From Classical Rationality to Quantum Cognition

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Abstract

This article highlights the difficulties of "classical" rationality, grounded on classical set-based logic and classical probability calculus, and explains how they can be overcome. Classical rationality may have gone some way towards realizing the age-old project of mechanizing thought, thus making possible the dazzling development of artificial intelligence we are witnessing. However, as shown from experimental data, it suffers from many biases that make it incapable of reliably modeling mental processes. This article shows that mental processes can be more reliably modelled within a generalized probability theory grounded on the vector formalism of quantum theory. The reason is that such a quantum-like approach to cognition is capable of accounting for the contextual, order and interference effects inherent to most of mental processes. An important consequence of this necessary shift, from classical to quantum cognition, is that Bayes' rule, which plays a fundamental role in categorization tasks, must be replaced by a generalized probabilistic rule capable of accounting for order effects. It is shown that implementing this new probabilistic rule could significantly improve the current deep learning algorithms of artificial emotional intelligence.

Keywords: Classical rationality; Quantum cognition; Bayes' rule; Emotional Artificial Intelligence

Introduction: From the language of human thought to classical logic

Leibniz's project was to create a system of ideographic signs, the *lingua characteristica universalis* in direct contact with the ideas they express and in which human reasoning could be translated into the form of a calculation [1]. In this language of human thoughts, inspired from Chinese ideography, ideas could be broken down into simple ones, forming the alphabet of human thought, while the combination of its "letters" would enable invention, not just deduction. Following this idea, Boole proposed in his *Laws of Thought* [2] to study "the fundamental laws of the operations of the mind, expressing them in the symbolic language of calculus". His "algebra of human thoughts" clearly reduces the language of human thought to *classical propositional logic*. A reduction which echoes with more recent proposals, like Fodor's hypothesis of a mental language, the "Mentalese", whose terms would be structured as logical statements, using the classically defined logical connectors [3]. In the same vein, let us finally mention Frege's seminal work on predicate classical logic. If in his *Begriffsschrift* [4], he set out to take up Leibniz's unfinished project of developing a universal ideographic language of human thoughts, his "formal language of pure thought constructed on the model of arithmetic" is nothing but the *classical predicate calculus*.

The connection between this symbolic approach to thought in terms of classical logic and the project of its mechanization was finalized by the work of Turing and Church. According to Church-Turing's thesis, "computable" functions (in intuitive sense) are those whose values can be calculated by a Turing machine [5] [6]. Moreover, Turing showed that there exists a universal Turing Machine which can have as input (written on the tape) the description of the functioning of any Turing Machine and the input of the latter, which is the theoretical model of modern computers. Mental processes, understood as calculation governed by classical logic, could thus be mechanized. However, the project of mechanisation of thought has been realized thanks to a clear shift in the search for the "universal language of human thought": Leibniz' initial idea of a universal *ideographic*

language, as a direct and reliable expression of human thoughts, has clearly been reduced to the idea that mental processes can be modeled by *classical logic*.

It could be argued that, more recently, this project of mechanization of thought has been realized within an alternative approach to mental processes which does not rely on the symbolic approach to thought. According to this alternative, explicitly materialist approach, thought would *emerge* from the brain, as its biological product. This emergentist or "connectionist" approach inspired by the functioning of the brain overcomes several limitations of the symbolic approach, like the fragility and the rigidity of its programs, and made it possible to mechanize important cognitive tasks, such as pattern recognition, learning or visual perception. However, *realizing these tasks appeals to Bayes' inference rule, whose validity still relies on the relevance of classical logic and classical probability calculus for modeling mental processes.*

This article emphasizes several difficulties of the modelling of human cognition by the classical rationality, grounded on classical logic and classical probability calculus (section 1). It then briefly presents the quantum-like approach to cognition where all these issues can find elegant and satisfactory solutions (section 2). Section 3 deals with the key ingredient which is currently used for integrating uncertainty in cognition and decision-making, namely Bayes' rule. It is recalled that the latter is valid only if the order in which the events considered is indifferent. A new, probabilistic rule capable of accounting for order effects is then shown within quantum cognition (section 3.2) and the way it can be implemented in the field of artificial emotional intelligence is explained in section 4.

1. The failure of classical rationality in the current models of mental processes

As emphasized by many authors, including Cruz, Baratgin, Bruza, Busemeyer, Wang and Aerts [7-10], several "fallacies" in human reasoning seem to occur when cognition and decision-making are modeled according to classical rationality. These apparent fallacies of human reasoning mainly regard conjunctive, disjunctive and conditional inferences, as much as decision making under uncertainty. Several apparent "fallacies" of human reasoning have been analyzed in detail and attempts to explain or overcome them have been provided in the previously mentioned references [7-10]. However, for a question of length of the article, and as suggested by an anonymous referee, we will here essentially focus on a few of them, those that involve *typically non-classical features of human rationality*, namely *context effects*, referring to works including those of Hampton and Uzan [15] [16], *order effects*, in agreement with Wang and Busemeyer's more formal presentation [10] *and interference effects*, referring to the works of Aerts, Busemeyer and Uzan [8-9] [16]. Note that a particular focus on the study of order effects will introduce to the *non-classical Bayesian modelling* developed in section 3.3 and its application in the field of emotional intelligence.

1.1. The conjunction fallacy.

The conjunction fallacy refers to the fact that, *in contradiction with the classical probability calculus*, human subjects often assign a probability of occurrence of the conjunction of two events greater than the probability of the occurrence of each of them: $P(A \wedge B)$ could be greater than $P(A)$ and $P(B)$ while, as a direct consequence of Kolmogorov's axioms of classical probability calculus, the probability $P(A \wedge B)$ cannot exceed either $P(A)$

or P (B) [1]. This apparent "fallacy" of human reasoning about conjunction emphasizes *the role of the context* where these events occur: the belief that B is true when A is known to be true can be greater than the belief that B is true without knowing that A is true because A can play the role of a context in which the conjunction $A \wedge B$ becomes more probable than A and B.

This emphasis on contextuality questions the thesis of compositionality of meaning whose roots lie in the principle of verifunctionality of classical logic. According to the thesis of compositionality of meaning, which was for example supported by Fodor [11], the meaning of an utterance would be determined exclusively by those of its components and the way in which they combine. However, everyday experience shows us that the meaning that we attribute to a linguistic expression depends on the whole sentence or even the whole text in which it is used, and this meaning is thus defined by its contribution to this global linguistic context. This property of *contextuality* of natural language has been clarified and generalized by several authors, including Stalnacker [12], following the founding works of Austin [13] and Wittgenstein [14].

The failure of the thesis of compositionality of meaning, and then that of the principle of verifunctionality of classical logic, has been illustrated by an experimental study conducted by Hampton [15]. This experiment quantifies the role played by the context in a very simple case, by studying the way we assign meaning to a concept obtained by composing the two concepts "food" and "plant" and gives very significant results. A study of this experiment and of its possible quantum-like interpretation has been developed by Uzan in reference [16].

1.2. Order effects in decision-making

In a survey realized in 1997 (September 6-7) and involving 1002 respondents, half of the participants were asked the two questions 'is Clinton honest and trustworthy?', noted as A hereafter, *and then* 'is Gore honest and trustworthy?', noted as B hereafter, while the other half were asked the same pair of questions in the opposite order. As reported by Moore [17], the list of answers for the two groups shows that Clinton received *50% agreement when asked first* (which defines the "non-comparative" context) but *57% when asked second* (which defines the "comparative" context because this answer can be influenced by the first one). It also shows that Gore received *68% when asked first and 60% when asked second*. This difference in the frequencies of the respondents' answers shows that *the order in which the questions are asked is significant* since the frequency of the positive answers to the same question depends on whether this question is asked first or second. Focusing for example on positive answers for both questions A and B, respectively noted as Ay and By, this order effect can be expressed by the following difference: $P(Ay\ By) \neq P(By\ Ay)$, where P (Ay By) is the probability of responding "yes" to question A followed by "yes" to question B, and P (By Ay) is the probability of obtaining the same answer to these questions asked in the inverse order.

Moore calls this type of question order effect "consistency effect" to denote the fact that the difference between the probabilities of positive answers for questions A and B decreases from the non-comparative context to the comparative context, which is here the case since in the non-comparative context P (By) – P (Ay) = 18% while in the comparative context P (Ay/n By) – P (By/n Ay) = 3%. Note that other types of order effects in decision making

[1] The reason of this constraint is that the classical probability calculus is isomorphic to the Boolean algebra constituted by the set of parts of a set ordered by the inclusion relation and provided with the operations of complementation, intersection and union.

have been observed in other similar survey experiments, for example a "contrast" order effect showing that, unlike the previous consistency effect, the difference between these probabilities is amplified in the comparative context.

1.3. Order effects in the domain of emotions

Order effects do not only occur in decision making but in *all* mental processes where subjective experience is involved. In particular, significant order effects can be observed and quantified in the domain of emotions, and should then be taken into account in the cutting-edge research in artificial intelligence (see section 4). As can be observed, and in contrast with the possible assignation of intrinsic properties to physical objects in classical physics, experienced emotions cannot be regarded as intrinsic features of a person since they are continuously changing according to our life experience. Their nature and their intensity are highly *contextual* since they strongly depend on our personal past and present experience of life, on our social environment and even on what we felt just a moment before. For example, asking a subject about her degree of happiness and asking the same question after reminding her of a sad event in her life generally provides different results. As was the case for the previous example of surveys with two successive questions, the order effects relative to emotions can be evaluated from data on successive measurements of the intensities of emotions experienced by subjects. These intensities can be collected by asking them to report discrete values on a graduate scale or to report them continuously, using a continuous response digital interface on which the subject moves a stylus or finger [18]. A precise study of the non-commutativity of emotional observables has been provided by Uzan in reference [19] and is briefly reported in section 2.1.1, after introduction of the appropriate quantum-like formalism. In this reference, a method for computing their degree of non-commutativity is provided. This computation is explicitely developed for the couple of observables Anger and Disgust

Moreover, emotional observables do not generally commute neither with their physiological correlates nor with their behavioral correlates. In a similar manner, this can be established from data about the joint measurement of emotional observables and their physiological or behavioral correlates reported in the literature [20-23]. For example, Kassam's and Mendes's article [22] shows on experimental basis, that the very act of reporting one's own emotional state can drastically change one's physiological and behavioral "responses", this effect being particularly significant for subjects conditioned in angry state. In this experiment, the subjects are conditioned in such an angry state by delivering them a negative feedback to a difficult task they have done -for example by telling them that they are incompetent. The observed physiological responses are here evaluated by the values of cardiovascular observables, like heart rate and pre-ejection period, which can be measured almost continuously. The behavioral responses are evaluated by external experimenters through videos showing the participants performing the required tasks, by noting for example their facial expression and their body movements. Kassam's and Mendes' study clearly shows that for these subjects, conditioned in angry state, the changes in the values of the cardiovascular and behavioral observables are significantly different depending on whether or not they report their emotional state (of anger, in this example). This tells us that the successive measurement of emotional and physiological or behavioral observables gives rise to order effects because if these observables were commuting their values would not be inter-dependent, as is actually the case.

1.4. Ellsberg's paradox of the classical decision-making theory

Tversky and Shafir [24] have demonstrated on experimental grounds a significant violation of the principle of consistency of possible choices in classical decision theory, *which questions the validity of the law of total*

probabilities of the classical probability calculus. This violation gives rise to Ellsberg's paradox regarding the behavior of economic agents [25].

The principle of consistency of possible choices, known in the literature as the "sure thing principle", formulated as follows by Savage [26]:

> "If A is preferred to B when X is realized and if A is preferred to B when X is not realized, then A is preferred to B even if it is not known whether X is or is not realized."

This principle was tested by Tversky and Shafir in the following experiment: participants are (actually) offered to play a game where the chance of winning $200 is the same as the chance of losing $100, that is, 1/2. Participants are offered the game a first time and asked if they would like to play a second time in the following three cases (the population being divided into three groups of equal size): (a) they know they won the first time, (b) they know they lost the first time and (c) they don't know whether they won or lost the first time. The results are as follows: 69% of participants in group (a) decide to play again, 59% of participants in group (b) decide to play again, while only 36% of participants in group (c), whose participants don't know whether they won or lost, decide to play again. This result is at odds with the "sure thing" principle mentioned above, since if the latter applied, the probability to play again for group (c) would be 64 %. More precisely, according to the law of total probability of the classical probability calculus, this probability would be equal to half the sum of the probabilities measured for groups (a) and (b) respectively, which define exclusive sequences of events:

$$\begin{aligned} P(\text{to play again}) &= P(\text{to have won the first time and to play again}) \\ &\quad + P(\text{to have lost the first time and to play again}) \\ &= (1/2)\,[P(\text{to play again knowing that you won the first time}) \\ &\quad + P(\text{to play again knowing you lost the first time})]. \end{aligned}$$

This *violation of classical probability calculus*, which at first glance seems to indicate that human beings would be prone to "fallacious" reasoning, is in fact a typical example of how they "reason" and make decisions.

2. How overcoming the failure of classical rationality?

The previous developments show some important limitations in the classical approach to mental processes and thus corresponding limitations in the development of artificial intelligence-whose first aim is to simulate these mental processes. Many proposals have been done to overcome these limitations according to two different ways. The first one involves pragmatic or heuristic considerations on real human cognition processes, but they do not question the validity of classical rationality.

For example, Grice [27] has proposed a purely pragmatic, socially-based, explanations of the disjunction fallacy, according to which disjunctive statement is estimated to be less probable than one of its component statements. This author has suggested that it would be inappropriate, even misleading, for a person to endorse in a conversation the disjunction of two statements when one of its components is regarded as more informative about the situation considered. Gigerenzer and Todd [28] have suggested that, in contrast to what is generally supposed in logical modelling of human reasoning, the human is not endowed with supernatural powers of reasoning, limitless knowledge, and endless time but that "decisions in the real world requires a more psychologically plausible notion of bounded rationality", namely the use of *"tool boxes" of heuristics* that shorten and greatly simplify and accelerate the decision making process and preserve the validity of classical rationality. In order to illustrate such considerations, these authors mention the simple decision tree often used by physicians for quickly

classifying incoming heart attack patients into high and low risk patients. In the same order of ideas, Tversky and Kahneman [29] mention three mental operations generally used in decision making under uncertainty, which lead to systematic and predictable errors of reasoning and should then better understood in order to improve judgments and decisions in situations of uncertainty while preserving classical rationality.

A very different approach to deal with these apparent "fallacies" of human cognition consists in questioning the logical basis of classical rationality. For example, in the field of three-valued logics, initially proposed by Łukasiewicz and further developed by Reichenbach, Frege and other famous logicians, de Finetti [30] has proposed a three-valued treatment of conditional sentences in order to characterize uncertainty in judgements. In contrast with the classical truth-value assignation of conditional, Finetti assigns the truth-value "undefined" to a conditional inference when the antecedent is false. More accurately, fuzzy logic [31], a form of many-valued logic in which the truth value of variables may be any real number between 0 and 1, can be used to quantify the indetermination or the uncertainty of a judgment, since it captures the idea that our reasoning can only be "approximate" and introduces a notion of "degree of truth" of a statement –which can be defined as a probability. Let us also mention the development of non-monotonic logics [32] capable of dealing with the process of restriction of knowledge due to the reception of a new piece of information –a property characteristic of our daily reasoning, which cannot be accounted by classical logic.

We do not here question the fecundity of these non-classical logics, neither the utility of pragmatic and heuristic approaches to cognition, which are or could be used successfully to model very *specific* decision-making tasks. However, we will present here a possible, elegant solution, *capable of comprehensively dealing with all these supposed "fallacies" of human reasoning and decision-making processes.* Pursuing the project of developing a logical approach to thought and cognitive processes, we believe that these require, rather than a panoply of pragmatic-heuristic "recipes" specific to each situation studied, a unifying modeling, which supposes *the development of a single appropriate logic*. This synthetic solution will be explained by first setting its theoretical framework, which is different from the classical, set theoretical framework, and by then proposing solutions for the previously presented difficulties of the classical approach to cognition and decision-making. This new approach to human cognition, called "quantum cognition" in the literature, can deal with the important properties of cognitive processes that cannot be tackled by the classical models of cognition, namely contextuality, order effects and interferences effects.

2.1. A survey of quantum cognition

"Quantum cognition" deals with mental processes within the same *mathematical framework* than that of quantum theory but it has *a priori* nothing to do with physics. It does not at all refer to the "physicality" of the world, through its parts, like elementary particles or black holes, or to physical concepts, like those of energy, mass or velocity. Quantum cognition focus on the properties of mental states and the rules that govern mental processes, independent of any physical reference. It has been developed for a few decades by several authors, including Busemeyer and Bruza [8], Aerts *et al.* [9], Aerts and Sozzo [9]. Its basic idea is to represent geometrically the cognitive state of a subject (which can also be understood as her "mental state" or her "belief state") by a vector of an appropriate Hilbert space spanned by all her possible cognitive states and to represent its transition into a new cognitive state by its projection onto the vector subspace associated to this new cognitive state. Like for physical observables in quantum theory, mental observables, which are the properties of any

cognitive state that can be evaluated or "measured", are represented by Hermitian operators forming a *non-commutative* algebra and whose (real) eigenvalues are the possible results of their measurement. In Dirac notations, the transition of the cognitive state $|\psi\rangle$ to the new one obtained by measuring[2] the observable **A** with the outcome A_i is represented by the projection $\mathbb{P}_{Ai}|\psi\rangle$, where $\mathbb{P}_{Ai}$ is the projector $|A_i\rangle\langle A_i|$ onto the eigenspace associated with A_i, which, *for sake of simplicity of presentation*[3], has been assumed to be one-dimensional, meaning that it is spanned by the vector $|A_i\rangle$ of H. This transition can be geometrically illustrated as follows:

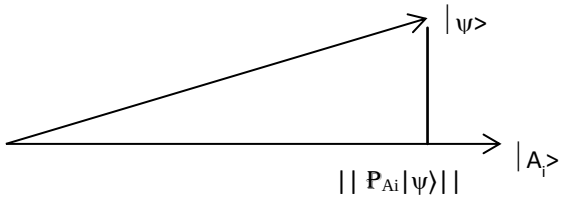

Fig 1. A geometrical representation of the measurement of the observable **A** with result A_i

Now, using Born's rule, which gives the only probability measure that can be assigned to this Hilbert space, the probability for a subject in state $|\psi\rangle$ to transit into the new state $\mathbb{P}_{Ai}|\psi\rangle$, is computed as the square of the modulus of its projection onto the subspace associated with the result A_i :

$$P(A) = \| \mathbb{P}_A |\psi\rangle \|^2 = \langle\psi| \mathbb{P}_A|\psi\rangle.$$

As will be shown hereafter, the previously mentioned difficulties of classical cognition and classical decision-making can find fruitful representations and satisfactory solutions within this quantum-like approach. Only some of them are briefly presented hereafter.

2.2. Evaluation of the degree of complementarity for emotions

A fruitful representation of the order effect in the domain of emotions and its behavioural and physiological correlates can be provided within the theoretical framework of quantum cognition. Such an evaluation of the degree of non-commutativity (or "complementarity") of emotions is required for improving the capacity of machines (robots) to simulate emotional intelligence (see section 4).

Uzan [19] has provided an estimation of the degree of non-commutativity of emotions from experimental data reported in Prkachin *et al.*'s article [20], which report the average intensity of five emotions experienced by subjects conditioned in target emotional by Lang's method [33]. Conditional probabilities of experiencing an emotion A for a subject conditioned in an emotional state B can be evaluated by the ratio of the average intensity of experienced emotion A to the sum of the average intensities for the five basic emotions. In accordance with the presentation of the previous section, a geometrical representation of the corresponding vector-states $|A\rangle$ and $|B\rangle$ is provided. In this representation, each considered emotion E is associated with the two-dimensional, orthonormal E-basis $\{|E\rangle, |\neg E\rangle\}$, the vector $|E\rangle$ representing a state of extreme emotion E (with a rating of 7) and the vector $|\neg E\rangle$ a state where no emotion E is felt (rating 0). The reference [19] provides a detailed

[2] "Measuring" in a very general sense, which can be understood as observing or even experiencing in the case of emotional observables (see section 2.1.2).

[3] Of course, the eigenspace associated with a particular eigenvalue of **A** is generally multi-dimensional. This point is important since, as recalled by Boyer-Kassem [34], only multi-dimensional state-spaces (relative to what he calls "degenerate" situations) can rigorously explain the conjunction fallacy, thus showing the superiority of quantum cognition on classical rationality in this case.

evaluation of the commutator of the observables Anger and disgust, respectively noted as **A** and **D**, by first computing the probability amplitude <D|A> of the transition from the state of pure anger to the state of pure disgust and then computing the outer product **D** = |D><D| in the basis{|A>, |¬A>}. The norm of the commutator [**A**, **D**], which evaluates the degree on complementarity of the observables Anger and Disgust is equal to 0.449, which shows a strong order effect in the evaluation of the intensity of the successive experiences of Anger and Disgust. More generally, it can be shown that all the other couples of emotions considered in Prkachin *et al.*'s data [20] do not commute and that many other examples of couples of non-commutative emotional observables can be computed from the relevant experimental data reported in the literature [21-23].

2.3. Solving Ellsberg's paradox

This generalized probabilistic framework can also be applied to deal with decision- making in uncertain situation. For example, Ellsberg's paradox [26] bearing on the behavior of economic agents in a situation of uncertain knowledge (see section 1.4) can be solved within this mathematical framework. As mentioned in section 1.4, Ellsberg's paradox puts into question the law of total probabilities of classical probability calculus. According to the quantum-like approach to cognition and decision making presented here, the paradoxical difference between the probability of the agent's choice computed by classical probability calculus and the experimental result comes from the *interference term* between the possible belief states of a participant in the uncertain situation c) defined in section 1.4 as the uncertainty about the outcome of the first game –if she won or she lost. This violation of the classical calculus of probability, which at first glance seems to indicate that human beings are prone to "fallacious" reasoning, is, in fact, a typical example of how we "reason" and make decisions in uncertain situations.

Several authors, including Busemeyer and Bruza [8], Aerts and Sozzo [36] and al-Nowaihi *et al.* [37] provide clear presentations of the quantum decision theory where Ellsberg's paradox can be solved. However, for a question of place, we will here present a simple solution that highlights the key-ingredient of the quantum-like approach to decision-making under uncertainty, namely the interference effects responsible for this human "fallacy". Consider the two possible sequences of mental states h1 and h2 (or possible "stories") leading a participant to the decision to play again. History h1 is defined by the sequence of states (I, G, J) and history h2 is defined by the sequence of states (I, ¬G,J), where I is the "initial" cognitive state of the subject,
G is her mental state when she learned that she won the first time, ¬G is her mental state when she learned that she lost the first time and J is her mental state when she makes the decision to play again (see Figure 2 below).

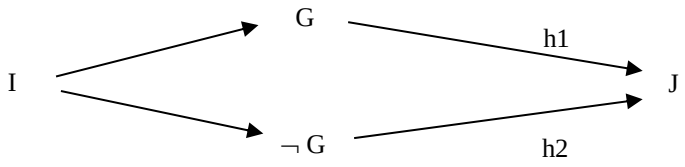

Figure 2. The two possible stories leading to the decision to play again.

The law of total probabilities of the classical probability calculus tells us that:

$$P(J) = (1/2)\,[P(J/G) + P(J/\neg G)],$$

which is, as we have just seen, in contradiction with the experimental data. On the other hand, this paradox can be resolved if we assume here that, contrary to what would be required by a presupposed "realism of mental

states", the participant who does not know whether or not she won the first time *is himself in a state of superposition of states* G and ⌝G. Let $|g>$ this state of superposition, which is represented by a unit vector of the configuration space of mental states (Hilbert space), and can be written as:

$$|g> = (1/\sqrt{2})\,(|G> + |\neg G>),$$

where $|G>$ and $|\neg G>$ are the representative unit vectors of the states G and ⌝G, while the normalization coefficient $(1/\sqrt{2})$ is calculated considering the equality of the probabilities of winning and losing the first time. Applying now Born's rule to the transition from $|g>$ to the state $|J>$ describing the decision to play again, we can calculate the value of the probability of replaying according to this model (see **Appendix 1**). This probability contains an *interference term* expressing the fact that the initial sate $|g>$ is a state of superposition. The presence of this term can explain with accuracy the difference between the experimental results reported above (section 1.4) and what we should find by applying the law of total probabilities.

3. Bayesian models of cognition revisited

3.1. Bayesian rationality

Bayesian rationality is the idea that cognitive processes must be modeled in probabilistic terms, by updating the prior distribution when a new event occurs -instead of applying rigid rules of deduction [38-39]. The current Bayesian models of cognition, which are developed in order to model high-level cognitive processes along the connectionist approach, appeal to Bayes' inference in order to update the prior probabilities. This can be checked in the range of perception, categorization, language processing and emotion recognition [40-46].

Bayes' inference is thus regarded as the key ingredient used for integrating uncertainty in cognition and decision-making. It is used to model many areas of human activities, like finance (for modelling risk), medicine (for diagnostic and decision making) or meteorology (for weather forecasting), and the success of Bayesian models of cognition seems certain, as well in reasoning, learning or making decision. Accordingly, Bayesian networks, are used in machine learning whose applications have been developed in image processing, neuroscience and medical diagnostics, among other domains. Bayes' inference extends the framework of binary classical logic by taking into account the uncertainty in the knowledge of premises and the acquisition of information, which is evaluated in terms of probabilities. And even when probabilities are interpreted subjectively, as degrees of belief [47], Bayesian reasoning still satisfies the rules of the classical probability calculus, as shown by Cox-Jaynes theorem [48], which strengthens the idea that Bayesian inference would be totally appropriate for modeling mental processes. It is thus supposed to correctly represent the way we reason, we learn and make decision in uncertain situation, and therefore to be a key-ingredient for developing artificial intelligence. However, as will be recalled hereafter, Bayes' rule is valid under the assumption that the order in which are evaluated the considered observables does not matter *while, as shown in section 1, most of mental processes do not satisfy this condition of commutativity.*

3.2. Bayes' rule

Bayes' rule allows to calculate how a priori probabilities are updated when new information is gathered: the posterior probability P (A/E) of the event A given the evidence E, which can denote in particular the observation of some feature of the situation under consideration, is calculated from the prior probability P (A), estimated

before the occurrence (or the knowledge of the occurrence) of E, and the "likelihood" P (E/A), which is the conditional probability of observing E when A is realized:

(1) $$P(A/E) = [P(A).\ P(E/A)] / P(E),$$

where P(E) is the probability of occurrence of the event E, which is a priori estimated independently of A.

Bayes' rule is derived from the definition of conditional probability in the classical probability calculus:

$$P(A/E) = P(A \text{ and } E) / P(E),$$

where *"A and E" has no temporal connotation*, meaning that A and E can occur in any temporal order or be simultaneous. This order-independent definition of P (A and B), the joint probability of the conjunction of A and B, gives rise to the "rule of multiplication" of the classical probability calculus:

(2) $$P(A \text{ and } E) = P(A/E).\ P(E) = P(E/A).\ P(A),$$

and Bayes'rule (1) is then straightforwardly obtained by dividing the two terms of the second equality of (2) by P(E). The relation (2) is thus valid on the condition that the value of the joint probability P (A and E) is independent from the order of occurrence of A and E. But in the case P (A and E) depends on the order of occurrence of A and E, one must clearly distinguish the calculation of P (A and then E) = P (E/A). P (A) from that of P (E and then A) = P (A/E). P (E). In this case Bayes' rule (1) is not verified since (2) is no longer valid. This means that *Bayes' rule is valid only on condition that the order in which the events under consideration occur does not matter.*

A careful study of the paradigmatic applications of Bayes' rule given in academic textbooks and in the relevant literature shows that the events considered in the presented experimental situations are regarded as purely intrinsic features of reality, independent of each other and independent of our means of observation. For example, in the paradigmatic application of the drawing of balls in two urns, noted as I and II, Bayes'rule is used to evaluate the conditional probability that a ball of a certain color was drawn from urn I or urn II. However, in this situation the colors of the balls are intrinsically and once and for all defined, and consequently the observables measuring the proportions of balls of a given color in each urn can be regarded as intrinsic properties of the physical world. As a consequence, the order in which they are evaluated does not matter and Bayes' rule can be successfully applied. Similarly, in the paradigmatic case of diagnostic testing [48], the involved observables, which respectively measure the state of health of a patient and its contamination rate can be regarded as independent, intrinsic properties of reality and are then assumed to commute. However, as shown previously (section 1), it is not the case for mental processes: *the condition of commutativity of the relevant observables is generally not fulfilled –which seriously questions the reliability of the current Bayesian models of cognition.*

3.3. A new probabilistic rule of inference

Fortunately, Bayes' rule can be generalized in order to account for the non-commutativity of mental observables which, as explained in section 1, is involved in most of mental processes. This generalization, which can be done within the quantum-like framework presented in section 2, is justified by the will to continue working in the paradigm of Bayesian rationality, while making it capable to deal with order effects. Moreover, such a generalization of Bayes' rule seems to be quite necessary in the field of artificial emotional intelligence (see section 4).

To compute the conditional probability P (E / F) for the cognitive events E and F, let us first compute within the quantum-like framework presented here the difference of the probabilities of the two opposite sequences of events E and F:

(3) $$P(E\,F) - P(F\,E) = \langle\psi|\mathbf{P}_E\,\mathbf{P}_F\,\mathbf{P}_E\,|\psi\rangle - \langle\psi|\mathbf{P}_F\,\mathbf{P}_E\,\mathbf{P}_F\,|\psi\rangle$$
$$= \langle\psi|\mathbf{P}_E\,\mathbf{P}_F\,\mathbf{P}_E - \mathbf{P}_F\,\mathbf{P}_E\,\mathbf{P}_F\,|\psi\rangle$$
$$= \langle\psi|[\mathbf{P}_E, \mathbf{P}_F]\,(\mathbf{P}_E + \mathbf{P}_F - \mathbf{I})\,|\psi\rangle,$$

where the last equality has been obtained by factorizing the expression between the bra $\langle\psi|$ and the ket $|\psi\rangle$, and by using the definition of the commutator $[\mathbf{P}_E, \mathbf{P}_F] = \mathbf{P}_E\,\mathbf{P}_F - \mathbf{P}_F\,\mathbf{P}_E$. Defining the operator $\mathbf{Q}$ as:

$$\mathbf{Q} =_{df} [\mathbf{P}_E, \mathbf{P}_F]\,(\mathbf{P}_E + \mathbf{P}_F - \mathbf{I}),$$

the difference P (E F) - P (F E) can be written as the expectation value of Q in the mental state $|\psi\rangle$:

(4) $$P(E\,F) - P(F\,E) = \langle\psi|\,\mathbf{Q}\,|\psi\rangle \equiv \langle\mathbf{Q}\rangle_\psi.$$

Using relation (4) and the definition of conditional probabilities P (F/E) and P (F/E), one can show the following new rule of probabilistic inference:

(5) $$P(F/E) = [P(E/F) \times P(F) + \langle\mathbf{Q}\rangle_\psi] / P(E).$$

This new generalized probabilistic rule of inference computes the conditional probability of occurrence of the event F given the occurrence (or the knowledge of the occurrence) of the event E for a subject in the mental state $|\psi\rangle$. Its *classical limit*, when the projectors associated to the event E and F are commuting, and then when $\langle\mathbf{Q}\rangle_\psi = 0$, is of course nothing but Bayes' rule (1): P (F/E) = [P (E/F) x P (F)] / P (E).

4. The quantum-like approach to cognition implemented in emotional AI

The most impressive tasks of artificial intelligence, like learning, categorizing or perceiving, which are presently developed according to the connectionist approach (artificial neural networks), utilize probabilistic algorithms and thus appeal to Bayes' rule in order to compute the update the probabilities that can be assigned to new data. In order to take into account the order effects inherent to most of mental processes, these algorithms should then be revisited. Still keeping the probabilistic, Bayesian approach to cognition, such a change only regards the inference rule that must be used to update the probabilities of realization of some assumptions when new information is gathered. These models should use the general probabilistic rule (5) instead of Bayes' rule (1) insofar as the latter cannot account for order effects. Let us focus in the following on the important case of *emotional* artificial intelligence, which play a crucial role in any aspect of life [49-50].

Emotional intelligence involves verbal and non-verbal communication, like reading persons' face expression, observing body movements and postures, and physiological manifestations. Simulating emotional intelligence algorithmically is now an important subject of research in the field of artificial intelligence, namely for improving human-machine interaction. It first requires *emotion recognition* on which we will focus hereafter.

Emotion "recognition" is a classification task from multimodal sensory, behavioral or physiological data [51]. It uses several types of sensors that detect speech signal, voice tone, facial expressions and body language, and appeal to data on previously observed correlations (called above "the common-view correspondence") between, on the one hand, the nature and intensities of emotions, and, on the other hand, the values of physiological and behavioral observables. These data are analyzed by deep learning algorithms that compute the most probable emotional state that can be assigned to a subject by analyzing her behavior, like her

facial expression or her body language and posture [52-53], which seems the most informative observations even realized from afar and from any angle of view.

This task requires computing the conditional probabilities, noted as $P(E_k / \{B_j\})$, that the emotional observable experienced by a subject takes the value E_k given the observation of a set of behavioral observables $\{B_j\}$, like her facial expression or her body language. However, as explained in section 1.3, emotional observables do not commute with each other and do neither commute with their behavioral and physiological correlates. Consequently, classifying emotions experienced by a subject from the knowledge of her behavioral features thus requires considering these order effects, *which means that the generalized probabilistic rule (5) must be used instead of Bayes' classical inference rule.* This requires to first compute the commutators $[\mathbf{E_k}, \mathbf{B_j}]$ of the relevant observables $\mathbf{E_k}$, $\mathbf{B_j}$, which, as mentioned above (section 2.3), can be computed from data about their joint measurement. These data can be found, for example, in the very comprehensive study published in the articles by Duran and Fernandez-Dols [54] and by Barrett *et al.* [21]. Such a computation, which allows asserting the relevance of the non-classical Bayesian approach introduced in section 3, is detailed in **Appendix 2.**

Revisiting the emotion recognition task thus consists in changing the current deep learning algorithms by grounding the computation of the most probable emotion given a facial expression on the generalized probabilistic rule (5) instead on Bayes' rule, which requires a preliminary computation of the commutators of type $[\mathbf{E_i}, \mathbf{F_k}]$, for all considered couples of emotional and behavioral observables, and then the computation of the term $< \mathbf{Q} >_\psi$ in order to correctly evaluate the conditional probabilities $P(E_i / F_k)$ by rule (5).

Moreover, let us notice that emotion recognition is only the first step to achieve in order to "understand and to reason with emotions". In particular, in order for a subject to respond appropriately to a situation, she must be able to make *predictions* about the emotional state of her social environment given her present emotional state and new data she can gather, like information about the change of their face expression, their voice tone or even their physiological changes [55]. *This requires again the use of the generalized probabilistic rule (5) capable of accounting for the non-commutativity of the relevant emotional / behavioral and emotional / physiological couples of observables.*

As a final remark, one can ask whether the aforementioned changes of the algorithms that are currently used for simulating emotional intelligence would apply for the simulation of *all* aspects of mental activity insofar as emotions are involved in them to varying degrees. For example, as is well known, "negative" emotions, like anger or sadness, can disturb our concentration and our capacity of memorization, while "positive" emotions, like a feeling of happiness, can improve our ability to perform these same tasks[4]. Following this idea, quantum cognition would be the fundamental ingredient for modeling mental processes and thus for improving the algorithms of artificial intelligence.

[4] This well-known point has been analyzed by Wang and Ross [58] and reported in the field of psychoanalysis [59].

Appendix 1: Interference of cognitive states in Tversky's and Shaffir's experiment [25]

Referring to Figure 2 of section 2.1, the probability to play again for a subject in the state of superposition $|g>$ can be computed within quantum cognition as follows, Π (J) being the projector onto the state $|J>$ corresponding to her decision to play again:

$$P_Q(J) = <g|\Pi(J)|g>$$

$$= (1/2)\,[<G|\Pi(J)|G> + <\neg G|\Pi(J)|\neg G> + <G|\Pi(J)|\neg G> + <\neg G|\Pi(J)|G>]$$

$$= (1/2)\,[P(J/G) + P(J/\neg G)] + ||<\neg G|\Pi(J)|G>|| \cos w,$$

where to the first term of this sum (which is the result P_{class} (J) according to classical probability calculus) is added the interference term:

$$Int = ||<\neg G|\Pi(J)|G>|| \cos w,$$

where w is the argument of the complex number $<\neg G|\Pi(J)|G>$.

The presence of this term can thus explain the difference between the experimental result reported above (see section 1.4)and what we should find by applying the law of total probabilities, that is, P_{class} (J)**.**This interference term can be evaluated from the conditional probabilities P(J/G) and P(J/$\neg$G):

$$Int = P(J/G).\,P(J/\neg G))^{1/2}.\cos w.$$

For w = 116°, we compute that Int = – 0.28 and therefore:

$$P_Q(J) = P_{class}(J) - 0.2 = 0.36,$$

which is exactly Tversky's and Shaffir's experimental result.

The angle w, which determines the magnitude of the interference term, therefore evaluates the psychological factor characterizing the (average) sensitivity of the participants to the information that tells them (or not) that they have won or lost in the first game. In other words, w measures their ability to modify their decision to play again upon receipt of this information.

Appendix 2: Dealing with the emotion recognition task within quantum cognition.

To show that the generalized probabilistic rule (5) of section 3.2, which accounts for order effects, must be applied (instead of Bayes' rule) in emotion recognition tasks, one has to demonstrate that the difference P (E F_E) – P (F_E E) is effectively equal to the average value of the operator **Q** for an initial, neutral state of the subject. Following the method presented in reference [19], one can compute the commutators [$\mathbf{E_i}$, $\mathbf{F_k}$] involved in this task and show the expected equality. The data that are used in this computation are drawn from Barrett *et al.*'s article [21] (see section 4.1).

The first required experimental conditional probability P (E/ F_E) can be found on page 36 in Barrett *et al.'s* article [21]. For example, on the first line of Fig. 11, we can read that a subject presented with the "common view" facial expression of Anger labels it as an expression of Anger with the rate 39.92 / 71.92 = 0. 555. In this rate, 39.92 is the average number of subjects (average over the entire population of subjects tested) who reported an emotion of Anger given the presentation of its common view facial expression, while 71.92 is the sum of the values corresponding to the subject's possible answers. That is, P (A / F_A) = 0.555.

Since the same common view facial expression of Anger is presented to all the tested subjects, we can write, *in this specific experiment*, that its prior probability is P (F_A) = 1, and that, consequently:

$$P(F_A\,A) = P(A/F_A) = 0.555.$$

The second conditional probability P (F_E/E) can be found on page 19, figure 6, of Barrett *et al.*'s article [21]. This experiment provides, for each of the six basic emotions, the average proportion of subjects *conditioned in this emotional state* who move their face according to the corresponding common view facial expression of this emotion. This experiment thus evaluates the average correlation degree between each of the six experienced emotions they are conditioned and their respective "common view" facial expression. Let us again focus on Anger, whose common view facial expression is characterized by brows furrowed, eyes wide, lips tightened and pressed together –see figure 2 A of Barrett *et al.'s* article [21]. Figure 6 shows that for Anger this conditional experimental probability is P (F_A / A) = 0.22.

Again, since in this experiment the subjects are all conditioned in a same specified emotion (of anger), one can say that the event A is certain, that is, its prior probability of occurrence is P (A) = 1. The conditional probability P (F_A / A) is then equal, in this specific experiment, to the sequential probability P (A F_A) of being first conditioned in an emotional state A *and then* showing the facial expression state F_A. That is, P (A F_A) = P (F_A / A) = 0.22. According to Bayes' rule, P (F_A / A) and P (A / F_A) would here be identical, *which is wrong*. *Bayes' rule cannot thus explain the difference between the two sequential probabilities* P (F_A A) and P (A F_A). By contrast, this difference can be computed by using the new, probabilistic rule derived in section 3.2, which generalizes Bayes' rule in order to account for the complementarity of the observables considered. To show this, one has to first compute the commutator [**A, F_A**] of these two observables in order to evaluate the term $<\mathbf{Q}>_\psi$ that appears in this rule, for an initial "neutral" cognitive state $|\psi>$ of the subject, and to check that:

(6) $$P(A\,F_A) - P(F_A\,A) = <\mathbf{Q}>_\psi$$

For computing the expectation value $<\mathbf{Q}>_\psi$ of the operator $\mathbf{Q}$ in neutral state, one have to first decompose the state $|F_A>$ on the two-dimensional basis $\{|A>, |\neg A>\}$:

$$|F_A> = (0.555)^{1/2}|A> + (1 - 0.555)^{1/2}|\neg A>.$$

The matrix of the projector $\mathbf{F_A}$ in this two-dimensional state space representation can then be computed, as the outer product $|F_A><F_A|$:

$$\mathbf{F_A} = \begin{pmatrix} 0.555 & 0.497 \\ 0.497 & 0.445 \end{pmatrix}$$

while he matrix of the observable $\mathbf{A} = |A><A|$ in this basis is $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.

The commutator [**A, F_A**] = **A F_A- F_AA** can then be computed:

$$[\mathbf{A}, \mathbf{F_A}] = 0.497 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

and the operator $\mathbf{Q}$ is then:

$$\mathbf{Q} = [\mathbf{A}, \mathbf{F_A}](\mathbf{A} + \mathbf{F_A} - \mathbf{I}) = \begin{pmatrix} 0.247 & -0.276 \\ -0.276 & -0,247 \end{pmatrix}$$

The subject's "neutral" cognitive state can be reasonably defined by the equiprobable superposition of the six basic emotions (which are those considered in Barrett's article) and can thus be written as:

$$|\psi> = (1/6)^{1/2}|A> + (5/6)^{1/2}|\neg A>.$$

Consequently, one can compute $<\mathbf{Q}>_\psi$:

$$<\mathbf{Q}>_\psi = <\psi|\mathbf{Q}|\psi> \approx 0.370.$$

Within the quantum-like approach presented above, one can thus find that 1) the difference P (A F_A) – P (F_AA) is not null (contrary to what would be derived from Bayes' rule) and that 2) a theoretical evaluation of this difference from the generalized Bayes' rule of section 3.2, which is derived within the framework of quantum cognition, gives the value 0.370. This result is not so far from the *experimental* difference:

$$P_{exp} (A\ F_A) - P_{exp} (F_A A) = 0.555 - 0.22 = 0.335,$$

since in this case the value of $P_Q (A/F_A) = 0.555 - 0.370 = 0.185$ computed with relation (4) is situated *within the confidence interval (error bar) of the average experimental value* (0.22) reported in Barrett et al.'s article [21], on figure 6.

Of course, he adequacy of the probabilistic rule (5) can be checked for the other basic emotions considered in Barrett et al.'s article [21]. For example, regarding Disgust, one can find on figure 11, p. 36, of Barrett *et al.*'s article (2019) that P (F_D D) = 0.406, while from figure 6, p. 19 of this article, one has P (D F_D) = 0.24. According to a quite similar method to that used above, one can compute, for a neutral subject's state, that $<Q>_\psi = <\psi|Q|\psi> \approx 0.165$, which is *very close* from the experimental difference P (F_D D) – P (D F_D) = 0.166.

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