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PAPER

A game to unfold decision risks under uncertain probabilities of extreme events

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Abstract

Making decisions in an uncertain environment is the daily routine of most decision makers (if not most human beings). This process naturally defines the notion of risk, that comes from the occurrence of hazards. One key illustration is linked to investments in large infrastructures in a climate change context, which alters the nature and properties of hazards. Idealized settings help understanding the nature of decision risks, when the physical and statistical properties of hazards change, e.g. due to climate change. This paper describes the mathematics of a game that illustrates the risks behind decisions. We explain that, in some cases, optimal gain and the risk of ruin do not necessarily lead to the same choices, especially when the hazard probabilities are uncertain. The goal of this paper is to explain to a general audience how to use available information for proper decision making. Hence, this paper serves as an advanced guide to teachers or climate researchers who are interested in outreach, and explaining the theory behind the game.

Key words: Climate Extremes, Decisions, Risk, Hazards

Introduction

Extreme climate events have huge impacts on society. Assessing their probabilities has been deemed important for decision makers. It has been a major endeavor of extreme event attribution [EEA: National Academies of Sciences Engineering and Medicine, 2016]. Major climate modeling efforts have been undertaken to sample the probability distribution of climate variability, in order to assess the probability of extremes [Bevacqua et al., 2023]. Statistical theories (Extreme Value Theory) have been adapted and applied to climate variables to model the extremes and be able to simulate them [Coles, 2001]. The seminal work of Taleb on “black swan” events suggests that such attempts might be vain [Taleb, 2010]. Many decision makers have to balance decades in advance strategies of adaptation to climate change (and its extremes) and the general welfare of population. This balance requires some anticipation of future extremes. In many cases, anticipation of the future is based on past observations, which is the main use of return levels and return periods [Coles, 2001]. As there might be discrepancies between empirical probabilities (obtained from observations) and “real” probabilities (obtained from the mathematical laws of physics), there is always a risk of failure of any adaptation strategy. In such cases, the risk can lead to a form of ruin, i.e. the loss of all assets [Embrechts et al., 1997]. On the other hand, in a world with finite and bounded income, paying for overprotection is often regarded as

poor planning and might lead to political backlash. There is no obvious optimal solution to achieve this balance.

For decision makers, information on the probabilities of climate hazards is a first step. The essence of decision is to estimate the necessary and sufficient investments into adaptation, with the risk of losing everything when the investments are underestimated. Therefore, decision makers have to consider two questions: what is the risk of ruin for a given adaptation plan? and what is the optimal residual gain (after the adaptation investment)? The answers to those two simple questions might not lead to the same investment plan.

Climate sciences have made progresses to assess the probabilities of extreme events, and the changes of probabilities due to climate change [Seneviratne et al., 2021]. Those probabilities are generally small numbers, with relatively large uncertainties [National Academies of Sciences Engineering and Medicine, 2016]. Therefore, knowing those probabilities (and uncertainties) is in principle useful for decision making for adaptation. Uncertainties in the changes of those probabilities can lead to surprising effects, and also need to be anticipated.

Most long term investors (or decision makers) can have sophisticated tools from economy or finance [Embrechts et al., 1997] to estimate risks, but they rarely have reliable and quantitative estimates of hazard probabilities, and have to rely on qualitative descriptions [Ranasinghe et al., 2021, IPCC, 2021]. Therefore, the sophistication of such tools might be rather vain, due the major uncertainty stemming from climate

variability, as suggested by Taleb [2010]. Therefore, there is a need to illustrate the risks linked to climate extremes on adaptation decisions for the climate change to come.

This paper presents a game, adapted from a game developed by the Red Cross's Climate Center, to illustrate the dilemmas of decision makers that face climate hazards that can be devastating [de Suarez et al., 2012]. The game is played with a set of dice and is an idealized version of the decisions made by planners. We aim at exploring the mathematics behind the game to identify optimal strategies. The players are certainly not aware of such mathematics (and should not be, before playing). Although this game was designed for decision makers, its format has many educational issues for high school to graduate level, in order to illustrate probabilities and risk. In particular, it illustrates some of the dilemmas faced by scientists when communicating to the public on extreme event attribution.

Presentation of the game

Decisions for seasons is a competition game with several players, which emphasizes the role of decision making for adaptation in uncertain context. The general setting is that each player (or group of players) is the (assumed elected) governor of a region for 10 years (or rounds). There are three games of 10 rounds, of increasing difficulty. At the beginning of each round, each player receives an initial capital of 10 smileys, which can be used to buy protections against extreme events. Each protection costs 1 smiley. When an extreme event occurs and a player has no protection against it, the player loses all smileys for the game. If an extreme event occurs and the player has at least one protection, this protection is ticked off, but the player keeps the smileys. The simple goal is to end the game with more smileys than the other players.

The players can invest between 0 and 10 *smileys* into protections. The remaining smileys represent the "gain" after the set (which can be re-invested elsewhere and generate a reward). Before the game starts, the players have to bet on the minimum number of protections that ensure the maximum reward in smileys. This can be conceived as a competitive game with assigning the objective of keeping more smileys than the other players.

The players have little time to decide on an adaptation strategy. The game master can keep talking and making noise during this reflection time, as it is often the case in real life.

First game: One type of hazard

During each round of the first game, an extreme precipitation event can occur, and destroy everything, leading to the loss of all smiley capital of the region. To prevent from such a disastrous loss, the player can buy protections (e.g. dams), which are destroyed during the extreme event, but save the wealth of the region for one round.

The first game is run by rolling a 6-sided die 10 times, and noting the outcome at each round. The players need to have written down on a sheet of paper the number of protections and the remaining smileys. The game master explains briefly the procedure of the game and gives, say, 60 seconds to the players to write down their strategy. Then the game begins. At the end of the game, there can be 0 smileys if not enough protections were purchased (or a player used all smileys to buy protections), or the number of remaining smileys. The player(s) with the most remaining smileys win the first game.

Second game: Two types of hazards

The second game emulates a form of climate change: deadly droughts occur when the die shows a 1. The player starts again with a capital of $K = 10$ smileys. The remaining smileys of the previous game were used for the welfare of the region. As in game 1, the players can buy protections against major droughts, i.e., buckets (or the controversial "mega-bassines" in France)¹.

Therefore when a 1 or a 6 appear: if a player has neither protection against drought or precipitation, s/he cries "Oh no!", loses all smileys, and goes bankrupt for game 2. If a player has a corresponding protection, it is ticked off, but the player keeps her/his smileys for the next round.

The game master rolls a 6-sided die 10 times and writes down the outcome. The players must have written down the number and type of protections, and remaining smileys. Again, the game master explains briefly the game procedure, and the players are allowed 60 seconds to write their strategy. The game then starts. The player(s) with the most remaining smileys win the second game.

As the initial capital is still 10 smileys, it is obvious that the players will have to buy more protections than in Game 1 to avoid ruin.

Third game: The black swan

We emulate further the impact of climate change on extremes by a subtle game changer. Droughts occur when a 1 appears. But scientists claim that the probability of heavy precipitation have changed by a factor between 1 and 3 [e.g. Luu et al., 2018]. They are not sure about this probability ratio. We emulate such an uncertain change by using either a 6-sided die (as before), an 8-sided, 10 sided, 12 sided, or 20-sided die. An extreme precipitation occurs when the value is 6 or above. The players do not know which die is used. The game master might decide at random, or decide in advance which one (and make no change). In this game, the probabilities of extreme precipitation obviously increase, while the probability of droughts decrease.

The black swan is due to the absence of precise information from the players on the probability of extreme precipitation. The strategy they built for the two first games was based on the information that the game master rolls a 6-sided die. As we will see below, the optimal strategies are fairly easy to derive for those two games. This is not the same for game 3, although the rules are similar.

Again, the game master explains briefly the game procedure, and the players are allowed 60 seconds to write down their strategy. The game then starts. The game master can write for her/himself the outcome of the die, but should not show it to the players, for obvious reasons. The player(s) with the most remaining smileys win the third game.

The sum of smileys of the three games is made. The player with the largest sum wins.

Mechanisms of strategy

A first strategy consists of balancing the expected gain (original capital minus protection investment) with the risk of losing everything (not enough protection). If a player buys 10 protections, then she/he ends with zero smileys (and no gain). If a player buys 0 protection, then she/he faces the risk of losing

¹ The first author of this paper does not support the idea of "mega-bassines" by any means. This is just for illustration purposes.

everything (hence no gain either). Therefore, a trade-off has to be found.

We will present some the mathematics behind the game, which can help an objective evaluation of the odds. We will then discuss how the competitive nature of the game and the interpretation of the first rounds can increase the variance of decisions. We emphasize that this game is to be played only once, so that nothing prevents an optimal strategy on average from failing on a particularly unlucky streak of events.

This game can be played with any kind of audience. It is very unlikely (with all due respect) that players can make the computations that are detailed below within the duration of the game. Even if they could (because they have some knowledge in probability theory), the game master would not give them enough time to perform any computation. Therefore most decisions taken by players are based on their intuition (which could be right for wrong reasons). One of the authors of this paper has played this game to various audiences, from junior high school to graduate students, including secondary school teachers. The players who obtained the largest number of smileys were never those who had any reasonable background in statistics or probabilities.

Probabilities and decision

In this section, we explain how strategies can be objectively be designed. The mathematical formulas might look complex (for nonspecialists or high school pupils), but they are anecdotal. It is just important to know that things can actually be computed. High school professors could replace those complex formulations by programming exercises, with the same result.

Game 1

The probability of extreme precipitation is emulated with a 6-sided die. With 1–5, nothing happens and no protection is required. A 6 gives an extreme precipitation. Therefore, during each of the $N = 10$ rounds, the probability of an extreme is $p_0 = 1/6$. During a game of $N = 10$ rounds, the probability that h extremes (the letter h is for "hazard") occur follows a binomial distribution:

$$\Pr(H = h) = \binom{N}{h} p_0^h (1 - p_0)^{N-h}, \quad (1)$$

where $\binom{N}{h} = \frac{N!}{h!(N-h)!}$. The expected number of extremes among N rounds is $E(H) = Np_0$, and the variance is $\sigma^2(H) = Np_0(1 - p_0)$.

The probability that at most h extremes occur is:

$$\Pr(H \leq h) = \sum_{j=0}^h \binom{N}{j} p_0^j (1 - p_0)^{N-j}. \quad (2)$$

Let K be the initial capital (e.g. $K = 10$ smileys), I be the investment into protections ($I \in \{0, \dots, K\}$, H the (random) number of extremes, and N is the number of rounds. For convenience, we equate the number of hazards to their cost in smileys, and $K = N$. In real life, there is a transfer function from the extreme event occurrence to its cost. In the equations given below, K is not necessarily equal to N . The game makes this hypothesis for convenience so that fewer variables are considered. G is the gain after the game of N rounds, with

an investment I , after H extreme events have occurred:

$$G = \begin{cases} K - I, & \text{if } I \geq H, \\ 0, & \text{if } I < H. \end{cases} \quad (3)$$

This means that the player's gain is $K - I$ smileys if less than I extremes have occurred within the N rounds. The player loses everything if more than I extremes have occurred: $G = 0$. In this simple version of the game, one cannot lose more than the initial capital K .

For a given investment I taken on the initial capital K , we deduce from Eq. (2) that the probability $\pi(K, I)$ of a gain $G = K - I$ is the probability that there are at most I extreme events during the N rounds of the game:

$$\pi(K, I) \equiv \Pr(G = K - I) = \sum_{j=0}^I \binom{N}{j} (1 - p_0)^{N-j} p_0^j. \quad (4)$$

Note that this probability does not depend on K , but only on the expected number of hazards (reflected by the number of purchased protections).

For a given protection investment I , the expected value of the gain G is:

$$E(G|I) = (K - I)\pi(K, I), \quad (5)$$

because the player either gets $K - I$ smileys, or loses everything. The player is naturally interested in finding an investment I that maximizes the gain G .

We can also be interested in the probability of ruin for a given investment I :

$$\bar{\pi}(K, I) \equiv \Pr(G = 0|I) = 1 - \pi(K, I). \quad (6)$$

This probability $\bar{\pi}(K, I)$ is naturally a decreasing function of I : the higher the investment, the lower the odds of ruin (Figure 1a). A player wants to find an investment I for which the risk of ruin is perceived as negligible (i.e. this probability is lower to some threshold).

Figure 1b shows the average gain G for protection investment $I \in \{0, \dots, 10\}$. The optimum protection investment is $I = 3$. The average number of extreme precipitation events is ≈ 1.67 , with a standard deviation of ≈ 1.18 . Those two quantities can be intuited without major computations. Without knowing the result of Figure 1b, a reasonable choice is to invest into 2 protections. A safe choice would be to take 3 protections, due to variability. Competitive players would opt for only 1 protection, although the probability of losing everything is 0.5.

From Figure 1b, one sees that buying more than 4 protections is not really useful, as the probability of drawing four sixes is very small.

In general, even without performing any of the computations explained before, most players chose between 1 and 3 protections, and end up with a gain between 7 and 9 smileys.

Game 2

In this game, two types of hazards can occur (h_1 for precipitation and h_2 for droughts), and hence two types of protections can be purchased by the players. The probability for both hazards is still $p_{h_1} = p_{h_2} = p_0 = 1/6$.

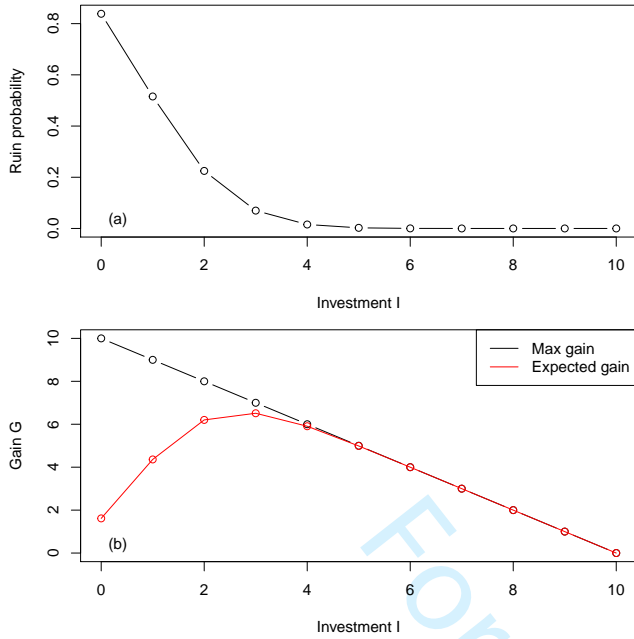


Fig. 1. Key quantities of game 1, as a function of investment into protections against extreme precipitations. Panel (a): Ruin probability as a function of protection investment. Panel (b): Expected value of gain (red line) and maximum gain (black line) as a function of protection investment.

During a game of $N = 10$ rounds, the probability $P(h_1, h_2)$ that h_1 events and h_2 events occur (i.e. h_1 sixes and h_2 ones) is a variation over a binomial distribution:

$$P(h_1, h_2) \equiv \Pr(H_1 = h_1 \& H_2 = h_2) = \binom{N}{h_1} p_0^{h_1} (1 - p_0)^{N-h_1} \binom{N-h_1}{h_2} p_0^{h_2} (1 - p_0)^{N-h_1-h_2}. \quad (7)$$

This is the probability of having h_1 extreme precipitations among N draws, and having h_2 droughts in the remaining $N - h_1$ draws. The constraint for a proper definition is obviously that $h_1 + h_2 \leq N$.

The probability that at most h_1 precipitation events and h_2 drought events occur is:

$$\Pr(H_1 \leq h_1 \& H_2 \leq h_2) = \sum_{i=0}^{h_1} \sum_{j=0}^{h_2} P(i, j). \quad (8)$$

For an investment $I = I_1 + I_2$ of protections against precipitation events I_1 and drought events I_2 , the probability of a gain $K - I$ is hence:

$$\pi(K, I_1, I_2) = \Pr(H_1 \leq I_1 \& H_2 \leq I_2). \quad (9)$$

The expected gain after N rounds is:

$$E(G|I_1 \& I_2) = (K - I_1 - I_2) \pi(K, I_1, I_2). \quad (10)$$

The probability of ruin during N rounds is:

$$\bar{\pi}(K, I_1, I_2) = 1 - \pi(K, I_1, I_2) \quad (11)$$

because one either gains $K - I_1 - I_2$ smileys, or loses everything.

Displaying the expected gain G of the probability of ruin $\bar{\pi}$ requires 3-dimensional figures. One can simplify the visualisation by noticing that the probabilities of the two hazards are the same ($p_1 = p_2 = 1/6$), so that one is actually interested in the case when $I_1 = I_2$. This is reflected in Figure 2.

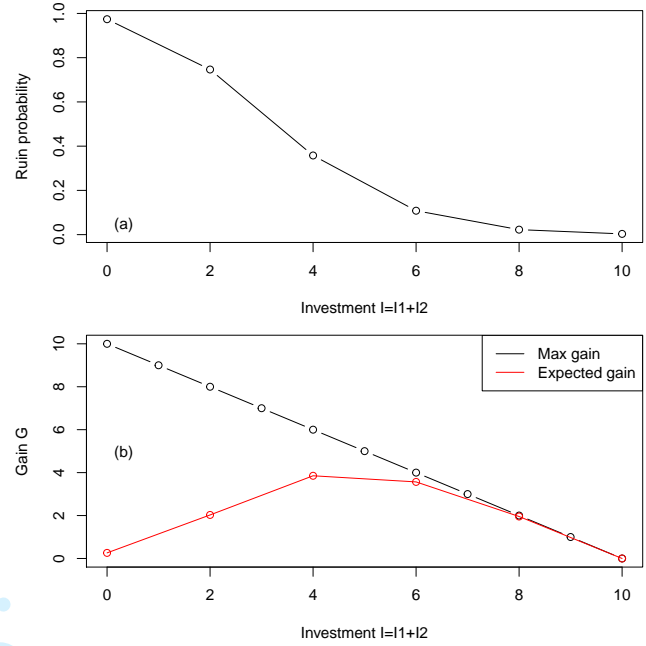


Fig. 2. Key quantities of game 2, as a function of equal investments into protections against extreme precipitations and droughts. Panel (a): Ruin probability as a function of protection investment. Panel (b): Expected value of gain (red line) and maximum gain (black line) as a function of protection investment.

From Figure 2a, the ruin probability becomes very small $\bar{\pi} \approx 0.1$ for protection investments $I = I_1 + I_2 \geq 6$. The optimal gain is obtained with a protection investment of $I = 4$ (i.e., 2 of each type), although the risk of ruin is close to $\bar{\pi} = 0.4$. Game 2 is an example where the policy to maximize the mean expected gain also comes with a high probability of ruin. Players who want to make a safe decision would consider 3 protections of each type, hence a global investment of $I = 6$ smileys, and an average expected gain of 4 smileys. Making a positive gain difference with respect to this safe decision comes with a risk that increases fast with respect to the maximum possible gain.

Game 3

The third game is a variation of Game 2, but the probabilities of droughts and precipitations are not longer the same. Equation (7) can be rewritten as:

$$P'(h_1, h_2) \equiv \Pr(H_1 = h_1 \& H_2 = h_2) = \binom{N}{h_1} p_0^{h_1} (1 - p_0)^{N-h_1} \binom{N-h_1}{h_2} p_1^{h_2} (1 - p_1)^{N-h_1-h_2}. \quad (12)$$

where $p_1 = (D - 6 + 1)/D$ is the probability of extreme precipitation, and $p_2 = 1/D$ is the probability of drought, and D is the number of sides of the playing die.

The mathematical expressions for the mean expected gain G and the probability of ruin are the same as in Eqs. (10) and (11), except that the hazard probabilities are different.

The difficulty is that the players do not know the value of $D \geq 6$. What should be obvious is that $p_2 \leq 1/6$, and that $p_1 \geq 1/6$. Therefore one should choose to buy more protections against extreme precipitations than against droughts.

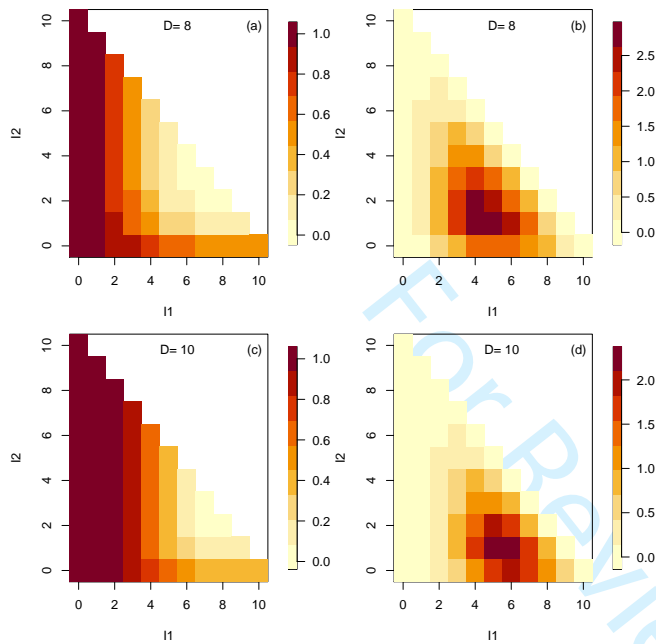


Fig. 3. Key quantities of game 3, as functions of investments into protections against extreme precipitations (I_1) and droughts (I_2). Panels (a, c): Ruin probability as functions of I_1 and I_2 . The upper panels are for an 8-sided die. The lower panels are for a 10-sided die. Panels (b, d): Expected value of gain as functions of protection investment I_1 and I_2 .

Figure 3 reports the ruin probabilities and mean expected gain for varying investments in precipitation (I_1) and drought (I_2) protections, for an 8 sided die and a 10-sided die. As expected, it seems unnecessary to buy more than 1 protection against drought to avoid ruin, as the probability of drought overall decreases from Game 2 to Game 3 (from $1/6$ to $1/8$ or $1/10$). Between 5 and 6 protections against precipitation are necessary to avoid ruin. The optimal expected gain (3 smileys or 2.5 smileys) is obtained for lower protection investments. This means that such expected gains are obtained with a fairly high risk of ruin.

Running the game

The players are given no chance to perform any of the statistical computations presented in this paper. Even if they knew in advance the probability formulas, those are sufficiently tedious to require the use of a computer to obtain the figures shown above. In practise, nobody is able to assess precisely the value of ruin probability nor mean expected gain (even in Game 1).

When the first author reaches game 3, he generally takes an 8-sided die (and sticks to it), so that the odds do not change drastically from game 2. Of course, the players do not know this. If a particularly "unlucky" streak of extreme precipitations (or droughts) occurs within the first 5 rounds,

the game master asks the audience whether they think that the probabilities have changed (due to climate change), and asks to argue for or against a change of dice. It is interesting to make a survey at this point of the game, as a drama factor. Of course, the decisions which were taken before the game started cannot be changed. This discussion illustrates the difficulty of communicating to the public for climate scientists about extreme event attribution.

The authors have played this game with various types of audiences. "Higher level" audiences (e.g. high school professors or undergraduate university students) do not perform more efficiently than "lower academic level" audiences (e.g. junior high school students). From an admittedly non significant statistical sample, it even appears that mathematics teachers perform rather poorly in estimating ruin probabilities, as they have seemed to take more risks than their colleagues.

Learning from one game to the next can be misleading, because of the slight rule changes. The outcome of the die roll for Games 1 and 2 can be written down on a blackboard. As illustrated by Kahneman [2011], many players tend to use this information to help their choices for Games 2 and 3. For instance, if an unlikely streak of 1s (i.e. more than 3) or an unlikely low number of 6s (i.e. one or less) occur in Game 1, many players seem to use this information for their decision in Game 2, although both games are independent. This means that some players are prone to base their decisions on empirical probabilities from a rather low number of samples, although they are informed of the actual probabilities of extremes. When the players go from Game 2 (where probabilities are known, although not obvious) to Game 3 (where probabilities are no longer known), then they are even more prone to using empirical information from the previous games. Only a few players realise that $p_1 > p_2$. Those players have a better chance of avoiding ruin in Game 3.

This Game is a "one shot" experience. Repeating it with the same audience has a fairly low utility. It illustrates that even though climate or meteorological prediction is based on probabilities, the outcome (or observed climate) is unique, and could even have a fairly low probability. A sophistication can be added to the game by stating that a ruin can lead to a negative capital (players "owe" smileys), as is described by the "Lottery of Babylon" [Borges, 1941]. In such a case, the equations for the expected gain can be rewritten easily, and the optimal strategies give more weight to the probabilities of ruin.

The competition against other players is an incentive to think about risks: the ones with the lower gains after games 1 and 2 are prone to lower the investments to have a chance to "catch up". The ones with the higher gains after games 1 and 2 might be tempted to "play it safe"...or not. Proposing a reward to the winners is also an incentive to take risks.

Conclusion

This very idealized game illustrates the process of decision under random hazards, and uncertainty in the randomness of hazards. Beyond the computations exposed in this paper, storytelling is essential to make this game useful. The game is more efficient when teams of 2-3 players are organized, so that discussions are encouraged. This game is an idealized version of ClimaRisq, which yields many types of hazards and opportunities of adaptation.

One potential story is that the teams compete for a local election. The citizens might be happy when the gain G is maximized at the end of the mandate, as it translates into local welfare, but they are extremely unhappy when everything is lost. Ending a 10 round game with too many protections is rarely seen as good planning. Thus, each game empirically tests policies that balance hazard protection versus welfare gain, with the constraint that overall wealth is finite. This is done only once, so that "luck" becomes the essence in the competition between policies.

Although very simplified, this game is analogous to "real life" decision making, which is often based on empirical probabilities (of extreme events) computed from a fairly low period of observations (a few decades), and which rarely account for changes in probabilities (as in Game 3).

Game 3 illustrates the appearance of a "black swan" [Taleb, 2010]: the 6-sided die paradigm has changed to something else, and can no longer explain the outcome of the die rolls. Extreme event attribution tries to assess how extreme event probabilities change with climate change. The author has generally given a lecture on EEA before proposing playing the game.

It is very important to spend some time for debriefing after the game, in order to let the players analyze the processes that lead to each decision [de Suarez et al., 2012]. Explaining how probabilities can be computed is left for a more advanced audience (e.g. mathematics professors in high school), who could then promote the game to others. The game can easily be emulated on a computer with no real loss of information, if the notion of mathematical expectancy (or conditional means) are above the level of the audience.

To conclude, knowing p_1 and p_2 is obviously important to estimate the probability of ruin or the mean gain, for a given protection investment. If one only has access to empirical estimates of p_1 and p_2 from previous die rolls, then the uncertainty of such empirical estimates might be misleading for safe decisions, or just "win the game".

Competing interests

No competing interest is declared.

Author contributions statement

P.Y. devised the mathematical analyses around the game and wrote the manuscript. P.S. and J.M.sS conceived the Game of Seasons and its setting, and reviewed the manuscript.

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