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A misleading naming convention: de Sitter ‘tachyonic’ scalar fields

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We revisit the so-called ‘tachyonic’ scalar fields in de Sitter (dS) spacetime. Through a rigorous group-theoretical analysis, we argue that labeling these fields as ‘tachyonic’ lacks physical relevance, frequently resulting in misinterpretations in scholarly discourse, such as the misconception of faster-than-light travel associated with the corresponding particles.

Let us begin by recalling the most comprehensive action on the dS background, resulting in a linear equation of motion for a dS scalar field φ :

$$S(\varphi) = \frac{1}{2} \int_{\text{dS}} (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m_H^2 \varphi^2) d\Omega, \quad (1)$$

where $g_{\mu\nu}$, with $\mu, \nu = 0, 1, 2, 3$, stands for the dS metric, $d\Omega$ for the invariant measure on dS, $R = 12H^2$ for the Ricci (or curvature) scalar, m_H for the ‘mass’ term. Note that the latter parameter, strictly speaking, m_H^2 , encompasses the coupling term to the curvature, which is conventionally denoted in literature by ξR , where ξ denotes the coupling constant. The equation of motion derived from this action reads as:

$$(\square_H + m_H^2) \varphi = 0. \quad (2)$$

By definition (see, for instance, Refs. [1, 2]), the ‘tachyonic’ scalar fields in $(1+3)$ -dimensional dS spacetime are characterized by their squared ‘masses’ being negative and assuming an infinite set of discrete values, as outlined below:

$$m_H^2 = -H^2 n(n+3), \quad (3)$$

where $n = 0, 1, 2, \dots$. Note that the so-called dS ‘massless’ minimally coupled scalar field stands as a particular example of such fields, distinguished by $n = 0$, and hence, $m_H = 0$. As already pointed out, the commonly accepted interpretation for the emergence of the real negative squared ‘masses’ attributed to the dS ‘tachyonic’ fields is a would-be particle traveling faster than light (see, for instance, Refs. [1–9]).

This manuscript raises a critical question regarding the interpretation outlined above: does the quantity m_H , as defined in Eq. (3), truly signify a meaningful mass within the framework of dS relativity? Clearly, if it does not, any conclusions drawn from this parameter would lack physical relevance. The remainder of this manuscript endeavors to address this very question, while maintaining simplicity by adhering to units where $c = 1 = \hbar$,

representing the speed of light and the Planck constant, respectively.

To begin, we highlight two well-established facts regarding the modern formulation of elementary particles:

- In Minkowski quantum field theory (QFT), the concept of ‘free’ particles emerges as a consequence of Wigner’s classification of the unitary irreducible representations (UIRs) of the Poincaré group, that is, the group of motions in flat Minkowski spacetime [10, 11]. In this context, the rest mass m and the spin s of an (Einsteinian) elementary particle stand as the two invariants that characterize the corresponding UIR of the Poincaré group.
- Introducing a specific curvature to spacetimes is the sole method for deforming the Poincaré group. This deformation gives rise to the dS and anti-dS relativity groups of motion, as elaborated in Refs. [12, 13]. This distinctive situation confers a unique status upon dS and anti-dS spacetimes, establishing them as the singular family of curved spacetimes where, to a certain extent, the extension of the particle concept aligns with Wigner’s framework (see Refs. [14, 15], and references therein); UIRs of the dS and anti-dS groups, similar to their common Poincaré (null-curvature) contraction limit, are generally distinguished by two invariant parameters of the spin and energy scales.

Note that the latter fact reminds us of the Fronsda principle (1965) [16]: “*A physical theory that treats spacetime as Minkowskian flat must be obtainable as a well-defined limit of a more general physical theory, for which the assumption of flatness is not essential.*”

With this foundation, let us delve deeper into dS relativity. The $(1+3)$ -dimensional dS spacetime can be effectively represented as a hyperboloid situated within a $(1+4)$ -dimensional ambient Minkowski spacetime, denoted as \mathbb{R}^5 :

$$\text{dS} = \{x \in \mathbb{R}^5 ; (x)^2 \equiv \eta_{\alpha\beta} x^\alpha x^\beta = -H^{-2}\}, \quad (4)$$

where $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1, -1)$, with $\alpha, \beta = 0, 1, 2, 3, 4$, and H refers to the Hubble constant. The latter determines the rate of expansion for the spatial sections of dS. In this context, the dS metric is obtained

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by inducing the natural metric of \mathbb{R}^5 onto the dS hyperboloid:

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta \Big|_{(x)^2 = -H^{-2}} = g_{\mu\nu} dX^\mu dX^\nu, \quad (5)$$

where X^μ ($\mu, \nu = 0, 1, 2, 3$) denotes the four local space-time coordinates of the dS hyperboloid.

The dS relativity group is $SO_0(1, 4)$ (representing the connected subgroup of the identity in $O(1, 4)$) or any of its covering groups. The Lie algebra corresponding to this group is manifested through the linear span of the (ten) Killing vectors:

$$K_{\alpha\beta} = x_\alpha \partial_\beta - x_\beta \partial_\alpha. \quad (6)$$

On the level of representation, these Killing vectors are represented by (essentially) self-adjoint operators within the Hilbert space of functions that are (spinor-)tensor-valued, and square-integrable under some invariant inner product of Klein-Gordon type (or similar), defined on the dS spacetime manifold:

$$K_{\alpha\beta} \mapsto L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta}, \quad (7)$$

where the orbital component is expressed as $M_{\alpha\beta} = -i(x_\alpha \partial_\beta - x_\beta \partial_\alpha)$, while the spinorial component $S_{\alpha\beta}$ operates on the indices of the specified (spinor-)tensor-valued functions in a specific permutational manner. The generator representatives $L_{\alpha\beta}$ adhere to the commutation rules specified by the expression:

$$[L_{\alpha\beta}, L_{\gamma\delta}] = -i(\eta_{\alpha\gamma} L_{\beta\delta} + \eta_{\beta\delta} L_{\alpha\gamma} - \eta_{\alpha\delta} L_{\beta\gamma} - \eta_{\beta\gamma} L_{\alpha\delta}). \quad (8)$$

Within this framework, two Casimir operators emerge:

$$Q^{(1)} = -\frac{1}{2} L_{\alpha\beta} L^{\alpha\beta}, \quad Q^{(2)} = -W_\alpha W^\alpha, \quad (9)$$

where the $W_\alpha = -\frac{1}{8} \varepsilon_{\alpha\beta\gamma\delta\rho} L^{\beta\gamma} L^{\delta\rho}$, while $\varepsilon_{\alpha\beta\gamma\delta\rho}$ referees to the five-dimensional totally antisymmetric Levi-Civita symbol. The Casimir operators, by construction, commute with all generator representatives $L_{\alpha\beta}$, and hence, they remain constant across all states within a given dS UIR. Consequently, the classification of dS UIRs can be efficiently accomplished based on the respective eigenvalues of these Casimir operators [17–21].

To elaborate this classification, let us focus more technically on the specific context of dS scalar UIRs (relevant to the current study), realized in the Hilbert space of Klein-Gordon square-integrable functions $\varphi(x)$ defined on the dS spacetime manifold. In this context, the quartic Casimir operator $Q_0^{(2)1}$ vanishes and the quartic one explicitly reads as:

$$Q_0^{(1)} = -\frac{1}{2} M_{\alpha\beta} M^{\alpha\beta} = -H^{-2} \square_H, \quad (10)$$

where $\square_H = \partial_\alpha \partial^\alpha$ is the d'Alembertian operator. We symbolize its action on the states $\varphi(x)$ carrying a specified dS scalar UIR as:

$$Q_0^{(1)} \varphi(x) = (-H^{-2} \square_H \varphi(x)) = \langle Q_0^{(1)} \rangle \varphi(x), \quad (11)$$

where $\langle Q_0^{(1)} \rangle$ represents the corresponding Casimir eigenvalue. Accordingly, the classification of dS scalar UIRs can be effectively carried out based on the corresponding Casimir eigenvalues $\langle Q_0^{(1)} \rangle$ [19, 20]:

- The principal series $U_{0,\nu}$, characterized by $\langle Q_0^{(1)} \rangle = \frac{9}{4} + \nu^2$, where $\nu \in \mathbb{R}$. Note that, for a given ν , two representations $U_{0,\nu}$ and $U_{0,-\nu}$ are equivalent. In the null-curvature limit, these representations contract to the massive scalar UIRs of the Poincaré group, comprehensively covering the entire set of the latter and earning the designation of dS massive scalar representations [22, 23].
- The complementary series $V_{0,\nu}$, characterized by $\langle Q_0^{(1)} \rangle = \frac{9}{4} - \nu^2$, where $\nu \in \mathbb{R}$ and $0 < |\nu| < \frac{3}{2}$. Akin to the principal case, for any given ν within the allowed range, the representations $V_{0,\nu}$ and $V_{0,-\nu}$ are equivalent. Note that $V_{0,\nu=\frac{1}{2}}$ denotes the massless scalar representation of the dS group, characterized by its unique extension to the conformal group scalar UIR [24, 25]. Notably, this extension perfectly corresponds to the conformal extension of the massless scalar UIR of the Poincaré group [24, 25].
- The discrete series $\Pi_{p,0}$, characterized by $\langle Q_0^{(1)} \rangle = (p+2)(-p+1)$, where $p = 1, 2, \dots$.

Remark 1: All dS UIRs, not falling strictly into the massive or massless categories, either possess a nonphysical Poincaré contraction limit or lack such a limit altogether.

Remark 2: Practically speaking, in this group-theoretical context, the Hilbert space carrying a given dS scalar UIR is densely generated by all Klein-Gordon square-integrable eigenfunctions of the eigenvalue equation (11), specifically adapted to the respective Casimir eigenvalue. As a result, Eq. (11) assumes a fundamental role as the ‘field (wave) equation’.

Now, let us consider what serves as a universal replacement for the notion of mass within the framework of dS relativity. In addressing this significant query, we turn our attention to the Garidi mass formula [26]. This formula, formulated in terms of the invariant parameters that characterize the UIRs of the dS group, offers a consistent and precise definition of mass in dS relativity. It enables us to meaningfully distinguish between dS massive and massless fields by aligning them with their counterparts in flat Minkowski spacetime. Additionally, it holds the advantage of comprehensively encompassing all previously introduced mass formulas within the dS context (for further elucidation, see Refs. [14, 26]).

¹ Note that the subscript ‘0’ indicates that the carrier space is composed of scalar functions.

In technical terms, for a given spin- s dS UIR/field, the Garidi mass formula precisely represents the difference between the eigenvalue of the quadratic Casimir operator associated with the UIR/field and the eigenvalue of the quadratic Casimir operator for the corresponding massless spin- s scenario (up to the physical dimension). Let us delve deeper into this formula by exploring the case of the dS scalar, which holds particular relevance to our study:

$$\mathbf{m}_H^2 = H^2 \left(\langle Q_0^{(1)} \rangle - \langle Q_0^{(1)} \rangle_{\text{massless}} \right), \quad (12)$$

where $\langle Q_0^{(1)} \rangle$ denotes the Casimir eigenvalue linked with the scalar representation/field under scrutiny, and $\langle Q_0^{(1)} \rangle_{\text{massless}} = 2$ signifies the eigenvalue corresponding to the massless scalar representation $V_{0,\nu=\frac{1}{2}}$.

The Garidi mass definition, exemplified by the scalar case (12), holds significance across all strictly dS massless and massive UIRs/fields. By construction, it trivially reduces to zero for all massless UIRs/fields, and remains positive for any massive UIR/field, such that, in the null-curvature limit, consistently converges to the Minkowski mass m of the respective scalar, positive energy, Poincaré massive UIR $\mathcal{P}_{0,m}^>$. The latter point can be easily checked for the massive scalar case, by substituting $\langle Q_0^{(1)} \rangle = \frac{9}{4} + \nu^2$ ($\nu \in \mathbb{R}$) into the mass definition (12):

$$\mathbf{m}_H = H \left(\nu^2 + \frac{1}{4} \right)^{\frac{1}{2}} = H|\nu| \left(1 + \frac{1}{4\nu^2} \right)^{\frac{1}{2}}. \quad (13)$$

Then, by defining the asymptotic relation between the dS-invariant parameter ν and the Poincaré-Minkowski mass m as $m \sim H|\nu|$ and letting $H \rightarrow 0$ and $\nu \rightarrow \infty$, we derive the following under the null-curvature limit:

$$\mathbf{m}_H \rightarrow m. \quad (14)$$

Thus, at the limit, the Garidi mass \mathbf{m}_H arises as the proper mass m of the elementary particle under consideration.

As a matter of fact, if we accept Einstein's assertion that the proper mass of an elementary particle remains unaffected by the curvature of spacetime, we should regard the Garidi mass \mathbf{m}_H as equal to the proper mass m , regardless of the curvature. Essentially, there should be no discernible distinction between inertial and gravitational mass. This requirement implies that the asymptotic relationship mentioned above between the dS-invariant parameter ν and the Poincaré-Minkowski mass m , expressed as $m \sim H|\nu|$, should consistently be stated as:

$$\mathbf{m}_H = H|\nu| \left(1 + \frac{1}{4\nu^2} \right)^{\frac{1}{2}} = m. \quad (15)$$

See Ref. [27] for more detailed discussions.

Considering the context provided above, it is also tempting to briefly discuss the status of the Garidi mass

formula for those dS scalar UIRs/fields that do not strictly fall into the massive or massless categories. In the case of the complementary scalar UIRs/fields, excluding the strictly massless one $V_{0,\nu=\frac{1}{2}}$ where $\mathbf{m}_H = 0$, the bounded nature of the parameter ν labeling these UIRs/fields renders the definition of a Poincaré contraction limit technically unfeasible. Similarly, the same holds true for the discrete scalar UIRs/fields, as the labeling parameter p for each UIR/field remains fixed. This observation exemplifies another crucial point in our reasoning, as we elucidate below.

Remark 3: *The Garidi mass definition is significant as a universal proper mass only for dS UIRs/fields that hold meaning from the Minkowskian perspective. This includes both the dS massive and massless UIRs/fields. However, for those dS UIRs/fields lacking a meaningful Minkowskian interpretation, one can still apply the Garidi mass formula, albeit without a physical interpretation.*

With all the preceding information in mind, we are now poised to address the initial question raised concerning the 'tachyonic' interpretation of the scalar fields delineated by the (squared) 'mass' term (3). A direct comparison between the field equation (2), incorporating the (squared) 'mass' term (3), and its counterpart (11) (the latter derived within the context of dS group representations) promptly illustrates that the 'tachyonic' scalar fields align with the discrete scalar UIRs $\Pi_{p,0}$, where $p = 1, 2, \dots$, with the dS 'massless' minimally coupled scalar field corresponding to the 'lowest limit' of these representations $\Pi_{1,0}$. The Garidi (squared) mass for these UIRs/fields takes the following negative values:

$$\mathbf{m}_H^2 = H^2((p+2)(-p+1)-2), \quad (16)$$

where $p = 1, 2, \dots$. However, as already pointed out, apart from those dS UIRs/fields labeled as strictly massive or massless, the Garidi mass for all UIRs/fields does not represent a genuine physical mass in the sense given by Wigner, as it is not associated (either through contraction or conformal extension) with the ('true') mass invariant parameter that labels the Poincaré scalar UIRs. Frankly speaking, in these cases, it serves just as a parameter in the theory, devoid of further significance. Consequently, any physical interpretation derived from the real negative squared of this parameter, such as faster-than-light travel, is totally irrelevant. The same argument holds for the naming convention 'massless' minimally coupled scalar field, as its corresponding representation $\Pi_{1,0}$ deviates from the aforementioned strictly massless category of the dS UIRs — it is not, in fact, a massless field.

Here, it is crucial to underscore that the preceding statement does not imply that dS UIRs/fields without a Minkowskian counterpart hold no physical significance. On the contrary, it is entirely valid to explore all dS UIRs/fields within a unified framework, considering both mathematical (group representation) and physical (field quantization) perspectives, albeit without referenc-

ing concepts, such as mass, whose very existence depends on Poincaré-Minkowski relativity.

In the end, it is tempting to compare the two definitions of mass \mathbf{m}_H and m_H , particularly in the context of the dS scalar case. A straightforward evaluation of Eqs. (2), (11), and (12) unveils the following relationship:

$$\mathbf{m}_H^2 = m_H^2 - 2H^2. \quad (17)$$

Evidently, in the strictly massless scalar case $V_{0,\nu=\frac{1}{2}}$, where $\mathbf{m}_H = 0$, the nonvanishing nature of the ‘mass’ parameter m_H is misleading; notably, m_H diminishes to

zero in the scenario of the ‘massless’ minimally coupled scalar field!

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