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► To cite this version:

Fábris Kossoski, Martial Boggio-Pasqua, Pierre-François Loos, Denis Jacquemin. Reference Energies for Double Excitations: Improvement and Extension. 2024. hal-04526063

HAL Id: hal-04526063

<https://hal.science/hal-04526063>

Preprint submitted on 29 Mar 2024

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Reference Energies for Double Excitations: Improvement and Extension

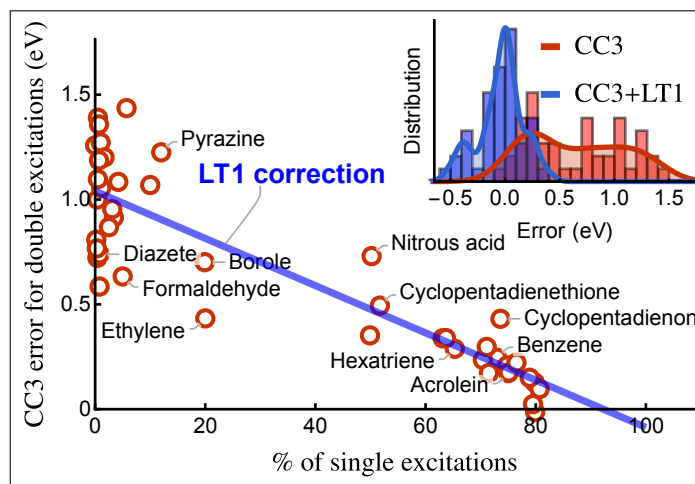
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In the realm of photochemistry, the significance of double excitations (also known as doubly-excited states), where two electrons are concurrently elevated to higher energy levels, lies in their involvement in key electronic transitions essential in light-induced chemical reactions as well as their challenging nature from the computational theoretical chemistry point of view. Based on state-of-the-art electronic structure methods (such as high-order coupled-cluster, selected configuration interaction, and multiconfigurational methods), we improve and expand our prior set of accurate reference excitation energies for electronic states exhibiting a substantial amount of double excitations [Loos et al. *J. Chem. Theory Comput.* **2019**, *15*, 1939]. This extended collection encompasses 47 electronic transitions across 26 molecular systems that we separate into two distinct subsets: (i) 28 “genuine” doubly-excited states where the transitions almost exclusively involve doubly-excited configurations and (ii) 19 “partial” doubly-excited states which exhibit a more balanced character between singly- and doubly-excited configurations. For each subset, we assess the performance of high-order coupled-cluster (CC3, CCSDT, CC4, and CCSDTQ) and multiconfigurational methods (CASPT2, CASPT3, PC-NEVPT2, and SC-NEVPT2). Using as a probe the percentage of single excitations involved in a given transition ($\%T_1$) computed at the CC3 level, we also propose a simple correction that reduces the errors of CC3 by a factor of 3, for both sets of excitations. We hope that this more complete and diverse compilation of double excitations will help future developments of electronic excited-state methodologies.



I. INTRODUCTION

When discussing optical excitations in molecules, it is often useful to specify the number of electrons involved in a particular excitation. In the simple orbital picture, a “single” excitation refers to the promotion of a single electron from an occupied orbital, or hole (h) state, to a vacant orbital, or particle (p) state, resulting in a “singly-excited” (or 1h1p) configuration. In contrast, a “double” excitation involves the promotion of a pair of electrons with opposite or same spins from the same or different

occupied orbitals to vacant ones, leading to a “doubly-excited” (or 2h2p) configuration.

In the framework of many-body perturbation theory, singly-excited states correspond to bound states between an optically excited electron and the hole left behind due to the excitation. This quasiparticle state is known as an exciton. On the other hand, doubly-excited states correspond to the appearance of a biexciton, or a pair of excitons, which consists of two holes and two excited electrons bound together. At the many-body level, biexcitons involve four-body correlations, which explains the difficulty behind their theoretical treatment, whereas experiments also struggle to characterize them.

These multiple excited states can serve as rapid relaxation pathways after the absorption of highly energized photons to prevent damage¹ or utilize the extra energy deposited in nanocrystal quantum dots by highly energetic photons.² These multiple exciton states exhibit correla-

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tions between their constituent electrons and holes and can be observed in non-linear multidimensional optical spectroscopy.^{3,4} Direct spectroscopic investigation of biexcitons is challenging because the optical transitions are formally forbidden, hence their classification as “dark” states. However, these states can be indirectly probed through techniques like photoluminescence⁵ or spectrally resolved four-wave mixing experiments.⁶

Because this assignment is only meaningful within a given wave function method and set of reference orbitals, the concept of single or double excitations is not precisely defined, and it is more accurate to refer to double excitations as “*electronic states with doubly-excited character*”. Classifying doubly-excited states is a tedious task,⁷ that has sparked controversies in recent years, particularly concerning the lowest dark state of butadiene.^{7–10}

Doubly-excited states are ubiquitous in many photochemical mechanisms, such as the photochemistry of polyenes,^{11–23} in singlet fission²⁴ where they can play a major role in the formation of the triplet-pair state,²⁵ in thermally activated delayed fluorescence (TADF)^{26,27} where their predominant interaction with singlet states can produce molecules with an inverted singlet-triplet gap,^{28–35} i.e., where the lowest singlet excited state is lower in energy than the lowest triplet state.

Double excitations pose significant challenges for adiabatic time-dependent density-functional theory (TD-DFT),^{36–39} which only considers explicitly the 1h1p configurations. To capture doubly-excited states, one must go beyond the adiabatic approximation which is a non-trivial task.^{15,16,20,22,23,40–44} The Bethe-Salpeter equation (BSE) formalism^{45–48} encounters a similar challenge, requiring frequency-dependent kernels to compute doubly-excited states.^{46,49–55} This is why more advanced theories should be used. However, second-order wave function methods like CIS(D)^{56,57} and EOM-CC2^{58,59} do not explicitly account for the 2h2p configurations, while ADC(2)^{60–62} and EOM-CCSD^{63–68} do not provide sufficient accuracy for double excitations. In short, the most computationally effective theories are inadequate to tackle doubly-excited states.

Although ADC(3) includes the 2h2p configurations,^{61,62,69,70} it struggles to deliver accurate transition energies for doubly-excited states.⁷¹ A recent study have demonstrated that ADC(4), which additionally incorporates explicitly the 3h3p configurations, is better suited for this purpose.⁷²

Spin-flip methods offer an alternative approach where ground and doubly-excited states are accessed through single excitations from the lowest triplet state.^{73–79} Pushing the excitation degree to triple or quadruple excitations within coupled-cluster (CC) theory^{80–85} also enables to capture double excitations.^{86–89}

Multiconfigurational self-consistent field methods, such as complete-active-space self-consistent field (CASSCF), and their second-order perturbatively corrected variants, CASPT2^{90–93} and NEVPT2,^{94–97} naturally handle single and double excitations on an equal footing.^{98–100}

All of these methods have well-documented strengths and weaknesses, and it is beneficial to utilize each of them in their optimal scenarios, capitalizing on their respective advantages. For example, CASPT2 and NEVPT2 suffer from exponential growth in computational cost with the number of active electrons and orbitals. Careful selection of the active space can provide access to doubly-excited states at a manageable expense, albeit with limitations imposed by the active space choice. In practical applications, accuracy in the 0.1–0.2 eV range can be achieved by employing appropriate active spaces.

EOM-CC methods offer a systematically improvable pathway to high accuracy, exemplified by the series of methods EOM-CC3,^{101–105} EOM-CCSDT,^{106–109} EOM-CC4,^{110–113} and EOM-CCSDTQ.^{114–118} Although the cost increases with the maximum excitation degree, it remains polynomial. By including quadruples, chemical accuracy (i.e., an absolute error below 0.043 eV) can be reached for double excitations.^{119,120}

For small systems, one can also compute full CI (FCI) excitation energies that are usually obtained with selected CI (SCI) methods for computational efficiency.^{121–133} The “*Configuration Interaction using a Perturbative Selection made Iteratively*” (CIPSI) algorithm^{121,122,134–140} is particularly effective in computing vertical excitation energies in small molecules.^{87–89,112,141–145} Other SCI methods, such as semistochastic heatbath CI (SHCI),^{123,126} Monte Carlo CI (MCCI),^{133,146} iterative CI (iCI),¹⁴⁷ and adaptive sampling CI (ASCI),¹⁴⁸ have also been successfully employed for computing excited states.

In a 2019 paper, we presented reference energies for double excitations in a collection of 14 small- and medium-sized molecules, encompassing a set of 20 vertical transitions.⁸⁸ For the majority of these calculations, we relied on FCI computations to establish our “theoretical best estimates” (TBEs). However, for the largest molecules in the dataset, we employed high-order EOM-CC or multiconfigurational methods to achieve accurate estimates.

These reference energies for doubly-excited states have proven to be valuable to the scientific community, serving as benchmarks and test cases for new computational methods, including orbital-optimized DFT,^{149–151} ensemble DFT,¹⁵² quantum Monte Carlo,^{153–155} EOM-CC,^{156,157} and others,^{10,72,99,100,158–160} and have now been incorporated into QUEST, a comprehensive and diverse database of highly accurate vertical excitation energies for small- and medium-sized molecules.^{71,142}

In this work, using the same computational protocol,^{87–89,112,141–143} we broaden and enhance this set of reference data. Firstly, we significantly augment our collection of double excitations by reporting 47 vertical excitation energies for electronic states exhibiting substantial and more balanced doubly-excited character. We classify these states as “genuine” (where transition primarily involves doubly-excited determinants) and “partial” (where the transition involves a more balanced character between singly- and doubly-excited determinants). Sec-

only, we refine the previous TBEs by incorporating new FCI, high-order CC, and multiconfigurational (NEVPT2, CASPT2, and CASPT3) calculations to improve their accuracy.

II. COMPUTATIONAL DETAILS

The ground-state geometries are extracted from Ref. 88 or optimized using the same protocol, that is, at the CC3/aug-cc-pVTZ level of theory for most systems. These geometries can be found in the supporting information (SI). Calculations are performed for all systems using three Gaussian atomic basis sets: 6-31+G*, aug-cc-pVDZ, and aug-cc-pVTZ, which are sometimes referred to as Pop, AVDZ, and AVTZ from here on. All excited-state calculations were performed using the frozen-core approach (except for the Be atom).

Vertical excitation energies were obtained using FCI calculations employing the CIPSI algorithm.^{121,122,134,144,161} All calculations are performed using QUANTUM PACKAGE¹⁴⁰ following the same protocol as our previous studies.^{87–89} Here, the extrapolated FCI estimate obtained with the CIPSI calculations is referred to as exFCI.¹⁶² Extrapolation errors are estimated following the procedure outlined in Ref. 142.

Most CC calculations were conducted using CFOUR,¹⁶³ which offers an efficient implementation of high-order CC methods up to quadruples.¹⁶⁴ For CC3 calculations, we utilize DALTON,¹⁶⁵ while MRCC¹⁶⁶ is employed for CC calculations that go beyond quadruples.

Multiconfigurational calculations are performed using MOLPRO¹⁶⁷ following the protocol described in Ref. 88. The SI provides additional details and a description of the active spaces for each symmetry representation for each system and transition. We performed NEVPT2 calculations in the partially contracted (PC) and in the strongly-contracted (SC) schemes, as well as CASPT2 and CASPT3 calculations,^{100,168} both with and without an IPEA shift.^{169–171} To differentiate the two cases, throughout the text, we indicate in parenthesis when the IPEA shift is adopted: CASPT2(IPEA) uses it and CASPT2 does not. The IPEA shift is set to its default value of $0.25 E_h$. Unless otherwise stated, all CASPT2 and CASPT3 calculations have been performed with a level shift of 0.3 a.u. Note that the Fock operator used in the zeroth-order Hamiltonian was systematically computed using the state-specific density.

To obtain a basic definition of the singly- or doubly-excited character, we use the $\%T_1$ diagnostic computed at the CC3/AVTZ level, where one sums the weights of all singly-excited configurations. A “genuine” double excitation corresponds to $\%T_1 \approx 0\%$, whereas a “partial” double excitation, where the transition is dominated by single excitations, has intermediate values, up to $\%T_1 \approx 85\%$. Transitions characterized by $\%T_1 \gtrsim 85\%$ correspond to singly-excited states and are not addressed here. With few exceptions (discussed below), the classification based

on the $\%T_1$ diagnostic computed at the CC3 level agrees with that based on inspection of the CIPSI wave function or the amplitudes in high-order CC calculations.

Here we address 47 excited states having doubly-excited character, encompassing the 26 molecules depicted in Fig. 1. We have classified 19 as “partial” double excitations, 14 of them are new compared to Ref. 88 and we have performed additional calculations for the others. The other 28 states were classified as “genuine” double excitations, 13 of them are new in comparison to Ref. 88, but we also performed new calculations for the others. In comparison to Ref. 88, the present set thus substantially increases the pool of states possessing doubly-excited character, from 15 to 28 genuine double excitations, and, most noticeably, from 5 to 19 partial double excitations. This allows us to draw more general conclusions about the performance of various methodologies in describing doubly-excited states. In line with the latest contributions to the QUEST database,^{35,172–174} all excitation energies reported here are given with three decimal places.

We provide a TBE for each basis set (Pop, AVDZ, and AVTZ). The result for the largest one, TBE/AVTZ, should be taken as our most accurate estimate of the excitation energy. For the sake of conciseness, the TBE/AVTZ is referred to as TBE from hereon, whereas the smaller ones are explicitly stated (TBE/Pop and TBE/AVDZ). When a given TBE is chemically accurate (estimated uncertainty less than 1 kcal/mol or 0.043 eV), we label it as safe. Otherwise, we referred to the TBE as unsafe. For borderline cases, we conservatively assign them as unsafe.

Starting with the smallest 6-31+G* basis set, the most accurate level of theory we are able to employ yields the TBE/Pop. When increasing the basis set, the same level of theory is often impractical, and we adopt instead less demanding and more approximate methods. This happens in the majority of cases, where we resort to composite models to compute the TBE. The result from a smaller basis set and accurate level of theory, say CCSDTQ in the 6-31+G* basis set (CCSDTQ/Pop), is combined with that from a larger basis set and less accurate level of theory, say CC4 in the AVDZ basis set (CC4/AVDZ), to yield a $TBE/AVDZ = CCSDTQ/Pop + [CC4/AVDZ - CC4/Pop]$. Here we refer to the term in brackets as a basis set correction. We often employ two different levels of theory for each basis set correction. Following on the previous example, CCSDT can be used to provide the AVTZ basis set correction, such that $TBE = TBE/AVDZ + [CCSDT/AVTZ - CCSDT/AVDZ]$.

It is not always straightforward to choose which method yields the most accurate excitation energy for a given basis set, and, similarly, which method is more trustworthy for the basis set correction. This is further complicated by the fact that the performance of the methods can significantly depend on whether the doubly-excited state has a genuine or partial character. Here we adopted the following guiding principles. When CIPSI calculations yield an extrapolation error smaller than the chemical accuracy, this result is taken as a safe TBE for that basis

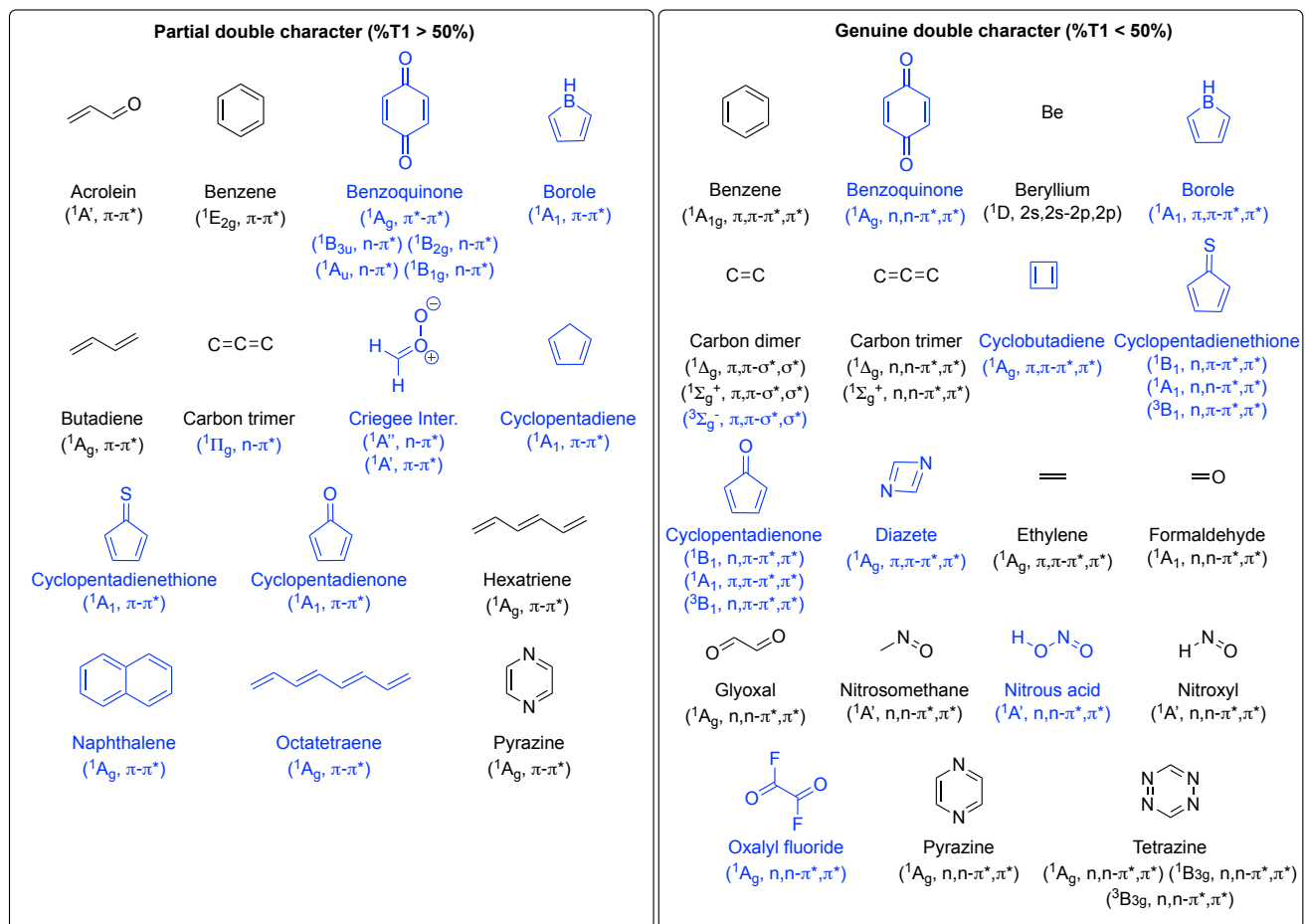


FIG. 1. List of molecules considered in the present study having states with partial double excitation character (left) and genuine double excitation character (right). In comparison to Ref. 88, the additional systems and states are highlighted in blue.

set. We achieved this level of convergence in CIPSI calculations only for relatively small systems (such as carbon trimer in the AVTZ basis) and/or basis set (for instance, the 1B_1 state of cyclopentadienone in the 6-31+G* basis). When larger extrapolation errors are obtained, the upper and lower bounds of the CIPSI result can guide the choice of a higher-order CC or multiconfigurational method for the TBE. When available, CCSDTQ or CC4 is preferred over multiconfigurational methods, which is justified *a posteriori* by their overall superior performance, as discussed below. CCSDT remains more accurate than multiconfigurational alternatives for partial double excitations, while the opposite is found for genuine double excitations, which is also justified *a posteriori*. In the latter case, selecting the most reliable multiconfigurational method is less trivial. The extrapolation error of CIPSI, the very likely overestimated energies produced by CC, and the global statistics, all provide valuable bounds and insights to determine which method is the most reliable to produce the TBE. In light of the present results, we further elaborate on the choice of method for the basis set correction in Sec. V.

III. DOUBLE EXCITATIONS: IMPROVEMENT

Table I contains the vertical excitation energies for the 47 excited states considered here, as obtained with various levels of theory and basis sets, along with the TBEs for each basis and the methods employed to obtain the TBEs. The TBEs are gathered in Table II, along with a comparison with previously reported TBEs.^{88,120,142,175}

TABLE I: Vertical excitation energies (in eV) computed at various levels of theory, and for the 6-31+G*, aug-cc-pVDZ, and aug-cc-pVTZ basis sets (abbreviated to Pop, AVDZ, and AVTZ) along with the TBEs for each basis and the methods employed to obtain the TBEs. Each state is labeled by its spatial and spin symmetries and by its %T₁ value obtained at the CC3/AVTZ level of theory.

	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
Acrolein, ¹ A', %T ₁ = 75%			
TBE	8.034 ^b	7.940 ^g	7.928 ^m
FCI	8.00(3)		
CCSDTQ	8.034		
CC4	8.035	7.941	
CCSDT	8.110	8.024	8.012
CC3	8.206	8.106	8.078
CASPT2	7.634	7.626	7.528
CASPT2(IPEA)	8.028	8.036	7.961
CASPT3	7.976	7.969	7.906
CASPT3(IPEA)	8.048	8.046	7.979
SC-NEVPT2	8.082	8.085	8.006
PC-NEVPT2	7.909	7.930	7.846
Benzene, ¹ E _{2g} , %T ₁ = 73.3%			
TBE	8.259 ^c	8.211 ^c	8.190 ^m
CC4	8.259	8.211	
CCSDT	8.424	8.380	8.359
CC3	8.504	8.439	8.381
CASPT2	7.986	7.901	7.816
CASPT2(IPEA)	8.408	8.374	8.314
CASPT3	8.251	8.220	8.162
CASPT3(IPEA)	8.342	8.325	8.263
SC-NEVPT2	8.623	8.606	8.555
PC-NEVPT2	8.584	8.564	8.512
Benzene, ¹ A _{1g}			
TBE	10.531 ^c	10.360 ^e	10.315 ^e
CASPT2	9.698	9.463	9.326
CASPT2(IPEA)	10.501	10.343	10.236
CASPT3	10.531	10.360	10.315
CASPT3(IPEA)	10.676	10.528	10.468
SC-NEVPT2	10.633	10.479	10.381
PC-NEVPT2	10.349	10.179	9.995
Benzoquinone, ¹ A _g , %T ₁ = 63%			
TBE	6.339 ^c	6.381 ^h	6.351 ⁿ
CC4	6.339		
CCSDT	6.600	6.642	
CC3	6.677	6.701	6.671
CASPT2	5.960	5.934	5.849
CASPT2(IPEA)	6.341	6.354	6.287
CASPT3	6.355	6.348	6.303
CASPT3(IPEA)	6.390	6.394	6.343
SC-NEVPT2	6.471	6.492	6.431
PC-NEVPT2	6.435	6.452	6.390
Benzoquinone, ¹ B _{3u} , %T ₁ = 80%			
TBE	5.819 ^c	5.670 ^h	5.656 ⁿ
CC4	5.819		
CCSDT	5.959	5.810	
CC3	5.947	5.769	5.755
CASPT2	5.211	5.100	5.002
CASPT2(IPEA)	5.895	5.841	5.774
CASPT3	6.051	5.985	5.954
CASPT3(IPEA)	6.121	6.073	6.036
SC-NEVPT2	6.114	6.074	6.020
PC-NEVPT2	6.048	5.999	5.944
Benzoquinone, ¹ B _{2g} , %T ₁ = 76%			
TBE	5.898 ^c	5.766 ^h	5.764 ⁿ
CC4	5.898		
CCSDT	6.104	5.972	
CC3	6.108	5.937	5.935
CASPT2	5.213	5.102	5.005
CASPT2(IPEA)	5.882	5.828	5.761
CASPT3	6.046	5.982	5.950
CASPT3(IPEA)	6.113	6.066	6.028
SC-NEVPT2	6.099	6.059	6.004
PC-NEVPT2	6.037	5.988	5.932
Benzoquinone, ¹ A _u , %T ₁ = 75%			
TBE	6.200 ^c	6.109 ^h	6.083 ⁿ
CC4	6.200		
CCSDT	6.463	6.372	
CC3	6.408	6.295	6.269
CASPT2	5.742	5.650	5.547
CASPT2(IPEA)	6.315	6.281	6.210
CASPT3	6.462	6.402	6.352
CASPT3(IPEA)	6.521	6.477	6.422

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	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
SC-NEVPT2	6.617	6.603	6.548
PC-NEVPT2	6.581	6.558	6.502
Benzoquinone, ¹ B _{1g} , %T ₁ = 70%			
TBE	6.529 ^c	6.495 ^h	6.469 ⁿ
CC4	6.529		
CCSDT	6.773	6.739	
CC3	6.764	6.707	6.681
CASPT2	5.693	5.596	5.479
CASPT2(IPEA)	6.336	6.301	6.234
CASPT3	6.581	6.523	6.492
CASPT3(IPEA)	6.547	6.503	6.451
SC-NEVPT2	6.641	6.625	6.574
PC-NEVPT2	6.606	6.581	6.530
Benzoquinone, ¹ A _g , %T ₁ = 0%			
TBE	4.665 ^f	4.593 ^f	4.566 ^f
CC4	4.771		
CCSDT	5.792	5.834	
CC3	5.923	5.935	6.017
CASPT2	4.429	4.345	4.301
CASPT2(IPEA)	4.583	4.509	4.472
CASPT3	4.827	4.729	4.696
CASPT3(IPEA)	4.792	4.703	4.672
SC-NEVPT2	4.657	4.590	4.565
PC-NEVPT2	4.665	4.593	4.566
Beryllium, ¹ D, %T ₁ = 31%			
TBE	8.038 ^a	7.225 ^a	7.151 ^a
FCI	8.038(0)	7.225(0)	7.151(0)
CCSDTQ	8.038	7.225	7.151
CC4	8.035	7.225	7.151
CCSDT	8.038	7.225	7.152
CC3	8.041	7.234	7.158
CASPT2	8.037	7.209	7.129
CASPT2(IPEA)	8.036	7.212	7.137
CASPT3	8.043	7.214	7.139
CASPT3(IPEA)	8.042	7.215	7.142
SC-NEVPT2	8.042	7.208	7.133
PC-NEVPT2	8.042	7.208	7.133
Borole, ¹ A ₁ , %T ₁ = 81%			
TBE	6.579 ^b	6.489 ^g	6.484 ^m
FCI	6.626(112)		
CCSDTQ	6.579		
CC4	6.584	6.494	
CCSDT	6.637	6.546	6.541
CC3	6.675	6.584	6.571
CASPT2	6.201	6.022	5.979
CASPT2(IPEA)	6.782	6.668	6.650
CASPT3	6.738	6.630	6.622
CASPT3(IPEA)	6.867	6.771	6.760
SC-NEVPT2	6.628	6.506	6.484
PC-NEVPT2	6.519	6.391	6.368
Borole, ¹ A ₁ , %T ₁ = 20%			
TBE	4.711 ^b	4.702 ^g	4.708 ^o
FCI	4.730(56)		
CCSDTQ	4.711		
CC4	4.718	4.709	
CCSDT	5.069	5.098	5.211
CC3	5.412	5.460	5.517
CASPT2	4.628	4.566	4.580
CASPT2(IPEA)	4.819	4.774	4.798
CASPT3	4.765	4.747	4.753
CASPT3(IPEA)	4.801	4.781	4.792
SC-NEVPT2	4.800	4.758	4.785
PC-NEVPT2	4.779	4.733	4.760
Butadiene, ¹ A _g , %T ₁ = 75%			
TBE	6.556 ^b	6.506 ^g	6.515 ^m
FCI	6.560(47)	6.530(46)	6.633(186)
CCSDTQ	6.556		
CC4	6.558	6.508	
CCSDT	6.632	6.589	6.598
CC3	6.731	6.678	6.671
CASPT2	6.496	6.443	6.384
CASPT2(IPEA)	6.791	6.777	6.736
CASPT3	6.672	6.654	6.627
CASPT3(IPEA)	6.753	6.748	6.718
SC-NEVPT2	6.830	6.818	6.780
PC-NEVPT2	6.754	6.738	6.700
Carbon Dimer, ¹ Δ _g , %T ₁ = 0%			
TBE	2.292 ^a	2.213 ^a	2.091 ^a
FCI	2.292(0)	2.213(0)	2.091(0)

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	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
CCSDTQPH	2.292		
CCSDTQP	2.292	2.214	2.091
CCSDTQ	2.316	2.240	2.127
CC4	2.413	2.341	2.209
CCSDT	2.694	2.632	2.567
CC3	3.100	3.107	3.053
CASPT2	2.457	2.418	2.277
CASPT2(IPEA)	2.434	2.404	2.271
CASPT3	2.308	2.242	2.096
CASPT3(IPEA)	2.322	2.267	2.126
SC-NEVPT2	2.347	2.281	2.144
PC-NEVPT2	2.330	2.255	2.119
Carbon Dimer, $^1\Sigma_g^+$, %T ₁ = 0%			
TBE	2.509 ^a	2.503 ^a	2.420 ^a
FCI	2.509(1)	2.503(1)	2.420(0)
CCSDTQPH	2.510		
CCSDTQP	2.511	2.505	2.423
CCSDTQ	2.524	2.521	2.447
CC4	2.598	2.602	2.515
CCSDT	2.847	2.874	2.861
CC3	3.231	3.283	3.256
CASPT2	2.627	2.650	2.518
CASPT2(IPEA)	2.646	2.682	2.558
CASPT3	2.515	2.513	2.397
CASPT3(IPEA)	2.544	2.556	2.439
SC-NEVPT2	2.585	2.596	2.478
PC-NEVPT2	2.541	2.542	2.423
Carbon Dimer, $^3\Sigma_g^-$, %T ₁ = 0%			
TBE	1.394 ^a	1.315 ^a	1.258 ^a
FCI	1.394(1)	1.315(1)	1.258(2)
CCSDTQPH	1.394		
CCSDTQP	1.395	1.317	1.261
CCSDTQ	1.431	1.355	1.308
CCSDT	1.899	1.827	1.825
CC3	2.394	2.399	2.388
CASPT2	1.586	1.547	1.461
CASPT2(IPEA)	1.587	1.559	1.476
CASPT3	1.435	1.365	1.279
CASPT3(IPEA)	1.459	1.402	1.318
SC-NEVPT2	1.484	1.420	1.328
PC-NEVPT2	1.432	1.357	1.266
Carbon Trimer, $^1\Pi_g$, %T ₁ = 77%			
TBE	4.042 ^a	4.009 ^a	4.008 ^a
FCI	4.042(3)	4.009(3)	4.008(4)
CCSDTQPH	4.043		
CCSDTQP	4.051	4.017	4.014
CCSDTQ	4.071	4.035	4.031
CC4	4.128	4.093	4.095
CCSDT	4.264	4.224	4.204
CC3	3.779	3.699	3.654
CASPT2	3.994	3.958	3.934
CASPT2(IPEA)	4.011	3.968	3.969
CASPT3	4.044	4.014	4.009
CASPT3(IPEA)	4.087	4.077	4.063
SC-NEVPT2	4.059	4.043	4.027
PC-NEVPT2	4.059	4.043	4.027
Carbon Trimer, $^1\Delta_g$, %T ₁ = 1%			
TBE	5.263 ^a	5.222 ^a	5.230 ^a
FCI	5.263(6)	5.222(4)	5.230(17)
CCSDTQPH	5.270		
CCSDTQP	5.353	5.312	5.330
CCSDTQ	5.412	5.376	5.373
CC4	5.854	5.816	5.899
CCSDT	6.653	6.646	6.681
CC3	4.881	4.760	4.744
CASPT2	5.081	5.015	5.027
CASPT2(IPEA)	5.158	5.107	5.123
CASPT3	5.198	5.156	5.172
CASPT3(IPEA)	5.213	5.191	5.209
SC-NEVPT2	5.255	5.236	5.255
PC-NEVPT2	5.255	5.236	5.255
Carbon Trimer, $^1\Sigma_g^+$, %T ₁ = 1%			
TBE	5.924 ^a	5.897 ^a	5.908 ^a
FCI	5.924(6)	5.897(5)	5.908(11)
CCSDTQPH	5.932		
CCSDTQP	6.021	5.996	6.016
CCSDTQ	6.096	6.073	6.071
CC4	6.520	6.493	6.571
CCSDT	7.196	7.194	7.238
CC3	5.602	5.521	5.506
CASPT2	5.817	5.769	5.776
CASPT2(IPEA)			

Continued on next page

	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
CASPT3	5.860	5.813	5.826
CASPT3(IPEA)	5.919	5.884	5.896
SC-NEVPT2	5.977	5.974	5.993
PC-NEVPT2	5.966	5.969	5.986
Criegee's Intermediate, $^1A''$, %T ₁ = 80%			
TBE	2.474 ^a	2.443 ^a	2.403 ⁱ
FCI	2.474(6)	2.443(3)	2.377(57)
CCSDTQ	2.469	2.442	
CC4	2.471	2.445	2.405
CCSDT	2.471	2.446	2.408
CC3	2.460	2.426	2.384
CASPT2	2.318	2.291	2.234
CASPT2(IPEA)	2.391	2.371	2.318
CASPT3	2.424	2.391	2.339
CASPT3(IPEA)	2.428	2.400	2.348
SC-NEVPT2	2.449	2.440	2.388
PC-NEVPT2	2.501	2.488	2.440
Criegee's Intermediate, $^1A'$, %T ₁ = 80%			
TBE	3.807 ^a	3.747 ^a	3.715 ⁱ
FCI	3.807(8)	3.747(12)	3.721(54)
CCSDTQ	3.808	3.742	
CC4	3.804	3.738	3.706
CCSDT	3.829	3.765	3.737
CC3	3.831	3.760	3.725
CASPT2	3.766	3.703	3.645
CASPT2(IPEA)	3.977	3.932	3.881
CASPT3	3.918	3.863	3.825
CASPT3(IPEA)	3.963	3.914	3.870
SC-NEVPT2	3.971	3.923	3.866
PC-NEVPT2	3.856	3.801	3.746
Cyclobutadiene, 1A_g , %T ₁ = 1%			
TBE	4.073 ^b	4.046 ^g	4.036 ^o
FCI	4.084(12)	4.04(3)	4.03(9)
CCSDTQ	4.073		
CC4	4.067	4.040	
CCSDT	4.311	4.327	4.429
CC3	4.658	4.711	4.777
CASPT2	4.040	3.982	3.971
CASPT2(IPEA)	4.096	4.047	4.039
CASPT3	4.077	4.041	4.031
CASPT3(IPEA)	4.087	4.050	4.041
SC-NEVPT2	4.130	4.093	4.086
PC-NEVPT2	4.103	4.064	4.056
Cyclopentadiene, 1A_1 , %T ₁ = 79%			
TBE	6.522 ^b	6.459 ^g	6.451 ^m
FCI	6.572(46)		
CCSDTQ	6.522		
CC4	6.526	6.463	
CCSDT	6.597	6.531	6.523
CC3	6.672	6.595	6.570
CASPT2	6.383	6.303	6.249
CASPT2(IPEA)	6.755	6.721	6.686
CASPT3	6.619	6.591	6.558
CASPT3(IPEA)	6.706	6.688	6.650
SC-NEVPT2	6.899	6.878	6.848
PC-NEVPT2	6.872	6.848	6.817
Cyclopentadienethione, 1A_1 , %T ₁ = 52%			
TBE	5.366 ^c	5.363 ⁱ	5.329 ^o
FCI	5.584(490)		
CC4	5.366		
CCSDT	5.678	5.711	5.744
CC3	5.860	5.895	5.884
CASPT2	5.042	4.983	4.919
CASPT2(IPEA)	5.461	5.458	5.418
CASPT3	5.377	5.373	5.340
CASPT3(IPEA)	5.452	5.460	5.423
SC-NEVPT2	5.496	5.498	5.466
PC-NEVPT2	5.462	5.462	5.428
Cyclopentadienethione, 1B_1 , %T ₁ = 1%			
TBE	3.211 ^a	3.195 ⁱ	3.156 ^o
FCI	3.211(21)		
CC4	3.300		
CCSDT	3.842	3.929	4.036
CC3	4.266	4.349	4.399
CASPT2	3.020	2.966	2.900
CASPT2(IPEA)	3.197	3.185	3.137
CASPT3	3.182	3.166	3.127
CASPT3(IPEA)	3.222	3.216	3.178
SC-NEVPT2	3.222	3.214	3.169

Continued on next page

	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
PC-NEVPT2	3.211	3.200	3.154
Cyclopentadienethione, 1A_1 , % T_1 = 3%			
TBE	5.780 ^c	5.615 ^j	5.555 ^p
FCI	5.901(213)		
CC4	5.780		
CCSDT	6.376	6.318	6.365
CC3	6.694	6.627	6.622
CASPT2	5.414	5.156	4.995
CASPT2(IPEA)	5.879	5.747	5.679
CASPT3	6.110	5.966	5.942
CASPT3(IPEA)	6.182	6.057	6.032
SC-NEVPT2	5.601	5.451	5.397
PC-NEVPT2	5.513	5.348	5.288
Cyclopentadienethione, 3B_1 , % T_1 = 4%			
TBE	3.170 ^a	3.152 ⁱ	3.115 ^o
FCI	3.170(26)		
CCSDT	3.815	3.900	
CC3	4.254	4.349	4.385
CASPT2	2.984	2.929	2.867
CASPT2(IPEA)	3.163	3.149	3.105
CASPT3	3.141	3.124	3.087
CASPT3(IPEA)	3.183	3.176	3.140
SC-NEVPT2	3.189	3.179	3.136
PC-NEVPT2	3.178	3.165	3.121
Cyclopentadienone, 1A_1 , % T_1 = 50%			
TBE	6.850 ^c	6.798 ^j	6.714 ^p
FCI	7.005(183)		
CC4	6.850		
CCSDT	7.066	6.957	6.967
CC3	7.202	7.118	7.098
CASPT2	6.477	6.350	6.244
CASPT2(IPEA)	7.323	7.253	7.172
CASPT3	7.557	7.479	7.468
CASPT3(IPEA)	7.642	7.572	7.548
SC-NEVPT2	7.004	6.953	6.872
PC-NEVPT2	6.797	6.746	6.662
Cyclopentadienone, 1B_1 , % T_1 = 3%			
TBE	5.103 ^a	5.038 ⁱ	5.009 ^o
FCI	5.103(22)		
CC4	5.098		
CCSDT	5.665	5.689	5.824
CC3	6.057	6.072	6.125
CASPT2	4.777	4.691	4.649
CASPT2(IPEA)	5.006	4.944	4.914
CASPT3	5.102	5.037	5.008
CASPT3(IPEA)	5.107	5.047	5.018
SC-NEVPT2	5.111	5.062	5.041
PC-NEVPT2	5.102	5.046	5.025
Cyclopentadienone, 1A_1 , % T_1 = 74%			
TBE	5.826 ^c	5.819 ⁱ	5.795 ^o
FCI	5.750(348)		
CC4	5.826		
CCSDT	6.119	6.113	6.126
CC3	6.257	6.231	6.205
CASPT2	5.720	5.664	5.612
CASPT2(IPEA)	6.044	6.019	5.983
CASPT3	5.940	5.933	5.910
CASPT3(IPEA)	6.011	6.007	5.979
SC-NEVPT2	6.103	6.088	6.057
PC-NEVPT2	6.071	6.053	6.022
Cyclopentadienone, 3B_1 , % T_1 = 10%			
TBE	4.910 ^a	4.844 ⁱ	4.821 ^o
FCI	4.910(39)		
CCSDT	5.553	5.574	
CC3	5.979	5.990	6.046
CASPT2	4.669	4.579	4.542
CASPT2(IPEA)	4.905	4.839	4.814
CASPT3	4.986	4.920	4.897
CASPT3(IPEA)	4.997	4.936	4.913
SC-NEVPT2	5.002	4.948	4.933
PC-NEVPT2	4.988	4.929	4.913
Diazete, 1A_1 , % T_1 = 1%			
TBE	6.726 ^b	6.635 ^g	6.605 ^o
FCI	6.727(47)	6.651(76)	6.655(115)
CCSDTQ	6.726		
CC4	6.722	6.631	
CCSDT	7.097	7.080	7.192
CC3	7.465	7.455	7.509
CASPT2	6.729	6.623	6.584

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	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
CASPT2(IPEA)	6.824	6.731	6.699
CASPT3	6.787	6.702	6.672
CASPT3(IPEA)	6.796	6.712	6.684
SC-NEVPT2	6.846	6.765	6.732
PC-NEVPT2	6.776	6.689	6.649
Ethylene, 1A_g , % T_1 = 20%			
TBE	13.387 ^a	13.068 ^a	12.899 ^k
FCI	13.387(3)	13.068(10)	12.898(29)
CCSDTQP	13.386		
CCSDTQ	13.390	13.068	12.899
CC4	13.391	13.076	12.906
CCSDT	13.497	13.204	13.080
CC3	13.820	13.569	13.420
CASPT2	13.418	13.163	13.093
CASPT2(IPEA)	13.474	13.230	13.163
CASPT3	13.475	13.230	13.177
CASPT3(IPEA)	13.499	13.257	13.205
SC-NEVPT2	13.570	13.334	13.262
PC-NEVPT2	14.353	13.418	13.109
Formaldehyde, 1A_1 , % T_1 = 5%			
TBE	10.859 ^a	10.422 ^a	10.426 ^a
FCI	10.859(1)	10.422(34)	10.426(12)
CCSDTQP	10.860		
CCSDTQ	10.872	10.435	10.428
CC4	10.884	10.552	10.434
CCSDT	11.102	10.784	10.785
CC3	11.492	11.219	11.200
CASPT2	10.923	10.500	10.400
CASPT2(IPEA)	10.948	10.522	10.420
CASPT3	11.182	10.788	10.718
CASPT3(IPEA)	11.175	10.782	10.713
SC-NEVPT2	10.865	10.398	10.296
PC-NEVPT2	10.840	10.370	10.265
Glyoxal, 1A_g , % T_1 = 1%			
TBE	5.638 ^a	5.532 ^g	5.502 ^o
FCI	5.638(39)	5.513(75)	
CCSDTQ	5.670		
CC4	5.699	5.593	
CCSDT	6.243	6.222	6.353
CC3	6.735	6.706	6.763
CASPT2	5.406	5.270	5.211
CASPT2(IPEA)	5.544	5.421	5.372
CASPT3	5.689	5.580	5.549
CASPT3(IPEA)	5.672	5.564	5.535
SC-NEVPT2	5.678	5.581	5.546
PC-NEVPT2	5.660	5.557	5.518
Hexatriene, 1A_g , % T_1 = 65%			
TBE	5.471 ^c	5.457 ⁱ	5.435 ^o
CC4	5.471		
CCSDT	5.617	5.627	5.652
CC3	5.759	5.755	5.747
CASPT2	5.277	5.222	5.171
CASPT2(IPEA)	5.617	5.603	5.571
CASPT3	5.508	5.494	5.472
CASPT3(IPEA)	5.586	5.582	5.556
SC-NEVPT2	5.702	5.695	5.667
PC-NEVPT2	5.673	5.664	5.636
Naphthalene, 1A_g , % T_1 = 72%			
TBE	6.814 ^e	6.767 ^e	6.748 ^e
CCSDT	6.993	6.935	
CC3	6.987	6.909	6.868
CASPT2	6.201	6.101	5.942
CASPT2(IPEA)	6.888	6.852	6.793
CASPT3	6.814	6.767	6.748
CASPT3(IPEA)	6.887	6.858	6.809
SC-NEVPT2	7.038	7.011	6.958
PC-NEVPT2	6.985	6.953	6.900
Nitrosomethane, $^1A'$, % T_1 = 3%			
TBE	4.861 ^a	4.816 ^a	4.732 ^l
FCI	4.861(2)	4.816(12)	4.764(34)
CCSDTQ	4.895	4.848	
CC4	4.926	4.878	4.794
CCSDT	5.260	5.258	5.293
CC3	5.729	5.749	5.757
CASPT2	4.908	4.862	4.775
CASPT2(IPEA)	4.918	4.873	4.787
CASPT3	4.859	4.820	4.742
CASPT3(IPEA)	4.866	4.825	4.748
SC-NEVPT2	4.938	4.901	4.815

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	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
PC-NEVPT2	4.923	4.881	4.794
Nitrous Acid, $^1A'$, % T_1 = 50%			
TBE	8.185 ^b	8.081 ^b	7.970 ^l
FCI	8.170(40)	8.052(69)	7.999(131)
CCSDTQ	8.185	8.081	
CC4	8.198	8.104	7.993
CCSDT	8.500	8.500	8.524
CC3	8.915	9.174	9.105
CASPT2	8.097	8.089	7.970
CASPT2(IPEA)	8.089	8.084	7.967
CASPT3	8.064	8.052	7.945
CASPT3(IPEA)	8.059	8.047	7.939
SC-NEVPT2	8.106	8.109	7.992
PC-NEVPT2	8.126	8.121	8.001
Nitroxyl, $^1A'$, % T_1 = 0%			
TBE	4.511 ^a	4.397 ^a	4.333 ^a
FCI	4.511(1)	4.397(1)	4.333(1)
CCSDTQP	4.513	4.399	
CCSDTQ	4.535	4.424	4.364
CC4	4.562	4.454	4.380
CCSDT	4.819	4.756	4.785
CC3	5.278	5.247	5.257
CASPT2	4.549	4.447	4.358
CASPT2(IPEA)	4.554	4.454	4.366
CASPT3	4.527	4.428	4.361
CASPT3(IPEA)	4.530	4.432	4.364
SC-NEVPT2	4.585	4.485	4.398
PC-NEVPT2	4.563	4.457	4.371
Octatetraene, 1A_g , % T_1 = 64%			
TBE	4.709 ^e	4.694 ^e	4.680 ^e
CCSDT	4.907	4.929	
CC3	5.048	5.048	5.042
CASPT2	4.424	4.375	4.326
CASPT2(IPEA)	4.780	4.771	4.741
CASPT3	4.709	4.694	4.680
CASPT3(IPEA)	4.775	4.772	4.751
SC-NEVPT2	4.843	4.840	4.814
PC-NEVPT2	4.816	4.810	4.783
Oxalyl Fluoride, 1A_1 , % T_1 = 2%			
TBE	9.056 ^e	8.976 ^e	8.923 ^e
CC4	9.206	9.126	
CCSDT	9.861	9.874	10.028
CC3	10.257	10.252	10.305
CASPT2	8.813	8.718	8.643
CASPT2(IPEA)	8.963	8.878	8.814
CASPT3	9.056	8.976	8.923
CASPT3(IPEA)	9.029	8.949	8.899
SC-NEVPT2	9.075	9.009	8.958
PC-NEVPT2	9.070	8.997	8.942
Pyrazine, 1A_g , % T_1 = 71%			
TBE	8.582 ^c	8.560 ^c	8.480 ^o
FCI	8.656(107)		
CC4	8.582	8.560	
CCSDT	8.858	8.775	8.693
CC3	8.879	8.771	8.697
CASPT2	8.419	8.311	8.208
CASPT2(IPEA)	8.884	8.824	8.746
CASPT3	8.754	8.704	8.624
CASPT3(IPEA)	8.800	8.759	8.676
SC-NEVPT2	9.163	9.118	9.051
PC-NEVPT2	9.119	9.066	8.999
Pyrazine, 1A_g , % T_1 = 12%			
TBE	8.049 ^c	7.986 ^c	7.904 ^o
CC4	8.049	7.986	
CCSDT	8.794	8.697	8.813
CC3	9.274	9.172	9.168
CASPT2	7.842	7.687	7.584
CASPT2(IPEA)	8.074	7.938	7.845
CASPT3	8.147	8.031	7.949
CASPT3(IPEA)	8.145	8.028	7.945
SC-NEVPT2	8.275	8.151	8.067
PC-NEVPT2	8.251	8.120	8.037
Tetrazine, 1A_g , % T_1 = 1%			
TBE	5.035 ^e	4.991 ^e	4.951 ^e
FCI	5.029(73)		
CC4	5.063	4.970	
CCSDT	5.856	5.858	5.958
CC3	6.222	6.218	6.209
CASPT2	4.497	4.428	4.318

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	Basis Set		
	6-31+G*	aug-cc-pVDZ	aug-cc-pVTZ
CASPT2(IPEA)	4.724	4.666	4.566
CASPT3	5.035	4.991	4.951
CASPT3(IPEA)	5.009	4.963	4.917
SC-NEVPT2	4.823	4.778	4.688
PC-NEVPT2	4.749	4.699	4.608
Tetrazine, $^1B_{3g}$, % T_1 = 1%			
TBE	6.282 ^e	6.250 ^e	6.215 ^e
FCI	6.227(178)		
CC4	6.308	6.246	
CCSDT	7.300	7.302	7.434
CC3	7.640	7.618	7.617
CASPT2	5.437	5.340	5.218
CASPT2(IPEA)	5.997	5.945	5.853
CASPT3	6.282	6.250	6.215
CASPT3(IPEA)	6.279	6.247	6.201
SC-NEVPT2	6.299	6.273	6.200
PC-NEVPT2	6.255	6.223	6.149
Tetrazine, $^3B_{3g}$, % T_1 = 6%			
TBE	5.919 ^e	5.866 ^e	5.848 ^e
FCI	5.914(131)		
CCSDT	6.944	6.931	
CC3	7.355	7.331	7.347
CASPT2	5.079	4.968	4.861
CASPT2(IPEA)	5.542	5.471	5.388
CASPT3	5.919	5.866	5.848
CASPT3(IPEA)	5.909	5.857	5.829
SC-NEVPT2	5.688	5.642	5.570
PC-NEVPT2	5.632	5.579	5.506

^aexFCI.^bCCSDTQ.^cCC4.^dCCSDT.^eCASPT3.^fPC-NEVPT2.^gTBE/Pop + CC4/AVDZ - CC4/Pop.^hTBE/Pop + CCSDT/AVDZ - CCSDT/Pop.ⁱTBE/Pop + CASPT3/AVDZ - CASPT3/Pop.^jTBE/Pop + PC-NEVPT2/AVDZ - PC-NEVPT2/Pop.^kTBE/AVDZ + CCSDTQ/AVTZ - CCSDTQ/AVDZ.^lTBE/AVDZ + CC4/AVTZ - CC4/AVDZ.^mTBE/AVDZ + CCSDT/AVTZ - CCSDT/AVDZ.ⁿTBE/AVDZ + CC3/AVTZ - CC3/AVDZ.^oTBE/AVDZ + CASPT3/AVTZ - CASPT3/AVDZ.^pTBE/AVDZ + PC-NEVPT2/AVTZ - PC-NEVPT2/AVDZ.

A. Beryllium, Carbon Dimer, and Carbon Trimer

Because it is a very small, four-electron system, the 1D ($2s, 2s \rightarrow 2p, 2p$) genuine doubly-excited states of beryllium is accurately described by all methods considered here. For this reason, beryllium is discarded in the evaluation of the global statistics discussed in Sec. V. Our safe TBE of 7.151 eV matches the previous one,¹⁴² and is consistent with the 7.059 eV value obtained from explicitly correlated calculations,¹⁷⁶ which remains the most accurate theoretical estimate for this transition.

The two genuine doubly-excited states of carbon dimer (C_2), $^1\Delta_g$ and $^1\Sigma_g^+$, both of ($\pi, \pi \rightarrow \sigma^*, \sigma^*$) character, are theoretically very challenging for a 12-electron system. From the well-converged CIPSI calculations, we report safe TBE values of 2.091 eV ($^1\Delta_g$) and 2.420 eV ($^1\Sigma_g^+$). While they match the previous values,¹⁴² here we reach a level of accuracy of 0.001 eV. The carbon dimer has an additional $^3\Sigma_g^-$ ($\pi, \pi \rightarrow \sigma^*, \sigma^*$) genuine doubly-excited state, not addressed before in the QUEST database.¹⁴² This genuine double excitation has the lowest excitation energy from the present set. Our CIPSI/AVTZ calculations

TABLE II. Present TBEs (in eV), the composite method used to compute them, the symmetry and doubly-excited character of the state (P stands for partial and G for genuine), whether they are considered to be safe (uncertainty less than 0.043 eV), and their comparison to previous TBEs, ordered by the number of valence electrons. States with blank entries in the previous and difference columns represent new additions to the QUEST database. For the sake of conciseness, the 6-31+G*, aug-cc-pVDZ, and aug-cc-pVTZ basis sets are labeled P, D, and T, respectively.

No. e^-	Molecule	State	P/G	Safe	New	Prev.	Diff.	Method
4	Beryllium	1D	P	Y	7.151	7.15 ¹⁴²	+0.001	exFCI/T
8	Carbon dimer	$^1\Delta_g$	G	Y	2.091	2.09 ¹⁴²	+0.001	exFCI/T
		$^1\Sigma_g^+$	G	Y	2.420	2.42 ¹⁴²	0.000	exFCI/T
		$^3\Sigma_g^-$	G	Y	1.258			exFCI/T
		$^1A'$	G	Y	4.333	4.33 ¹⁴²	+0.003	exFCI/T
12	Nitroxyl	1A_1	G	Y	10.426	10.35 ¹⁴²	+0.076	exFCI/T
12	Formaldehyde	1A_g	G	Y	12.899	12.92 ¹⁴²	-0.021	exFCI/T
12	Carbon trimer	$^1\Pi_g$	P	Y	4.008			exFCI/T
		$^1\Delta_g$	G	Y	5.230	5.22 ¹⁴²	+0.010	exFCI/T
		$^1\Sigma_g^+$	G	Y	5.908	5.91 ¹⁴²	-0.002	exFCI/T
		$^1A''$	P	Y	2.403			exFCI/D + CC4/T - CC4/D
18	Criegee's intermediate	$^1A'$	P	Y	3.715			exFCI/D + CC4/T - CC4/D
		$^1A'$	G	Y	4.732	4.76 ¹⁴²	-0.028	exFCI/D + CASPT3/T - CASPT3/D
18	Nitrosomethane	1A_1	G	N	7.970			CCSDTQ/D + CC4/T - CC4/D
18	Nitrous acid	1A_1	G	Y	6.605			CCSDTQ/P + CC4/D - CC4/P + CASPT3/D - CASPT3/P
20	Diazete	1A_g	G	Y	4.036	4.038 ¹⁷⁵	-0.002	CCSDTQ/P + CC4/D - CC4/P + CASPT3/T - CASPT3/D
20	Cyclobutadiene	1A_g	P	Y	6.515	6.50 ¹⁴²	+0.015	CCSDTQ/P + CC4/D - CC4/P + CCSDT/T - CCSDT/D
22	Butadiene	$^1A'$	P	Y	7.928	7.929 ¹²⁰	-0.001	CCSDTQ/P + CC4/D - CC4/P + CCSDT/T - CCSDT/D
22	Acrolein	1A_g	G	N	5.502	5.61 ¹⁴²	-0.108	exFCI/P + CC4/D - CC4/P + CASPT3/T - CASPT3/D
24	Borole	1A_1	G	N	4.708			exFCI/P + CASPT3/T - CASPT3/P
		1A_1	P	Y	6.484			CCSDTQ/P + CC4/D - CC4/P + CCSDT/T - CCSDT/D
26	Cyclopentadiene	1A_1	P	Y	6.451	6.452 ¹²⁰	-0.001	CCSDTQ/P + CC4/D - CC4/P + CCSDT/T - CCSDT/P
30	Benzene	$^1E_{2g}$	P	N	8.190	8.28 ¹⁴²	-0.090	CC4/D + CCSDT/T - CCSDT/D
		$^1A_{1g}$	G	N	10.315	10.55 ¹⁴²	-0.235	CASPT3/T
30	Cyclopentadienethione	1B_1	G	N	3.156	3.154 ¹⁴²	+0.002	exFCI/P + CASPT3/T - CASPT3/P
		1A_1	P	N	5.329	5.428 ¹⁴²	+0.127	CC4/P + CASPT3/T - CASPT3/P
		1A_1	G	N	5.555			CC4/P + PC-NEVPT2/T - PC-NEVPT2/P
		3B_1	G	N	3.115	3.121 ¹⁴²	-0.006	exFCI/P + CASPT3/T - CASPT3/P
30	Cyclopentadienone	1B_1	G	N	5.009	5.025 ¹⁴²	-0.016	exFCI/P + CASPT3/T - CASPT3/P
		1A_1	G	N	5.795	6.087 ¹⁴²	-0.292	CC4/P + CASPT3/T - CASPT3/P
		1A_1	P	N	6.714	6.662 ¹⁴²	-0.052	CC4/P + PC-NEVPT2/T - PC-NEVPT2/P
		3B_1	G	N	4.821	4.913 ¹⁴²	-0.092	exFCI/P + CASPT3/T - CASPT3/P
30	Pyrazine	1A_g	G	N	7.904	8.037 ¹⁴²	-0.133	CC4/D + CASPT3/T - CASPT3/D
		1A_g	P	N	8.480	8.697 ¹⁴²	-0.217	CC4/D + CASPT3/T - CASPT3/D
30	Tetrazine	1A_g	G	N	4.951	4.608 ¹⁴²	+0.343	CASPT3/T
		$^1B_{3g}$	G	N	6.215	6.149 ¹⁴²	+0.066	CASPT3/T
		$^3B_{3g}$	G	N	5.848	5.506 ¹⁴²	+0.342	CASPT3/T
32	Hexatriene	1A_g	P	N	5.435	5.459 ¹²⁰	-0.024	CC4/P + CASPT3/T - CASPT3/P
34	Oxalyl fluoride	1A_g	G	N	8.923			CASPT3/T
40	Benzoquinone	1A_g	G	N	4.566	4.566 ¹⁴²	0.000	PC-NEVPT2/T
		1A_g	P	N	6.351			CC4/P + CCSDT/D - CCSDT/P + CC3/T - CC3/D
		$^1B_{3u}$	P	Y	5.656	5.796 ¹⁴²	-0.140	CC4/P + CCSDT/D - CCSDT/P + CC3/T - CC3/D
		$^1B_{2g}$	P	Y	5.764	5.970 ¹⁴²	-0.206	CC4/P + CCSDT/D - CCSDT/P + CC3/T - CC3/D
		1A_u	P	Y	6.083	6.346 ¹⁴²	-0.263	CC4/P + CCSDT/D - CCSDT/P + CC3/T - CC3/D
		$^1B_{1g}$	P	N	6.469			CC4/P + CCSDT/D - CCSDT/P + CC3/T - CC3/D
42	Octatetraene	1A_g	P	N	4.680	4.901 ¹⁴²	-0.221	CASPT3/T
48	Naphthalene	1A_g	P	N	6.748	6.874 ¹⁴²	-0.126	CASPT3/T

provide a safe TBE of 1.258 eV, with an uncertainty of 0.002 eV only.

For the three states of C_2 , we are able to include up to sextuple excitation in CC (CCSDTQPH) in the 6-31+G* basis, which yields results within the tiny (0.001 eV) error bars of the exFCI results with the same basis set. In turn, CCSDTQP overestimates the TBEs minimally, by 0.001 eV to 0.002 eV, whereas CCSDTQ is chemically accurate but has larger errors since it overestimates the TBEs by 0.025 eV on average.

By interpolating the potential energy curves obtained with SHCI in the cc-pV5Z basis from Ref. 123 to the bond length employed here, we obtain 2.075 eV ($^1\Delta_g$), 2.412 eV ($^1\Sigma_g^+$), and 1.266 eV ($^3\Sigma_g^-$), which deviate from our TBEs

by less than 0.02 eV.

The carbon trimer (C_3) is another system with two theoretically challenging genuine doubly-excited states, of $^1\Delta_g$ and $^1\Sigma_g^+$ symmetries, and of $(n, n \rightarrow \pi^*, \pi^*)$ character. (As first mentioned in Ref. 10, these two transitions were wrongly assigned as $(\pi, \pi \rightarrow \sigma^*, \sigma^*)$ in Ref. 88.) We report safe TBEs of 5.230 eV ($^1\Delta_g$) and 5.908 eV ($^1\Sigma_g^+$), matching the previous estimates.¹⁴² They are obtained from CIPSI calculations with associated extrapolation errors between 0.004 eV and 0.017 eV. Among all the states considered here, these two doubly-excited states of C_3 are arguably the most challenging for the higher-order CC methods. CC4 and CCSDTQ respectively overestimate

the TBEs by 0.16 eV and 0.10 eV on average, and are thus not chemically accurate for these states. One has to go up to CCSDTQP to approach the exFCI values, which remain overestimated by around 0.01 eV.

We address another state of carbon trimer, a $^1\Pi_g$ ($n \rightarrow \pi^*$) partial doubly-excited state, also not considered in Ref. 88. Our CIPSI/AVTZ calculations directly yield a safe TBE of 4.008 eV. Since the doubly-excited character is not dominant, CC methods converge much faster than seen for genuine doubles. We are not aware of previous works for this state to compare with.

B. Nitroxyl and Nitrosomethane

Nitroxyl has a low-lying $^1A'$ ($n, n \rightarrow \pi^*, \pi^*$) genuine doubly-excited state. We report a safe TBE of 4.333 eV, also matching the previous one of 4.33 eV,¹⁴² but here given with an uncertainty associated with the exFCI extrapolation error, of 0.001 eV only. With the AVDZ basis set, CCSDTQP overestimates exFCI by 0.002 eV, similar to what was observed with the smaller 6-31+G* basis.⁸⁸ Our TBE (4.333 eV) agrees perfectly with available diffusion Monte Carlo (DMC) results [4.32(1) eV].¹⁵⁴

An analogous $^1A'$ ($n, n \rightarrow \pi^*, \pi^*$) genuine doubly-excited state is present in nitrosomethane. The previous TBE of 4.76 eV, obtained from CIPSI calculations, had an error bar of 0.04 eV, making it barely safe.⁸⁸ In the present CIPSI calculation, we are able to considerably reduce the extrapolation error for both the 6-31+G* (from 0.01 eV to 0.002 eV) and AVDZ (from 0.02 eV to 0.012 eV) basis sets, though not as much for the AVTZ basis (from 0.04 eV to 0.034 eV). The present TBE is based on the updated exFCI/AVDZ estimate and a CC4 basis set correction. Our new value, of 4.732 eV, appears 0.028 eV below the previous estimate.⁸⁸ Because the combined extrapolation error of exFCI and the basis set correction error remains below chemical accuracy, the present TBE of nitrosomethane is considered safe.

C. Ethylene and Formaldehyde

A genuine doubly-excited state can be found at relatively high energy in ethylene. The previously reported TBE for this 1A_g ($\pi, \pi \rightarrow \pi^*, \pi^*$) state was 12.92 eV.⁸⁸ It was obtained from CIPSI calculations, which was assigned as safe despite the relatively large extrapolation error of 0.06 eV. Here, we update the TBE to 12.899 eV, thanks to new CIPSI calculations in the AVDZ basis (with a small extrapolation error of 0.01 eV) and CCSDTQ basis set correction. It is also consistent with our exFCI/AVTZ result, of 12.898(29) eV. The new TBE is considered safe and appears below the previous estimate by a mere 0.02 eV.¹⁴²

Formaldehyde also has a genuine double excitation, of 1A_1 symmetry, and ($n, n \rightarrow \pi^*, \pi^*$) character. The previous TBE, of 10.35 eV, was directly obtained from exFCI

results.⁸⁸ Here, we performed new CIPSI calculations and were able to reproduce this value for the fourth state of the 1A_1 symmetry, which, however, is a clear singly-excited state. We find that the doubly-excited state corresponds instead to the fifth state, slightly higher in energy, at 10.425 eV. The TBE is therefore updated to 10.425 eV, which has an associated uncertainty of 0.019 eV and can thus be considered safe. The previous misassignment of the correct state explains the large shift of 0.08 eV from the previous to the present TBE.

The doubly-excited state of ethylene displays the largest basis set effects from the present set of systems. This is not too surprising as the present TBE of 12.899 eV is greater than the vertical ionization threshold, reported between 10.75 eV and 10.95 eV.^{177–179} The state is therefore a resonance, but its interaction with the ionization continuum is neglected in our calculations. It is also worth mentioning that increasing the basis set from 6-31+G* to AVDZ has a large effect on the doubly-excited state of formaldehyde. Despite being close to, our TBE (10.425 eV) appears below the vertical ionization energy, estimated experimentally at 10.9 eV,¹⁸⁰ and obtained with exFCI calculations in the same basis and geometry as ours at 10.90 eV.¹⁷⁴ For the multiconfigurational methods, the largest basis set errors are found for ethylene and formaldehyde, as further discussed in Sec. V.

D. Butadiene, Glyoxal, and Acrolein

Butadiene has a well-known 1A_g ($\pi \rightarrow \pi^*$) state with a significant doubly-excited character. The former TBE of 6.50 eV was based on the exFCI/AVDZ result and a CCSDT basis set correction. Here, while we adopt the same AVTZ basis set correction, we change the protocol for the first term. Indeed, we rely on the CCSDTQ/Pop result combined with a AVDZ basis set correction obtained at the CC4 level, which should have a significantly smaller uncertainty than the extrapolation error of exFCI/AVDZ. The present TBE of 6.515 eV is considered safe, and blueshifts the previous one by 0.015 eV only.¹⁴²

Glyoxal has a genuine 1A_g doubly-excited state of ($n, n \rightarrow \pi^*, \pi^*$) character, which is relatively low-lying in energy. The previous exFCI/Pop and exFCI/AVDZ, of 5.60 eV and 5.48 eV, had a reported error bar not higher than 0.01 eV,⁸⁸ which was probably underestimated. The latter value, supplemented with a CCSDT basis set correction, yielded a TBE of 5.61 eV, which was considered to be safe due to the small extrapolation errors.¹⁴² We performed new CIPSI calculations with the 6-31+G* and AVDZ basis sets and found considerably larger extrapolation errors, of around 0.04 eV and 0.07 eV, respectively. To establish a new TBE, we employ the exFCI/Pop result, followed by a AVDZ basis set correction with CC4, and then by a AVTZ basis set correction with CASPT3. The latter basis set correction is indeed more sound with CASPT3 than with CCSDT, as the CCSDT would suggest an upshift of the transition energy. As discussed

in more detail in Sec. V, CCSDT (as well as CC3) are not reliable methods to correct for basis set effects in the case of genuine doubly-excited states. Our new protocol yields 5.502 eV for the TBE of glyoxal, which is unsafe because of the significant extrapolation error of exFCI/Pop (0.04 eV). The revised TBE significantly redshifts (-0.11 eV) the previous estimate of 5.61 eV,¹⁴² mostly reflecting our more reliable basis set correction. It also underestimates (-0.128 eV) the available DMC result of 5.63(1) eV.¹⁵⁴

The $^1A'$ ($\pi \rightarrow \pi^*$) state of acrolein has a partial doubly-excited character. The first TBE reported for this state, of 7.87 eV, was obtained from exFCI/Pop results and a CC3 basis set correction.^{88,142} It was later revised to 7.929 eV, from CC4/AVDZ and CCSDT/AVTZ basis set corrections.¹²⁰ Here, we were able to perform CCSDTQ/Pop calculations, which yield an excitation energy only 0.001 eV below the CC4 result in the same basis. Using the same protocol for the basis set corrections, the new TBE is essentially unchanged at 7.928 eV, and is considered to be safe.

E. Cyclopentadiene, Benzene, and Hexatriene

The ($\pi \rightarrow \pi^*$) 1A_1 partial doubly-excited state ($\%T_1 = 79\%$) of cyclopentadiene was not considered in Ref. 88, but was later addressed and extensively discussed in Refs. 89 and 120. In the former contribution,⁸⁹ a TBE of 6.523 eV was reported, based on CCSDT/AVTZ calculations. The latter publication¹²⁰ updated it to a safe value of 6.452 eV, based on the CC4/Pop result and CCSDT basis set corrections, a significant reduction of 0.07 eV with respect to the former. Here, we were able to perform both CCSDTQ/Pop and CC4/AVDZ calculations. Combining these results with the CCSDT basis set correction, the TBE of cyclopentadiene moves negligibly, to 6.451 eV. This is a safe estimate.

Benzene has a ($\pi \rightarrow \pi^*$) $^1E_{2g}$ partial doubly-excited state ($\%T_1 = 73\%$). The previous TBE, of 8.28 eV, was determined using the exFCI/Pop result and a CC3 basis set correction.^{88,142} It was considered to be safe because of the small extrapolation error of exFCI. Here, we performed new CIPSI calculations but could not locate this state with a sufficiently small uncertainty, which makes us believe that the previous extrapolation error was optimistically too small.⁸⁸ The present CC4/AVDZ calculations and CCSDT basis set correction allow us to revise the TBE to 8.190 eV, which is below the previous value by 0.09 eV. Due to the sizeable gap of 0.18 eV between CC4/AVDZ and CCSDT/AVDZ and the significant double component ($\%T_1 = 73\%$), we now assign the TBE of benzene as unsafe.

Benzene also has a ($\pi, \pi \rightarrow \pi^*, \pi^*$) $^1A_{1g}$ genuine doubly-excited state, here described with multiconfigurational methods only, since high-order CC or CIPSI calculations cannot clearly identify it. CASPT3 yields excitation energies in-between the CASTP2(IPEA) and SC-NEVPT2

results, whereas PC-NEVPT2 appears to underestimate the true value. Our unsafe TBE of 10.315 eV is thus based on CASPT3/AVTZ, which shifts the previous and also unsafe estimate,¹⁴² based on extended multistate (XMS-)CASPT2, by -0.235 eV. Similarly to ethylene, the genuine doubly-excited state of benzene lies above its vertical ionization threshold, theoretically reported at 9.448 eV.¹⁸¹

Hexatriene has a 1A_g ($\pi \rightarrow \pi^*$) excitation with a significant partial doubly-excited character ($\%T_1 = 65\%$). The first and unsafe estimate of its TBE, reported at 5.619 eV, was based on CCSDT/AVDZ results and a basis set correction at the CC3 level of theory.¹⁴² It was later revised to 5.459 eV, which was considered a safe estimate (though an upper limit), as obtained from a CC4/Pop calculation plus a CC3 basis set correction.¹²⁰ The present CCSDT results indicate a AVTZ – Pop basis set effect of a different sign than the one obtained with CC3, suggesting that neither of them is a reliable option for hexatriene. Here, instead, we employ CASPT3 for the basis set correction, on top of the CC4/Pop value, which yields a new TBE of 5.435 eV, shifted by -0.02 eV with respect to the former value.¹²⁰ Our revised TBE is also consistent with a second estimate from Ref. 120, of 5.43 eV, which was put forward based on a comparison between results of hexatriene and butadiene. Although its uncertainty is likely below 0.05 eV, we prefer to be conservative and assign the present TBE of hexatriene as unsafe.

F. Pyrazine and Tetrazine

We address two close-lying states with doubly-excited character of pyrazine, a 1A_g ($n \rightarrow \pi^*$) partial double and a 1A_g ($n, n \rightarrow \pi^*, \pi^*$) genuine double. The previous unsafe TBE (8.037 eV) for the latter was the PC-NEVPT2/AVTZ estimate.¹⁴² The present CC4 calculations yield excitation energies significantly below PC-NEVPT2, whereas CASPT3 appears closer to CC4 than PC-NEVPT2 does. For these reasons, here we rely on the CC4/AVDZ result and a CASPT3 basis set correction to provide an improved TBE for the genuine double of pyrazine at 7.904 eV. The new value displaces the previous estimate by -0.13 eV, yet remains unsafe. We notice that the impact of increasing the basis set from 6-31+G* to AVDZ differs substantially with CC4 (-0.06 eV) and multiconfigurational methods (-0.13 eV to -0.12 eV). In addition, the partial and genuine doubly-excited states are strongly mixed at some levels of CC, which could cause unusual basis set effects. The present TBE is therefore subject to significant uncertainty.

The previously reported TBE for the 1A_g ($n \rightarrow \pi^*$) partial doubly-excited state of pyrazine stemmed from CC3/AVTZ calculations.¹⁴² The value of 8.697 eV was labeled unsafe because of its significant partial doubly-excited character ($\%T_1 = 71\%$). Here, we rely on CC4/AVDZ calculations and basis set correction from

CASPT3 to improve the TBE to 8.480 eV, lying, as expected, significantly below (-0.22 eV) the previous estimate. While likely significantly more accurate than the previous TBE, we still assign it as being unsafe. Had we used CCSDT for the basis set correction, or instead a CASPT3 calculation with no Rydberg orbitals in the active space, the TBE would change very little, by only 0.002 eV and 0.006 eV, respectively, hinting that the error associated with the basis set correction is probably small.

Tetrazine has three genuine ($n, n \rightarrow \pi^*, \pi^*$) doubly-excited states. The lowest-lying 1A_g state is formed by exciting two electrons from the same non-bonding orbital into the same π^* orbital, whereas in the $^3B_{3g}$ and $^1B_{3g}$ states, each electron is promoted to a different π^* orbital. The previous TBEs, of 4.608 eV (1A_g), 5.506 eV ($^3B_{3g}$), and 6.149 eV ($^1B_{3g}$), were obtained from PC-NEVPT2 calculations,⁸⁹ and were estimated to have an error bar of ± 0.1 eV (thus unsafe).¹²⁰

Starting with the 1A_g state in the 6-31+G* basis, we find consistent excitation energies among CASPT3 (5.03 eV), CC4 (5.06 eV), and exFCI [5.03(7) eV], whereas PC-NEVPT2 provides a significantly lower value (4.75 eV), close to the CASPT2 one (4.72 eV). Although the 0.07 eV extrapolation error of the present exFCI is not small enough to define a safe TBE, it allows us to confidently rule out PC-NEVPT2 and SC-NEVPT2 as safe choices. Similar trends are observed with the AVDZ basis set, even though we do not have an exFCI estimate in this basis. From the overall consistency among the CASPT3, CC4, and exFCI, and since we have results for the three bases with CASPT3, this is the method of choice to provide the TBE for the 1A_g state. The same reasoning holds for both the $^3B_{3g}$ and $^1B_{3g}$ doubly-excited states, and we therefore also rely on CASPT3 to provide the TBEs for these states. For the $^1B_{3g}$ state, we notice that PC-NEVPT2 performs better than for the other states, although still underestimating the excitation energy.

The new TBEs, of 4.951 eV (1A_g), 5.848 eV ($^3B_{3g}$), and 6.215 eV ($^1B_{3g}$), are certainly more reliable than the previous values, which change substantially, by 0.34 eV, 0.34 eV, and 0.07 eV, respectively. Although the revised TBEs of tetrazine are still considered to be unsafe, the overall consistency between the various methodologies indicate that the errors are small, probably of the order of 0.1 eV or less. We also underline that the present TBE for the 1A_g doubly-excited state, of 4.951 eV, is reasonably close to the available DMC value of 4.99(1) eV.¹⁵⁴

IV. DOUBLE EXCITATIONS: EXTENSION

A. Criegee's Intermediate

Here, we address the first two singlet excited states of the simplest Criegee's intermediate (CH_2OO), both having a partial doubly-excited character ($\%T_1 = 80\%$). For the lowest-lying $^1A'$ ($n \rightarrow \pi^*$) state, we report a safe TBE of 2.403 eV, based on the CIPSI/AVDZ result and a

CC4 basis set correction. In comparison to the most recent theoretical data, our TBE is substantially lower than MRCI/AVTZ (2.70 eV)¹⁸² and closer to CASPT2/AVDZ (2.43 eV)¹⁸³ results. The latter agrees well with our own CASPT2/AVDZ value of 2.371 eV.

The second singlet state, $^1A'$ ($\pi \rightarrow \pi^*$), is optically bright and has received significant attention due to its involvement in atmospheric chemistry.^{182,184–186} Previous calculations find this state at 3.89 eV (with MRCI/AVTZ)¹⁸² and at 3.74 eV (with CASPT2/AVDZ).¹⁸³ Our own CASPT2/AVDZ result yields 3.932 eV. The most accurate available calculation was performed by Dawes et al.,¹⁸⁶ who reported an excitation energy of 3.751 eV obtained at the MRCI-F12/cc-pVTZ level of theory.

Here, we employ CIPSI/AVDZ and a CC4 basis set correction to provide a safe TBE of 3.715 eV for the $^1A'$ partial double of this Criegee's intermediate. Our value deviates only by -0.036 eV from the result of Ref. 186. The present TBE of 3.715 eV also fits the measured photoabsorption spectra,^{184,185} which reported the maximum of the absorption band at 335 nm (3.70 eV)¹⁸⁴ and at 340 nm (3.65 eV),¹⁸⁵ though we recall that such comparisons between vertical transition energies and measured peak maxima always come with significant approximations.

B. Nitrous acid

Despite its small size, nitrous acid is a challenging system that deserves a dedicated discussion. We are interested in its $^1A'$ ($n, n \rightarrow \pi^*, \pi^*$) genuine doubly-excited state, which appears as the third and/or fourth state in the $^1A'$ symmetry, and with a more genuine or partial character, depending on the level of theory and basis set.

In the 6-31+G* basis set, there is a strong mixing between the ($n, n \rightarrow \pi^*, \pi^*$) doubly-excited and a ($n \rightarrow 3s$) Rydberg singly-excited configurations, giving rise to two close-lying adiabatic states (third and fourth in the $^1A'$ symmetry). This is observed in our well-converged CIPSI calculations, and at the CCSDTQ and multiconfigurational levels alike, and is thus taken as the qualitatively correct picture for this basis. In this sense, there is no single adiabatic state with a well-defined genuine doubly-excited state, but rather two states with a doubly-excited configuration, combined in-phase or out-of-phase with a Rydberg configuration. If the 3s Rydberg orbital is not included in the active space of multiconfigurational calculations, a single genuine double excitation is described. Similarly, by downgrading to CC4 and CCSDT, the two configurations are no longer strongly mixed, and one can find a genuine doubly-excited state and a Rydberg singly-excited state. Surprisingly, the mixing returns at the CC3 level, but for the wrong reason, given the opposite trend observed in CCSDT and CC4. In fact, at the CC3 level, the doubly-excited configuration mixes with a different, higher-lying configuration.

Clearly, the close proximity and strong coupling between the two states prove to be very challenging for CC methods. Once the CCSDTQ level is reached, the excitation energies (7.979 eV and 8.185 eV) are in very good agreement with our CIPSI results [7.979(40) eV and 8.170(41) eV]. The singly- and doubly-excited configurations are important for both states, which are combined in-phase for the lower-lying state, and out-of-phase for the higher-lying one. The mixing is weak at the CASSCF level but becomes pronounced after the perturbative treatment.

Augmenting the basis sets to AVDZ or to AVTZ, the $(n \rightarrow 3s)$ Rydberg configuration is largely stabilized in comparison to the $(n, n \rightarrow \pi^*, \pi^*)$ one, which suppresses the strong mixing observed in the small basis. Indeed, 6-31+G* is too compact to properly model the Rydberg configuration, which appears too high in energy and artificially mixes with the doubly-excited configuration. In the AVDZ and AVTZ bases, the fourth $^1A'$ state can be unambiguously assigned as the genuine double excitation across all methods, despite the relatively high value of $\%T_1$ (50%) obtained at the CC3 level. The multiconfigurational methods are overall consistent among themselves, with the CASPT3 energy virtually matching the CCSDTQ one in the AVDZ basis, while the NEVPT2 values are overestimated by around 0.1 eV. In the AVDZ basis, CCSDTQ yields an excitation energy (8.081 eV) within the CIPSI error bar [8.052(69) eV]. Here, we take the CCSDTQ/AVDZ result and a CC4 basis set correction to obtain an unsafe TBE of 7.970 eV. The same value would be obtained with a CASPT3 basis set correction. We notice that this is a substantial basis set effect, which decreases the excitation energies by around 0.11 eV.

To the very best of our knowledge, there is no available experimental or theoretical data on the doubly-excited state of nitrous acid.

C. Cyclobutadiene and Diazete

The $^1A_g (\pi, \pi \rightarrow \pi^*, \pi^*)$ genuine doubly-excited state of cyclobutadiene has been extensively discussed in Ref. 175. There, the authors provide a TBE of 4.038 eV. Here, we employ CASPT3 results to provide the AVTZ basis set correction, instead of PC-NEVPT2 used in Ref. 175. This change is motivated by the very close agreement between CASPT3 and the TBE/Pop and TBE/AVDZ values. The updated and safe TBE shifts minimally (-0.002 eV), to 4.036 eV.

The diamond-shaped diazete has a 1A_1 genuine doubly-excited state of $(\pi, \pi \rightarrow \pi^*, \pi^*)$ character, analogous to the one of cyclobutadiene. To the best of our knowledge, neither this nor other excited states have been previously investigated using high-level wave function approaches. We employ the same composite method as for cyclobutadiene to provide a TBE of 6.605 eV for the genuine double excitation of diazete. We believe this is probably a safe estimate due to the small energy gap between CC4/Pop and CCSDTQ/Pop, of 0.004 eV, the fact that CCSDTQ/Pop

yields an excitation energy (6.726 eV) in excellent agreement with CIPSI, at 6.727(47) eV, and the overall parallel between diazete and the safe case of cyclobutadiene.

D. Borole

We address two excited states of borole with significant double contributions, a lower-lying genuine $^1A_1 (\pi, \pi \rightarrow \pi^*, \pi^*)$ state, and a higher-lying $^1A_1 (\pi \rightarrow \pi^*)$ state with partial double character.

For the latter, we provide a TBE of 6.484 eV. This is based on the CCSDTQ/Pop result and basis set corrections done at the CC4/AVDZ and CCSDT/AVTZ levels of theory. The CC series rapidly converges in the 6-31+G* basis, while basis set effects are consistent across various methods. Further taking into account the relatively high value of $\%T_1$ (81%), we conclude that our TBE is safe for this partial double excitation of borole.

Similarly, based on CCSDTQ/Pop with additional CC4 and CASPT3 basis set corrections for AVDZ and AVTZ, respectively, we provide a TBE of 4.708 eV for the genuine doubly-excited state. This estimate is assigned as unsafe, though, as we could not reduce the CIPSI/Pop incertitude below 0.056 eV.

We have not found any previous studies regarding the excited states of borole, except for a recent work of our groups that evaluated state-specific CC methods to describe its genuine double excitation.¹⁸⁷

E. Cyclopentadienone and Cyclopentadienethione

Cyclopentadienone has four low-lying excited states with relevant double characters. Here, we investigate the $^1A_1 (\pi \rightarrow \pi^*)$ partial double and the $^1A_1 (\pi, \pi \rightarrow \pi^*, \pi^*)$ genuine double, in addition to the $^1B_1 (n, \pi \rightarrow \pi^*, \pi^*)$ and $^3B_1 (n, \pi \rightarrow \pi^*, \pi^*)$ genuine doubles.

Starting with the latter two, we provide TBEs based on CIPSI/Pop and CASPT3 basis set corrections. The present values, 5.009 eV (1B_1) and 4.821 eV (3B_1), change by -0.016 eV (1B_1) and -0.092 eV (3B_1) with respect to the previous unsafe estimate, which relied on PC-NEVPT2/AVTZ.¹⁴² Although more accurate, the present TBEs are still deemed unsafe, because of the sizeable CIPSI extrapolation errors (0.02 eV and 0.04 eV) and the additional CASPT3 basis set error, which averages at 0.015 eV for the present set (see the SI). These are borderline cases though, and the present errors are probably below 0.1 eV.

Due to their strong mixing, the two 1A_1 excited states of cyclopentadienone are perhaps the most challenging systems from the present set. As discussed in Ref. 154 and also confirmed by our calculations, both $(\pi, \pi \rightarrow \pi^*, \pi^*)$ doubly-excited and $(\pi \rightarrow \pi^*)$ singly-excited configurations are important for the two states, with a dominance of the doubly-excited one in the lower-lying state and of the singly-excited one in the higher-lying one. It is thus

reasonable to label the first as the genuine double and the second as the partial double, and an analysis of the CC4 data supports such an assignment. This is in contrast to what the CC3 $\%T_1$ values suggest, based on a lower value ($\%T_1 = 50\%$) for the partial double and a higher one ($\%T_1 = 74\%$) for the genuine double.

For the 1A_1 genuine, we rely on our CC4/Pop result and CASPT3 basis set corrections to obtain a new yet still unsafe TBE of 5.795 eV, which significantly changes (-0.292 eV) the previous unsafe estimate from PC-NEVPT2/AVTZ.¹⁴² Our value (5.795 eV) is somewhat below the available DMC result [5.90(1) eV],¹⁵⁴ deviating from it by -0.105 eV.

To revise the TBE for the partial double, we also employ the CC4/Pop result, but in combination with a PC-NEVPT2 basis set correction, as CASPT3 yields too high excitation energies for this state. Our unsafe value of 6.714 eV is redshifted (-0.052 eV) with respect to the previous and also unsafe value, which was based on CCSDT/AVDZ and a CC3 basis set correction.¹⁴² This brings our TBE further lower in energy in comparison to the DMC result [6.89(1) eV],¹⁵⁴ by -0.176 eV.

Cyclopentadienethione also presents four doubly-excited states analogous to those of cyclopentadienone. For the 1B_1 ($n, \pi \rightarrow \pi^*, \pi^*$) and 3B_1 ($n, \pi \rightarrow \pi^*, \pi^*$) genuine doubly-excited states, we provide TBEs of 3.156 eV and 3.115 eV, respectively, based on the exFCI/Pop results and CASPT3 basis set corrections. In spite of the small exFCI extrapolation errors, around 0.02 eV and 0.03 eV, we prefer to be conservative here and label these TBEs as unsafe. It is worth noticing the overall consistency among CASPT3, CASPT2(IPEA), and the two variants of NEVPT2 for these two transitions.

We further address two 1A_1 states of cyclopentadienethione with strongly mixed ($\pi \rightarrow \pi^*$) and ($\pi, \pi \rightarrow \pi^*, \pi^*$) characters. The singly-excited configuration dominates in the lower-lying state (the partial double), whereas the doubly-excited configuration is more pronounced in the higher-lying one (the genuine double). This is the opposite of what we find for cyclopentadienone. Here, we update the TBE for the partial double to 5.329 eV, thus changing the previous value by -0.099 eV.¹⁴² For that, we use CC4/Pop and a CASPT3 basis set correction, while PC-NEVPT2/AVTZ was used in Ref. 142.

The higher-lying 1A_1 genuine double has not yet been considered in the QUEST database. Excluding the less reliable CASPT2 result, the other multiconfigurational methods display excitation energies spanning a wide range of approximately 0.7 eV. The CC4/Pop calculation yields 5.780 eV, which is expected to be slightly overestimating the true value for this basis set. For this genuine double, we employ CC4/Pop and the PC-NEVPT2 basis set correction to provide a TBE of 5.555 eV.

F. Oxalyl fluoride

Oxalyl fluoride is structurally similar to glyoxal, with hydrogen atoms replaced by fluorines. In close analogy to glyoxal, oxalyl fluoride has a 1A_g ($n, n \rightarrow \pi^*, \pi^*$) genuine doubly-excited state, though much higher in energy due to the inductive effect of the halogen atoms.

All our multiconfigurational calculations yield comparable excitation energies, with the exception of CASPT2(IPEA), too low by 0.1 eV, and CASPT2 which is not very reliable as expected. CASPT3, CASPT3(IPEA), PC-NEVPT2, and SC-NEVPT2 results differ among themselves by no more than 0.046 eV, 0.060 eV, and 0.035 eV, in the 6-31+G*, AVDZ, and AVTZ basis sets. Meanwhile, CC4 yields a value higher than the multiconfigurational methods by 0.15 eV on average, in both 6-31+G* and AVDZ basis. This is a typical error of CC4 for genuine doubles, albeit in the higher range of the expected error bar, as discussed in Sec. V. We thus rely on CASPT3 to provide the TBE, which yields energies slightly above CASPT3(IPEA) and slightly below the two NEVPT2 variants. CASPT3/AVTZ calculation provides a TBE of 8.923 eV for the genuine doubly-excited state of oxalyl fluoride. In the absence of more definitive results, we prefer to assign this TBE as unsafe, even though the overall consistency among the different methodologies suggests a reasonable estimate.

Besides our recent investigation that targeted this genuine double excitation with state-specific CC methods,¹⁸⁷ we are not aware of other studies for this compound.

G. Benzoquinone, Octatetraene, and Naphthalene

The excited states of benzoquinone have been addressed before in the QUEST database,¹⁴² but they have not been extensively discussed. Here, we revisit six states having a relevant doubly-excited component: the genuine 1A_g ($n, n \rightarrow \pi^*, \pi^*$) state, the partial 1A_g ($\pi \rightarrow \pi^*$), and the partial $^1B_{3u}$, $^1B_{2g}$, 1A_u , and $^1B_{1g}$ states, the latter four having a dominant ($n \rightarrow \pi^*$) character.

For the 1A_g genuine double, our CC4/Pop result is slightly below CASPT3 and above PC-NEVPT2, which suggests the latter method is probably closer to the true value. The unsafe TBE reported in Ref. 142, obtained from PC-NEVPT2/AVTZ calculations, thus remains unchanged, at 4.566 eV. For the 1A_g ($\pi \rightarrow \pi^*$) and the $^1B_{1g}$ ($n \rightarrow \pi^*$) states presenting a partial double character, we adopt CC4/Pop, a AVDZ basis set correction with CCSDT, and a AVTZ basis set correction with CC3, to yield TBEs of 6.351 eV (1A_g) and 6.469 eV ($^1B_{1g}$). Due to their relatively small $\%T_1$ (64% and 70%), these TBEs are unsafe. For the three remaining partial doubles, the previous TBEs were based on CCSDT/AVDZ and a CC3 basis set correction.¹⁴² Here, we revise them with CC4/Pop results plus a AVDZ basis set correction with CCSDT and a AVTZ basis set correction with CC3, i.e., we use the same approach for all partial doubly-excited states. This proto-

col yields 5.656 eV ($^1B_{3u}$), 5.764 eV ($^1B_{2g}$), and 6.083 eV (1A_u), all considered to be safe, given the relatively large $\%T_1$ (80%, 76%, and 75%, respectively). They are significantly below the previous estimates,¹⁴² by -0.14 eV, -0.21 eV, and -0.26 eV. The shift is attributed largely to the quadruple excitations of CC4, proven to be significant even for states with significant values of $\%T_1$ (from 75% to 80%), consistently with the findings of Ref. 120.

The excited states of benzoquinone have been the subject of numerous theoretical studies.^{188–192} Our TBE for the $^1B_{3u}$ state (5.656 eV) is comparable to the reference value of 5.60 eV recommended in Ref. 190, based on CASPT2(IPEA)/TZVP calculations. Our own CASPT2(IPEA) calculations in the AVTZ basis yield 5.774 eV. Here, we do not perform an exhaustive comparison for the other states, which have been described with more approximate methods before, like CASPT2 without IPEA shift^{188,189} and TD-DFT.¹⁹² Transitions to the states of benzoquinone addressed here have not been spectroscopically identified,¹⁹² as they are optically dark or have very low oscillator strengths.^{188–192}

In parallel to butadiene and hexatriene, octatetraene also has a 1A_g ($\pi \rightarrow \pi^*$) state with partial doubly-excited character. It appears lower in energy than the analogous state of hexatriene, which in turn is lower than in butadiene. Octatetraene is the smallest polyene where the dark 1A_g state is lower lying than the bright 1B_u ($\pi \rightarrow \pi^*$) singly-excited state.^{190,193} The previous unsafe TBE for the 1A_g state, of 4.901 eV, relied on the CCSDT/Pop result and a CC3 basis set correction.¹⁴² Because of its partial double character ($\%T_1 = 64\%$), this previous estimate should be too large, appearing above the TBE for the 1B_u state, reported at 4.78 eV following the same protocol.¹⁴² Here, we opt for CASPT3/AVTZ to provide an updated TBE of 4.680 eV, substantially redshifted (-0.221 eV) with respect to the previous one. While probably more accurate, it remains an unsafe estimate. With this revised value, we obtain the correct ordering of 1A_g and 1B_u states. Our TBE is very close to the reference value of 4.66 eV chosen by Thiel and coworkers,¹⁹⁰ obtained from multireference Møller-Plesset perturbation theory and basis set extrapolation.¹⁹³

Naphthalene has a 1A_g ($\pi \rightarrow \pi^*$) state with partial double character. This is the last and the largest system in the present set. CCSDT probably overestimates the real excitation energy, whereas PC-NEVPT2 yields comparable results, and CASPT3 yields somewhat lower values. For this reason, our reported TBE of 6.748 eV results from the CASPT3/AVTZ calculation. The new value, considered unsafe, is below the previous estimate by -0.126 eV, which was based on CCSDT/Pop and CC3 basis set correction.¹⁴²

Our revised TBE is closer to the reference value reported in Ref. 190 (6.71 eV), which was based on CASPT2(IPEA) calculations in the TZVP basis set. The latter value is consistent with our own CASPT2(IPEA)/AVTZ result of 6.793 eV, the main difference being the presence of diffuse functions in our case.

For a comprehensive comparison with more approximate theoretical results on naphthalene, we refer the interested reader to Refs. 194 and 195. Here, we highlight that the inclusion of a second set of diffuse basis functions at the CC2 level of theory was found to have a significant stabilization effect (-0.21 eV) for the 1A_g excitation energy even though this is a valence state.¹⁹⁴ It is not clear at this point if such a pronounced effect also occurs at a higher level of theory, or if it is an artifact of CC2. Finally, our TBE (6.748 eV) significantly overestimates the reported value from two-photon spectroscopy measurements, at 6.05 eV.¹⁹⁶

V. STATISTICS

Once established the TBEs for the vertical excitation energy of each state and for the three basis sets, we computed their energy differences with respect to the CC and multireference results. Table III gathers the usual statistical measures, for the ensemble of states and is also separated by partial and genuine doubly-excited states. For this purpose, we take into account the TBEs for the three bases, though limited to those labeled as safe. To obtain the statistics for a given method, we do not discard the states where such a method is used to define the TBE. We do mention below, however, the effect of removing such states from the statistics, which can be relevant for CCSDTQ and CC4. The underlying distribution of errors, for both safe and unsafe cases, is presented in the SI.

In all cases, the mean absolute errors (MAEs) decrease as we move towards higher-order CC, which was expected. Similarly, accounting for higher-order excitations, crucial for properly correlating doubly-excited states, systematically decreases the mean signed errors (MSEs). They always remain positive though, reflecting the well-known bias of CC towards the ground state.

We find that partial double excitations are much better described than genuine doubles, across all levels of CC, by a factor of 5 to 8. To further investigate this strong dependence on the character of the excited state, we plot in Fig. 2 the errors with respect to the TBEs (accounting for the three basis sets) as a function of the $\%T_1$ value obtained from CC3/AVTZ. For the vast majority of states, the value of $\%T_1$ usually depends very little on the choice of basis set. It is clear that CC3 and CCSDT yield progressively smaller errors as $\%T_1$ increases, i.e., as the excited state assumes an increasingly singly-excited and less doubly-excited character. Despite the large dispersion of errors in the regime of genuine doubles (small $\%T_1$), an overall linear trend is apparent. We will come back to this point in Sec. VI. Similarly, CC4 and CCSDTQ perform better for the partial than for the genuine doubles. However, due to the fewer data points, and since these methods are often used to define the TBEs, it is not currently possible to infer how the errors evolve between the two groups of states.

Looking at how the errors behave along the CC hierar-

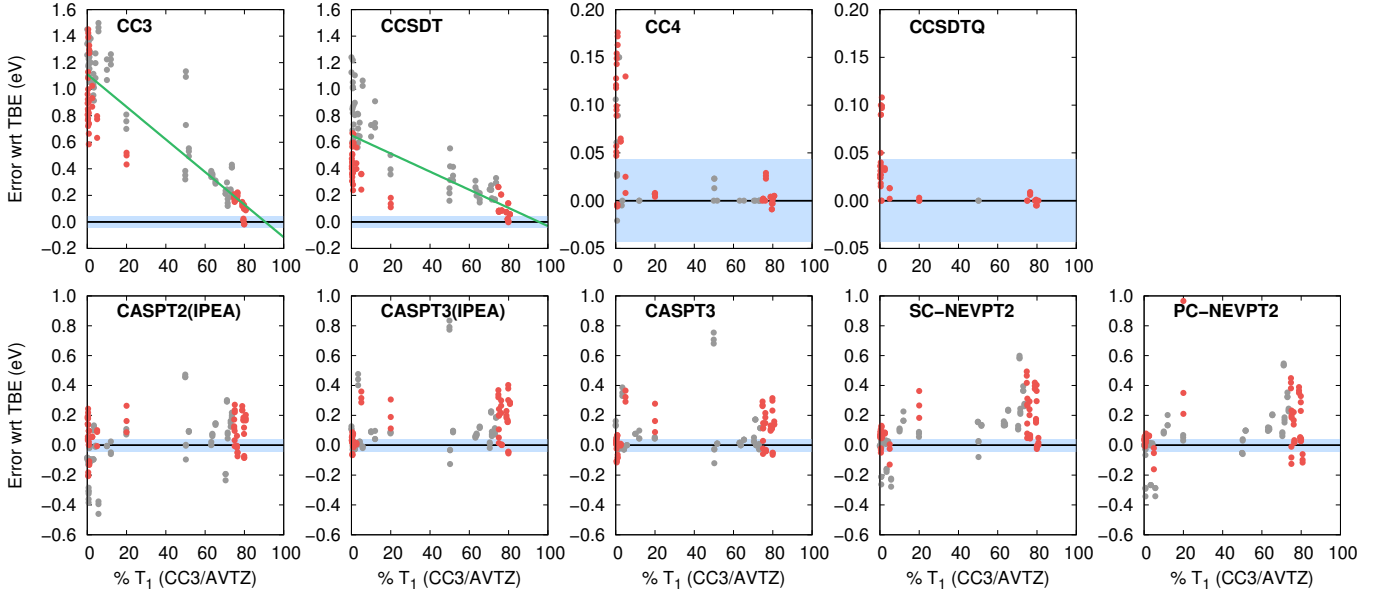


FIG. 2. Errors with respect to the TBEs, for various CC (top) and multiconfigurational (bottom) methods, as functions of the $\%T_1$ value obtained from CC3/AVTZ calculations, for both safe (red) and unsafe (gray) states. A linear fit to both safe and unsafe data points is shown as a green line, whereas chemical accuracy (0.043 eV) is represented by the blue region.

chy, starting with the partial doubles, we find a relatively small MAE of 0.13 eV at the CC3 level, which decreases in CCSDT to 0.09 eV. At the CC4 level, the errors are significantly reduced and reach chemical accuracy (MAE of only 0.01 eV). CCSDTQ yields an even lower MAE, of 0.003 eV. Excluding the partial doubles where CCSDTQ is used to generate the TBE increases its MAE to 0.006 eV, while doing the same for CC4 does not change its MAE.

Turning to the genuine doubly-excited states, CC3 only provides a qualitative description, and a large MAE of 0.90 eV is obtained. This is in sharp contrast with the very small MAE (around 0.03 eV) obtained for states having a strong single-excitation character.¹⁴² Moving to CCSDT decreases the MAE by a factor of two, down to 0.42 eV. The most significant improvement appears at the CC4 level, with a 5-fold smaller MAE of 0.08 eV. Some improvement is observed with CCSDTQ, which yields a MAE of 0.04 eV, rendering this method barely chemically accurate for genuine doubles. (Excluding the states where CCSDTQ is used to generate the TBE does not change its MAE.) For a given pair of successive methods in the CC hierarchy, we observe a similar level of improvement for both partial and genuine doubles.

As for the multiconfigurational methods, we find more consistent results between partial and double excitations, in comparison to more evident variations encountered in the CC methods. There are, however, statistically meaningful differences. As can be seen in Fig. 2, the genuine doubles are more accurately described than the partial doubles, across all the multireference methods, contrasting with the trend observed for the CC methods. The MAEs range from 0.08 eV to 0.12 eV for the genuine dou-

bles, and from 0.13 eV to 0.21 eV for the partial doubles (excluding the less accurate CASPT2 method without an IPEA shift). In addition, all multiconfigurational methods tend to overestimate the energies of partial doubly-excited states, with MSEs ranging from 0.10 eV to 0.21 eV, while for the genuine doubles the MSEs are overall closer to zero, ranging from 0.04 eV to 0.07 eV (again, excluding CASPT2 without IPEA shift).

Comparing all the multiconfigurational methods, CASPT2(IPEA) and CASPT3 are the most accurate ones for the partial doubles, with respective MAEs of 0.13 eV and 0.14 eV, comparable to that of CC3 (0.13 eV), though larger than that of CCSDT (0.09 eV). CASPT3 was also found to outperform the other multiconfigurational methods for singly-excited states.¹⁰⁰ As for the genuine doubles, we find that all multiconfigurational methods, except for CASPT2 and CASPT2(IPEA), yield surprisingly low and comparable MAEs, around 0.08 eV, the same as obtained with CC4.

Applying an IPEA shift is a common practice in CASPT2 calculations, as it tends to improve the computed excitation energies.^{99,100,197} Here, we find that the IPEA shift has a much more pronounced effect for the partial doubles, where the MAE plummets from 0.34 eV to 0.13 eV, than for the genuine doubles, where the reduction is less impressive, from 0.15 eV to 0.12 eV. Singly-excited states appear in-between, with respective MAEs of 0.27 eV and 0.11 eV according to comprehensive benchmarks reported in Refs. 99 and 100.

In contrast to CASPT2, an IPEA shift does not change the overall performance of CASPT3 in the case of genuine doubles (MAEs of 0.08 eV), and is detrimental in the case of partial doubles (MAE increases from 0.14 eV

TABLE III. Mean signed error (MSE), mean absolute error (MAE), root-mean-square error (RMSE), and standard deviation of the error (SDE), in units of eV, with respect to the TBEs, for various CC and multiconfigurational methods, including all excitations labeled as safe, and the subsets of genuine and partial doubly-excited states, while accounting for the three basis sets.

Method	#	MSE	MAE	RMSE	SDE
All safe excitations					
CCSDTQ	39	+0.03	0.03	0.04	0.03
CC4	48	+0.05	0.05	0.07	0.06
CCSDT	60	+0.27	0.27	0.34	0.20
CC3	63	+0.53	0.53	0.69	0.44
SA-CASSCF	63	+0.47	0.50	0.66	0.46
CASPT2	63	-0.17	0.24	0.32	0.26
CASPT2(IPEA)	63	+0.08	0.12	0.15	0.12
CASPT3(IPEA)	63	+0.12	0.13	0.18	0.13
CASPT3	63	+0.08	0.11	0.14	0.12
SC-NEVPT2	63	+0.14	0.15	0.20	0.14
PC-NEVPT2	63	+0.10	0.13	0.20	0.18
Genuine doubly-excited states					
CCSDTQ	28	+0.04	0.04	0.05	0.03
CC4	28	+0.08	0.08	0.10	0.06
CCSDT	33	+0.42	0.42	0.45	0.14
CC3	33	+0.90	0.90	0.94	0.26
SA-CASSCF	33	+0.43	0.49	0.68	0.52
CASPT2	33	-0.01	0.15	0.20	0.20
CASPT2(IPEA)	33	+0.06	0.12	0.14	0.12
CASPT3(IPEA)	33	+0.07	0.08	0.12	0.11
CASPT3	33	+0.04	0.08	0.12	0.12
SC-NEVPT2	33	+0.07	0.09	0.11	0.08
PC-NEVPT2	33	+0.07	0.09	0.19	0.17
Partial doubly-excited states					
CCSDTQ	11	+0.00	0.00	0.00	0.00
CC4	20	+0.00	0.01	0.01	0.01
CCSDT	27	+0.09	0.09	0.11	0.07
CC3	30	+0.13	0.13	0.15	0.07
SA-CASSCF	30	+0.51	0.51	0.63	0.37
CASPT2	30	-0.34	0.34	0.40	0.21
CASPT2(IPEA)	30	+0.10	0.13	0.15	0.11
CASPT3(IPEA)	30	+0.18	0.19	0.23	0.13
CASPT3	30	+0.11	0.14	0.16	0.11
SC-NEVPT2	30	+0.21	0.21	0.26	0.16
PC-NEVPT2	30	+0.14	0.17	0.22	0.17

to 0.19 eV). A slight worsening effect had also been observed for singly-excited states, with MAEs oscillating from 0.09 eV to 0.11 eV.¹⁰⁰ In light of this collection of results, applying an IPEA shift in CASPT3 calculations is probably not advisable. While this represents an interesting advantage of CASPT3 over CASPT2(IPEA), there is no improvement for partial doubles (the MAE oscillates from 0.13 eV to 0.14 eV), and a modest improvement for genuine doubles (the MAE decreases from 0.12 eV to 0.08 eV). For singly-excited states, going from CASPT2(IPEA) to CASPT3 also has a marginal effect on the MAE, which decreases from 0.11 eV to 0.09 eV.¹⁰⁰

As for the two NEVPT2 approaches, we find that PC-NEVPT2 is somewhat more accurate than SC-NEVPT2

for the partial doubles (MAEs of 0.17 eV and 0.21 eV, respectively), whereas both schemes are equally accurate for genuine doubles, with MAEs of 0.09 eV. A small preference in favor of PC-NEVPT2 was also pointed for singly-excited states, with respective MAEs of 0.13 eV and 0.15 eV.⁹⁹

We also gauge how the basis set effect computed with a given method compares to that obtained with our TBEs. The results for all the methods considered here, and for the three basis set differences, AVDZ – Pop, AVTZ – AVDZ, and AVTZ – Pop, are presented in the SI. The genuine doubly-excited state of ethylene has the most pronounced basis set effects, with a difference of 0.49 eV between TBE/AVTZ and TBE/Pop. For this reason, this state is excluded in the calculation of average errors regarding the basis set correction.

The errors associated with CC3 have an overall positive sign in the regime of genuine double excitations. This means that the CC3/AVDZ – CC3/Pop difference is smaller than the TBE/AVDZ – TBE/Pop reference value, such that CC3 systematically underestimates the basis set effect in the case of genuine doubles. Using CC3 to evaluate the basis set correction would introduce typical errors of around 0.08 eV for the AVDZ – Pop basis set correction, 0.05 eV for the AVTZ – AVDZ difference, and a large 0.13 eV error for the AVTZ – Pop difference. This explains some of the main discrepancies, mentioned in Sec. III and IV, regarding the present and previous TBEs for genuine doubles. The present findings clearly demonstrate that basis set corrections performed at the CC3 level introduce too large errors in the case of genuine double excitations.

This issue persists in CCSDT. Somewhat surprisingly, the CCSDT average error for the AVTZ – AVDZ difference is greater (0.08 eV) than the corresponding CC3 average error (0.05 eV). Neither CC3 nor CCSDT are therefore recommended to perform a basis set correction for genuine doubly-excited states. Conversely, CC4 and CCSDTQ can be safely employed, as they introduce errors below 0.02 eV and 0.01 eV, respectively. The genuine doubly-excited state of formaldehyde is a notable exception, though, where CC4 yields an absolute error of around 0.1 eV for the AVTZ – AVDZ and AVDZ – Pop basis set corrections. Even though this large error does not pose a problem in defining our TBEs for formaldehyde (which rely on exFCI results), it highlights that even CC4 is not always reliable for the basis set correction.

The performance of multiconfigurational methods is comparable to that of CC4, with CASPT3 delivering the smallest basis set correction errors, between 0.01 eV and 0.02 eV. Comparing different methods for the basis set corrections can thus be useful to identify the consistency of the correction or the presence of a more problematic state.

In contrast to the case of genuine doubles, the basis set correction obtained with CC methods is much more well-behaved for the partial doubly-excited states. CC3 tends to slightly underestimate the basis set effect. The

absolute errors lie within the region of chemical accuracy, averaging at 0.01 eV for the AVTZ – AVDZ and AVDZ – Pop differences and at 0.02 eV for the AVTZ – Pop difference. CCSDT and higher levels of CC are very safe for the basis set correction of partial doubles, with errors below 0.01 eV. Multiconfiguration methods have errors ranging from 0.02 eV to 0.05 eV and should therefore be avoided if CC3 results are available. We underline that the average errors for multiconfigurational methods have opposite signs in the AVDZ – Pop and AVTZ – AVDZ basis set effects, which partially cancels out in the AVTZ – Pop difference, which ends up having smaller absolute errors.

Despite being small in absolute terms, the errors stemming from the basis set correction should be considered when evaluating the TBEs. If chemical accuracy is desired, the calculation performed with the smaller basis should be more stringent than usual, depending on the typical errors associated with the basis set correction.

VI. CORRECTION BASED ON $\%T_1$

We mentioned before that a linear trend (see Fig. 2) can be observed for the errors of CC3 and CCSDT when plotted as functions of the $\%T_1$ value (computed at the CC3/AVTZ level). Based on this observation, we propose a simple additive correction to the computed excitation energies, denoted as +LT1 (which stands for linear $\%T_1$). When applied to CC3, for instance, it gives rise to the CC3+LT1 model. By fitting our full set of data points (safe and unsafe results, and the three basis sets) to the linear function $a \times \%T_1/100 + b$ (the green lines in Fig. 2), the computed excitation energy ΔE is shifted to $\Delta E(+LT1) = \Delta E - [a \times \%T_1/100 + b]$. The fitting parameters a and b are presented in the SI.

This type of correction can be useful when the errors of a given method are relatively consistent, or, in other words, if its standard deviation of error (SDE) is smaller than its MAE. This is evident for CC3, which yields systematically overestimated excitation energies for the genuine doubles, with a dispersion in energy (SDE of 0.26 eV) smaller than the overall absolute errors (MAE of 0.90 eV) by a factor of around 3.5. Similarly, such a correction can be helpful to correct the genuine doubles computed with CCSDT, which has a corresponding SDE (0.14 eV) smaller than its MAE (0.42 eV) by a factor of 3. When applied to CC3 and CCSDT, the +LT1 correction is therefore a simple attempt to recover from the lack of higher-order excitations that would correlate the genuine doubly-excited states. Because the fitting procedure accounts for both genuine and partial doubles, and the latter also has smaller SDEs than MAEs, both types of states are impacted by the correction.

In Fig. 3, we show the distribution of errors obtained with CC3, CCSDT, and their +LT1 corrected counterparts. Safe states are represented in red and unsafe states in gray. We find that the +LT1 correction significantly

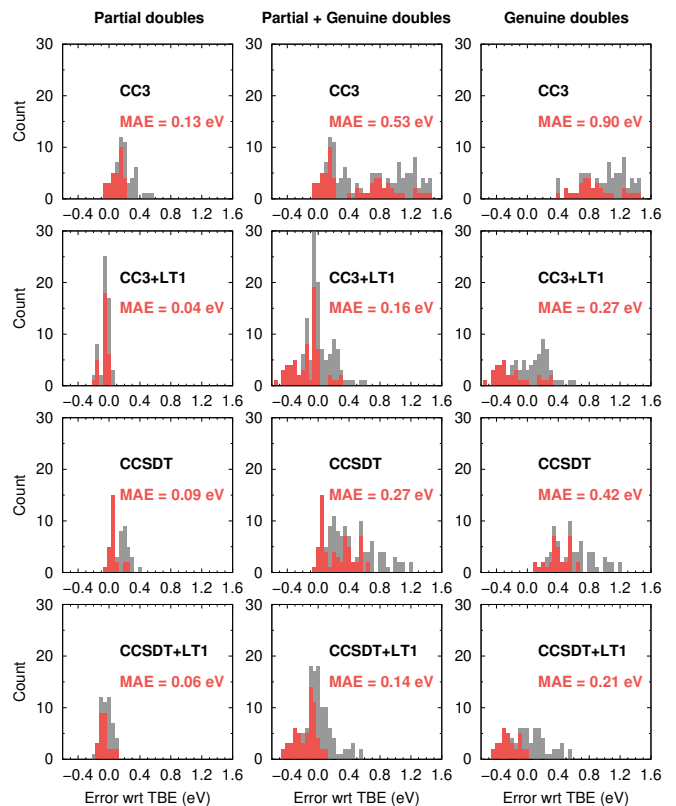


FIG. 3. Distribution of errors with respect to the TBEs (of all basis sets), computed at the CC3, CC3+LT1, CCSDT, and CCSDT+LT1 levels of theory (see text for explanation of the LT1 correction), restricted to partial (left) or genuine (right) doubles, and for both partial and doubles (center), including safe (red) and both safe and unsafe (gray) states.

reduces the MAEs of CC3, by a factor ranging between 3 and 3.5. The relative improvement is comparable for both partial and genuine doubles. From CC3 to CC3+LT1, the MAE for the partial doubles decreases from 0.13 eV to 0.04 eV, whereas, for the genuine doubles, it reduces from 0.90 eV down to 0.27 eV. While these values concern the safe states, the same conclusions also hold when accounting for the unsafe states. The observed difference on their underlying distribution of errors reflects the system size in each subset: safe states comprise mostly smaller systems (thus smaller errors), whereas unsafe states include larger systems (and larger errors).

The conclusions obtained when comparing CCSDT and CCSDT+LT1 are similar to their CC3 analogs. The main difference is that the LT1 correction reduces the errors of CCSDT by a factor ranging between 1.5 and 2, whereas in CC3, this factor is found to range between 3 and 3.5. For the partial doubles, the MAE decreases from 0.09 eV to 0.06 eV, and for the genuine doubles they shift from 0.42 eV to 0.21 eV. Interestingly, we find that CCSDT+LT1 and CC3+LT1 have comparable performances, for both partials and doubles. When accounting for all the present states, they share a MAE of 0.14 eV,

compared to 0.27 eV of CCSDT and 0.53 eV of CC3. Here, we find that CC3+LT1 also outperforms CCSDT, despite being a cheaper approach.

Because there are fewer CC4 and CCSDTQ data points, especially in the intermediate $\%T_1$ region, and considering that these methods are often used to define the TBEs, we cannot conclude that a linear trend would still be observed. Despite that, the +LT1 correction (based on the available CC4 data) brings down the overestimated energies for the genuine doubles computed with CC4, reducing its MAE from 0.08 eV to 0.05 eV. For the multiconfigurational methods, the correction can be applied to improve the energies of the partial double excitations, which appear generally overestimated (see Fig. 2), while they have virtually no effect on the genuine doubles. The overall improvements are more modest than those observed for CC3 and CCSDT. The MAE decreases from 0.14 eV in CASPT3 to 0.09 eV in CASPT3+LT1, for instance. The full set of results are reported in the SI.

We further recall that our fitting procedure accounts for safe and unsafe states. In this way, CC3+LT1 and CCSDT+LT1 tend to underestimate the excitation energies of the safe ones, while overestimating the unsafe ones. Had we included only the states whose TBEs are considered safe, their errors would certainly be smaller, at the expense of larger errors for the unsafe ones. Naturally, one could generalize this simple +LT1 correction to also account for the system size, but we did not pursue this here.

VII. CONCLUSION

We have performed a systematic investigation on the vertical excitation energies of molecular states that have strong doubly-excited character. The present set comprises 28 genuine doubles, which are dominated by a doubly-excited character, plus 19 partial doubles, where this character is not dominant but is still important. Based on CIPSI, high-order CC, and multiconfigurational calculations, TBEs for this set of 47 states are reported.

Compared to our previous benchmark study on doubly-excited states,⁸⁸ the more substantial number of states considered here (47 against 20), especially on partial doubles, in addition to the more reliable TBEs, allow us to draw solid conclusions about the performance of different methodologies. CC methods are found to be progressively more accurate as the doubly-excited character becomes less prominent, from the genuine doubles to the partial doubles and finally to the singly-excited states. Multiconfigurational methods, in contrast, are similarly accurate for genuine doubles and singly-excited states, but less accurate for the partial doubles.

To obtain chemically accurate excitation energies of genuine doubles, one needs at least CCSDTQ (MAE of 0.04 eV), whereas CC4 is already very accurate for the partial doubles (MAE of 0.01 eV). If the desired accuracy is less stringent, CC4, CASPT3, or NEVPT2 are reasonable

choices for genuine doubles, all having MAEs of around 0.08 eV. For the partial doubles, CCSDT and CC3 are surprisingly accurate (MAE of 0.09 eV and 0.13 eV, respectively). In contrast, multiconfigurational methods struggle more in these cases, with the smaller MAEs provided by CASPT2(IPEA) (0.13 eV) and CASPT3 (0.14 eV).

CC3 is the lowest-order CC method providing a qualitatively correct description of doubly-excited states. However, it yields systematically overestimated excitation energies. We have demonstrated that the $\%T_1$ value computed with CC3 can reasonably predict the method's inaccuracy in describing doubly-excited states. This motivated us to introduce a simple correction (labeled +LT1) that can be employed after a CC3 calculation, significantly improving estimates of excitation energies. Such a correction reduces the MAEs obtained with CC3 by a factor of 3, from 0.90 eV down to 0.27 eV for the genuine doubles, and from 0.13 eV down to 0.04 eV for the partial doubles, whereas we do not recommend it for the singly-excited states.

To conclude, the comprehensive and accurate set of TBEs reported here helps in assessing the performance of known excited-state methods. Likewise, we hope they will be helpful for gauging novel methodologies aiming to describe doubly-excited states.

ACKNOWLEDGMENTS

This work was performed using the HPC resources from CALMIP (Toulouse) under allocation 2024-18005, and of the GLiCiD/CCICP computational center installed in Nantes. This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant agreement No. 863481).

SUPPORTING INFORMATION

MAEs for the excitation energies, MAEs for the basis set effect, errors of the basis set effects as functions of $\%T_1$, distribution of errors on the excitation energies, specific orbitals involved in each electronic transition, additional details about the multiconfigurational calculations, raw data on the excitation energies, and additional statistical results.

REFERENCES

- ¹D. Abramavicius, D. V. Voronine, and S. Mukamel, "Double-quantum resonances and exciton-scattering in coherent 2d spectroscopy of photosynthetic complexes," *Proc. Natl. Acad. Sci. U.S.A.* **105**, 8525–8530 (2008).
- ²V. I. Klimov, A. A. Mikhailovsky, S. Xu, A. Malko, J. A. Hollingsworth, C. A. Leatherdale, H.-J. Eisler, and M. G. Bawendi, "Optical gain and stimulated emission in nanocrystal quantum dots," *Science* **290**, 314–317 (2000).

- ³K. W. Stone, D. B. Turner, K. Gundogdu, S. T. Cundiff, and K. A. Nelson, "Exciton-exciton correlations revealed by two-quantum, two-dimensional fourier transform optical spectroscopy," *Acc. Chem. Res.* **42**, 1452–1461 (2009).
- ⁴S. Biswas, J. Kim, X. Zhang, and G. D. Scholes, "Coherent two-dimensional and broadband electronic spectroscopies," *Chem. Rev.* **122**, 4257–4321 (2022).
- ⁵R. C. Miller, D. A. Kleinman, A. C. Gossard, and O. Munteanu, "Biexcitons in gaas quantum wells," *Phys. Rev. B* **25**, 6545–6547 (1982).
- ⁶B. F. Feuerbacher, J. Kuhl, and K. Ploog, "Biexcitonic contribution to the degenerate-four-wave-mixing signal from a $\text{gaas}/\text{al}_x\text{ga}_{1-x}$ quantum well," *Phys. Rev. B* **43**, 2439–2441 (1991).
- ⁷M. T. do Casal, J. M. Toldo, M. Barbatti, and F. Plasser, "Classification of doubly excited molecular electronic states," *Chem. Sci.* **14**, 4012–4026 (2023).
- ⁸Y. Shu and D. G. Truhlar, "Doubly Excited Character or Static Correlation of the Reference State in the Controversial 2^1A_g State of *trans*-Butadiene?" *J. Am. Chem. Soc.* **139**, 13770–13778 (2017).
- ⁹G. M. J. Barca, A. T. B. Gilbert, and P. M. W. Gill, "Excitation Number: Characterizing Multiply Excited States," *J. Chem. Theory Comput.* **14**, 9–13 (2018).
- ¹⁰F. Kossoski and P.-F. Loos, "State-specific configuration interaction for excited states," *J. Chem. Theory Comput.* **19**, 2258–2269 (2023).
- ¹¹L. Serrano-Andrés, M. Merchán, I. Nebot-Gil, R. Lindh, and B. O. Roos, "Towards an accurate molecular orbital theory for excited states: Ethene, butadiene, and hexatriene," *J. Chem. Phys.* **98**, 3151–3162 (1993).
- ¹²R. J. Cave and E. R. Davidson, "Theoretical investigation of several low-lying states of *trans*, *trans*-1, 3,5-hexatriene," *J. Phys. Chem.* **92**, 614–620 (1988).
- ¹³J. Lappe and R. J. Cave, "On the Vertical and Adiabatic Excitation Energies of the 2^1A_g State of *trans*-1,3-Butadiene," *J. Phys. Chem. A* **104**, 2294–2300 (2000).
- ¹⁴M. Boggio-Pasqua, M. J. Bearpark, M. Klene, and M. A. Robb, "A computational strategy for geometry optimization of ionic and covalent excited states, applied to butadiene and hexatriene," *J. Chem. Phys.* **120**, 7849–7860 (2004).
- ¹⁵N. T. Maitra, R. J. C. F. Zhang, and K. Burke, "Double excitations within time-dependent density functional theory linear response," *J. Chem. Phys.* **120**, 5932 (2004).
- ¹⁶R. J. Cave, F. Zhang, N. T. Maitra, and K. Burke, "A dressed TDDFT treatment of the 2^1A_g states of butadiene and hexatriene," *Chem. Phys. Lett.* **389**, 39–42 (2004).
- ¹⁷M. Wanko, M. Hoffmann, P. Strodel, A. Koslowski, W. Thiel, F. Neese, T. Frauenheim, and M. Elstner, "Calculating Absorption Shifts for Retinal Proteins: Computational Challenges," *J. Phys. Chem. B* **109**, 3606–3615 (2005).
- ¹⁸J. H. Starcke, M. Wormit, J. Schirmer, and A. Dreuw, "How much double excitation character do the lowest excited states of linear polyenes have?" *Chem. Phys.* **329**, 39–49 (2006).
- ¹⁹J. Catalán and J. L. G. de Paz, "On the photophysics of all-*trans* polyenes: Hexatriene versus octatetraene," *J. Chem. Phys.* **124**, 034306 (2006).
- ²⁰G. Mazur and R. Włodarczyk, "Application of the dressed time-dependent density functional theory for the excited states of linear polyenes," *J. Comput. Chem.* **30**, 811–817 (2009).
- ²¹C. Angeli, "An analysis of the dynamic σ polarization in the V state of ethene," *Int. J. Quantum Chem.*, NA–NA (2010).
- ²²G. Mazur, M. Makowski, R. Włodarczyk, and Y. Aoki, "Dressed TDDFT study of low-lying electronic excited states in selected linear polyenes and diphenylpolyenes," *Int. J. Quantum Chem.* **111**, 819–825 (2011).
- ²³M. Huix-Rotlant, A. Ipatov, A. Rubio, and M. E. Casida, "Assessment of dressed time-dependent density-functional theory for the low-lying valence states of 28 organic chromophores," *Chem. Phys.* **391**, 120–129 (2011).
- ²⁴M. B. Smith and J. Michl, "Singlet fission," *Chem. Rev.* **110**, 6891–6936 (2010).
- ²⁵M. E. Sandoval-Salinas and D. Casanova, "The doubly excited state in singlet fission," *ChemPhotoChem* **5**, 282–293 (2020).
- ²⁶A. Endo, M. Ogasawara, A. Takahashi, D. Yokoyama, Y. Kato, and C. Adachi, "Thermally activated delayed fluorescence from sn^{4+} -porphyrin complexes and their application to organic light emitting diodes - a novel mechanism for electroluminescence," *Adv. Mater.* **21**, 4802–4806 (2009).
- ²⁷H. Uoyama, K. Goushi, K. Shizu, H. Nomura, and C. Adachi, "Highly efficient organic light-emitting diodes from delayed fluorescence," *Nature* **492**, 234–238 (2012).
- ²⁸P. de Silva, "Inverted singlet-triplet gaps and their relevance to thermally activated delayed fluorescence," *J. Phys. Chem. Lett.* **10**, 5674–5679 (2019).
- ²⁹J. Sanz-Rodrigo, G. Ricci, Y. Olivier, and J. C. Sancho-García, "Negative singlet-triplet excitation energy gap in triangle-shaped molecular emitters for efficient triplet harvesting," *J. Phys. Chem. A* **125**, 513–522 (2021).
- ³⁰G. Ricci, E. San-Fabián, Y. Olivier, and J. C. Sancho-García, "Singlet-triplet excited-state inversion in heptazine and related molecules: Assessment of td-dft and ab initio methods," *ChemPhysChem* **22**, 553–560 (2021).
- ³¹Y. Olivier, B. Yurash, L. Muccioli, G. D'Avino, O. Mikhnenko, J. C. Sancho-García, C. Adachi, T.-Q. Nguyen, and D. Beljonne, "Nature of the singlet and triplet excitations mediating thermally activated delayed fluorescence," *Phys. Rev. Mater.* **1**, 075602 (2017).
- ³²Y. Olivier, J.-C. Sancho-García, L. Muccioli, G. D'Avino, and D. Beljonne, "Computational design of thermally activated delayed fluorescence materials: The challenges ahead," *J. Chem. Theory Comput.* **9**, 6149–6163 (2018).
- ³³J. C. Sancho-García, E. Brémond, G. Ricci, A. J. Pérez-Jiménez, Y. Olivier, and C. Adamo, "Violation of hund's rule in molecules: Predicting the excited-state energy inversion by TD-DFT with double-hybrid methods," *J. Chem. Phys.* **156**, 034105 (2022).
- ³⁴K. Curtis, O. Adeyiga, O. Suleiman, and S. O. Odoh, "Building on the strengths of a double-hybrid density functional for excitation energies and inverted singlet-triplet energy gaps," *J. Chem. Phys.* **158**, 024116 (2023).
- ³⁵P.-F. Loos, F. Lipparini, and D. Jacquemin, "Heptazine, cyclazine, and related compounds: Chemically-accurate estimates of the inverted singlet-triplet gap," *J. Phys. Chem. Lett.* **14**, 11069–11075 (2023).
- ³⁶E. Runge and E. K. U. Gross, "Density-functional theory for time-dependent systems," *Phys. Rev. Lett.* **52**, 997–1000 (1984).
- ³⁷M. Petersilka, U. J. Gossmann, and E. K. U. Gross, "Excitation energies from time-dependent density-functional theory," *Phys. Rev. Lett.* **76**, 1212 (1996).
- ³⁸M. E. Casida, "Time-dependent density functional response theory for molecules," (World Scientific, Singapore, 1995) pp. 155–192.
- ³⁹C. Ullrich, *Time-Dependent Density-Functional Theory: Concepts and Applications*, Oxford Graduate Texts (Oxford University Press, New York, 2012).
- ⁴⁰B. G. Levine, C. Ko, J. Quenneville, and T. J. Martinez, "Conical intersections and double excitations in time-dependent density functional theory," *Mol. Phys.* **104**, 1039–1051 (2006).
- ⁴¹D. J. Tozer and N. C. Handy, "On the determination of excitation energies using density functional theory," *Phys. Chem. Chem. Phys.* **2**, 2117–2121 (2000).
- ⁴²P. Elliott, S. Goldson, C. Canahui, and N. T. Maitra, "Perspectives on double-excitations in TDDFT," *Chem. Phys.* **391**, 110–119 (2011).
- ⁴³P.-F. Loos and D. Jacquemin, "Chemically accurate 0-0 energies with not-so-accurate excited state geometries," *J. Chem. Theory Comput.* **15**, 2481–2491 (2019).
- ⁴⁴N. T. Maitra, "Memory: History, initial-state dependence, and double-excitations," in *Fundamentals of Time-Dependent Density Functional Theory*, Vol. 837, edited by M. A. Marques,

- N. T. Maitra, F. M. Nogueira, E. Gross, and A. Rubio (Springer Berlin Heidelberg, Berlin, Heidelberg, 2012) pp. 167–184.
- ⁴⁵E. E. Salpeter and H. A. Bethe, “A relativistic equation for bound-state problems,” *Phys. Rev.* **84**, 1232 (1951).
 - ⁴⁶G. Strinati, “Application of the Green’s functions method to the study of the optical properties of semiconductors,” *Riv. Nuovo Cimento* **11**, 1–86 (1988).
 - ⁴⁷X. Blase, I. Duchemin, and D. Jacquemin, “The bethe–salpeter equation in chemistry: relations with td-dft, applications and challenges,” *Chem. Soc. Rev.* **47**, 1022–1043 (2018).
 - ⁴⁸X. Blase, I. Duchemin, D. Jacquemin, and P.-F. Loos, “The bethe–salpeter equation formalism: From physics to chemistry,” *J. Phys. Chem. Lett.* **11**, 7371–7382 (2020).
 - ⁴⁹M. Rohlfing and S. G. Louie, “Electron-hole excitations and optical spectra from first principles,” *Phys. Rev. B* **62**, 4927–4944 (2000).
 - ⁵⁰P. Romaniello, S. Guyot, and L. Reining, “The self-energy beyond GW: Local and nonlocal vertex corrections,” *J. Chem. Phys.* **131**, 154111 (2009).
 - ⁵¹D. Sangalli, P. Romaniello, G. Onida, and A. Marini, “Double excitations in correlated systems: A many-body approach,” *J. Chem. Phys.* **134**, 034115 (2011).
 - ⁵²P.-F. Loos and X. Blase, “Dynamical correction to the bethe–salpeter equation beyond the plasmon-pole approximation,” *J. Chem. Phys.* **153**, 114120 (2020).
 - ⁵³J. Authier and P.-F. Loos, “Dynamical kernels for optical excitations,” *J. Chem. Phys.* **153**, 184105 (2020).
 - ⁵⁴S. J. Bintrim and T. C. Berkelbach, “Full-frequency dynamical bethe–salpeter equation without frequency and a study of double excitations,” *J. Chem. Phys.* **156**, 044114 (2022).
 - ⁵⁵E. Monino and P.-F. Loos, “Connections and performances of green’s function methods for charged and neutral excitations,” *J. Chem. Phys.* **159**, 034105 (2023).
 - ⁵⁶M. Head-Gordon, R. J. Rico, M. Oumi, and T. J. Lee, “A doubles correction to electronic excited states from configuration interaction in the space of single substitutions,” *Chem. Phys. Lett.* **219**, 21–29 (1994).
 - ⁵⁷M. Head-Gordon, D. Maurice, and M. Oumi, “A perturbative correction to restricted open-shell configuration-interaction with single substitutions for excited-states of radicals,” *Chem. Phys. Lett.* **246**, 114–121 (1995).
 - ⁵⁸O. Christiansen, H. Koch, and P. Jørgensen, “The second-order approximate coupled cluster singles and doubles model cc2,” *Chem. Phys. Lett.* **243**, 409–418 (1995).
 - ⁵⁹C. Hättig and F. Weigend, “Cc2 excitation energy calculations on large molecules using the resolution of the identity approximation,” *J. Chem. Phys.* **113**, 5154–5161 (2000).
 - ⁶⁰A. Trofimov and J. Schirmer, “Polarization propagator study of electronic excitation in key heterocyclic molecules i. pyrrole,” *Chem. Phys.* **214**, 153–170 (1997).
 - ⁶¹A. Dreuw and M. Wormit, “The algebraic diagrammatic construction scheme for the polarization propagator for the calculation of excited states,” *Wiley Interdiscip. Rev. Comput. Mol. Sci.* **5**, 82–95 (2015).
 - ⁶²J. Schirmer, *Many-Body Methods for Atoms, Molecules and Clusters* (Springer, 2018).
 - ⁶³G. P. Purvis III and R. J. Bartlett, “A full coupled-cluster singles and doubles model: The inclusion of disconnected triples,” *J. Chem. Phys.* **76**, 1910–1918 (1982).
 - ⁶⁴G. E. Scuseria, A. C. Scheiner, T. J. Lee, J. E. Rice, and H. F. Schaefer, “The closed-shell coupled cluster single and double excitation (CCSD) model for the description of electron correlation. A comparison with configuration interaction (CISD) results,” *J. Chem. Phys.* **86**, 2881–2890 (1987).
 - ⁶⁵H. Koch and P. Jørgensen, “Coupled cluster response functions,” *J. Chem. Phys.* **93**, 3333–3344 (1990).
 - ⁶⁶H. Koch, H. J. A. Jensen, P. Jørgensen, and T. Helgaker, “Excitation energies from the coupled cluster singles and doubles linear response function (CCSDLR). Applications to Be, CH⁺, CO, and H₂O,” *J. Chem. Phys.* **93**, 3345–3350 (1990).
 - ⁶⁷J. F. Stanton and R. J. Bartlett, “The equation of motion coupled-cluster method. a systematic biorthogonal approach to molecular excitation energies, transition probabilities, and excited state properties,” *J. Chem. Phys.* **98**, 7029–7039 (1993).
 - ⁶⁸J. F. Stanton, “Many-body methods for excited state potential energy surfaces. I. General theory of energy gradients for the equation-of-motion coupled-cluster method,” *J. Chem. Phys.* **99**, 8840–8847 (1993).
 - ⁶⁹A. B. Trofimov, G. Stelter, and J. Schirmer, “Electron excitation energies using a consistent third-order propagator approach: Comparison with full configuration interaction and coupled cluster results,” *J. Chem. Phys.* **117**, 6402–6410 (2002).
 - ⁷⁰P. H. P. Harbach, M. Wormit, and A. Dreuw, “The third-order algebraic diagrammatic construction method (ADC(3)) for the polarization propagator for closed-shell molecules: Efficient implementation and benchmarking,” *J. Chem. Phys.* **141**, 064113 (2014).
 - ⁷¹P.-F. Loos, A. Scemama, and D. Jacquemin, “The quest for highly accurate excitation energies: A computational perspective,” *J. Phys. Chem. Lett.* **11**, 2374–2383 (2020).
 - ⁷²J. Leitner, A. L. Dempwolff, and A. Dreuw, “The fourth-order algebraic diagrammatic construction scheme for the polarization propagator,” *J. Chem. Phys.* **157**, 184101 (2022).
 - ⁷³M. Huix-Rotllant, B. Natarajan, A. Ipatov, C. Muhavini Wawire, T. Deutsch, and M. E. Casida, “Assessment of noncollinear spin-flip Tamm–Dancoff approximation time-dependent density-functional theory for the photochemical ring-opening of oxirane,” *Phys. Chem. Chem. Phys.* **12**, 12811 (2010).
 - ⁷⁴A. I. Krylov, “Spin-flip configuration interaction: An electronic structure model that is both variational and size-consistent,” *Chem. Phys. Lett.* **350**, 522–530 (2001).
 - ⁷⁵Y. Shao, M. Head-Gordon, and A. I. Krylov, “The spin-flip approach within time-dependent density functional theory: Theory and applications to diradicals,” *J. Chem. Phys.* **118**, 4807–4818 (2003).
 - ⁷⁶F. Wang and T. Ziegler, “Time-dependent density functional theory based on a noncollinear formulation of the exchange-correlation potential,” *J. Chem. Phys.* **121**, 12191 (2004).
 - ⁷⁷F. Wang and T. Ziegler, “Use of noncollinear exchange-correlation potentials in multiplet resolutions by time-dependent density functional theory,” *Int. J. Quantum Chem.* **106**, 2545–2550 (2006).
 - ⁷⁸N. Minezawa and M. S. Gordon, “Optimizing Conical Intersections by Spin-Flip Density Functional Theory: Application to Ethylene,” *J. Phys. Chem. A* **113**, 12749–12753 (2009).
 - ⁷⁹E. Monino and P.-F. Loos, “Spin-conserved and spin-flip optical excitations from the bethe–salpeter equation formalism,” *J. Chem. Theory Comput.* **17**, 2852–2867 (2021).
 - ⁸⁰J. Čížek, “On the Correlation Problem in Atomic and Molecular Systems. Calculation of Wavefunction Components in Ursell-Type Expansion Using Quantum-Field Theoretical Methods,” *J. Chem. Phys.* **45**, 4256–4266 (1966).
 - ⁸¹J. Čížek, “On the Use of the Cluster Expansion and the Technique of Diagrams in Calculations of Correlation Effects in Atoms and Molecules,” in *Adv. Chem. Phys.* (John Wiley & Sons, Ltd, Chichester, England, UK, 1969) pp. 35–89.
 - ⁸²J. Paldus, “Coupled Cluster Theory,” in *Methods in Computational Molecular Physics* (Springer, Boston, MA, Boston, MA, USA, 1992) pp. 99–194.
 - ⁸³T. D. Crawford and H. F. Schaefer, “An Introduction to Coupled Cluster Theory for Computational Chemists,” in *Reviews in Computational Chemistry* (John Wiley & Sons, Ltd, 2000) pp. 33–136.
 - ⁸⁴R. J. Bartlett and M. Musiał, “Coupled-cluster theory in quantum chemistry,” *Rev. Mod. Phys.* **79**, 291–352 (2007).
 - ⁸⁵I. Shavitt and R. J. Bartlett, *Many-Body Methods in Chemistry and Physics: MBPT and Coupled-Cluster Theory* (Cambridge University Press, 2009).
 - ⁸⁶M. A. Watson and G. K.-L. Chan, “Excited states of butadiene to chemical accuracy: Reconciling theory and experiment,” *J.*

- Chem. Theory Comput. **8**, 4013–4018 (2012).
- ⁸⁷P. F. Loos, A. Scemama, A. Blondel, Y. Garniron, M. Caffarel, and D. Jacquemin, “A mountaineering strategy to excited states: Highly-accurate reference energies and benchmarks,” *J. Chem. Theory Comput.* **14**, 4360 (2018).
 - ⁸⁸P.-F. Loos, M. Boggio-Pasqua, A. Scemama, M. Caffarel, and D. Jacquemin, “Reference energies for double excitations,” *J. Chem. Theory Comput.* **15**, 1939–1956 (2019).
 - ⁸⁹P. F. Loos, F. Lipparini, M. Boggio-Pasqua, A. Scemama, and D. Jacquemin, “A mountaineering strategy to excited states: Highly-accurate energies and benchmarks for medium sized molecules,” *J. Chem. Theory Comput.* **16**, 1711–1741 (2020).
 - ⁹⁰K. Andersson, P. A. Malmqvist, B. O. Roos, A. J. Sadlej, and K. Wolinski, “Second-order perturbation theory with a casscf reference function,” *J. Phys. Chem.* **94**, 5483–5488 (1990).
 - ⁹¹K. Andersson, P.-A. Malmqvist, and B. O. Roos, “Second-order perturbation theory with a complete active space self-consistent field reference function,” *J. Chem. Phys.* **96**, 1218–1226 (1992).
 - ⁹²B. O. Roos, M. Fülcher, P.-Å. Malmqvist, M. Merchán, and L. Serrano-Andrés, *Quantum Mechanical Electronic Structure Calculations with Chemical Accuracy* (Springer Netherlands, Dordrecht, 1995) pp. 357–438.
 - ⁹³S. Battaglia, I. F. Galván, and R. Lindh, “Multiconfigurational quantum chemistry: The CASPT2 method,” in *Theoretical and Computational Photochemistry* (Elsevier, 2023) pp. 135–162.
 - ⁹⁴C. Angeli, R. Cimiraglia, and J.-P. Malrieu, “N-electron valence state perturbation theory: A fast implementation of the strongly contracted variant,” *Chem. Phys. Lett.* **350**, 297–305 (2001).
 - ⁹⁵C. Angeli, R. Cimiraglia, S. Evangelisti, T. Leininger, and J.-P. Malrieu, “Introduction of n -electron valence states for multireference perturbation theory,” *J. Chem. Phys.* **114**, 10252–10264 (2001).
 - ⁹⁶C. Angeli, R. Cimiraglia, and J.-P. Malrieu, “ N -electron valence state perturbation theory: A spinless formulation and an efficient implementation of the strongly contracted and of the partially contracted variants,” *J. Chem. Phys.* **117**, 9138–9153 (2002).
 - ⁹⁷C. Angeli, B. Bories, A. Cavallini, and R. Cimiraglia, “Third-order multireference perturbation theory: The n -electron valence state perturbation-theory approach,” *J. Chem. Phys.* **124**, 054108 (2006).
 - ⁹⁸I. Schapiro, K. Sivalingam, and F. Neese, “Assessment of n -electron valence state perturbation theory for vertical excitation energies,” *J. Chem. Theory Comput.* **9**, 3567–3580 (2013).
 - ⁹⁹R. Sarkar, P.-F. Loos, M. Boggio-Pasqua, and D. Jacquemin, “Assessing the performances of CASPT2 and NEVPT2 for vertical excitation energies,” *J. Chem. Theory Comput.* **18**, 2418–2436 (2022).
 - ¹⁰⁰M. Boggio-Pasqua, D. Jacquemin, and P.-F. Loos, “Benchmarking CASPT3 vertical excitation energies,” *J. Chem. Phys.* **157**, 014103 (2022).
 - ¹⁰¹O. Christiansen, H. Koch, and P. Jørgensen, “Response functions in the cc3 iterative triple excitation model,” *J. Chem. Phys.* **103**, 7429–7441 (1995).
 - ¹⁰²H. Koch, O. Christiansen, P. Jørgensen, and J. Olsen, “Excitation energies of BH, CH₂ and Ne in full configuration interaction and the hierarchy CCS, CC2, CCSD and CC3 of coupled cluster models,” *Chem. Phys. Lett.* **244**, 75–82 (1995).
 - ¹⁰³H. Koch, O. Christiansen, P. Jørgensen, A. M. Sanchez de Merás, and T. Helgaker, “The cc3 model: An iterative coupled cluster approach including connected triples,” *J. Chem. Phys.* **106**, 1808–1818 (1997).
 - ¹⁰⁴K. Hald, P. Jørgensen, J. Olsen, and M. Jaszuński, “An analysis and implementation of a general coupled cluster approach to excitation energies with application to the B2 molecule,” *J. Chem. Phys.* **115**, 671–679 (2001).
 - ¹⁰⁵A. C. Paul, R. H. Myhre, and H. Koch, “New and Efficient Implementation of CC3,” *J. Chem. Theory Comput.* **17**, 117–126 (2021).
 - ¹⁰⁶J. Noga and R. J. Bartlett, “The full ccsdt model for molecular electronic structure,” *J. Chem. Phys.* **86**, 7041–7050 (1987).
 - ¹⁰⁷G. E. Scuseria and H. F. Schaefer, “A new implementation of the full CCSDT model for molecular electronic structure,” *Chem. Phys. Lett.* **152**, 382–386 (1988).
 - ¹⁰⁸J. D. Watts and R. J. Bartlett, “The inclusion of connected triple excitations in the equation-of-motion coupled-cluster method,” *J. Chem. Phys.* **101**, 3073–3078 (1994).
 - ¹⁰⁹S. A. Kucharski, M. Wloch, M. Musiał, and R. J. Bartlett, “Coupled-cluster theory for excited electronic states: The full equation-of-motion coupled-cluster single, double, and triple excitation method,” *J. Chem. Phys.* **115**, 8263–8266 (2001).
 - ¹¹⁰M. Kállay and J. Gauss, “Calculation of excited-state properties using general coupled-cluster and configuration-interaction models,” *J. Chem. Phys.* **121**, 9257–9269 (2004).
 - ¹¹¹M. Kállay and J. Gauss, “Approximate treatment of higher excitations in coupled-cluster theory,” *J. Chem. Phys.* **123**, 214105 (2005).
 - ¹¹²P.-F. Loos, M. Comin, X. Blase, and D. Jacquemin, “Reference energies for intramolecular charge-transfer excitations,” *J. Chem. Theory Comput.* **17**, 3666–3686 (2021).
 - ¹¹³P.-F. Loos and P. Romaniello, “Static and dynamic bethe-salpeter equations in the t-matrix approximation,” *J. Chem. Phys.* **156**, 164101 (2022).
 - ¹¹⁴S. A. Kucharski and R. J. Bartlett, “Recursive intermediate factorization and complete computational linearization of the coupled-cluster single, double, triple, and quadruple excitation equations,” *Theor. Chim. Acta* **80**, 387–405 (1991).
 - ¹¹⁵M. Kállay and P. R. Surján, “Higher excitations in coupled-cluster theory,” *J. Chem. Phys.* **115**, 2945–2954 (2001).
 - ¹¹⁶S. Hirata, “Higher-order equation-of-motion coupled-cluster methods,” *J. Chem. Phys.* **121**, 51–59 (2004).
 - ¹¹⁷M. Kállay, J. Gauss, and P. G. Szalay, “Analytic first derivatives for general coupled-cluster and configuration interaction models,” *J. Chem. Phys.* **119**, 2991–3004 (2003).
 - ¹¹⁸M. Kállay and J. Gauss, “Analytic second derivatives for general coupled-cluster and configuration-interaction models,” *J. Chem. Phys.* **120**, 6841–6848 (2004).
 - ¹¹⁹P.-F. Loos, D. A. Matthews, F. Lipparini, and D. Jacquemin, “How accurate are EOM-CC4 vertical excitation energies?” *J. Chem. Phys.* **154**, 221103 (2021).
 - ¹²⁰P.-F. Loos, F. Lipparini, D. A. Matthews, A. Blondel, and D. Jacquemin, “A mountaineering strategy to excited states: Revising reference values with EOM-CC4,” *J. Chem. Theory Comput.* **18**, 4418–4427 (2022).
 - ¹²¹E. Giner, A. Scemama, and M. Caffarel, “Using perturbatively selected configuration interaction in quantum monte carlo calculations,” *Can. J. Chem.* **91**, 879–885 (2013).
 - ¹²²E. Giner, A. Scemama, and M. Caffarel, “Fixed-node diffusion monte carlo potential energy curve of the fluorine molecule f2 using selected configuration interaction trial wavefunctions,” *J. Chem. Phys.* **142**, 044115 (2015).
 - ¹²³A. A. Holmes, C. J. Umrigar, and S. Sharma, “Excited states using semistochastic heat-bath configuration interaction,” *J. Chem. Phys.* **147**, 164111 (2017).
 - ¹²⁴B. Mussard and S. Sharma, “One-Step Treatment of Spin–Orbit Coupling and Electron Correlation in Large Active Spaces,” *J. Chem. Theory Comput.* **14**, 154–165 (2018).
 - ¹²⁵N. M. Tubman, D. S. Levine, D. Hait, M. Head-Gordon, and K. B. Whaley, “An efficient deterministic perturbation theory for selected configuration interaction methods,” *arXiv* (2018), 10.48550/arXiv.1808.02049, 1808.02049.
 - ¹²⁶A. D. Chien, A. A. Holmes, M. Otten, C. J. Umrigar, S. Sharma, and P. M. Zimmerman, “Excited States of Methylene, Polyenes, and Ozone from Heat-Bath Configuration Interaction,” *J. Phys. Chem. A* **122**, 2714–2722 (2018).
 - ¹²⁷N. M. Tubman, C. D. Freeman, D. S. Levine, D. Hait, M. Head-Gordon, and K. B. Whaley, “Modern Approaches to Exact Diagonalization and Selected Configuration Interaction with the Adaptive Sampling CI Method,” *J. Chem. Theory Comput.* **16**, 2139–2159 (2020).

- ¹²⁸P.-F. Loos, Y. Damour, and A. Scemama, "The performance of cipsi on the ground state electronic energy of benzene," *J. Chem. Phys.* **153**, 176101 (2020).
- ¹²⁹Y. Yao, E. Giner, J. Li, J. Toulouse, and C. J. Umrigar, "Almost exact energies for the Gaussian-2 set with the semistochastic heat-bath configuration interaction method," *J. Chem. Phys.* **153**, 124117 (2020).
- ¹³⁰Y. Damour, M. V  ril, F. Kossoski, M. Caffarel, D. Jacquemin, A. Scemama, and P.-F. Loos, "Accurate full configuration interaction correlation energy estimates for five- and six-membered rings," *J. Chem. Phys.* **155**, 134104 (2021).
- ¹³¹Y. Yao and C. J. Umrigar, "Orbital optimization in selected configuration interaction methods," *J. Chem. Theory Comput.* **17**, 4183–4194 (2021).
- ¹³²H. R. Larsson, H. Zhai, C. J. Umrigar, and G. K.-L. Chan, "The Chromium Dimer: Closing a Chapter of Quantum Chemistry," *J. Am. Chem. Soc.* **144**, 15932–15937 (2022).
- ¹³³J. P. Coe, A. Moreno Carrascosa, M. Simmermacher, A. Kirrander, and M. J. Paterson, "Efficient computation of two-electron reduced density matrices via selected configuration interaction," *J. Chem. Theory Comput.* **18**, 6690–6699 (2022).
- ¹³⁴B. Huron, J. P. Malrieu, and P. Rancurel, "Iterative perturbation calculations of ground and excited state energies from multi-configurational zeroth-order wavefunctions," *J. Chem. Phys.* **58**, 5745–5759 (1973).
- ¹³⁵S. Evangelisti, J.-P. Daudey, and J.-P. Malrieu, "Convergence of an improved cipsi algorithm," *Chem. Phys.* **75**, 91–102 (1983).
- ¹³⁶R. Cimiraglia, "Second order perturbation correction to CI energies by use of diagrammatic techniques: An improvement to the CIPSI algorithm," *J. Chem. Phys.* **83**, 1746–1749 (1985).
- ¹³⁷R. Cimiraglia and M. Persico, "Recent advances in multireference second order perturbation CI: The CIPSI method revisited," *J. Comput. Chem.* **8**, 39–47 (1987).
- ¹³⁸F. Illas, J. Rubio, and J. M. Ricart, "Approximate natural orbitals and the convergence of a second order multireference many-body perturbation theory (CIPSI) algorithm," *J. Chem. Phys.* **89**, 6376–6384 (1988).
- ¹³⁹A. Povill, J. Rubio, and F. Illas, "Treating large intermediate spaces in the CIPSI method through a direct selected CI algorithm," *Theor. Chem. Acc.* **82**, 229–238 (1992).
- ¹⁴⁰Y. Garniron, K. Gasperich, T. Applencourt, A. Benali, A. Fert  , J. Paquier, B. Pradines, R. Assaraf, P. Reinhardt, J. Toulouse, P. Barbaresco, N. Renon, G. David, J. P. Malrieu, M. V  ril, M. Caffarel, P. F. Loos, E. Giner, and A. Scemama, "Quantum package 2.0: a open-source determinant-driven suite of programs," *J. Chem. Theory Comput.* **15**, 3591 (2019).
- ¹⁴¹P.-F. Loos, A. Scemama, M. Boggio-Pasqua, and D. Jacquemin, "Mountaineering strategy to excited states: Highly accurate energies and benchmarks for exotic molecules and radicals," *J. Chem. Theory Comput.* **16**, 3720–3736 (2020).
- ¹⁴²M. V  ril, A. Scemama, M. Caffarel, F. Lipparini, M. Boggio-Pasqua, D. Jacquemin, and P.-F. Loos, "Questdb: A database of highly accurate excitation energies for the electronic structure community," *WIREs Comput. Mol. Sci.* **11**, e1517.
- ¹⁴³P.-F. Loos and D. Jacquemin, "A mountaineering strategy to excited states: Highly accurate energies and benchmarks for bicyclic systems," *J. Phys. Chem. A* **125**, 10174–10188 (2021).
- ¹⁴⁴Y. Garniron, A. Scemama, E. Giner, M. Caffarel, and P. F. Loos, "Selected configuration interaction dressed by perturbation," *J. Chem. Phys.* **149**, 064103 (2018).
- ¹⁴⁵E. Giner, A. Scemama, J. Toulouse, and P. F. Loos, "Chemically accurate excitation energies with small basis sets," *J. Chem. Phys.* **151**, 144118 (2019).
- ¹⁴⁶A. W. Prentice, J. P. Coe, and M. J. Paterson, "A systematic construction of configuration interaction wavefunctions in the complete CI space," *J. Chem. Phys.* **151**, 164112 (2019).
- ¹⁴⁷N. Zhang, W. Liu, and M. R. Hoffmann, "Iterative configuration interaction with selection," *J. Chem. Theory Comput.* **16**, 2296–2316 (2020).
- ¹⁴⁸J. B. Schriber and F. A. Evangelista, "Adaptive configuration interaction for computing challenging electronic excited states with tunable accuracy," *J. Chem. Theory Comput.* **13**, 5354–5366 (2017).
- ¹⁴⁹D. Hait and M. Head-Gordon, "Excited state orbital optimization via minimizing the square of the gradient: General approach and application to singly and doubly excited states via density functional theory," *J. Chem. Theory Comput.* **16**, 1699–1710 (2020).
- ¹⁵⁰D. Hait and M. Head-Gordon, "Orbital optimized density functional theory for electronic excited states," *J. Phys. Chem. Lett.* **12**, 4517–4529 (2021).
- ¹⁵¹J. Zhang, Z. Tang, X. Zhang, H. Zhu, R. Zhao, Y. Lu, and J. Gao, "Target state optimized density functional theory for electronic excited and diabatic states," *J. Chem. Theory Comput.* **19**, 1777–1789 (2023).
- ¹⁵²T. Gould, Z. Hashimi, L. Kronik, and S. G. Dale, "Single excitation energies obtained from the ensemble "homo–lumo gap": Exact results and approximations," *J. Phys. Chem. Lett.* **13**, 2452–2458 (2022).
- ¹⁵³L. Otis, I. M. Craig, and E. Neuscamman, "A hybrid approach to excited-state-specific variational monte carlo and doubly excited states," *J. Chem. Phys.* **153**, 234105 (2020).
- ¹⁵⁴S. Shepard, R. L. Panad  s-Barrueta, S. Moroni, A. Scemama, and C. Filippi, "Double excitation energies from quantum monte carlo using state-specific energy optimization," *J. Chem. Theory Comput.* **18**, 6722–6731 (2022).
- ¹⁵⁵L. Otis and E. Neuscamman, "A promising intersection of excited-state-specific methods from quantum chemistry and quantum monte carlo," *WIREs Comput. Mol. Sci.* (2023), 10.1002/wcms.1659.
- ¹⁵⁶M. Ravi, Y. choon Park, A. Perera, and R. J. Bartlett, "The intermediate state approach for doubly excited dark states in EOM-coupled-cluster theory," *J. Chem. Phys.* **156**, 201102 (2022).
- ¹⁵⁷V. Rishi, M. Ravi, A. Perera, and R. J. Bartlett, "Dark doubly excited states with modified coupled cluster models: A reliable compromise between cost and accuracy?" *J. Phys. Chem. A* **127**, 828–834 (2023).
- ¹⁵⁸D. R. Dombrowski, T. Schulz, M. Kleinschmidt, and C. M. Marian, "R2022: A DFT/MRCI ansatz with improved performance for double excitations," *J. Phys. Chem. A* **127**, 2011–2025 (2023).
- ¹⁵⁹D. S. King, M. R. Hermes, D. G. Truhlar, and L. Gagliardi, "Large-scale benchmarking of multireference vertical-excitation calculations via automated active-space selection," *J. Chem. Theory Comput.* **18**, 6065–6076 (2022).
- ¹⁶⁰M. Wang, W.-H. Fang, and C. Li, "Assessment of state-averaged driven similarity renormalization group on vertical excitation energies: Optimal flow parameters and applications to nucleobases," *J. Chem. Theory Comput.* **19**, 122–136 (2023).
- ¹⁶¹Y. Garniron, A. Scemama, P.-F. Loos, and M. Caffarel, "Hybrid stochastic-deterministic calculation of the second-order perturbative contribution of multireference perturbation theory," *J. Chem. Phys.* **147**, 034101 (2017).
- ¹⁶²H. G. A. Burton and P.-F. Loos, "Rationale for the extrapolation procedure in selected configuration interaction," *J. Chem. Phys.* **160**, 104102 (2024).
- ¹⁶³D. A. Matthews, L. Cheng, M. E. Harding, F. Lipparini, S. Stopkowicz, T.-C. Jagau, P. G. Szalay, J. Gauss, and J. F. Stanton, "Coupled-cluster techniques for computational chemistry: The CFOUR program package," *J. Chem. Phys.* **152**, 214108 (2020).
- ¹⁶⁴D. A. Matthews and J. F. Stanton, "Non-orthogonal spin-adaptation of coupled cluster methods: A new implementation of methods including quadruple excitations," *J. Chem. Phys.* **142**, 064108 (2015).
- ¹⁶⁵K. Aidas, C. Angeli, K. L. Bak, V. Bakken, R. Bast, L. Boman, O. Christiansen, R. Cimiraglia, S. Coriani, P. Dahle, E. K. Dalskov, U. Ekstr  m, T. Enevoldsen, J. J. Eriksen, P. Ettenhuber, B. Fern  ndez, L. Ferrighi, H. Fliegl, L. Frediani, K. Hald, A. Halkier, C. H  ttig, H. Heiberg, T. Helgaker, A. C. Hennum, H. Hettema, E. Hjerten  s, S. H  st, I.-M. H  yv  k, M. F. Iozzi,

- B. Jansík, H. J. A. Jensen, D. Jonsson, P. Jørgensen, J. Kauczor, S. Kirpekar, T. Kjærgaard, W. Klopper, S. Knecht, R. Kobayashi, H. Koch, J. Kongsted, A. Krapp, K. Kristensen, A. Ligabue, O. B. Lutnæs, J. I. Melo, K. V. Mikkelsen, R. H. Myhre, C. Neiss, C. B. Nielsen, P. Norman, J. Olsen, J. M. H. Olsen, A. Osted, M. J. Packer, F. Pawłowski, T. B. Pedersen, P. F. Provasi, S. Reine, Z. Rinkevicius, T. A. Ruden, K. Ruud, V. V. Rybkin, P. Salek, C. C. M. Samson, A. S. de Merás, T. Saue, S. P. A. Sauer, B. Schimmelpfennig, K. Sneskov, A. H. Steindal, K. O. Sylvester-Hvid, P. R. Taylor, A. M. Teale, E. I. Tellgren, D. P. Tew, A. J. Thorvaldsen, L. Thøgersen, O. Vahtras, M. A. Watson, D. J. D. Wilson, M. Ziolkowski, and H. Ågren, "The dalton quantum chemistry program system," *WIREs Comput. Mol. Sci.* **4**, 269–284 (2014).
- ¹⁶⁶M. Kállay, P. R. Nagy, D. Mester, Z. Rolik, G. Samu, J. Csontos, J. Csóka, P. B. Szabó, L. Gyevi-Nagy, B. Hégyi, I. Ladjanskzi, L. Szegedy, B. Ladóczki, K. Petrov, M. Farkas, P. D. Mezei, and Á. Ganyecz, "The MRCC program system: Accurate quantum chemistry from water to proteins," *J. Chem. Phys.* **152**, 074107 (2020).
- ¹⁶⁷H.-J. Werner, P. J. Knowles, F. R. Manby, J. A. Black, K. Doll, A. Heßelmann, D. Kats, A. Köhn, T. Korona, D. A. Kreplin, Q. Ma, T. F. Miller, A. Mitrushchenkov, K. A. Peterson, I. Polyak, G. Rauhut, and M. Sibaev, "The molpro quantum chemistry package," *J. Chem. Phys.* **152**, 144107 (2020).
- ¹⁶⁸H.-J. Werner, "Third-order multireference perturbation theory the caspt3 method," *Mol. Phys.* **89**, 645–661 (1996).
- ¹⁶⁹K. Andersson and B. O. Roos, "Multiconfigurational second-order perturbation theory: a test of geometries and binding energies," *Int. J. Quantum Chem.* **45**, 591–607 (1993).
- ¹⁷⁰K. Andersson, "Different forms of the zeroth-order hamiltonian in second-order perturbation theory with a complete active space self-consistent field reference function," *Theor. Chim. Acta* **91**, 31–46 (1995).
- ¹⁷¹G. Ghigo, B. O. Roos, and P.-Å. Malmqvist, "A modified definition of the zeroth-order hamiltonian in multiconfigurational perturbation theory (caspt2)," *Chem. Phys. Lett.* **396**, 142–149 (2004).
- ¹⁷²D. Jacquemin, F. Kossoski, F. Gam, M. Boggio-Pasqua, and P.-F. Loos, "Reference vertical excitation energies for transition metal compounds," *J. Chem. Theory Comput.* **19**, 8782–8800 (2023).
- ¹⁷³P.-F. Loos and D. Jacquemin, "A mountaineering strategy to excited states: Accurate vertical transition energies and benchmarks for substituted benzenes," (2024), [arXiv:2401.13809](https://arxiv.org/abs/2401.13809) [physics.chem-ph].
- ¹⁷⁴A. Marie and P.-F. Loos, "Reference energies for valence ionizations and satellite transitions," (2024), [arXiv:2402.13877](https://arxiv.org/abs/2402.13877) [physics.chem-ph].
- ¹⁷⁵E. Monino, M. Boggio-Pasqua, A. Scemama, D. Jacquemin, and P.-F. Loos, "Reference energies for cyclobutadiene: Automerization and excited states," *J. Phys. Chem. A* **126**, 4664–4679 (2022).
- ¹⁷⁶F. J. Gálvez, E. Buendía, and A. Sarsa, "Excited states of beryllium atom from explicitly correlated wave functions," *J. Chem. Phys.* **117**, 6071–6082 (2002).
- ¹⁷⁷D. Holland, D. Shaw, M. Hayes, L. Shpinkova, E. Rennie, L. Karlsson, P. Baltzer, and B. Wannberg, "A photoabsorption, photodissociation and photoelectron spectroscopy study of c2h4 and c2d4," *Chem. Phys.* **219**, 91–116 (1997).
- ¹⁷⁸E. R. Davidson and A. A. Jarzęcki, "Zero point corrections to vertical excitation energies," *Chem. Phys. Lett.* **285**, 155–159 (1998).
- ¹⁷⁹M. Musia and R. J. Bartlett, "Eom-ccsd study of the low-lying ionization potentials of ethylene, acetylene and formaldehyde," *Chem. Phys. Lett.* **384**, 210–214 (2004).
- ¹⁸⁰W. von Niessen, G. Bieri, and L. Åsbrink, "30.4-nm he (ii) photoelectron spectra of organic molecules: Part iii. oxo-compounds (c, h, o)," *J. Electron Spectros. Relat. Phenomena* **21**, 175–191 (1980).
- ¹⁸¹M. S. Deleuze, L. Claes, E. S. Kryachko, and J.-P. François, "Benchmark theoretical study of the ionization threshold of benzene and oligoacenes," *J. Chem. Phys.* **119**, 3106–3119 (2003).
- ¹⁸²Q. Meng and H.-D. Meyer, "A full-dimensional multilayer multi-configuration time-dependent Hartree study on the ultraviolet absorption spectrum of formaldehyde oxide," *J. Chem. Phys.* **141**, 124309 (2014).
- ¹⁸³V. J. Esposito, T. Liu, G. Wang, A. Caracciolo, M. F. Vansco, B. Marchetti, T. N. V. Karsili, and M. I. Lester, "Photodissociation dynamics of ch2oo on multiple potential energy surfaces: Experiment and theory," *J. Phys. Chem. A* **125**, 6571–6579 (2021).
- ¹⁸⁴J. M. Beames, F. Liu, L. Lu, and M. I. Lester, "Ultraviolet spectrum and photochemistry of the simplest criegee intermediate ch2oo," *J. Am. Chem. Soc.* **134**, 20045–20048 (2012).
- ¹⁸⁵W.-L. Ting, Y.-H. Chen, W. Chao, M. C. Smith, and J. J.-M. Lin, "The uv absorption spectrum of the simplest criegee intermediate ch2oo," *Phys. Chem. Chem. Phys.* **16**, 10438–10443 (2014).
- ¹⁸⁶R. Dawes, B. Jiang, and H. Guo, "Uv absorption spectrum and photodissociation channels of the simplest criegee intermediate (ch2oo)," *J. Am. Chem. Soc.* **137**, 50–53 (2015).
- ¹⁸⁷Y. Damour, A. Scemama, D. Jacquemin, F. Kossoski, and P.-F. Loos, "State-specific coupled-cluster methods for excited states," (2024), [arXiv:2401.05048](https://arxiv.org/abs/2401.05048) [physics.chem-ph].
- ¹⁸⁸R. Pou-Amérgo, M. Merchán, and O. Enrique, "Theoretical study of the electronic spectrum of p-benzoquinone," *J. Chem. Phys.* **110**, 9536–9546 (1999).
- ¹⁸⁹J. Weber, K. Malsch, and G. Hohlneicher, "Excited electronic states of p-benzoquinone," *Chem. Phys.* **264**, 275–318 (2001).
- ¹⁹⁰M. Schreiber, M. R. Silva-Junior, S. P. A. Sauer, and W. Thiel, "Benchmarks for electronically excited states: Caspt2, cc2, cc3 and cc3," *J. Chem. Phys.* **128**, 134110 (2008).
- ¹⁹¹D. Bousquet, R. Fukuda, P. Maitarad, D. Jacquemin, I. Ciofini, C. Adamo, and M. Ehara, "Excited-state geometries of heteroaromatic compounds: A comparative td-dft and sac-ci study," *J. Chem. Theory Comput.* **9**, 2368–2379 (2013).
- ¹⁹²D. B. Jones, P. Limão-Vieira, M. Mendes, N. C. Jones, S. V. Hoffmann, R. F. da Costa, M. T. d. N. Varella, M. H. F. Bettge, F. Blanco, G. García, O. Ingólfsson, M. A. P. Lima, and M. J. Brunger, "An experimental and theoretical investigation into the electronically excited states of para-benzoquinone," *J. Chem. Phys.* **146**, 184303 (2017).
- ¹⁹³K. Nakayama, H. Nakano, and K. Hirao, "Theoretical study of the ???* excited states of linear polyenes: The energy gap between 11Bu+ and 21Ag? states and their character," *Int. J. Quantum Chem.* **66**, 157–175 (1998).
- ¹⁹⁴H. Fliegl and D. Sundholm, "Coupled-cluster calculations of the lowest 0–0 bands of the electronic excitation spectrum of naphthalene," *Phys. Chem. Chem. Phys.* **16**, 9859–9865 (2014).
- ¹⁹⁵V. Sauri, L. Serrano-Andrés, A. R. M. Shahi, L. Gagliardi, S. Vancoillie, and K. Pierloot, "Multiconfigurational second-order perturbation theory restricted active space (raspt2) method for electronic excited states: A benchmark study," *J. Chem. Theory Comput.* **7**, 153–168 (2011).
- ¹⁹⁶B. Dick and G. Hohlneicher, "Two-photon spectroscopy of the low-lying singlet states of naphthalene and acenaphthene," *Chem. Phys. Lett.* **84**, 471–478 (1981).
- ¹⁹⁷J. P. Zobel, J. J. Nogueira, and L. Gonzalez, "The ipea dilemma in caspt2," *Chem. Sci.* **8**, 1482–1499 (2017).