for aluminas and glass-ceramics:

$$K_C = Ea_0\Delta^{1/2} + K_0 \quad (19)$$

where E is the elastic modulus, a_0 is a characteristic atomic dimension and K_0 corresponds to the toughness value for the hypothetical material with a smooth fractured surface and depends on the 'families' investigated. One other form obtained by these authors is:

$$\ln(K_{IC}) = a + b \ln(\Delta). \quad (20)$$

(iii) Pande et al. [8,17], and Richards and Dempsey [44] find no correlation between the mechanical properties and Δ . Bouchaud et al. [12] find a fractal dimension increment of 0.2 for an aluminium alloy not withstanding the heat treatment and exclude systematic variations of the fractal dimension with the fracture toughness, as obtained by Mandelbrot et al. [10].

Is this disappearance related to the materials investigated (Table 1) or is it more fundamental? Theoretical approaches were made to answer this question. Different models have been proposed over the past 10 years to account for the variation of the toughness with the fractal dimension of the fracture surface. Some basic considerations can be made.

In Irwin's approach to fracture mechanics, the conventional critical crack extension force is written:

$$G_{IC,0} = 2\gamma \quad (21)$$

where γ represents the effective energy per surface unit (surface energy γ_s + strain energy γ_p).

If the fracture surface is fractal, then the crack area a_f is greater than the smooth one a_0 :

$$a_f = a_0 r^{1-D_f} \quad (22)$$

where, as seen before, r is a dimensional measurement scale called the yardstick.

The actual critical crack extension force becomes:

$$G_{IC,f} = 2\gamma(a_f/a_0) = G_{IC,0} r^{1-D_f} \quad (23)$$

and

$$K_{IC,f} = K_{IC,0} r^{(1-D_f)/2}. \quad (24)$$

Different conclusions can be drawn from these relations:

(i) By considering γ to be constant, Mu and Lung [37] obtained a linear relation as follows:

$$\ln(K_{IC}) = \ln(K_{IC,0}) + [1 - D_f]\ln(r_i) \quad (25)$$

where r_i , the yardstick length would be equal to the smallest length of a step of crack propagation. A linear and negative correlation is then obtained between the logarithm of fracture toughness and the fractal dimension and corresponds to experimental results on 30 MnSiNi₂A and 24 SiMnCrNi₂Mo steels under different heat treatments. The anisotropic behaviour and the corrosion influence were introduced latter by Jiang et al. [45]. If we suppose that the smallest yardstick for crack propagation is in direct relationship with the grain size:

$$r = kd \quad (26)$$

where $k = 1$ corresponds to transgranular fracture and $0 < k < 1$ to intergranular fracture, so:

$$G_{IC,f} = 2\gamma_s d^{1-D_f} k^{1-D_f} \quad (27)$$

and shows the influence of rolling on crack propagation [46].

Such theoretical negative correlation was also obtained by Lung and Zhang [32] by analysing the distance between two large inclusions and the number of grains over the distance.

(ii) Tanaka [47] recently studied the toughness of SiC, AlN ceramics, soda lime glass and WC-Co by indentation tests. The cracks were introduced by a Vickers diamond pyramid indenter and modelled by means of an initiator and a generator with respect to the Von Koch flake construction as described in Fig. 1. As a result of the theoretical approach, the logarithm of the indentation fracture toughness was expressed as follows:

$$\ln K_{IC} = 0.5 \ln \gamma_s + [(1 - D_f)/2] \ln \epsilon_r \quad (28)$$

where ϵ_r is the ratio of the lower and upper limits of the scale length between which a crack exhibits a fractal nature, ϵ_{\min} is assumed to be roughly equivalent to atomic spacing for brittle materials and ϵ_{\max} is related to the crack path or some microstructural features such as the grain size. As $\epsilon_r < 1$ this relation shows an increase in the fracture toughness when the fractal dimension rises, i.e. the fractured path becomes more tortuous. This relation is in fairly good agreement with the author's results and comply very well with the experimental data from Mecholsky et al. [42, 43].

Jiang et al. [48] modelled the micro-roughness of ductile fractured surfaces by introducing the coefficient M :

$$M = h/W \quad (29)$$

where h is the depth and W the width of the microvoids, and considering this geometry as fractal. As J_{IC} is related to M by Thomson and Ashby [49] they found a relation between J_{IC} and the fractal dimension of the surface:

$$J_{IC} = \frac{\sigma_0}{3} \ln \left[\frac{4^{(1-D_f^{-1})} - 1}{12f_p} \right] l_0^* \quad (30)$$

where σ_0 is the tensile strength of the material, f_p is the volume fraction of voids and l_0^* a characteristic distance. This relation shows that the toughness rises when the fractal dimension increases. If the true fracture area is fractal, it is greater than the conventional one (the fracture surface is commonly assumed to be a simple two-dimensional flat object), and Irwin's critical crack extension force is rising.

(iii) For Nagahama [50] Eq. (20) can be representative of both a positive or a negative correlation, whence the coefficients a and b depend on the fracture path. If $b > 0$, the fracture will be ductile and corresponds to the results of Mu et al. [39], if $b < 0$, the fracture is brittle and corresponds to the results of Mecholsky [42,43]. This empirical relation is consistent with theoretical Eq. (25) where, if $\log r > 0$ or $\log r < 0$, we find the experimental trend obtained, respectively, for ductile and brittle fracture.

(iv) Lei and Chen [51] discussed the Mu and Lung model [37] and have shown that Eq. (20) is invalid because $K_{IC,f} < K_{IC,0}$, and no correlation can be found. The fractal dimension is a measure of the entropy and thus is a state function. Since the crack propagation is a non-reversible process, the entropy increases during the crack propagation and the fractal dimension of the fracture path is not constant.

Another interesting approach by Li [52] considers that all toughening mechanisms like plastic deformation are just means of increasing the area of the fracture surface, and the toughness can still be the surface energy.

(v) In the last type of model the fractal dimension is found to have a universal value. The variation of the toughness with the fractal dimension is in contradiction with the model from De Arcangelis et al. [53] which predicts that the fractal dimension increment is independent of the microstructure (close to 0.1). In the same way, material failure was studied by Termonia and Meakin [54] by means of a molecular model based on the kinetic theory of fracture. It was shown that in a wide range of materials, the fractal dimension increment is found to have a universal value $\Delta = 0.27$. By observing the results recorded in Table 1, it can be seen that the fractal dimension increments are generally close to 0.2, with small variations apart from this mean value.

We will try to summarize the origin of the fractal character. Firstly, the self-similarity of the fracture surface was observed by different authors [8, 10, 55] and the linearity of the log–log plot obtained with different methods proves it on many decades of magnification.

Then, Mandelbrot et al. [10] wrote that, if the growth and coalescence of voids for example for ductile fracture, were independent of their neighbors, we would have a percolation. It is the same reasoning for the crack propagation, but, if it was a percolation, we would have a universal value for the fractal dimension. To these authors the model is too crude and the resulting process differs from the usual percolation. It was the starting point of different researchers. Some authors developed another model based on the crack branching. The crack branching irregularities in brittle materials depends on microstructure, mechanical behaviour, specimens, etc. Those irregularities can be modeled by fractal geometry and this new approach provides simplicity in the investigation of crack growth [56]. Hermann developed a

vectorial model [57] and explained that the origin of fractality is the competition between different directions of propagation. His model is roughly equivalent to experimental cracks.

A more simple model was developed by Xie [58]. If we consider the $\pi\beta$ angle between the branches and the initial crack, we obtain:

$$\log\left(\frac{K}{K_0}\right) = \frac{\log(3) - \log\left(2 \cos \frac{\pi\beta}{2}\right)}{2}. \quad (31)$$

If the branching angle $\pi\beta$ is small, the amount of strain energy dissipated by crack branching is small. So, the rise of toughness by crack branching can be explained through fractal geometry. This branching angle should be dependant on the density of microcracks [59]. These conclusions are contested by Lei and Chen [51, 60], however the model seems to be good [61] and the experimental results show that the increase or decrease in toughness are linked with microstructure and crack branching. A decrease in toughness can be explained by the great influence of the micromechanism. The mechanic equations also need to be defined in a non-integral dimensional space.

This approach was improved by Chelidze [62] by taking account of anisotropy effects and by modelizing the growth by percolation. It explains the large energy consumption of heterogeneous materials when compared to single-crystal values. All these approaches are very recent and so, more conclusions are actually not possible.

The microstructure of materials has a great importance on the fracture surface. Wang et al. [41] studied the fatigue in dual-phase steels and concluded that the fractal dimension D_f varies parabolically with volume fraction of martensite which is in keeping with the fractographic features. Hornbogen [63] said that martensitic alloys in which the microstructure is fractal, provides cleavage planes. So if the microstructure, like martensite, is fractal, so will be a-priori the obtained fracture surface, and relations may be developed [64–66] between the fractal dimension and microstructural features such as grain size, grain boundary profile, inclusion volumes, etc. Keustermans [16] summarizes all these experiments. By taking for example eutectic or dendritic alloys, he concluded that microstructures are correlated, at least, with the self-similarity of the surface. The relation between fractal dimension and microstructure is clear, but a correlation between the fractal dimension and the toughness is difficult to establish. Davidson [55] explained the lack of correlation in aluminium alloys reinforced with SiC by the fact that the fracture toughness is mainly related to the work done within the plastic zone during the growth process. Very little work was expended on the formation of the surface as compared to the work dissipated by plastic zone. Plasticity is a volumic damage with $\Delta \gg 0$ and cleavage fracture is a surfacic damage which gives $\Delta \approx 0$, but the results they obtained for the fractal increment varies between 0.1 and 0.2. It can be seen that the variations are generally small while the errors on the fractal dimension measurements rather important. This topic has been much debated upon [31, 67, 68], but the results are not sufficient to conclude about a correlation, more particularly because of the lack of experimental results. It seems cleverer to observe the evolution of the fractal dimension on a profile with the crack propagation than on the whole surface. So Keustermans [16] and Dauskardt et al. [69] concluded that a universal correlation between tenacity and Δ is illusive.

However Δ is a good indicator of the influence of the microstructure and of the different fracture modes on the surface's roughness.

The proposition of a universal value proposed by Mandelbrot et al. was resumed by different authors: Bouchaud et al. [12, 70, 71], Silberschmidt [72], Måloy et al. [73] and Schmittbuhl et al. [74]. They wrote that no correlation between fractal dimension and fracture morphology exists: the fractal increment is then a universal value. This value was however confirmed by numerical simulations [75, 76], but does not agree with some results presented before. Bouchaud said that the result suggests that the crack propagation exhibits scaling laws at large length scales. All these works show that self-affine fracture surfaces obtained by crack-propagation seem to be a more interesting point of view than self-similarity. This is confirmed by Gouyet [5]. Still, today there are no other arguments to confirm this hypothesis of self-affinity and of universal value.

4. Summary and conclusions

It is often reported that the fractal geometry, introduced by Mandelbrot is of great importance for the understanding of the fracture mechanic process. In fact, its main merit is that the self-similarity of the observed surfaces proves that the mechanisms involved in the fracture process are generally scale independent. These observations imply that the macroscopic scale is related to the atomic one. However, many problems arise when we wish to correlate the fractal dimension of the fractured surfaces with the mechanical properties.

At first the methods themselves are suspicious. Table 1 shows that the results depend on the method used to calculate the fractal dimension. The differences obtained between two techniques are generally greater than the variations with the mechanical properties; and the algorithms developed are not precise enough to give the right fractal dimension of known fractal profiles [22].

Secondly the variations of fractal dimension obtained with similar techniques are in some cases of the same order of magnitude as the measurement errors. So a great number of profiles have to be taken in order to calculate a fractal dimension. Our experiments confirm that the correlation is problematic and perhaps illusive because of the complexity of the mechanism and the lack of precision of the method.

Thirdly, when the profile of fractured specimens is recorded, experimental errors are masked. These errors correspond to a noise superimposed onto the material signal when using confocal microscopy or backscattered electron in SEM, or filtering when using profilometer technique or image analyses. It is then obvious that the fractal dimensions obtained with confocal microscopy are greater than the results obtained by the Minkowski sausages. For these reasons it seems that further research is necessary to improve the theoretical method and the derived algorithms before drawing correlations between the mechanical properties and the fractal dimension.

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