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## A very large-scale traffic network modeling based on the integration of the Bi-dimensional and the GSOM Traffic Flow models

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### Abstract

A continuum traffic modeling is proposed to be applicable in very large and dense networks, requiring a large volume of information difficult to recover and process. The aim of this paper is to merge both macroscopic models: bi-dimensional model (BIDIM) with the GSOM (Generic Second Order) model in order to overcome the missing data for large networks and to benefit from the advantages of each model. As well, the major advantage of the proposed approach is to reproduce a detailed description of traffic dynamics for the main arteries using the GSOM model. Indeed, the bi-dimensional model allows the simulation of areas where the measurements are missing or incomplete. The method consists in proposing an interface node model ensures the exchange between the BIDIM and the GSOM traffic models. The exchange between arteries of the GSOM model and cells edges of the bi-dimensional is assumed through the supply and demand concept. Moreover, the simulation is carried out using Godunov scheme and finite volume method. The dynamic of the developed simulation model reproduces the propagation of the congestion between both models. Applied to a synthetic network, the output results are very promising.

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**Keywords:** Large networks modeling, macroscopic model, Bi-dimensional model, GSOM model, traffic flow, Godunov scheme;

### 1. Introduction and Motivation

Traffic growth and limited road infrastructure capacity are causing many problems, which are mainly related to road congestion Roy et al. (2020); Litman (2021). Indeed, this phenomenon has many negative impacts in terms of

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quality of life: travel time Litman (2009), economic losses Weisbrod et al. (2003), air pollution Zhang and Batterman (2013), etc. However, in order to solve these problems several traffic models are proposed. In the literature, there are two main classes of traffic models: microscopic and macroscopic (for a review of traffic model evolution we refer to Wageningen-Kessels et al. (2015)). The microscopic model has a detailed representation, that each vehicle is a particle whose dynamics are based on ordinary differential equations (ODE). Whereas, macroscopic model describes road traffic as a fluid by analogy with known physical phenomena associated with fluid mechanics. Macroscopic models are applicable on much larger networks than microscopic models. Modeling a large network requires many data, which are difficult to obtain. As the existing traffic models are not suitable for large-scale networks, a bi-dimensional (will be denoted BIDIM in the paper) macroscopic traffic model has been developed which is less demanding in data. The transition from the microscopic model to the macroscopic model is done through the vehicle aggregation. However, the passage from macroscopic model to the bi-dimensional model is done by aggregating the traffic links.

Bi-dimensional modeling is not very precise in the sense that it avoids the traffic keen description on a network, where the measurement points are not sufficient in number to allow an exhaustive assessment of the traffic state Taguchi and Iri (1982). Indeed, to remedy this problem, the generic second order modeling (GSOM) could be integrated at the level of the main axes for which traffic data are available. It is from this perspective that the main idea of the paper interfacing between bi-dimensional and GSOM is relevant in order to overcome the missing data for large networks. In addition, macroscopic models at different modeling scales are rarely coupled.

The main contribution of the paper is to integrate both macroscopic models in different networks scales: GSOM in highway, whereas, BIDIM applicable in large dense networks. The network can be divided in macro-cells where length up to 50 km.

In the subsequent section, we present the Bi-dimensional model. The GSOM traffic flow model is introduced in section 3. The integration of the Bi-dimension model with the GSOM model is developed in section 4. This is the core of the paper. The application site is described in section 5. Next, simulation test and results are discussed. Finally, we end the paper with a concluding part dealing with next steps in section 7.

## 2. Summary description of the Bi- dimensional traffic flow model

The bi-dimensional model was applied to solve flow problems on large networks. At the start, a continuous bi-dimensional model was studied Beckmann (1952) in the case of static models. Since then many articles have been published such as : Vaughan (1987); Yang and Wong (2000); Jiang et al. (2010); Saumtally et al. (2013); Sossoe et al. (2015); Sossoe and Lebacque (2017); Sossoe (2017); M. Khoshyaran and Lebacque (2018); Herty et al. (2018); Mollier et al. (2018); M. Khoshyaran and Lebacque (2020) for dynamic and static traffic assignment problems. This type of bi-dimensional equations is commonly used in pedestrian modeling Jiang et al. (2010); Hoogendoorn and Bovy (2004)) as well as for the space economy (Puu, 2009). The bi-dimensional approach consists of aggregating the network in order to blur their structures and forms Taguchi and Iri (1982). At most, it is a question of altering the detailed geometric structure of the network and replaced by a two-dimensional space continuum.

Let us indicate by  $\mathcal{H}$  the network area that occupies an area of the Euclidian space  $\mathbb{R}^2$ .  $\mathcal{H}$  is decomposed into sub-areas  $\mathcal{H}_m, m = 1, \dots, M : \mathcal{H} = \bigcup_{p=1}^P \mathcal{H}_m$ , where  $\mathcal{H}_m$  is a polygonal domain of  $\mathbb{R}^2$ . Each zone  $\mathcal{H}_m$  is decomposed into elementary cells (see Fig.1).



Fig. 1. Anisotropic network - local and global basis Sossoe et al. (2015) : (a) Anisotropic network with a mesh of 2D cells in an orthonormal basis ( $e_1, e_2$ ) and a local basis ( $u, v$ ); (b) Zoom on a 2D elementary cell.

Numerically, to determine the flow rates, densities and speeds of traffic, the bi-dimensional model is based on a finite volume calculation method applied for each cell. This method is widely applied in the industrial field but little used in the traffic field. Using the bi-dimensional model requires knowledge of the boundary conditions at the edge mesh levels and the splitting rate from  $(i)$  to  $(j)$  direction at each cell. In particular, the incoming flows on each boundary edges are provided. The conservation dynamic bi-dimensional equation of road traffic at each cell is:

$$N_{c,i}(t + \delta t) = N_{c,i}(t) + (Q_{fc}(t) - R_{cg}(t))\delta t + (r_{c,i}(t^+) - q_{c,i}(t^+))\delta t \quad (1)$$

With  $(i)$  is the direction of propagation of flow inside the cell  $(c)$  ( $i=1, \dots, 4$ , refer to Fig. 2).  $(f)$  is the cell located at the left and  $(g)$  is the cell located at the right of the cell  $(c)$  (see Fig.2).

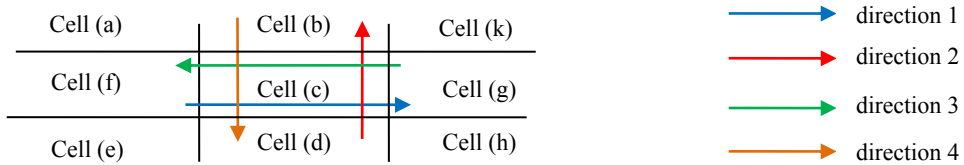


Fig. 2. The four considered directions within 2d elementary cells with its neighbour cells.

$N_{c,i}(t + \delta t)$  denotes the number of vehicles in cell  $(c)$  at the time  $(t + \delta t)$  with  $\delta t$  is the time step, and  $N_{c,i}(t)$  is the number of vehicles in cell  $(c)$  at the time  $(t)$ . The internal cell is governed by the intersection traffic flow model Lebacque and Khoshyaran (2005) where, outgoing vehicles flow  $q_{c,i}(t^+)$  and incoming vehicles flow  $r_{c,i}(t^+)$  at time  $t^+ = t + \delta t/2$  are solved using the below (2) linear-quadratic optimization problem :

$$\begin{aligned} & \max_{(q,r)} \left( \sum_{i=1}^4 \Phi(q_i) + \sum_{j=1}^4 \Psi(r_j) \right) \\ \text{s.t. } & \begin{cases} 0 \leq q_i \leq \Delta_{ci}^{t+1/2}, & \forall j \in \{1,2,3,4\}, \\ 0 \leq r_j \leq \Sigma_{cj}^{t+1/2}, & \forall j \in \{1,2,3,4\}, \\ -r_j + \sum_{i=1}^4 q_i \Gamma_{c,ij}^t = 0, & \forall j \in \{1,2,3,4\}. \end{cases} \end{aligned} \quad (2)$$

Functions  $\Phi(\vartheta_\ell) \doteq \Psi(\vartheta_\ell)$  are defined by:  $\Psi(\vartheta_\ell) \stackrel{\text{def}}{=} -\frac{1}{2}\vartheta_\ell^2 + \vartheta_\ell \cdot \vartheta_{\ell, \max}$ ,  $\vartheta_\ell$  referring to  $q_i$  or  $r_j$ . They are assumed equal, concave and increasing.  $\Delta_{ci}^{t+1/2}$  and  $\Sigma_{cj}^{t+1/2}$  are respectively the traffic demand in  $(c)$  to direction  $(i)$  and the traffic supply in direction  $j$  at time  $t^+ = t + 1/2$ . However, inflows  $Q_{fc}(t)$  and outflows  $R_{cg}(t)$  of the cell  $(c)$  at the time  $(t)$  are governed by the supply and demand concept:  $Q_{fc}(t) = \min(\delta_{f,f \rightarrow c}(t), \sigma_{c,f \rightarrow c}(t))$  and  $R_{cg}(t) = \min(\delta_{c,c \rightarrow g}(t), \sigma_{c,c \rightarrow g}(t))$ .  $\delta_{f,f \rightarrow c}(t)$  denotes the demand of cell  $(f)$  at the  $f \rightarrow c$  border,  $\sigma_{c,f \rightarrow c}(t)$  denotes the supply of cell  $(c)$  at the  $f \rightarrow c$  border.

### 3. Summary description of the GSOM Traffic Flow Model

The GSOM model proposed by Lebacque et al. (2007) is a generic model with several model families: first LWR model of Lighthill and Whitham (1955), Richards (1956) and second order model ARZ of Aw and Rascle (2000), Zhang (2002), as well as Colombo (2003). It combines the conservation equation of the macroscopic kinematic model with dynamic equation for attribute specific to the driver's behavior. The GSOM model equations are the following:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho I) + \partial_x(\rho v I) = -\rho f(I) \\ v = \mathfrak{I}(\rho, I) \end{cases} \quad (3)$$

Equation (3) is a system of conservation equations with source terms, consisting respectively of: conservation equation of vehicular traffic, conservation equation of attribute vector and attribute dependent fundamental diagram. The main variables of the GSOM model are density  $\rho(x,t)$ , flow  $q(x,t)$ , velocity  $v(x,t)$ , and attribute  $I(x,t)$ .  $x$  and  $t$  denotes the position and time. As well as the velocity is a function of density and the attribute  $I$  where  $\mathfrak{F}(\rho, I)$  denotes the velocity-density fundamental diagram (FD). The function  $f(I)$  is a relaxation function and the invariant  $I(x, t)$  is conserved along the vehicle trajectory. In our case  $f(I)=0$ .

#### 4. Integration of the Bi-dimensional & the GSOM model

The method consists in dividing the network into two-dimensional quadrangular cells such that the main roads constitute the borders of these cells. In fact, at these edges the GSOM is applied. We zoom in on the main edges of the bi-dimensional and we apply the GSOM at this level. The exchange between artery of the GSOM model and cells edges of the bi-dimensional is obtained by applying the supply and demand concept. Traffic supply and demand are basic ingredients of the Godunov scheme as applied to the GSOM model (refer to Lebacque et al. (2007)). Supply and demand also apply to the BIDIM model as seen above (section 2). However, the boundary conditions are imposed on the edges of the simulated network and inside the network (main roads: national roads, highways, etc.) by the GSOM. The main idea is to introduce a dummy node in order to ensure the exchange flows between the BIDIM and the GSOM network in both directions from BIDIM to GSOM and from GSOM to BIDIM, as reported in the Fig. 3, (a) and (b).

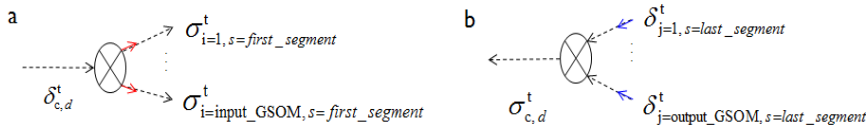


Fig. 3. (a) Flows from BIDIM to GSOM; (b) Flows from GSOM to BIDIM.

##### 4.1. Through-flows from Cell BIDIM to GSOM network : boundary flows

The outgoing flow from BIDIM cell to GSOM network will be distributed over the various inputs of the GSOM through dummy node. There is a privileged travel direction say ( $d$ ) corresponding to (BIDIM)  $\rightarrow$  (GSOM) traffic outside cell ( $c$ ). As illustrated in Fig. 3. (a).

For each dummy node, we define the boundary supplies and demands Lebacque et al. (2005) during time- step ( $t$ ) of BIDIM model:

- The total demand of the BIDIM cell is determined according to the density in cell ( $c$ ) defined as  $\rho_{c,d}^t$  and the number of lanes  $v_{c,d}^t$  :

$$\delta_{c,d}^t \stackrel{\text{def}}{=} v_d^c \Delta_e^c(\rho_{c,d}^t) \quad (4)$$

$\Delta_e^c(\rho_{c,d}^t)$  is the equilibrium demand function in cell ( $c$ ) upstream the dummy node at time ( $t$ ). It is represented by the well-known flow FD emphasize the equilibrium between the users' transport demand and the network supply.

- The supply of each input GSOM network ( $i$ ) is determined according to the density at first input segment link ( $s$ ) of the GSOM network defined as  $\rho_{i,s}^t$  and the number of lanes  $v_s^i$  :

$$\sigma_{i,s}^t \stackrel{\text{def}}{=} v_s^i \Sigma_e^c(\rho_{i,s}^t) \quad (5)$$

Where,  $\Sigma_e^c(\rho_{i,s}^t)$  is the equilibrium supply function of each downstream input link GSOM of the dummy node at time  $(t)$ . It is depicted by the flow fundamental diagram.

We obtain the out-flows  $Q_{out\_BIDIM}^n$  from cell BIDIM to GSOM network using the (min) function (see eqn. (6)):

$$Q_{out\_BIDIM}^n = \min[\sigma_{i,s}^t, q_{i,d}^t] \quad (6)$$

With  $n = \Delta t_{BIDIM} / \Delta t_{GSOM}$  corresponds to the time-step number and  $q_{i,d}^t$  the through-flows in direction  $(d)$  per input link GSOM  $(i)$  at time-step BIDIM model:

$$q_{i,d}^t = \alpha_i \delta_{c,d}^t \quad (7)$$

The global demand of the GSOM is split into several GSOM input. The turning coefficient  $\alpha_i$  denotes the fraction of the traffic (BIDIM)  $\rightarrow$  (GSOM) with destination  $(i)$ , with  $\sum_{i \in input\_GSOM} \alpha_i = 1$

Finally, by conservation of traffic we obtain:

$$Q_{out\_BIDIM} = \sum_{i \in (input\_GSOM)} q_{i,d}^t \quad (8)$$

#### 4.2. Outflows GSOM network -inflows Cell BIDIM : boundary flows

In this subsection we treat the reverse direction  $(r)$  of which  $(r)$  corresponds to the direction of flow from the bi-dimensional cell to the GSOM network: (BIDIM)  $\rightarrow$  (GSOM) as shown in indicated in the Fig.3.(b).

The incoming flow to the BIDIM cell corresponds to the aggregate of out flows from different exits of the GSOM network during a GSOM time-step  $\Delta t_{GSOM}$ . For each dummy node we define the boundary supplies and demands during time- step  $(\Delta t_{GSOM})$ :

- The supply of the BIDIM cell is calculated according to the density in cell  $(c)$  to the direction  $(d)$  defined as  $\rho_{c,r}^t$  and the number of lanes  $v_r^c$  :

$$\sigma_{c,d}^t \stackrel{\text{def}}{=} v_r^c \Sigma_e^c(\rho_{c,r}^t) \quad (9)$$

$\Sigma_e^c(\rho_{c,r}^t)$  is the equilibrium supply function in cell  $(c)$  and direction  $(r)$  upstream the dummy node at time  $t = \Delta t_{GSOM}$ . It is determined by the flow fundamental diagram.

- The demand of each output link GSOM  $(l)$  is determined according to the density at last output segment link  $(s)$  defined as  $\rho_{l,s}^t$  and the number of lanes  $v_s^l$  convergent to the dummy node:

$$\delta_{l,s}^t \stackrel{\text{def}}{=} v_s^l \Delta_e^l(\rho_{l,s}^t) \quad (10)$$

Note that,  $\Delta_e^l(\rho_{l,s}^t)$  is the equilibrium demand function of each output link GSOM  $(l)$  and the last segment  $(s)$  downstream the dummy node at time  $t = \Delta t_{GSOM}$ . It is determined by the flow fundamental diagram.

The total demand GSOM is the cumulative of the partial demand by link  $\delta_{l,s}^t$  during  $n$  time-step, with  $n = \Delta t_{BIDIM} / \Delta t_{GSOM}$  and  $\Delta t_{BIDIM} \ll \Delta t_{GSOM}$  :

$$q_{output\_GSOM}^n = \sum_{l \in (output\_GSOM)} \delta_{l,s}^t \quad (11)$$

Therefore, following the min principle Lebacque and Khoshyaran (2005) we obtain the through-flows  $Q_{in\_BIDIM}^n$  from GSOM network to cell BIDIM during  $n$  time-step as follows:

$$Q_{in\_BIDIM}^n = \min[\sigma_{c,r}^t, q_{output\_GSOM}^n] \quad (12)$$

## 5. Application site

The described traffic models are developed in C++ language and applied on the stretch traffic network reported in the Fig. 4. The considered site is inspired from the “Corridor Périphérique” of Paris where very dense traffic is observed during the peak and evening period of the day. This stretch is constituted by two main traffic components: the motorway part and the urban part. The BIDIM model is applied on the urban part and the GSOM on the motorway part. With respect to the urban part, two cells (C1 & C2) are considered covering 3 km by 3 km each. Each cell can include around 36 urban intersections. For the motorway part, the considered total length is equal to 6 km including 6 origins and 5 destinations. Except for the main origin of the motorway part (ORIG) and the main destination (DEST) (see Fig.4.), all off-ramps and on-ramps are connected to the dummy interface node where input and output flows are computed, as described in section 4, in both directions: GSOM to BIDIM and vice & versa.

Attention has been paid to the conservation equations in the interface node and in particular the  $Q_{in}$  flow (BIDIM) must be equal to the sum of the off-ramp flows (GSOM) on one hand, on the other hand, the  $Q_{out}$  flow (BIDIM) must be equal to the sum of on-ramp flows (GSOM) and no loss or generated vehicles must be observed.

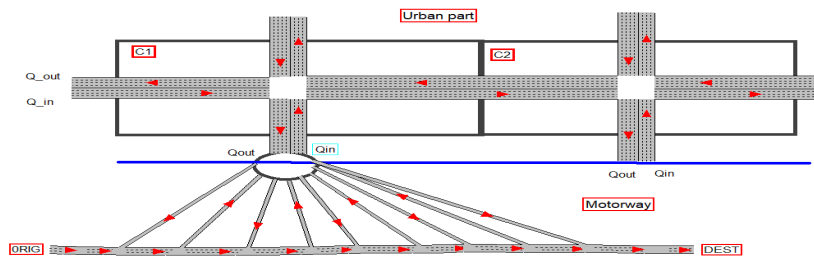


Fig. 4. Stretch of the test site.

## 6. Simulation tests and results

The fixed simulation parameters are the following: simulation time steps of the BIDIM: 30 sec and the GSOM: 10 sec, simulation horizon 2 hours, aggregate time of the data output (Traffic volume, density and speed): 60 sec, the following demand profile is applied to each origin of the BIDIM and the GSOM (see Fig.5.)

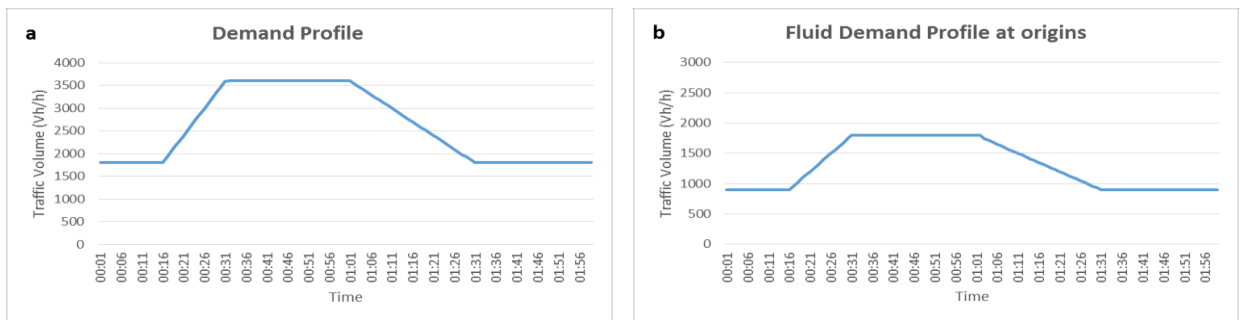


Fig. 5. Used demand profiles at the origins during the simulation of the tested site (a) congested ; (b) fluid.

Fig.5 depicts the fixed demand profile of two scenarios: fluid (b) and congested (a). In this figure, the considered number of lane is equal to 3. Consequently, this profile is used for the GSOM and adapted to the BIDIM according to the number of lanes at each border edge. With respect to the simulation models (GSOM and BIDIM) the same shape of the used fundamental diagram (FD) (triangular) model is used. For the motorway part, the characteristics of the FD per lane are: free speed:  $V_f = 80$  Km/h; critical density  $Ro\_Cr = 27$  Vh/km, Capacity = 2200 vh/h/lane, jam density = 180 Vh/Km/lane whereas for the urban part: free speed:  $V_f = 50$  Km/h; critical density  $Ro\_Cr = 30$  Vh/km, Capacity = 1500 Vh/h/lane, jam density= 140 Vh/Km/lane. In case of traffic volume computation of BIDIM in the direction of the GSOM, the splitting rates toward the 5 origins of the GSOM is equal to 0.2 ( $5 \times 0.2 = 1$ ).

In case of the GSOM model, the definition of the attribute can generate a specific second order model. For example, if the attribute is equal to the relative speed, the ARZ model will be used. In our case and for simplicity reasons, the Attribute I is set to 0. This means that the LWR is used for the BIDIM and the GSOM.

### 6.1. Fluid case

Fig.6. depicts the traffic volume exchange between both models: BIDIM and GSOM. In particular, the computed in flow to the GSOM and the out flow from the BIDIM and vice & versa are completely equal. This means that the conservation equation is preserved and no loss or generated vehicles is observed. Consequently, we can conclude that the modelling approach of the interface node is valid.

In the Fig. 6, the red colour corresponds to the out flow of the BIDIM of the Cell (C1) toward the direction 1 (GSOM), whereas, the blue is the sum of the inflow at each GSOM origin.

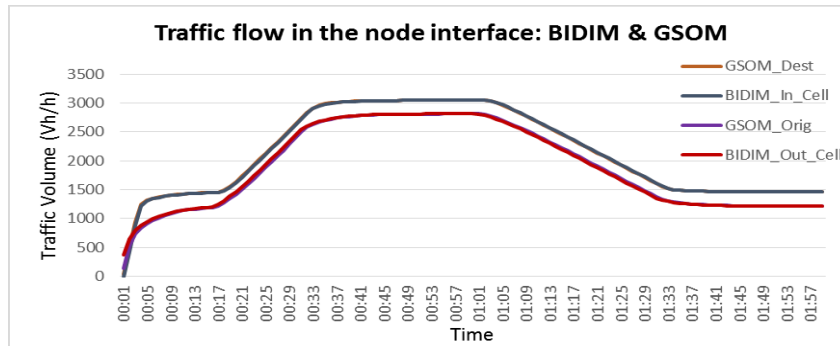


Fig. 6. Fluid: Time evolution of the computed Traffic volume in the interface node from the GSOM to the BIDIM and vice & versa.

### 6.2. Congested case

As described in the fluid case, the equality of the exchange flow between both models is observed in case of congested traffic. However, the time evolution of the traffic volume profiles is different compared to the fluid case. As a matter of fact, the congestion appears first in the GSOM in the middle of the motorway axis. This congestion is propagated in the BIDIM due to the limitation of the out flow of the BIDIM. The congestion propagation is observed in the Fig.7.(b).

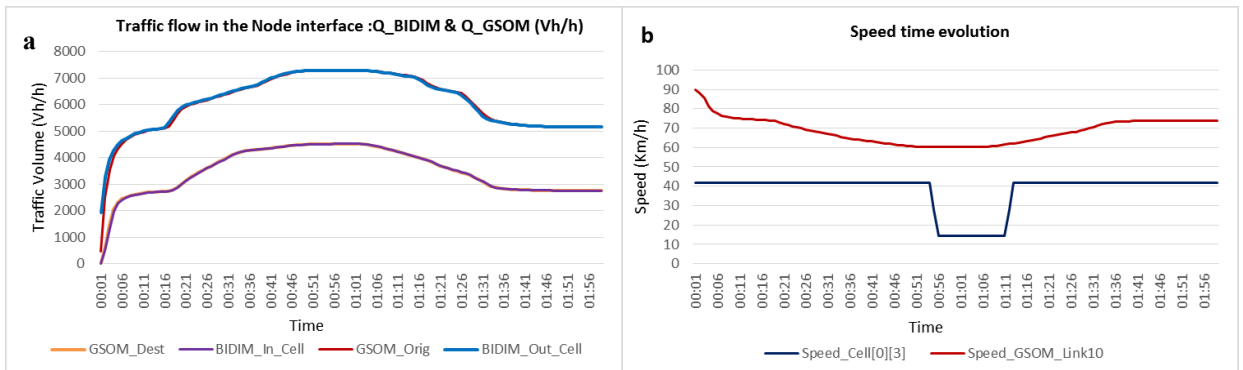


Fig. 7. (a) Congested: computed Traffic volume time evolution in the interface node from the GSOM to the BIDIM and Vice&versa; (b) Speed time evolution in C1 direction 1 and GSOM link.

In summary, the reported simulation results indicate that the developed node interface model is correct and coherent with respect to the congestion propagation in both directions.



## 7. Conclusion and next steps

The developed interface node model between the BIDIM and the GSOM traffic models is very promising. Qualitative and quantitative results obtained on the hypothetical simple test site are coherent. As a matter of fact, the connection between both models can be viewed as a loop with respect to the dynamic traffic evolution of a very large network. Providing a minimum input data for the BIDIM and connecting the GSOM model, vision of both dynamic traffic can be observed. However, investigation on the impact of the GSOM model will continue. In particular, the attribute must be set to the relative speed in order to test the ARZ model. On the other hand, calibration of both models must be investigated also. Consequently, simulated and real data will validate the developed model. **Further investigations should also include systematic comparison with other models.**

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