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# Friction Damping and Isolation Systems

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*This paper surveys the literature on the use of dry friction in passive damping and vibration isolation. Several analytical techniques are presented followed by applications from various areas. Despite difficulties in the analytical and experimental treatment of friction damped systems, numerous studies have endeavored to predict, measure, and/or enhance the energy dissipation and vibration isolation properties of dry friction in order to improve system performance. Undoubtedly, friction damping will continue to play an important role in many mechanical and structural systems.*

## 1 Introduction

While many engineers view dry (Coulombic) friction as being very detrimental to system performance, others view it as being very beneficial. Dry friction can result in a loss of accuracy in servomechanisms, a loss of efficiency in power generation units such as automotive engines, and could result in the wear of contacting parts. However, dry friction can also enhance system performance due to its damping and isolation properties. This is especially true in lightly damped flexible structures and in seismic isolation systems. This review will focus on the beneficial uses of dry friction and will survey prior research which has sought to enhance those beneficial features through design.

One of the major impediments to the analytical treatment of dry friction damped systems is that the mathematical models that attempt to describe it are highly nonlinear. Indeed, many of the models are discontinuous and/or non-smooth, rendering techniques such as perturbation methods and linearization in the vicinity of equilibrium impossible. Numerical treatment using time integration are also hampered by the non-smooth nature of the system models.

Experimental investigations have been hampered by a different aspect of the friction phenomenon; namely, its unrepeatability. The results of experimental studies of friction-damped systems have been known to vary with time, temperature, the history of the motion (a so-called memory effect), and even humidity. Furthermore, friction properties have been known to show a dependence on the systems in which the frictional interface is placed; this causes the frictional properties measured in test rigs to differ from those that exist in-situ.

The difficulties encountered in the analytical and experimental investigation of dry friction have not discouraged designers from relying on it to provide damping and isolation. For example, friction is one of the few feasible damping treatments for gas turbine blades due to the extreme temperatures encountered. In other situations, friction is already a major source of damping. In bolted frame and truss struc-

tures, for example, friction is thought to supply over 90 percent of the passive damping capacity. In the case of seismic isolation, friction is a natural choice because it can provide shock isolation in a relatively simple way.

Recently, three survey papers have appeared that have addressed friction in engineering systems. Ibrahim (1994a, 1994b) provides a thorough review of friction modeling and on friction-induced vibration. He also touches upon some of the major developments in *friction damping*, which is the focus of the present paper. A third survey paper of Armstrong-Hélouvy, Dupont, and Canudas de Wit (1994) review the analysis tools and compensation methods for the control of dry-friction damped systems. The reader is referred to these three papers for additional material on dry friction modeling and analysis.

This review will explore the role of friction damping in engineering systems and the techniques that have been applied to the analysis and design of friction damping components. The next section outlines some of the basic models of friction. Section 3 gives an overview of three solution techniques commonly applied to friction-damped systems. Section 4 discusses several application areas for friction damping, and Section 5 contains some concluding remarks.

## 2 Modeling Issues

A variety of mathematical models have been used to represent friction forces in the case of unlubricated contact. There are two main categories of friction models: *Sgn* models that are discontinuous with respect to the relative slip velocity, and “hysteretic” models that are continuous, hysteretic functions of slip displacement. A brief description of each is presented next.

**2.1 Sgn Friction Models.** Of the multitude of discontinuous friction models that have been proposed, the most frequently used for damping studies is Coulomb’s law. The force-velocity relation, shown in Fig. 1, can be stated as:

$$F_f = \begin{cases} \mu N & v_{rel} > 0 \\ -\mu N & v_{rel} < 0 \end{cases} \quad (1)$$

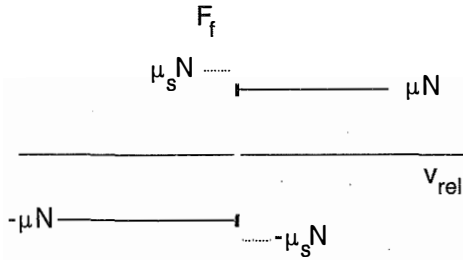


Fig. 1 Coulomb friction law

where  $F_f$  is the friction force,  $\mu$  is the sliding or “kinetic” coefficient of friction,  $N$  is the normal force acting to press the two bodies together, and  $v_{rel}$  is the slip velocity between the two contacting surfaces. During sticking,  $v_{rel} = 0$  for nonzero lengths of time. In this case,  $|F_f|$  is bounded by  $\mu_s N$ , where  $\mu_s$  is termed the static coefficient of friction. The magnitude and sign of  $F_f$  are dictated by force equilibrium considerations. For conciseness, Coulomb’s law is often abbreviated as

$$F_f = \mu N \text{Sgn}(v_{rel}) \quad (2)$$

where  $\text{Sgn}$  is the signum function defined to be  $+1$ ,  $-1$ , or  $0$  depending on whether its argument is positive, negative, or zero, respectively. Note that the  $\text{Sgn}$  notation is only correct in the case of sliding, and that sticking must be treated separately.

The dependence of the friction coefficient on slip velocity, normal load, and on the history of the motion (so called memory effects), has received considerable attention from researchers in the area of friction-induced vibration (Ibrahim, 1994b) and servomechanism control (Armstrong-Hélouvry et al., 1994). By comparison, the influence of various friction laws on the *passive damping* of systems has received only limited attention; most studies employ models having a constant friction coefficient. The simplest type of velocity dependence, the use of differing static and dynamic coefficients of friction, has been treated by several authors; e.g., Shaw (1986). Makris and Constantinou (1991b) examined the harmonic excitation of a single-degree-of-freedom (SDOF) system whose friction coefficient had an exponential dependence on slip velocity. Bindemann and Ferri (1995) compared several models of friction to determine their effect on the passive damping level of a flexible beam system. This work is described further in Section 4.2.

**2.2 Hysteretic Spring Friction Model.** A drawback of the  $\text{Sgn}$ -based friction laws is that they do not account for deformation that can occur prior to interfacial slip. One way to account for this phenomenon is to model the friction interface as an elastic spring in series with an ideal Coulomb damper element as shown in Fig. 2. The series spring/damper arrangement behaves like a spring with hysteretic stiffness as displayed in Fig. 3. Under the assumption of sinusoidal displacement for  $x$ , the hysteresis traces the closed “parallelogram” shape displayed in the figure. Due to the “straight-line” segments that make up the force-displacement relation, it is often termed a *bilinear hysteresis*. It should be added that this model is also prevalent in studies of elastic-plastic deformation where it is referred to as the *elastic/perfectly-plastic* model.

Several extensions/modifications have been developed for the hysteretic spring model of friction. For example, it was assumed above that  $\mu_s = \mu$ , however, several authors have considered the case of non-constant  $\mu$  (Wang and Shieh, 1991). The hysteretic spring model has also been extended to accommodate *microslip*, which is the situation where part of

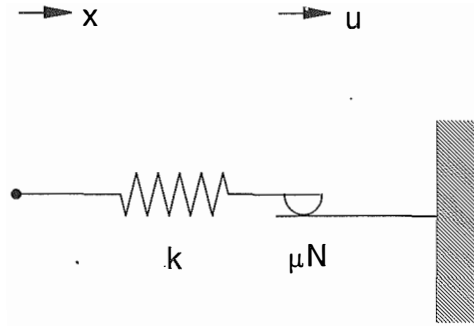


Fig. 2 Hysteretic spring model for friction

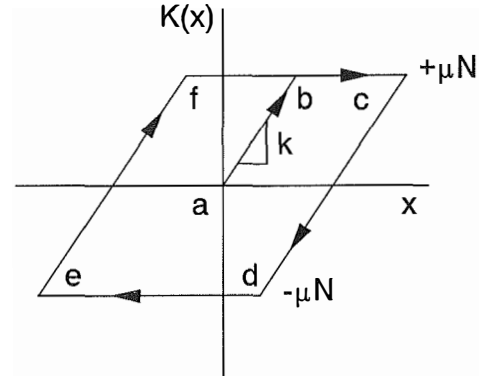


Fig. 3 Hysteretic spring relation

an interface is sliding while the other portion is stuck (Menq, Bielak, and Griffin, 1986). Finally, the hysteretic spring model has been extended recently to handle the case of two-dimensional slip motion (Menq et al., 1991).

**2.3 Motion-Dependent Normal Force.** It is difficult to design systems or to envision practical engineering systems for which the normal load across the frictional interface remains constant throughout the course of a system’s motion. This motion-dependence can be incorporated into either the  $\text{Sgn}$  friction models or the hysteretic spring model and has been found to have a significant effect on the system response. As discussed in the sections below, systems with motion-dependent friction forces exhibit significantly different behavior than systems with constant friction forces. Most importantly, the equivalent viscous damping ratio of such systems can resemble *viscous damping* in some instances. It has also been shown that an otherwise linear system with displacement-dependent friction forces can exhibit chaotic response when subjected to harmonic forcing. (Feeny, 1992; Feeny and Moon, 1994). Furthermore, recent work by Dupont has shown that displacement-dependent normal loads can have a significant impact on the “sliding-stability” of frictional surfaces (Dupont and Bapna, 1994).

Several researchers have been concerned about the effects of compliance and relative displacement in the direction normal to the frictional interface. Oden and his co-workers developed a normal-load-dependent model of friction that recovers many of the physically observed phenomena attributed to dry-friction damped systems (Oden and Martins, 1985; Tworzydło et al., 1994). The modeling of frictional contact between elastic bodies has received widespread attention in the finite-element and boundary-element communities; for example (Aliabadi and Brebbia, 1993). Other researchers have considered the effect of vibration normal to the frictional interface including Hess and Soom (1991) and Hess and Wagh (1992).

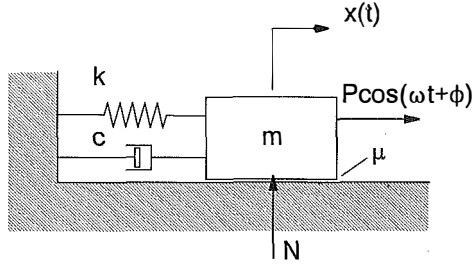


Fig. 4 SDOF dry friction damped system

### 3 Solution Techniques

A variety of solution techniques have been applied to the analysis of dry friction damped systems. This section briefly describes three of the more commonly applied techniques: the “exact” solution, the harmonic balance procedure, and the time integration technique.

**3.1 Exact Solution Technique.** One of the earliest analytical treatments of dry friction damped systems was performed by Den Hartog (1931). He exploited the piecewise-linear characteristic of the Coulomb damping friction law to obtain *exact* steady-state solutions for harmonic excitation. In order to review the exact solution technique, consider the single-degree-of-freedom (SDOF) system shown in Fig. 4. Using a Coulomb damping approximation for dry friction (2), the equation of motion can be written concisely as:

$$m\ddot{x} + c\dot{x} + kx + F_f \text{Sgn}(\dot{x}) = P \cos(\omega t + \phi) \quad (3)$$

For the case of stick-free motion, the nonlinear equation of motion (3) is equivalent to the following two linear equations:

$$\dot{x} > 0; \quad m\ddot{x} + c\dot{x} + kx = P \cos(\omega t + \phi) - F_f \quad (4)$$

$$\dot{x} < 0; \quad m\ddot{x} + c\dot{x} + kx = P \cos(\omega t + \phi) + F_f \quad (5)$$

For the case of free response in the absence of viscous damping,  $P = c = 0$ , it can be shown that the response will exhibit *linear envelopes of decay*. Furthermore, the system is not asymptotically stable to the origin, but will terminate with a displacement magnitude no greater than  $\mu N/k$ . For decades, linear envelopes of decay in free response have been interpreted by experimentalists as evidence that dry friction was present in their physical system.

For the forced case, the general solution to either (4) or (5) can be written in terms of two unknown constants of integration. Den Hartog solved for the steady-state motion by assuming that the fundamental period of the response was the same as that of the forcing function (namely,  $2\pi/\omega$ ) and that the second half-cycle of steady-state motion must match the negative of the first half-cycle (sometimes referred to as the *anti-periodicity* relation). In this way, he was able to find closed-form expressions for the amplitude and phase of steady-state response. (An equivalent result using a Fourier series representation for the friction force is also contained in that paper.) Den Hartog went further to develop an “exact” solution for steady-state motion with one stop interval per half-cycle; however, this case required the numerical solution of a transcendental equation.

The exact solution method has been used and even extended by a number of authors. Levitan (1960) used the Fourier series version of the exact solution technique to obtain the response of a system forced with harmonic base excitation. Yeh (1966) applied Den Hartog’s exact solution technique to a two-degree-of-freedom, base-excited system having a single friction interface between one of the masses and the ground. Westermo and Udawadia (1983) studied the response of a two-DOF, shear-type building structure model

to harmonic horizontal motion of the ground. Shaw (1986) revisited the system studied by Den Hartog with the modification that his Coulomb friction model was able to accommodate differing static and dynamic coefficients of friction, and Shaw treated both the case of positive *and negative* viscous damping. (Negative viscous damping can be viewed as a crude model of destabilizing aerodynamic forces, as discussed in Section 4.1 below.) Shaw not only solved for the steady-state response, but performed a dynamic stability and bifurcation analysis. A similar study by Karius (1985) applied the exact solution procedure to a system with general piecewise-linear damping. Karius also used a bifurcation analysis to find frequencies of excitation at which “nonsymmetric” response (i.e., response that was not antiperiodic) occurred and verified his results experimentally.

The exact solution procedure was also used by Beucke and Kelly (1985) for a SDOF system with a normal load that grew linearly with slip displacement—so called “linear Coulomb damping.” This type of system was also studied by Anderson and Ferri (1990) who pieced-together exact solutions to march forward in time. In particular, they investigated the influence of the displacement-dependent component of the friction force on the sticking conditions. A similar procedure was used by Makris and Constantinou (1991a). Since this method does not impose an anti-periodicity condition on the response, it can uncover nonsymmetric steady-state response; Makris and Constantinou were able to find occurrences of nonsymmetric response having only one stop per cycle.

**3.2 Harmonic Balance.** Due to the widespread use of the harmonic balance method in the analysis of dry friction damped systems, the method is outlined below for systems using the *Sgn* friction model and for systems using the hysteretic spring model of friction. For simplicity, only the first-order harmonic balance procedure (using a single harmonic) is presented; however, examples are cited in Section 4 that use the harmonic balance method with several harmonics.

It should be mentioned that the first-order harmonic balance technique is equivalent to other techniques that assume steady harmonic response. These include the sinusoidal describing function, techniques that equate the energy dissipated by an “equivalent viscous” damper, the method of averaging, and Galerkin’s procedure using harmonic bases functions.

**Sgn Friction Model.** Consider the SDOF system of Fig. 4 with governing Eq. (3). Harmonic balance assumes a response of the form:

$$x(t) = X \cos(\omega t) \quad (6)$$

Note that (6) assumes, without loss of generality, that time can be referenced to  $x(t)$ , which amounts to treating the phase angle  $\phi$  between the forcing and displacement as an unknown. Substitution of (6) into (3) and balancing harmonics (Ferri and Bindemann, 1992) leads to a quadratic equation for the response amplitude  $X$ . Inspection of this quadratic equation for  $X$ , shows that positive solutions for  $X$  are obtained only if  $P > 4\mu N/\pi$ . The quantity  $4\mu N/\pi$  is termed the *threshold force*,  $P_T$ , since no harmonic balance solutions exist for smaller harmonic force amplitudes. Based on a force balance argument, non-zero steady-state oscillations exist for  $P > \mu N$ ; thus the harmonic balance method overpredicts the true threshold force amplitude by about 27 percent. It can also be shown that, in the absence of viscous damping ( $c = 0$ ),  $X$  will be unbounded if forced at the natural frequency  $\omega_n^2 = k/m$ , provided that  $P > P_T$  (Den Hartog, 1931).

The harmonic balance method can also be used to obtain an equivalent viscous damping constant for Coulomb friction (Ferri and Bindemann, 1992):

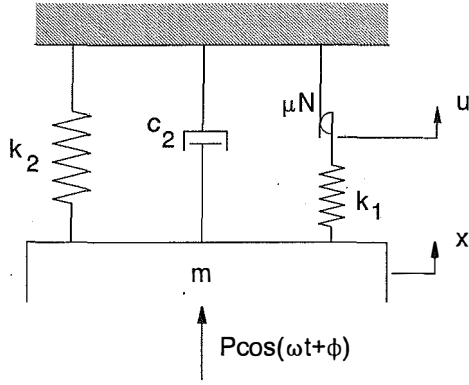


Fig. 5 SDOF system with series spring/dry-friction element

$$c_{eq} = \frac{4\mu N}{\pi\omega X} \quad (7)$$

This classic result suggests that the damping contribution from Coulomb friction is inversely proportional to the slip displacement amplitude (more precisely, the magnitude of the slip velocity,  $\omega X$ ). This inverse dependence explains why systems damped solely by dry friction can experience unbounded response at resonance.

**Hysteretic Spring Friction Model.** Harmonic balance can also be applied to systems that use the hysteretic spring model of friction. Consider the SDOF system shown in Fig. 5. This model is frequently used to model frictionally-damped turbine and compressor blades (see Section 4.1). Using the first-order harmonic balance procedure described above, expressions can be obtained for the equivalent stiffness and the equivalent viscous damping coefficient for the spring/dry-friction element:

$$k_{eq} = -\frac{4\mu N \cos(\omega t_1)}{\pi X} + \frac{k_1}{\pi} \left[ \omega t_1 - \frac{\sin(2\omega t_1)}{2} + 2 \cos(\omega t_1) - \frac{\pi}{2} \right] \quad (8)$$

$$c_{eq} = \frac{4\mu N \sin(\omega t_1)}{\pi\omega X} + \frac{k_1}{\pi\omega} \left[ \frac{3}{2} - \frac{\cos(2\omega t_1)}{2} - 2 \sin(\omega t_1) \right] \quad (9)$$

where

$$\omega t_1 = \sin^{-1} \left( 1 - \frac{2\mu N}{Xk_1} \right); \quad \frac{\pi}{2} \leq \omega t_1 \leq \frac{3\pi}{2} \quad (10)$$

The expressions are valid so long as  $X \geq \mu N/k_1$ . (Obviously, the damper reduces to a purely linear spring with  $k_{eq} = k_1$  and  $c_{eq} = 0$  when  $X < \mu N/k_1$ .) Examination of (8) and (9) shows that both the stiffness and damping of the hysteretic spring approach zero as the response amplitude increases. When the parallel spring  $k_2$  is considered, it is clear that the combined stiffness goes from  $k_1 + k_2$  at very small displacement amplitudes (or high levels of normal load) to  $k_2$  alone at very high displacement amplitudes (or low levels of normal load).

**3.3 Time Integration Computational Issues.** Many studies have been conducted that use time integration to obtain the free and/or forced response of single and multi-degree-of-freedom (MDOF) systems. Time integration of systems with hysteretic friction models is relatively straightforward (see, for example, Menq and Griffin, 1985). The discontinuous friction model, however, presents computational difficulties. In particular, when sticking occurs, the discontinuity in

the friction law causes the friction force to switch directions with every time step. This leads to very high computation times, especially when automatic step-size control techniques are used; when constant step size routines are used, “brute force” integration through a sticking interval usually leads to a significant loss of accuracy. To alleviate this problem, most studies make use of a sticking condition which is checked each time the slip velocity across the frictional interface crosses zero. If the static friction force is sufficient to prevent slipping, time integration is resumed using a reduced set of dynamics pertaining to the stuck system (see, for example, Ferri and Heck, 1992b). It should be noted that time integration can be very inefficient if one is primarily interested in the steady-state response to periodic excitation. In this case, one must integrate the system equations until the transient response has died out, which can take hundreds of forcing periods if the system is lightly damped.

There are significant difficulties involved in the implementation of sticking criteria within time integration routines, especially when there are multiple interfaces. The decision on whether a particular frictional interface is stuck or slipping depends on whether the sum of the forces acting tangent to a contact plane are sufficient to overcome the static friction force in an interface that is momentarily at rest. However, the normal and tangential forces themselves depend on the friction forces at other frictional interfaces. The analysis of these types of systems has received attention from a number of authors; for example (Klepp, 1991; Pfeiffer and Hajek, 1992; Pfeiffer, 1993).

The numerical stiffness problems associated with *Sgn*-based friction laws are exacerbated when the model of a particular system attempts to capture both the fast and slow dynamics. This problem has been addressed recently in Ferri and Heck (1994) where singular perturbation theory was extended to accommodate discontinuous dry friction damping. Their approach utilized the piecewise linearity of the system in order to prove that the errors incurred in the singular perturbation method were sufficiently small, even when trajectories crossed from one piecewise linear region to another.

## 4 Applications

**4.1 Friction Damping in Turbomachinery Systems.** An important application of dry friction is the passive damping of turbomachinery blades and bladed-disks. Dry friction is one of the few passive damping treatments that can withstand the hostile environment found in turbomachinery—especially gas turbines. The damping of turbine and compressor blades is crucial to suppressing vibration and flutter, which, in turn, extends the life of the blades. Furthermore, improved passive damping treatments for blades can result in the development of lighter, more flexible blades which could potentially increase the thrust-to-weight ratio of aircraft jet engines. Due to space limitations, only a sampling of the research in this area will be cited. A good reference for friction damping in turbomachinery is the series of review articles by Jones, the most recent one appearing in 1990.

There are three common ways that friction damping is introduced into turbine/compressor blade systems. Turbine and compressor blades are almost always attached to the rotor disk through sliding interfaces. Damping at the root of the blades occurs so long as relative slip between the blade and the disk is present. Often, however, slip is prevented from occurring due to the high centrifugally-induced normal loads acting across the blade root. The second location where friction can be introduced into turbomachinery blade systems is the platform damper (Griffin, 1980; Srinivasan and Cutts, 1982). The third potential location of friction damping in segmented interblade shrouds (Bielawa, 1978; Menq, Griffin,

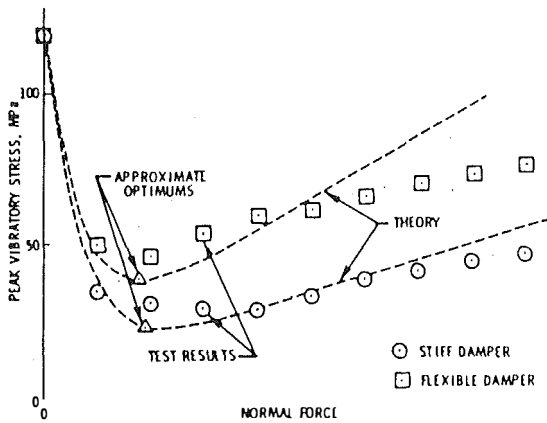


Fig. 6 Stress reduction versus damper normal load (from Griffin, 1980)

and Bielak, 1986). When the rotor is at rest, a small gap is present between the adjoining shroud segments. As the rotor's speed increases, the shroud segments are pressed together due to the centrifugally-induced "unwinding" of the pretwisted blades.

The study by Griffin (1980) highlights many of the design issues surrounding friction damping for turbine/compressor blades. Griffin investigated frictional platform dampers for gas turbine blades using analytical and experimental techniques. His blade/damper model was similar to the SDOF system shown in Fig. 5. Griffin determined the optimal normal force to minimize the resonant vibration amplitude of the blade. Figure 6 shows his theoretical and experimental results. Note that small normal forces produce an inadequate level of damping while overly high normal force levels do not allow a sufficient amount of slip to occur, again resulting in insufficient damping. Qualitatively similar results were obtained by Srinivasan and Cutts (1982) for the case of sinusoidal excitation and by Sinha (1987) for the case of random excitation.

Other studies that have considered the design of friction dampers for turbine/compressor blades include (Muszyńska and Jones, 1983b; Menq, Bielak and Griffin, 1986; Wang and Shieh, 1991; Pfeiffer and Hajek, 1992). The influence of dry friction on the mistuning phenomenon associated with bladed disks has also been studied (Muszyńska and Jones, 1983a; Griffin and Sinha, 1985; Wei and Pierre, 1989).

Several studies have examined the ability of friction to suppress flutter in turbine blade systems. Using negative viscous damping as a crude model for aerodynamic forcing, Sinha and Griffin (1983) and Ferri and Dowell (1985) showed that dry friction dampers can be designed to create a range of stable operation but that sufficiently large disturbances lead to unbounded response. The difficulty stems from the inverse dependence of friction damping on displacement amplitude. Sinha et al. (1986) investigated the ability of friction to suppress torsional blade flutter using a more refined aerodynamic model.

The role of displacement-dependent friction forces on the forced harmonic response of systems is a subject that is relevant to turbine/compressor blade vibration as well as to a number of other applications. Bielawa (1978) incorporated Coulomb friction into a finite element model of a bladed disk that allowed the normal stress to vary with the vibratory motion of the blades. A key finding of this study was that, the damping contribution from friction resembled that of *linear structural damping*. Similar findings were obtained by Menq, Griffin, and Bielak (1986) and by Whiteman and Ferri (1995). This result, which is consistent with the work on "linear Coulomb damping" cited in Section 3.1, suggests that dry friction may be more effective in controlling flutter than

previous studies have indicated.

It should be noted that both the hysteretic spring model of friction and the *Sgn* model of friction are consistent with Coulomb's Law when the damper is assumed to be massless. Recently, Ferri and Heck (1994) studied the influence of damper mass (associated with the coordinate  $u$  in Fig. 5) on the effectiveness of platform and shroud dampers. Through an extended singular perturbation theory, they show that the hysteretic spring model and the *Sgn* model can be viewed as zeroth-order singular perturbation models that assume very small damper mass and very large damper stiffness, respectively. They also develop a refined "hysteretic spring model" that includes the effects of damper mass to first order. It was seen that damper mass can have a surprisingly large effect on the damper's ability to limit resonant displacements.

A number of authors have studied frictionally-damped beam structures. These types of studies have direct relevance to the vibration of turbine/compressor blades with either platform or shroud dampers. Experiments were carried out by Dowell and Schwartz (1983) on a cantilever beam that contacted a friction damper at its tip. The results demonstrate clearly the complicated vibrational behavior that dry friction damped structures can exhibit. Namely, at low levels of excitation, the damper remained stuck and the observed behavior was nearly linear. Once the interface slipped, the resonant frequencies changed radically and the fundamental clamped-free mode appeared with a very high resonant amplitude.

Multi-harmonic solutions have been considered by several authors. Multi-harmonic analysis was used to capture the stick-slip motion of SDOF and two-DOF systems by Pierre et al. (1985). Ferri and Dowell (1988) applied a multi-harmonic balance technique to the Dowell and Schwartz cantilever-beam model (with a *Sgn* friction representation) and found that many of the experimentally observed phenomenon were attributable to stick-slip behavior at the frictional interface. In particular, it was seen that resonances could develop between the higher-harmonics of the stick-slip frictional forces and the higher modes of the elastic system. Multi-harmonic analysis of systems having hysteretic friction elements has also been considered (Wang and Chen, 1993; Ren and Beards, 1994). The alternating frequency-time (AFT) method, which is an efficient way of implementing multi-harmonic analyses, has also been applied to systems with hysteretic spring models of dry friction (Cameron and Griffin, 1989).

**4.2 Damping in Built-Up Structures.** It is well known that the passive damping level of "welded" or *monolithic structures* (i.e., structures made of one continuous piece of the same material) is quite low. Reported damping ratios for monolithic metallic structures are typically between 0.0001 and 0.001 (Lazan, 1968). However, the damping of built-up structures such as jointed truss structures is much higher (on the order of 0.01) with friction providing up to 90 percent of a built-up structure's passive damping (Ungar, 1973). This section will review some of the recent work in this broad area. Excellent references for this topic are the review articles written by Beards whose most recent update appeared in 1992.

There are two competing requirements on the joints of trusses, frames, and other types of built-up structures. On the one hand, it is desirable for them to be tight and stiff so that the structure will be as rigid and as strong as possible. On the other hand, it has been shown that reducing the clamping force can increase a structure's passive damping level and alleviate resonant stresses (Beards, 1983; Beards and Woowat, 1985; Beards, 1992). The level of passive damping can also be an important factor that determines the stability of structures when exposed to aerodynamic forces or active control forces.

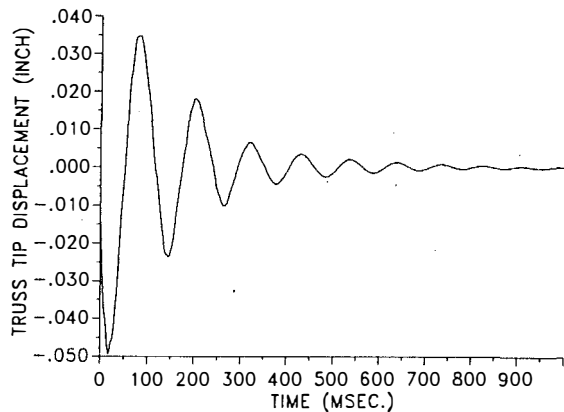


Fig. 7 Truss tip displacement versus time (from Folkman and Redd, 1990)

In the case of active vibration control, it is well known that instabilities can arise in the closed-loop system especially when the sensors and actuators are noncollocated; passive damping can reduce this tendency for instability and can lower the control efforts required for the force actuators in the control system.

**Truss and Frame Structures.** Several analytical studies have been performed that examined the effects of joint friction on the vibration of truss and frame structures. Hertz and Crawley (1984) analyzed a variety of pin, ball, and sleeve joints with displacement-dependent friction forces. A key finding of their analytical study was that the equivalent viscous damping ratio of systems with displacement-dependent friction forces was “viscous-like”; i.e., it was constant with structural displacement rather than inversely proportional to structural displacement as it is in the case of constant Coulomb friction. Crawley and Aubert (1986) used the force-state mapping technique to determine appropriate joint models from experimental measurements. Ferri (1988) developed a sleeve joint model that included displacement-dependent friction forces, impact damping, and clearances. Through numerical simulation of a beam-sleeve-joint system that was damped only by dry friction, Ferri showed that the free-response envelopes of decay were neither linear (indicating constant friction forces) nor exponential (viscous-like). The application of the log-decrement approach to the free response showed that the damping ratio, in fact, increased linearly with displacement in a manner that is similar to *air damping*, in which the damping or drag forces grow with the square of the velocity (Den Hartog, 1956).

An experimental investigation of the joint damping of truss structures was undertaken by Folkman and Redd (1990). Figure 7 shows the free response of the structure displaying exponential rather than linear envelopes of decay. A plot of log decrements vs average displacement amplitude for three different structural orientations (with respect to gravity) is shown in Fig. 8. (The figure compares the results obtained in the presence of air with those obtained when the entire truss structure was placed in a vacuum.) Their study shows clearly that the damping does not bear an inverse dependence on displacement amplitude. Except for the vertical orientation, the measured log decrement increases with displacement amplitude in a manner similar to the damping in the beam-sleeve system studied numerically by Ferri (1988). As mentioned above, displacement-dependent friction forces may account for this observed behavior.

In frame structures, a number of researchers have studied the role of bolted connections on system damping. Beards and co-workers studied the optimal clamping force to control the resonant amplitude of frame structures; see, for example, Beards and Woowat (1985), Beards (1992) and the references

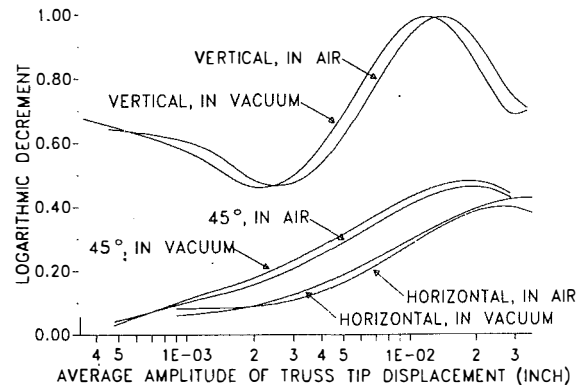


Fig. 8 Log decrement versus amplitude of truss tip displacement (from Folkman and Redd, 1990)

cited therein. It was shown through analysis and experiments that the damping of a bolted structure is low for low clamping forces and also low for very high clamping forces that prohibit interfacial slip. Between these extremes, an optimal clamping force was seen to exist. Design guidelines were found for proper selection of the clamping load.

The design of enhanced friction joints for vibration suppression in flexible systems was investigated by Ferri and co-workers using *semi-active friction elements*. In Ferri and Heck (1992b), the use of active control to vary the normal force in a pin joint was explored. The technology is termed semi-active because it uses active control to vary the properties of a passive element. It is shown that feeding back the absolute value of the relative angular velocity of the pin joint to the normal force effectively linearizes the frictional element. The linearization stems from the fact that the absolute value nonlinearity multiplied by the *Sgn* nonlinearity yields a purely linear term. Note that, since the normal force is proportional to the magnitude of the relative rotation rate, the control design precludes the occurrence of sticking. Ferri and Heck (1992b) also investigated the performance of purely passive joint geometries for which the normal load varied as a function of joint relative rotation. The occurrence of sticking in these joint designs was analyzed using variable structure control (VSC) theory. To the author's knowledge, this was the first application of VSC to Coulomb-damped systems and seems to offer many advantages in treating stick-slip motion. In Lane and Ferri (1992) it was shown that near-optimal energy removal could be achieved using semi-active friction joints and simple linear control designs.

**Beam and Plate Structures.** Some of the material concerning friction damping of beams was cited in Section 4.1 in reference to turbine blade vibration; e.g., (Dowell and Schwartz, 1983; Ferri and Dowell, 1985, 1988). Numerous other studies have been directed toward the damping properties of bolted or riveted connections in beam and plate-like structures. Much of this work was conducted in the 1960's in connection with aerospace structures and is reviewed in the survey papers (Ungar, 1973; Plunkett, 1980; Beards, 1992). For low amplitudes of displacement (microslip), the energy loss per cycle,  $\Delta E$ , from lap joints and from frictional interfaces in layered beams is seen to increase with the cube of the amplitude of displacement, implying that the damping ratio is linearly dependent on displacement amplitude. Additionally, it is seen that  $\Delta E$  decreases with increasing normal load and friction coefficient. However, for larger displacements (macroslip),  $\Delta E$  grows linearly with displacement amplitude and increases with normal load and friction coefficient in a manner consistent with (7). Many experimental studies report that the measured damping characteristics of built-up structures are “quite linear” leading Ungar to specu-

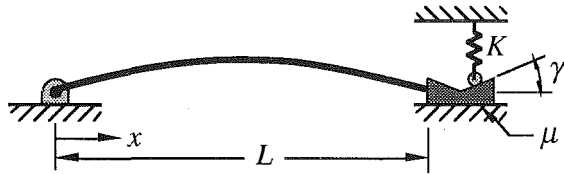


Fig. 9(a) Flexible beam system with normal load dependent on the in-plane slip displacement (from Ferri and Bindemann, 1992)

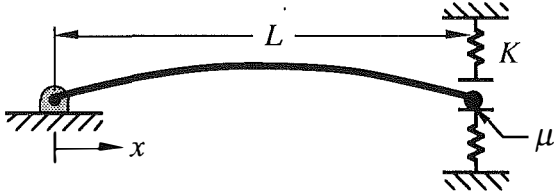


Fig. 9(b) Flexible beam system with normal load dependent on the transverse tip displacement (from Ferri and Bindemann, 1992)

late that Coulomb friction may not be the dominant mechanism of energy dissipation in these types of systems; other mechanisms such as “air pumping mechanisms” at riveted joints may contribute more to the overall damping level (Ungar, 1973). Another possibility, which has not yet received adequate attention, is that the omission of *displacement-dependent normal forces* in these analyses leads to the apparent contradiction.

The normal loads in bolted structures can be selected so as to maximize damping, or to minimize stresses and displacements. For a beam damped by a single transverse frictional interface, Earles and Beards (1970) found that the optimal normal load could be obtained by arranging for the slip amplitude at the contact surface to be one-half of the undamped displacement amplitude. The influence of normal load on damping levels in laminated plates was studied by Beards and Imam (1978) and later by Han (1985). It was found that an optimal clamping force exists which minimized the response amplitude. Shin et al. (1991) studied the influence of bolt clamping force on damping in a complicated experimental structure. They found that the structure’s damping level increased sharply when the bolt preloads were reduced below 40 percent of their proof loads.

Several studies have investigated the dependence of system damping on the in-plane or foreshortening displacement of beams and plates. Beards (1983) considered the influence of the clamping load on the measured damping ratio for rectangular plates with frictional supports. It was shown that it is possible to optimize the clamping force; clamping forces beyond a critical value result in no-slip at the friction interfaces and greatly reduced damping. Jézéquel (1983) showed that the equivalent viscous damping ratio of a circular plate with a constant clamping load around its edges exhibited a damping ratio that was invariant with respect to transverse displacement amplitude. In this sense, it is “viscous-like” in nature. The phenomenon stems from the fact that the foreshortening displacement depends on the square of the transverse displacement; thus, even though the damping is inversely proportional to the slip displacement, it is invariant with respect to transverse displacement. Dowell (1986) extended the work of Jézéquel by considering beams and plates with constant clamping loads at the edges. Dowell’s theoretical results were verified experimentally in Tang and Dowell (1986a, 1986b). The influence of variable normal load on the damping and nonlinear behavior of beams with in-plane slip was studied by Ferri and Bindemann (1992). Figures 9(a) and 9(b) show two types of frictional boundary supports that they considered. Using a single-beam-mode approximation and

first-order harmonic balance, they obtained closed-form expressions for the equivalent linear damping of the two systems. The equivalent linear damping ratio for the  $n$ th mode of the system in Fig. 9(a) was found to be of the form:

$$\zeta_n = \frac{\mu(c_1 + c_2 \tan \gamma A^2)}{\omega_n^2 + c_3 \tan \gamma + c_4 \tan^2 \gamma A^2} \quad (11)$$

where  $c_1$  through  $c_4$  are constants (that could depend on mode number  $n$ ),  $\mu$  is the coefficient of sliding friction,  $A$  is the modal amplitude of transverse displacement,  $\gamma$  is the ramp angle controlling the dependence of the normal load on slip displacement, and  $\omega_n$  is the  $n$ th natural frequency of the beam in the absence of friction. The case studied by Dowell and Jézéquel is recovered by setting  $\gamma = 0$  in (11). The case shown in Fig. 9(b) was found to have an equivalent damping ratio of:

$$\zeta_n = \frac{\mu \alpha_1 A}{\omega_n^2 + \alpha_2} \quad (12)$$

where  $\alpha_1$  and  $\alpha_2$  are constants. Note that in this case, the damping is linearly dependent on the amplitude of transverse displacement just like the beam-sleeve-joint system considered in Ferri (1988). The influence of alternative friction laws on the passive damping of a system such as the one shown in Fig. 9(a) has also been examined. In Bindemann and Ferri (1995), five different models of dry friction were compared. It was found that the refined friction models yielded passive damping results that were very close to those obtained using a simple constant- $\mu$  Coulomb Law.

**Damping Enhancing Devices.** The design and incorporation of supplemental frictional dampers into building structures has been discussed by a number of authors. Pall and Marsh (1982) proposed the design of a Friction Damped Braced Frame (FDBF) device. The inexpensive device, shown in Fig. 10, dissipates energy through dry friction at the center slip joint whenever diagonal tension forces are sufficiently high to overcome the static friction force. The study conducted by Filiatrault and Cherry (1988) reported that the isolation performance of the FDBF device was better, in many respects, than that of advanced base isolation systems. Effectiveness was evaluated by simulating the response of FDBF-equipped buildings to recorded ground-motion histories from the 1940 El Centro earthquake and the 1985 Mexico City earthquake. The experimental verification of the FDBF’s effectiveness as compared with six other passive damping devices was reported by Aiken et al. (1993). Examples of other interesting frictional joint designs can be found in Roik et al. (1988) and Filiatrault (1990). Several other friction-based energy dissipation elements are discussed in the August 1993 issue of the journal *Earthquake Spectra* (Vol. 9, No. 3). It would appear that these friction devices have application to a variety of structural systems outside of the civil engineering community.

**Stranded Cable.** As a final topic on damping in built-up structures, the role of friction in the damping properties of stranded cable is mentioned. Although used for years as a means of damping enhancement and isolation, no theory for predicting damping in stranded cables has been available until just recently. Hence, it has been difficult to improve damping performance except through experimentation and empirical results. Experiments reported in Pivovarov and Vinogradov (1987) were used to identify the parameters in a nonlinear hysteretic model of a Stockbridge damper made from stranded cable components. A model of friction damping in bent cable is developed in Vinogradov and Atatekin (1986). The model shows that friction losses increase with the helix angle and the tightness of the wires; furthermore, their

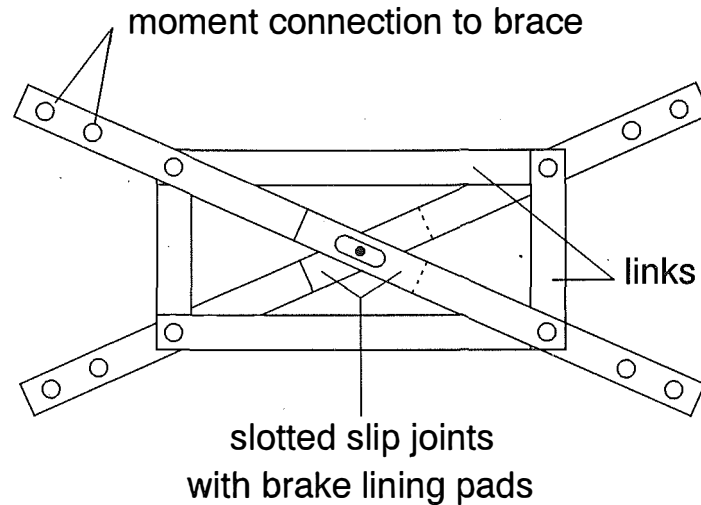


Fig. 10 Friction damped braced frame device

model suggests that the energy loss per cycle should grow with the amplitude of displacement. Tinker and Cutchins (1992) also proposed a model for the friction damping properties of stranded cable isolation systems. Their model accounts for the fact that the displacement of the cable can reduce the normal load between the strands.

**4.3 Aseismic Base Isolation Through Use of Friction Elements.** One of the major application areas for friction is that of earthquake isolation for civil engineering structures (Kelly, 1986). Rather than increasing the horizontal load-carrying capacity of buildings, this research seeks to reduce the damage to a building by allowing the ground to move beneath it. Some of the recent work in this area is reviewed next.

Constantinou and Tadjbakhsh (1984) studied the optimal design of a base isolation system with frictional elements. They found that the performance of traditional rubber-bearing isolators could be enhanced when friction elements are placed in parallel with them. The relative performance of two non-friction based isolation systems and two friction-based isolation systems were compared in Su et al. (1987). They determined that while all four isolators performed adequately when subjected to typical earthquakes such as the El Centro 1940 earthquake; earthquakes with a high amount of low frequency content, such as the Mexico City 1985 earthquake, were much easier to control with the friction-based systems. Tadjbakhsh and his co-workers proposed a displacement-dependent friction (DPF) base isolation system for which the normal loads increase with relative displacement and velocity through a system of cables (Tadjbakhsh and Lin, 1987; Su et al., 1989, 1990). It was shown that the acceleration transmissibilities of the system can be reduced over those achievable with constant normal loads. They also demonstrated through simulations using actual earthquake profiles that friction-based isolation systems are much less sensitive to severe variations in frequency content and amplitude of the ground acceleration than are the non-friction based isolation systems.

Other friction-based earthquake isolation devices include the Resilient-Friction Base Isolator (R-FBI) developed by Mostaghel and Khodaverdian (1988). Their system incorporates the advantages of sliding friction interfaces with a central rubber core. Another isolation device termed the Friction Pendulum System (FPS) has also demonstrated excellent performance in analytical and experimental studies (Mokha et al., 1991).

**4.4 Rail and Road Vehicle Systems.** The presence of friction in rail and road vehicles has long been recognized as an important factor in shock and vibration control. As an example, consider the friction devices designed to absorb longitudinal impact between coupled rail cars. These devices, termed *draft gears*, were first developed in the late 1800's and are still used today in rail transportation applications (Wallace, 1979; Garg and Dukkipati, 1984). Other examples are the leaf springs that are ubiquitous in train and truck suspensions (Cebon, 1986).

The role of friction damping in freight car vibration and stability has been studied by a number of authors. A survey of some of the early work in freight car dynamics, including a discussion of friction elements, was compiled by Law and Cooperrider (1974). Rail vehicles, both freight and passenger, typically consist of a carbody which rests on two "trucks." The trucks have two axles each and act to support the carbody and to guide it along the rails. Between the trucks and the carbody is a suspension system that resists lateral, longitudinal, and vertical translation and also the relative roll and yaw between the carbody and the trucks. In Cooperrider (1972) and Cooperrider et al. (1975), a nonlinear model of a truck that included the effects of nonlinear suspension characteristics was evaluated with respect to "hunting stability." (Hunting is characterized by violent lateral oscillations that can lead to limit-cycle oscillations or even derailment.) Of particular interest was the modeling of the yaw suspension, which was modeled by a series connection spring/dry-friction element. Cooperrider's study showed that friction in the yaw suspension may have the adverse effect of reducing the forward critical speed (the speed above which hunting occurs) in simple truck models. A generic suspension model, consisting of a parallel combination of a spring, a viscous damper, and a series-combination spring/Coulomb friction element was used in Fries et al. (1981). They concluded that increasing the frictional force in the suspension elements has the effect of increasing the suspension stiffness, which, in turn, increases the vehicle's stability on straight track. The analysis of friction elements in rail vehicle systems undergoing harmonic excitation was studied by Beaman and Hedrick (1980) using the multi-harmonic balance technique. In the case of random excitation, dry-friction-damped rail vehicle systems were analyzed using the statistical linearization procedure in Hedrick and Arslan (1980). A recent study by Diomin et al. (1994) also considered the role of friction elements on the hunting stability of rail vehicles. Their analysis procedure exploited the piecewise linearity of Coulomb

friction elements, and solved for the nonlinear response by piecing together the response of a sequence of linear systems.

The use of friction in automotive suspension was recently investigated in the context of semi-active suspension. An unfortunate trait of traditional frictional shock absorbers is their tendency to stick and slip intermittently, resulting in degraded ride quality. Using the semi-active approach mentioned in Section 4.2, Ferri and Heck (1992a) were able to eliminate the undesirable stick-slip behavior while retaining the beneficial attributes of dry friction-based isolation systems: namely, that the maximum friction force can be limited through the choice of normal load. The semi-active friction suspension was demonstrated experimentally in Lane et al. (1993). A similar semi-active friction device studied by Ryba (1993) was proposed as a means of improving the comfort of seats in truck cabs.

## 5 Concluding Remarks

Friction damping continues to play an important role in the design of structures that have improved vibrational properties. It is seen that friction need not produce damping that decreases with the displacement amplitude as predicted by the classic theory of Den Hartog. Depending on the interface design, the damping contribution from friction can be constant with displacement amplitude, or can increase with displacement amplitude. The level of damping obtained from dry friction depends strongly on the force normal to the frictional interface; for many systems, an optimal pressure load exists for which the damping is maximized.

In addition to supplying damping, dry friction is also viewed as an excellent means of attaining shock isolation because of its ability to limit transmitted forces across a frictional interface. Several researchers, especially in the earthquake-isolation area, have sought to improve upon the isolation performance of existing friction-based elements and to develop new friction-based isolation systems. The technology and analysis tools developed by the earthquake-isolation community may have significant applications to the base isolation of aerospace and mechanical systems.

The use of refined models for friction interfaces shows great promise for making better utilization of friction damping. A number of advances in this area have helped to further our understanding of these complicated nonlinear systems. For example, the incorporation of motion-dependence in mathematical models has been a major step forward. Further work in this area will most certainly enhance our ability to design improved friction damping elements. Although considerable progress has been made using simple constant- $\mu$  Coulomb approximations, the effect of advanced models of friction on damping and isolation performance warrants further examination.

It is evident that there are three major communities in which much of the friction damping research is being pursued: turbomachinery, building systems, and transportation systems. Many of the developments in friction damping in one area have direct applicability to the other areas, but there does not seem to be much cross-fertilization between the various communities. For example, a number of friction damping elements developed for use in building frame systems may have important applications to other types of structures as well. It is hoped that this review may help people become more fully aware of the broad range of research in the friction area.

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