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# Apparent temperature: demystifying the relation between quantum correlations, coherence, and heat flows

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Starting from the idea that energy exchanges are the essence of Thermodynamics, we show that out-of-equilibrium quantum systems are well characterised, at least when it comes to energy exchanges with external systems, by the introduction of apparent temperatures. We show that such apparent temperatures contain crucial information on the role of correlations and coherence in energy exchanges: both behave as populations. This can be traced back to the energy flows between out-of-equilibrium quantum systems. This counter-intuitive phenomena can affect dramatically the population balance and therefore the apparent temperatures. We show how seminal results can be re-obtained, offering an interesting alternative point of view. Our results show how genuine quantum effects can be used advantageously, finding applications in quantum thermal machine designs but also in collective-effect phenomena.

*Introduction.*—It has been a long standing question whether new properties of thermodynamics can arise from quantum phenomena and assist thermodynamic tasks. Answering this question could unlock quantum advantages in thermodynamics and potentially revolutionise energy science [1, 2]. Such perspective alimented intense research around the exploitation of two genuine quantum characteristics, namely quantum coherence and correlations. A theory of coherence as a resource was developed in [3–5]. More specifically, it has been reported that coherence can assist or enhance thermal engines [6–14] and work extraction [15–19]. Similarly, correlations can assist or enhance the performance of thermal machines [1, 2, 6–8, 20–28], quantum battery charging [29–32], work extraction [33–35], and energy transport [36]. A broader understanding of such phenomena requires to go back to the to the essence of Thermodynamics, namely energy exchanges, the common ground of any thermal machines. Some papers already looked at the relation between energy flows and correlations in some particular models of heat engines [10, 13], in a pair of two-level systems with cascaded bath interaction [37], and in a study of Landauer’s principle [38]. In a different context, a series of interesting works [39–43] investigate reversals of the natural heat flow (identified as the thermodynamic arrow of time) thanks to initial correlations *between* systems.

Here, we develop a broad framework valid for any system, and introduce the concept of apparent temperature which characterises the thermodynamic behaviour of out-of-equilibrium quantum systems and determine the ongoing heat flows. Its application to degenerated and many-body systems reveals how and why correlations and coherence present *within* the systems can impact and control the energy flows when interacting with other systems. Our results provide a deeper intuition and understanding of the interplay between correlations, coherence, and energy flows, essential for an efficient use of such resource.

*The model.*—We consider two systems  $S$  and  $R$ , possibly degenerated or composed of many (non-interacting) subsystems, with free Hamiltonian  $H_S$  and  $H_R$ . They interact through the coupling Hamiltonian  $H_{SR} = \lambda P_S P_R$ , where  $P_S$  and  $P_R$  are observables of  $S$  and  $R$  respectively, and  $\lambda$  characterises the strength of the coupling. For  $S$  and  $R$  initialised in *any states*, we want to investigate and characterise the energy exchange between them. This is a very challenging task in general. However, for short duration interactions, namely interaction time  $\tau$  much smaller than the evolution timescale generated by the coupling,  $|\lambda|^{-1}$ , the energy exchange can be put in a suitable form. In order to observe a significative evolution, we can consider repeated interactions between  $R$  and  $S$ , assuming that  $R$  is reinitialised in the same state or equivalently, is replaced by an identical system in the same initial state. This corresponds to the well-known collisional model [49, 50]. Such a model is well adapted to experimental descriptions [50] and is very versatile [49]. To derive the reduced dynamics of  $S$ , we use the eigenoperator decomposition [44] of the observables  $P_S$  and  $P_R$ , given by  $P_X = \sum_{\omega_X \in \mathcal{E}_X} A_X(\omega_X)$ , for  $X = S, R$ , where the eigenoperators (also called ladder operators)  $A_X(\omega_X)$  satisfies  $[H_X, A_X(\omega_X)] = -\omega_X A_X(\omega_X)$  ( $\hbar = 1$ ) and  $A_X(-\omega_X) = A_X^\dagger(\omega_X)$ . The frequency (positive and negative)  $\omega_X$  of  $\mathcal{E}_X$  represents the transition energy  $|\omega_X|$  associated to the ladder operator  $A_X(\omega_X)$ . We assume that although  $S$  and  $R$  can be complex systems, they still have discrete spectra. Then, for each transition frequency  $\omega_S$  of  $S$  we assume that for all transition frequencies  $\omega_R$  of  $R$  we have either  $|\omega_S - \omega_R|\tau \gg 1$ , either  $|\omega_S - \omega_R|\tau \ll 1$ . For the later situation we just consider that  $\omega_R = \omega_S$  (and if there are several such frequencies the ladder operator  $A_R(\omega_R)$  turns to denote the sum of all ladder operators of same frequency). We adopt the same convention for  $S$ . Under weak coupling conditions, namely  $|\lambda| \ll \min(|\omega_S|, |\omega_R|, |\omega_S - \omega_R|)$  for  $\omega_S \neq \omega_R$ , doing a second order expansion one can show (see details in

Supplemental Material [53]) that the reduced dynamics of  $S$  takes the form  $\rho_S(t + \tau) = (1 + \mathcal{L}_{t,\tau})\rho_S(t)$ , where

$$\begin{aligned} \mathcal{L}_{t,\tau}\rho_S &= -i\lambda\tau \sum_{\omega \in \mathcal{E}} [A_S(\omega), \rho_S] \langle A_R^\dagger(\omega) \rangle_{\rho_R^{Sc}(t)} e^{-i\omega t} \\ &\quad - \frac{\lambda^2 \tau^2}{2} \sum_{\omega, \omega' \in \mathcal{E}} \langle A_R^\dagger(\omega) A_R(\omega') \rangle_{\rho_R^{Sc}(t)} e^{i(\omega' - \omega)t} \\ &\quad \times [A_S(\omega) A_S^\dagger(\omega') \rho_S - A_S^\dagger(\omega') \rho_S A_S(\omega)] + \text{h.c.}, \end{aligned} \quad (1)$$

where  $\rho_S$  is the density matrix of  $S$  in the interaction picture,  $\rho_R^{Sc}(t)$  is the density matrix of  $R$  in the Schrodinger picture, and the sum runs over the frequencies  $\omega$  common to  $S$  and  $R$ ,  $\mathcal{E} = \mathcal{E}_R \cap \mathcal{E}_S$ . Considering an average of  $n = r\Delta t$  interactions with  $R$  during a time interval  $\Delta t \ll |\lambda|^{-1}$ , where  $r$  is the average interaction repetition rate, the coarse-grain derivative  $\dot{\rho}_S$  is

$$\dot{\rho}_S = \sum_{\omega \in \mathcal{E}} \frac{G(\omega)}{2} \left\{ [A_S(\omega)\rho_S, A_S^\dagger(\omega)] + [A_S(\omega), \rho_S A_S^\dagger(\omega)] \right\}, \quad (2)$$

where  $G(\omega) = r\lambda^2\tau^2 \text{Tr}[\rho_R A_R(\omega) A_R^\dagger(\omega)]$ . One should note that the effect of coherences between levels of different energy cancel out on average due to the random phase of such coherences at each interaction. In particular, the unitary contribution is null on average.

*Apparent temperatures.*—In the following we derive the expression of the energy flow between  $R$  and  $S$  using the dynamics (31). This brings us to introduce apparent temperatures. We define the internal energy of  $S$  as  $E_S := \text{Tr}\rho_S H_S$  (invariant from the Schrodinger picture to the interaction picture). The flow of energy from  $R$  to  $S$  is given by  $\dot{Q}_{S/R} := \dot{E}_S = \text{Tr}\dot{\rho}_S H_S$ , and is usually identified as heat since the unitary part of the evolution is not contributing. Using the expression (31) of  $\dot{\rho}_S$  the energy flow takes the form,  $\dot{Q}_{S/R} = -\sum_{\omega \in \mathcal{E}} \omega G(\omega) \langle A_S^\dagger(\omega) A_S(\omega) \rangle_{\rho_S}$ . The bracket  $\langle \mathcal{O} \rangle_{\rho_S}$  represents the expectation value of the operator  $\mathcal{O}$  taken in the state  $\rho_S$ . It is enlightening to consider first the situation where  $\rho_R$  is a thermal state at temperature  $T_R$ . Then, the following identity holds [53] (with  $\hbar = 1$ ,  $k_B = 1$ ),  $G(-\omega) = e^{-\omega/T_R} G(\omega)$ , and (31) takes exactly the form of a master equation generated by the interaction with a thermal bath. The corresponding equilibrium state for  $S$  is a thermal state at temperature  $T_R$  [44]. What happens when  $R$  is not in a thermal state? From the point of view of  $S$  the only difference is the value of the ratio  $G(-\omega)/G(\omega)$  which might change. This suggests introducing the parameters  $\mathcal{T}_R(\omega)$  defined through

$$e^{-\omega/\mathcal{T}_R(\omega)} := G(-\omega)/G(\omega) = \frac{\langle A_R^\dagger(\omega) A_R(\omega) \rangle_{\rho_R}}{\langle A_R(\omega) A_R^\dagger(\omega) \rangle_{\rho_R}}, \quad (3)$$

which acts as a temperature. Then, if  $S$  and  $R$  have only a single transition energy  $\omega$  in common, following the same reasoning as in the thermal case above,  $S$  equilibrates to a thermal state at a temperature *precisely*

equal to  $\mathcal{T}_R(\omega)$ . Seen from  $S$ ,  $R$  appears to be at the temperature  $\mathcal{T}_R(\omega)$ . For this reason, we call  $\mathcal{T}_R(\omega)$  the *apparent temperature* of  $R$ . If  $S$  and  $R$  have multiple transition energy in common, then an apparent temperature  $\mathcal{T}_R(\omega)$  is associated to each transition frequency  $\omega$  (taking note that  $\mathcal{T}_R(-\omega) = \mathcal{T}_R(\omega)$ ). Using this concept we can rewrite the energy flow in the form

$$\begin{aligned} \dot{Q}_{S/R} &= \sum_{\omega \geq 0} \omega G(\omega) \langle A_S(\omega) A_S^\dagger(\omega) \rangle_{\rho_S} \\ &\quad \times \left[ e^{-\omega/\mathcal{T}_R(\omega)} - e^{-\omega/\mathcal{T}_S(\omega)} \right], \end{aligned} \quad (4)$$

where we introduced, as for  $R$ ,  $\mathcal{T}_S(\omega) := \omega \left( \ln \frac{\langle A_S(\omega) A_S^\dagger(\omega) \rangle_{\rho_S}}{\langle A_S^\dagger(\omega) A_S(\omega) \rangle_{\rho_S}} \right)^{-1}$ , which defines, as for  $R$ , the apparent temperature of  $S$  (seen from  $R$ ) associated to the transition  $\omega$ . From (4), one can see the total energy flow between  $R$  and  $S$  as a sum of contributions from different channels of energy exchange  $\omega$ . Such channels correspond to the delocalised (i.e. from any level) absorption/emission of an energy  $\omega$  by  $S$  and  $R$ . For each channel, the sign of the energy flow from  $R$  to  $S$  is given by the sign of  $\mathcal{T}_R(\omega) - \mathcal{T}_S(\omega)$ , as it would be for the energy flow between two thermal states at temperature  $T = \mathcal{T}_R(\omega)$  and  $T = \mathcal{T}_S(\omega)$ , respectively. This is one more reason to consider  $\mathcal{T}_R(\omega)$  and  $\mathcal{T}_S(\omega)$  as the apparent temperature of the transition  $\omega$ . On top of that, when  $S$  or  $R$  is in a thermal state at temperature  $T$ , all apparent temperatures become equal to  $T$  (see SM [53]). Remarkably, the apparent temperature is the same as the one introduced in [45] to characterise the thermodynamic behaviour of systems powering thermal machines.

It is worth noticing that if one considers that  $S$  is interacting with a thermal bath  $B$  at temperature  $T_B$ , the corresponding master equation has the same form as (31) (see SM [53]). Consequently, the energy flow between  $B$  and  $S$  is the same as (4) substituting  $G(\omega)$  by the bath spectral density and  $\mathcal{T}_R$  by the bath temperature  $T_B$ . Then, the energy flows is determined by  $T_B$  and  $\mathcal{T}_S(\omega)$ . This provides another relevant situation involving the apparent temperature, on top of the one described above and the one presented in [45].

*Effects of coherence.*—We saw how heat exchanges are determined by the apparent temperature. We now investigate the relation between apparent temperatures and coherence. We focus our analysis on situations where  $S$  and  $R$  have only a single transition energy  $\omega$  in common. Consequently, the energy flow between  $S$  and  $R$  is characterised by only one apparent temperature  $\mathcal{T}_R$  and  $\mathcal{T}_S$  for each system (in the remainder of the text we drop the explicit dependence on  $\omega$  in  $\mathcal{T}_S$ ,  $\mathcal{T}_R$ , and  $A_R$ ). We consider in this paragraph a system  $X$  (it can be  $S$  or  $R$ ) of  $(N + 1)$  energy levels with degeneracy  $l_n$  for each level  $n$ ,  $n \in [0, N]$  (see Fig.1). In other words, the free Hamiltonian of  $X$  can be written as  $H_X = \sum_{n=0}^N \sum_{g=1}^{l_n} n\omega |n, g\rangle \langle n, g|$ , where the states  $|n, g\rangle$

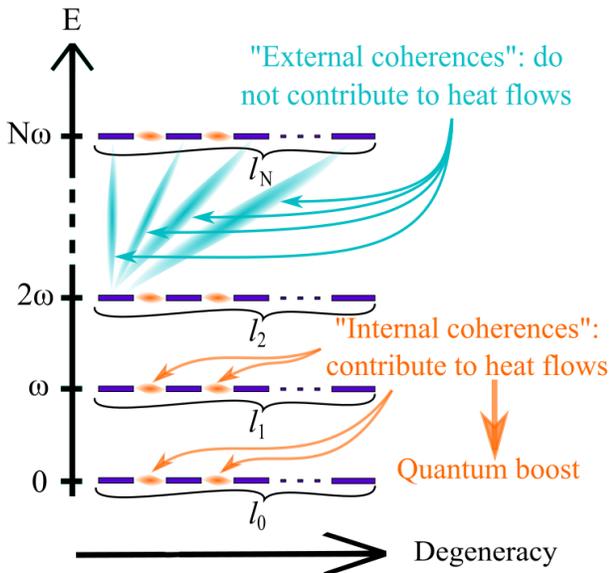


FIG. 1. Energy levels representation of a system of single transition energy  $\omega$ . The energy is represented vertically and the degeneracy, horizontally. “Internal coherences” (coherences between degenerated states) sum up to populations to contribute to heat flows, whereas “external coherences” (coherences between states of different energy) do not.

form an eigenbasis of  $H_X$ . The ladder operator  $A_X$  is of the form (using the expression given earlier),

$$A_X = \sum_{n=1}^N \sum_{g=1}^{l_{n-1}} \sum_{g'=1}^{l_n} \alpha_{n-1,n,g,g'} |n-1, g\rangle \langle n, g'|, \quad (5)$$

with  $\alpha_{n-1,n,g,g'} := \langle n-1, g | P_X | n, g' \rangle$ . In practice, the coefficients  $\alpha$  are often equal to 1 for all transitions and for simplicity we assume so in the remainder of the text as it does not change the nature of the results and simplify the expressions. Introducing (5) in (3), the apparent temperature is then equal to,

$$\mathcal{T}_X = \omega \left( \ln \frac{\sum_{n=1}^N l_n (\rho_{n-1} + c_{n-1})}{\sum_{n=1}^N l_{n-1} (\rho_n + c_n)} \right)^{-1}, \quad (6)$$

where  $\rho_n := \sum_{g=1}^{l_n} \langle n, g | \rho_X | n, g \rangle$  is the sum of the populations of the degenerated levels of energy  $n\omega$ , and  $c_n := \sum_{g \neq g' \in [1, l_n]} \langle n, g | \rho_X | n, g' \rangle$  is the sum of the coherences between these degenerated energy levels. We can learn two things from (6). First, *coherences behaves as populations* when it comes to heat exchange, and can change drastically the value of the apparent temperature since without them  $\mathcal{T}_X$  is reduced to  $\mathcal{T}_X = \omega \left( \ln \frac{\sum_{n=1}^N l_n \rho_{n-1}}{\sum_{n=1}^N l_{n-1} \rho_n} \right)^{-1}$  (see in [53] when coherences increase or reduce  $\mathcal{T}_X$ ). Secondly, only coherences between degenerated levels affect the apparent temperature (which could be expected since the contributions to the dynamics from coherence between levels of different

energy cancels out). In a different context the authors of [15] also observed a similar dichotomy: coherences between degenerated states, called “internal coherences”, affect the work extraction whereas coherences between states of different energy, called “external coherence”, do not. It would be interesting to investigate further how these two phenomena are related. As a consequence, coherences in non-degenerated systems cannot affect energy flows, but interesting effects of non-thermal population distributions can still persist (see [45]).

*Effects of correlations.*— We consider  $X$  (still designating either  $S$  or  $R$ ) as an ensemble of  $N$  non-interacting subsystems (not necessarily identical) of *same* transition energy  $\omega$ . One important condition we ask is that all the  $N$  subsystems experience the same environment, which usually requires confinement in a volume smaller than the typical variation length scale of that environment, like the electromagnetic emission wavelength in the Dicke model [46]. Upon the above conditions the ladder operator  $A_X$  is a *collective* ladder operator,  $A_X = \sum_{n=1}^N A_n$ , where  $A_n$  is the ladder operator of the subsystem  $n$  (with the same properties as above). Using the expression (3), the apparent temperature is,

$$\mathcal{T}_X = \omega \left( \ln \frac{\sum_{i=1}^m \langle A_i A_i^\dagger \rangle_{\rho_X} + c}{\sum_{i=1}^m \langle A_i^\dagger A_i \rangle_{\rho_X} + c} \right)^{-1}, \quad (7)$$

where  $c$  is the sum of the correlations between the  $N$  subsystems,  $c := \sum_{i \neq j=1}^m \langle A_i A_j^\dagger \rangle_{\rho_X} = \sum_{i \neq j=1}^m \langle A_i^\dagger A_j \rangle_{\rho_X}$ . As previously with the coherence, *correlations act as populations*. Their impact on the apparent temperature, and consequently on the energy flow, can be significant. To illustrate their effect we mention two examples. First, a pair of two-level atoms thermally entangled can have its temperature reduced by up to 50% due to the correlations (see [53]). Secondly, the correlations arising from a delocalised excitation in a cloud of  $N$  two-level atoms (Fig.2) rise the temperature of the cloud from  $\mathcal{T}_X = \frac{\omega}{\ln N-1} \simeq 0$  (for a localised excitation) to  $\mathcal{T}_X \simeq 1.5\omega$  for large  $N$  (whereas the thermal state of same temperature has an energy equal to  $N\omega/3$ ) [53]. One can also show that the correlations increase (reduce)  $\mathcal{T}_X$  if and only if  $c$  is positive (negative), given the individual subsystems are not in inverted-population states (otherwise it is the contrary) [53]. Taking forward the comparison with coherence, one can recast the states of the  $N$  subsystems in the same form as a single system with degenerated energy levels, see Fig.2 (exemplified with  $N$  two-level systems). Then, one can see that  $c$  is precisely the sum of coherences between levels of *same energy*, that is degenerated levels, recalling the previous observation on “internal coherences”. As a result, correlations between subsystems and internal coherences appear to be equivalent and have the same impact on apparent temperatures.

However, coherence is a genuine quantum feature, whereas correlations not necessarily (see for instance

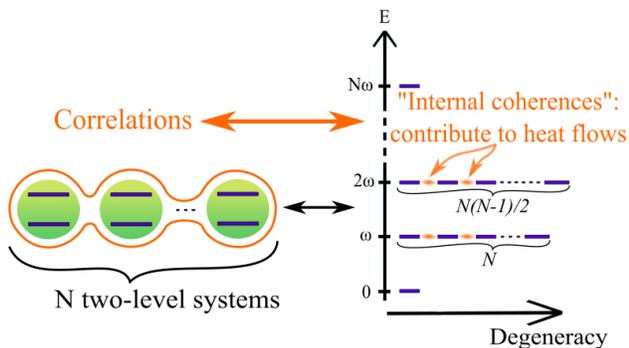


FIG. 2. Example of a cloud of  $N$  two-level systems. The cloud (left-hand side) can be described as a single system with degenerated energy levels (right-hand side). The correlations correspond to internal coherences in the energy levels representation, revealing an equivalence between them.

[8, 22]). Nevertheless, the alteration of energy flows due to correlations is a genuine quantum property. Indeed, for classical systems all operators commute,  $[A_X, A_X^\dagger] = 0$ , so that independently of the state of  $X$ ,  $\mathcal{T}_X$  is always equal to infinity (recovering a known property of classical systems [45, 48, 49]).

*Illustrative applications.*— In [22], a photonic engine is powered by a pair of two-level atoms. The pair plays the role of the system  $R$  repeatedly interacting with an optical cavity field (the working media of the engine) via the Tavis-Cummings coupling,  $H_{\text{int}} = -g(aS^\dagger + a^\dagger S^-)$ , with coupling constant  $g$ . The collective ladder operator  $S^-$  is given by  $S^- = |g\rangle_1\langle e| + |g\rangle_2\langle e|$ , where  $|e\rangle_i$  and  $|g\rangle_i$  are the excited and ground states of the atom  $i = 1, 2$ , respectively, and  $S^\dagger$  is given by the complex conjugated of  $S^-$ . The optical cavity mode is brought to equilibrium through repeated interactions with the pair of atoms. One central result of [22] is that the equilibrium temperature of the optical cavity is shown to be smaller when the pair of atoms is prepared in thermally entangled states. This is exploited to increase the efficiency of the engine.

Applying the expression (7) with  $A_i = |g\rangle_i\langle e|$ ,  $i = 1, 2$ , the apparent temperature of the pair of atoms is

$$\mathcal{T}_{\text{at}} = \omega \left( \ln \frac{\rho_g + \rho_d + \rho_{nd}}{\rho_e + \rho_d + \rho_{nd}} \right)^{-1}, \quad (8)$$

where  $\rho_g := \langle g, g | \rho | g, g \rangle$ ,  $\rho_e := \langle e, e | \rho | e, e \rangle$ ,  $\rho_d := (\langle e, g | \rho | g, e \rangle + \langle g, e | \rho | e, g \rangle) / 2$ , and  $\rho_{nd} := (\langle g, e | \rho | e, g \rangle + \langle e, g | \rho | g, e \rangle) / 2$ . The apparent temperature (8) is precisely the equilibrium temperature of the cavity mode found in [22] (where  $\langle e, g | \rho | g, e \rangle = \langle g, e | \rho | e, g \rangle$  and  $\langle g, e | \rho | e, g \rangle = \langle e, g | \rho | g, e \rangle$  due to the symmetric states used therein), and in [8] (when neglecting the cavity damping). This comes as a confirmation of our claim: correlations alter the heat flows bringing the cavity mode to *thermalise* at the altered apparent temperature (8). Furthermore, the correlation term for the kind

of states considered in [22] is  $c = -\frac{1}{Z} \sinh \lambda \beta$ , where  $\lambda$  is the coupling strength between the atoms,  $\beta$  is the underlying inverse temperature of the atoms, and  $Z$  is the partition function. Then  $c \leq 0$ , confirming our prediction that negative correlations decrease the apparent temperature.

Our second illustration concerns a three-level system in “ $\Lambda$ -configuration” as used in [18, 47]. The lower states  $|b\rangle$  and  $|c\rangle$  are (almost) degenerated and separated from the excited state  $|a\rangle$  by the transition energy  $\omega$ . The three-level system interacts repeatedly with an electromagnetic field of a cavity mode through the Jaynes-Cummings Hamiltonian,  $H_{RB} = \lambda(|b\rangle\langle a| + |c\rangle\langle a|)d^\dagger + \text{h.c.}$ , where  $d^\dagger$  is the creation operator of the cavity mode. The apparent temperature of the three-level system obtained from (6) with  $A_R = |b\rangle\langle a| + |c\rangle\langle a|$  is

$$\mathcal{T}_{\text{ph}} = \omega \left( \ln \frac{\rho_{bb} + \rho_{cc} + \rho_{bc} + \rho_{cb}}{2\rho_{aa}} \right)^{-1}, \quad (9)$$

where  $\rho_{xy} := \langle x | \rho_R | y \rangle$ ,  $x, y = a, b, c$ , and is increased by *negative* coherences (i.e.  $c_0 = \rho_{bc} + \rho_{cb} \leq 0$ ). In [8, 18, 47] the authors consider an ensemble of three-level atoms prepared in coherent states, the so-called phaseonium, interacting with an optical cavity mode. Their central result is that the steady state temperature of the cavity mode when reaching equilibrium with the phaseonium is affected by the existence of coherence between the state  $|b\rangle$  and  $|c\rangle$  of the atoms. By preparing such coherence with *negative* real part the steady state temperature of the cavity is significantly increased yielding an increase of the engine efficiency. This steady state temperature is precisely (9) (see [53]). Again, this confirms the predictions of our framework: even though the atoms are not in a thermal state, they appear to the cavity field at the apparent temperature (9) (increased by negative coherences), and the cavity field eventually thermalises at this temperature.

Additionally, the three-level system in “ $\Lambda$ -configuration” is also used in lasing without inversion [51]. One can find straightforwardly the essence of such phenomenon: the inversion of population usually required for lasers can be replaced by negative apparent temperature of the atoms since the effects of apparent temperatures are the same as usual temperatures. Then, as above, coherences prepared such that  $\rho_{bc} + \rho_{cb} < 0$  can assist atoms reaching an apparent negative temperature, or apparent inversion, realising lasing without inversion).

*Conclusion.*—Throughout this study we show that the concept of apparent temperature captures well the thermodynamic behaviour of out-of-equilibrium quantum systems when usual quantities like temperatures or free energy cannot be defined. In particular, it determines the heat flows between quantum systems. When applied to complex systems such as degenerated or many-body systems, the apparent temperature demystifies the tight relations between heat flows, correlations and coherence.

It reveals in particular that correlations and coherences between degenerated levels behave as populations when it comes to heat exchange, therefore modifying the populations balance and consequently the apparent temperatures. This is due to the collective interactions (promoted by the collective ladder operators  $A_X$ ) which make correlations and coherence between degenerated levels to contribute to heat flows. Such counter-intuitive phenomenon (correlations and coherences do not “bear” energy) is in sharp contrast with classical thermodynamics. Then, given that heat flows are at the heart of Thermodynamics, quantum Thermodynamics should be strikingly different from its classical counterpart. The dramatic impact of correlations and internal coherences on apparent temperatures is shown analytically and illustrated by two examples. Seminal results from the phaseonium and thermally entangled atoms are recovered by straight application of the concept of apparent temperature, confirming its relevance, and providing a unifying point of view different from usual interference-based interpreta-

tions.

Importantly, our results show that such phenomena are universal (not limited to a particular system, configuration, or platform). Our framework provides an intuitive understanding of the interplay between correlations, coherence, and energy flows, essential for the quest for quantum advantages in thermodynamic tasks, and potentially useful also in out-of-equilibrium thermodynamic problems and other correlation or coherence-based phenomena like superradiance or Electromagnetic-Induced-Transparency cooling [52].

## ACKNOWLEDGMENTS

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## SUPPLEMENTAL MATERIAL

### I. REPEATED INTERACTIONS/COLLISIONAL MODEL

At the instant of time  $t$  the system  $S$ , found in the state  $\rho_S^{Sc}(t)$ , interacts for a duration  $\tau$  with the system  $R$  in the state  $\rho_R^{Sc}(t)$ . The superscript  $Sc$  stands for Schrodinger picture. The evolution of  $SR$  between  $t$  and  $t + \tau$  is given by

$$\begin{aligned} \rho_{SR}^{Sc}(t + \tau) &= e^{-i\tau H_{\text{Tot}}} \rho_S^{Sc}(t) \rho_R^{Sc}(t) e^{i\tau H_{\text{Tot}}} \\ &= e^{-i\tau(H_S + H_R)} e^{-i\overleftarrow{T} \int_0^\tau du \tilde{H}_{SR}(u)} \rho_S^{Sc}(t) \rho_R^{Sc}(t) e^{i\overrightarrow{T} \int_0^\tau du \tilde{H}_{SR}(u)} e^{i\tau(H_S + H_R)} \end{aligned} \quad (10)$$

where  $H_{\text{Tot}} = H_S + H_R + H_{SR}$  is the total Hamiltonian,  $\overrightarrow{T}$  and  $\overleftarrow{T}$  are respectively the time ordering and anti-chronological ordering operators, and  $\tilde{H}_{SR}(u) = e^{iu(H_S + H_R)} H_{SR} e^{-iu(H_S + H_R)}$  stands for the interaction picture of  $H_{SR}$ . Assuming the duration of the interaction  $\tau$  is much smaller than the evolution timescale induced by the coupling  $H_{SR}$ , namely  $\tau \ll |\lambda|^{-1}$ , we can expand the time ordered integrals which yields

$$\begin{aligned} e^{i\tau(H_S + H_R)} \rho_{SR}^{Sc}(t + \tau) e^{-i\tau(H_S + H_R)} &= \rho_S^{Sc}(t) \rho_R^{Sc}(t) - i \int_0^\tau du [\tilde{H}_{SR}(u), \rho_S^{Sc}(t) \rho_R^{Sc}(t)] \\ &\quad - \int_0^\tau du_1 \int_0^{u_1} du_2 [\tilde{H}_{SR}(u_1), [\tilde{H}_{SR}(u_2), \rho_S^{Sc}(t) \rho_R^{Sc}(t)]] + \mathcal{O}(\tau^3 |\lambda|^3), \end{aligned} \quad (11)$$

and for the reduced dynamics of  $S$  (tracing out  $R$ ),

$$\begin{aligned} e^{i\tau H_S} \rho_S^{Sc}(t + \tau) e^{-i\tau H_S} &= \rho_S^{Sc}(t) - i \int_0^\tau du \text{Tr}_R [\tilde{H}_{SR}(u), \rho_S^{Sc}(t) \rho_R^{Sc}(t)] \\ &\quad - \int_0^\tau du_1 \int_0^{u_1} du_2 \text{Tr}_R [\tilde{H}_{SR}(u_1), [\tilde{H}_{SR}(u_2), \rho_S^{Sc}(t) \rho_R^{Sc}(t)]] + \mathcal{O}(\tau^3 |\lambda|^3), \end{aligned} \quad (12)$$

To provide a simpler expression for the reduce dynamics of  $S$  we rewrite the coupling Hamiltonian  $H_{SR} = \lambda P_S P_R$  in terms of the eigenoperators of  $S$  and  $R$  (also called ladder operators)  $A_S(\omega_S)$  and  $A_R(\omega_R)$  which verify the relations  $[H_X, A_X(\omega_X)] = -\omega_X A_X(\omega_X)$  ( $\hbar = 1$ ) and  $A_X(-\omega_X) = A_X^\dagger(\omega_X)$  for  $X = S, R$ . Such eigenoperators can be obtained from the operators  $P_S$  and  $P_R$  through [44]  $A_X(\omega_X) := \sum_{\epsilon' = -\epsilon = \omega_X} \pi_\epsilon P_X \pi_{\epsilon'}$ ,  $X = S, R$ , where  $\pi_\epsilon$  is the projector onto the eigenspace associated with the eigenvalue  $\epsilon$  of  $H_X$ . Then, the operator  $P_X$  can be written as  $P_X = \sum_{\omega_X \in \mathcal{E}_X} A_X(\omega_X) = \sum_{\omega_X \in \mathcal{E}_X} A_X^\dagger(\omega_X) = \sum_{\omega_X > 0} [A_X(\omega_X) + A_X^\dagger(\omega_X)]$ , where  $\mathcal{E}_X$  denotes the ensemble of frequencies equal to  $\pm\omega_X$  for each transition energy  $\omega_X$  of the system  $X = S, R$ . The coupling Hamiltonian can be

re-written as

$$H_{SR} = \lambda P_S P_R = \lambda \sum_{\omega_S \in \mathcal{E}_S, \omega_R \in \mathcal{E}_R} A_S(\omega_S) A_R(\omega_R) = \lambda \sum_{\omega_S \in \mathcal{E}_S, \omega_R \in \mathcal{E}_R} A_S(\omega_S) A_R^\dagger(\omega_R), \quad (13)$$

and in the interaction picture it is simply  $\tilde{H}_{SR}(u) = \lambda \sum_{\omega_S \in \mathcal{E}_S, \omega_R \in \mathcal{E}_R} e^{i(\omega_R - \omega_S)u} A_S(\omega_S) A_R^\dagger(\omega_R)$ . The term of first order in the expansion (12) becomes

$$-i \int_0^\tau du \text{Tr}_R[\tilde{H}_{SR}(u), \rho_S^{Sc}(t) \rho_R^{Sc}(t)] = -i\lambda \sum_{\omega_S \in \mathcal{E}_S} [A_S(\omega_S), \rho_S^{Sc}(t)] \sum_{\omega_R \in \mathcal{E}_R} \text{Tr}_R A_R^\dagger(\omega_R) \rho_R^{Sc}(t) \int_0^\tau du e^{i(\omega_R - \omega_S)u}. \quad (14)$$

The integral is equal to  $\int_0^\tau du e^{i(\omega_R - \omega_S)u} = \tau e^{i(\omega_R - \omega_S)\tau/2} \text{sinc}(\omega_R - \omega_S)\tau/2$  which is approximately equal to  $\tau$  when  $|\omega_R - \omega_S|\tau \ll 1$  and much smaller than  $\tau$  when  $|\omega_R - \omega_S|\tau \gg 1$ . Although we allow  $S$  and  $R$  to be complex systems we assume they have discrete spectra, and that there exists only two kinds of frequencies, those such that  $|\omega_R - \omega_S|\tau \ll 1$  and those such that  $|\omega_R - \omega_S|\tau \gg 1$ . For a given transition frequency  $\omega_S \in \mathcal{E}_S$ , all frequencies  $\omega_R \in \mathcal{E}_R$  such that  $|\omega_R - \omega_S|\tau \ll 1$  are assumed to be equal since they are not resolved by  $\tau$ , and the ladder operator  $A_R(\omega_R)$  turns to designate the sum of all ladder operators of same transition frequency  $\omega_R$ . We adopt the same convention for the transition frequencies of  $S$ , namely, for any given transition frequency  $\omega_R \in \mathcal{E}_R$ , all frequencies  $\omega_S \in \mathcal{E}_S$  such that  $|\omega_R - \omega_S|\tau \ll 1$  are assumed to be equal since they are not resolved by  $\tau$ , and the ladder operator  $A_S(\omega_S)$  turns to designate the sum of all ladder operators of same transition frequency  $\omega_S$ . Under such conventions, for any  $\omega_S \in \mathcal{E}_S$ , there exists at most one unique transition frequency  $\omega_R \in \mathcal{E}_R$  such that  $\omega_R = \omega_S$ . We denote the intersection of the two ensembles of frequencies by  $\mathcal{E} := \mathcal{E}_S \cap \mathcal{E}_R$  and by  $\omega$  the frequencies in it. The term of first order is then simplified to

$$-i \int_0^\tau du \text{Tr}_R[\tilde{H}_{SR}(u), \rho_S^{Sc}(t) \rho_R^{Sc}(t)] = -i\lambda\tau \sum_{\omega \in \mathcal{E}} [A_S(\omega), \rho_S^{Sc}(t)] \langle A_R^\dagger(\omega) \rangle_{\rho_R^{Sc}(t)}, \quad (15)$$

where  $\langle \mathcal{O} \rangle_\rho = \text{Tr} \mathcal{O} \rho$  denotes the expectation value of the operator  $\mathcal{O}$  taken in the state  $\rho$ .

The second order term appearing on the right-hand side of (12) gives

$$\begin{aligned} & - \int_0^\tau du_1 \int_0^{u_1} du_2 \text{Tr}_R[\tilde{H}_{SR}(u_1), [\tilde{H}_{SR}(u_2), \rho_S^{Sc}(t) \rho_R^{Sc}(t)]] \\ & = - \sum_{\omega_S, \omega'_S} \gamma_{\omega_S, \omega'_S}(\tau) [A_S(\omega_S) A_S^\dagger(\omega'_S) \rho_S^{Sc}(t) - A_S^\dagger(\omega'_S) \rho_S^{Sc}(t) A_S(\omega_S)] + \text{h.c.}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} \gamma_{\omega_S, \omega'_S}(t, \tau) &= \lambda^2 \sum_{\omega_R, \omega'_R} \langle A_R^\dagger(\omega_R) A_R(\omega'_R) \rangle_{\rho_R^{Sc}(t)} \int_0^\tau du_1 \int_0^{u_1} du_2 e^{i(\omega_R - \omega_S)u_1} e^{i(\omega'_R - \omega'_S)u_2} \\ &= \lambda^2 \sum_{\omega_R, \omega'_R} \langle A_R^\dagger(\omega_R) A_R(\omega'_R) \rangle_{\rho_R^{Sc}(t)} \frac{i\tau}{\omega'_R - \omega'_S} e^{i(\omega_R - \omega_S - \omega'_R + \omega'_S)\tau/2} \\ &\quad \times \left[ \text{sinc}(\omega_R - \omega_S - \omega'_R + \omega'_S)\tau/2 - e^{i(\omega'_R - \omega'_S)\tau/2} \text{sinc}(\omega_R - \omega_S)\tau/2 \right] \\ &= \frac{\tau^2 \lambda^2}{2} \langle A_R^\dagger(\omega_S) A_R(\omega'_S) \rangle_{\rho_R^{Sc}(t)}. \end{aligned} \quad (17)$$

The last line was obtained using the convention established above. Then, for each  $\omega_S$ , there is only one  $\omega_R$  such that  $\omega_R = \omega_S$ , and the others are such that  $|\omega_S - \omega_R|\tau \gg 1$ . It implies that for  $\omega_R$  and  $\omega'_R$  different from  $\omega_S$  and  $\omega'_S$ ,

$$\left| \int_0^\tau du_1 \int_0^{u_1} du_2 e^{i(\omega_R - \omega_S)u_1} e^{i(\omega'_R - \omega'_S)u_2} \right| \ll \frac{\tau}{|\omega'_R - \omega'_S|} \ll \tau^2, \quad (18)$$

yielding the last line of (17). Finally, the reduce dynamics of  $S$  becomes

$$\begin{aligned} e^{i\tau H_S} \rho_S^{Sc}(t + \tau) e^{-i\tau H_S} &= \rho_S^{Sc}(t) - i\lambda\tau \sum_{\omega \in \mathcal{E}} [A_S(\omega), \rho_S^{Sc}(t)] \langle A_R^\dagger(\omega) \rangle_{\rho_R^{Sc}(t)} \\ &\quad - \frac{\lambda^2 \tau^2}{2} \sum_{\omega, \omega' \in \mathcal{E}} \langle A_R^\dagger(\omega) A_R(\omega') \rangle_{\rho_R^{Sc}(t)} [A_S(\omega) A_S^\dagger(\omega') \rho_S^{Sc}(t) - A_S^\dagger(\omega') \rho_S^{Sc}(t) A_S(\omega)] + \text{h.c.} + \mathcal{O}(\tau^3 |\lambda|^3). \end{aligned} \quad (19)$$

It is now convenient to go to the interaction picture, which we defined as  $\rho_S(t) := e^{iH_S t} \rho_S^{Sc}(t) e^{-iH_S t}$ . In this picture, the reduced evolution of  $S$  for a time interval  $\tau$  can be written as

$$\rho_S(t + \tau) = (1 + \mathcal{L}_{t,\tau}) \rho_S(t), \quad (20)$$

where

$$\begin{aligned} \mathcal{L}_{t,\tau} \rho &= -i\lambda\tau \sum_{\omega \in \mathcal{E}} [A_S(\omega), \rho] \langle A_R^\dagger(\omega) \rangle_{\rho_R^{Sc}(t)} e^{-i\omega\tau} \\ &\quad - \frac{\lambda^2 \tau^2}{2} \sum_{\omega, \omega' \in \mathcal{E}} \langle A_R^\dagger(\omega) A_R(\omega') \rangle_{\rho_R^{Sc}(t)} e^{i(\omega' - \omega)\tau} [A_S(\omega) A_S^\dagger(\omega') \rho - A_S^\dagger(\omega') \rho A_S(\omega)] + \text{h.c.} \end{aligned} \quad (21)$$

So far, we expressed the evolution of  $S$  due to its interaction with  $R$  for a duration  $\tau$  (such that  $\tau \gg |\omega_R - \omega_S|^{-1}$  for all  $\omega_R \neq \omega_S$ ). We consider that the evolution of  $S$  is made of a sequence of interactions with  $R$  separated by interval of free evolution. Denoting by  $t_i$  the instant of time when  $S$  and  $R$  start interacting, and by  $\tau_i$  the subsequent free evolution time interval (so that  $t_{i+1} = t_i + \tau + \tau_i$ ), the state of  $S$  at  $t_n = t + \Delta t$  after  $n$  such sequences is

$$\rho_S(t + \Delta t) = (1 + \mathcal{L}_{t_{n-1},\tau}) \dots (1 + \mathcal{L}_{t_0,\tau}) \rho_S(t) \quad (22)$$

where  $t_0 = t$ . Note that the intermediary free evolution does not affect the dynamics in interaction picture,  $\rho_S(t + \tau + \tau_i) = \rho_S(t + \tau) = (1 + \mathcal{L}_{t,\tau}) \rho_S(t)$ . Since  $\tau$  is much smaller than  $\lambda^{-1}$  the action of the operators  $\mathcal{L}_{t_i,\tau}$  is small,  $|\mathcal{L}_{t_i,\tau} \rho_S(t)| \ll 1$ , so that

$$\begin{aligned} \rho_S(t + \Delta t) &\simeq (1 + \mathcal{L}_{t_{n-1},\tau} + \dots + \mathcal{L}_{t_0,\tau}) \rho_S(t) \\ &= \rho_S(t) - i\lambda\tau \sum_{\omega \in \mathcal{E}} [A_S(\omega), \rho_S(t)] \sum_{i=0}^{n-1} \langle A_R^\dagger(\omega) \rangle_{\rho_R^{Sc}(t_i)} e^{-i\omega t_i} \\ &\quad - \frac{\lambda^2 \tau^2}{2} \sum_{\omega, \omega' \in \mathcal{E}} [A_S(\omega) A_S^\dagger(\omega') \rho_S(t) - A_S^\dagger(\omega') \rho_S(t) A_S(\omega)] \sum_{i=0}^{n-1} \langle A_R^\dagger(\omega) A_R(\omega') \rangle_{\rho_R^{Sc}(t_i)} e^{i(\omega' - \omega) t_i} + \text{h.c.} \end{aligned} \quad (23)$$

The above expression can be simplified if we assume that  $R$  is always prepared in the same state  $\rho_R$ . This brings us to the problem of the phase: coherences between levels of different energy accumulated a phase (even if  $R$  is kept perfectly isolated). This phase depends on the "history" of each system, which appears in the terms  $\langle A_R^\dagger(\omega) \rangle_{\rho_R^{Sc}(t_i)}$  and  $\langle A_R^\dagger(\omega) A_R(\omega') \rangle_{\rho_R^{Sc}(t_i)}$ . Then, unless  $R$  is reinitialised with a well defined phase reference, or alternatively, identical systems  $R$  are prepared and maintained coherent (which both are not happening in "natural" systems and are highly demanding experimentally) the coherence phases are random. As a result the unitary contribution (term of first order) in (23) is null on average. Similarly, contributions from second order terms with  $\omega \neq \omega'$  are also null on average. One should note that it turns to be equivalent to assume that  $R$  is prepared in states such that  $\langle A_R(\omega_R) \rangle = 0$ . The above considerations provide a physical motivation for this assumption. We are thus left with

$$\rho_S(t + \Delta t) = \rho_S(t) - n \frac{\lambda^2 \tau^2}{2} \sum_{\omega \in \mathcal{E}} \langle A_R^\dagger(\omega) A_R(\omega) \rangle_{\rho_R} [A_S(\omega) A_S^\dagger(\omega) \rho_S(t) - A_S^\dagger(\omega) \rho_S(t) A_S(\omega)] + \text{h.c.} \quad (24)$$

On average  $S$  interacts  $n = r\Delta t$  times with  $R$  during a time interval  $\Delta t$ , where  $r$  is the rate of repetition of the interactions. The coarse-grain time derivative of  $\rho_S$  defined by  $\dot{\rho}_S = \frac{1}{\Delta t} [\rho_S(t + \Delta t) - \rho_S(t)]$  is then

$$\dot{\rho}_S = r \mathcal{L}_\tau \rho_S(t), \quad (25)$$

with the operator  $\mathcal{L}_\tau$  can be rewritten in the form

$$\mathcal{L}_\tau \rho_S(t) = \frac{\tau^2 \lambda^2}{2} \sum_{\omega \in \mathcal{E}} \langle A_R(\omega) A_R^\dagger(\omega) \rangle_{\rho_R} \left\{ [A_S(\omega) \rho_S, A_S^\dagger(\omega)] + [A_S(\omega), \rho_S A_S^\dagger(\omega)] \right\}. \quad (26)$$

This is the master equation indicated in the main text with  $G(\omega) := r\tau^2 \lambda^2 \text{Tr}_R [\rho_R A_R(\omega) A_R^\dagger(\omega)]$ . One should note that from Eq. (33) of Section III the relation used in the main text,

$$G(-\omega) = e^{-\omega/T_R} G(\omega), \quad (27)$$

comes out straightforwardly ( $\hbar = 1$ ,  $k_B = 1$ ).

## II. THERMAL BATHS

We consider here that  $S$  is interacting with a thermal bath  $B$  in an unitary way through the following Hamil-

tonian,

$$H_{Tot} = H_S + H_B + H_{SB}, \quad (28)$$

where  $H_S$  and  $H_B$  are the free Hamiltonians of  $S$  and  $H$ , respectively. Importantly,  $S$  can be composed of a single system or many non-interacting subsystems. The coupling term is of the form  $H_{SB} = \lambda P_S P_B$ ,  $P_S$  and  $P_B$  are operators of  $S$  and  $B$ , respectively, and  $\lambda$  is a real coupling constant. We consider that  $S$  has a discrete spectrum, and denoting by  $\epsilon$  its eigenvalues and by  $\pi_\epsilon$  the projector onto the eigenspace associated with  $\epsilon$  we can decompose  $P_S$  in a sum of eigenoperators of  $H_S$  [44],

$$P_S = \sum_{\omega} A_S(\omega), \quad (29)$$

where the ladder operators

$$A_S(\omega) := \sum_{\epsilon' - \epsilon = \omega} \pi_\epsilon P_S \pi_{\epsilon'}, \quad (30)$$

satisfy the commutation relation  $[H_S, A_S(\omega)] = -\omega A_S(\omega)$ , (setting  $\hbar = 1$ ) as mentioned in the main text. The sum runs over the transition frequencies  $\omega$  positive and negative. Furthermore, we assume that  $S$  and  $B$  are interacting weakly so that the Born and Markovian approximations are legitimate and following the usual procedure a Gorini-Kossakowski-Lindblad-Sudarshan (GKLS) quantum master equation [54] for the reduced dynamics of  $S$  is derived [44],

$$\begin{aligned} \dot{\rho}_S = \sum_{\omega} \Gamma(\omega) \left[ A_S(\omega) \rho_S A_S^\dagger(\omega) - A_S^\dagger(\omega) A_S(\omega) \rho_S \right] \\ + \text{h.c.}, \end{aligned} \quad (31)$$

where  $\rho_S$  is the density operator of  $S$  in the interaction picture (with respect to  $H_S$ ) and  $\dot{\rho}_S$  its time derivative. The complex function  $\Gamma(\omega)$  is given by  $\Gamma(\omega) := \lambda^2 \int_0^\infty ds e^{i\omega s} \text{Tr}[\rho_B P_B^I(s) P_B]$ , with  $P_B^I(s) = e^{iH_B s} P_B e^{-iH_B s}$  is the bath coupling operator in the interaction picture. This is the same master equation as Eq. (1) of the main text, substituting  $\frac{1}{2}G(\omega)$  by  $\Gamma(\omega)$ . As a conclusion, the energy flow between  $B$  and  $S$  is the same as for  $R$  (Eq. (3) of the main text) substituting  $G(\omega)$  by the bath spectral density  $G_{\text{bath}}(\omega) := \Gamma(\omega) + \Gamma^*(\omega)$  (which also satisfies (27), proof in the next Section) and the apparent temperature  $T_R$  by the bath temperature  $T_B$ .

### III. APPARENT TEMPERATURES FOR THERMAL STATES

When  $X = R$ ,  $S$  is in a thermal state at temperature  $T_X$ ,  $\rho_X = Z^{-1} e^{-H_X/T_X}$ , where  $Z$  is the partition function, one can derive the following identity,

$$\begin{aligned} e^{H_X/T_X} A_X(\omega) e^{-H_X/T_X} &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{T_X^n} \text{Ad}_{H_X}^n A_X(\omega) \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{T_X^n} (-\omega)^n A_X(\omega) \\ &= e^{-\omega/T_X} A_X(\omega), \end{aligned} \quad (32)$$

where  $\text{Ad}_M^n N := [M, \text{Ad}_M^{n-1} N]$  and  $\text{Ad}_M^0 N = N$ . We recall that the ladder operators are such that  $[H_X, A_X(\omega)] = -\omega A_X(\omega)$  so that  $\text{Ad}_{H_X}^n A_X(\omega) = (-\omega)^n A_X(\omega)$ . From (32) one obtains,

$$\begin{aligned} \langle A_X^\dagger(\omega) A_X(\omega) \rangle_{\rho_X} &= \text{Tr} \left[ Z^{-1} e^{-H_X/T_X} A_X^\dagger(\omega) A_X(\omega) \right] \\ &= Z^{-1} \text{Tr} \left[ A_X^\dagger(\omega) e^{-H_X/T_X} e^{H_X/T_X} A_X(\omega) \right. \\ &\quad \left. \times e^{-H_X/T_X} \right] \\ &= Z^{-1} \text{Tr} \left[ A_X^\dagger(\omega) e^{-H_X/T_X} e^{-\omega/T_X} A_X(\omega) \right] \\ &= e^{-\omega/T_X} \langle A_X(\omega) A_X^\dagger(\omega) \rangle_{\rho_X}. \end{aligned} \quad (33)$$

Substituting in the expression of the apparent temperature (Eq. (2) of the main text), one obtains  $T_X(\omega) = T_X$  for all transition  $\omega$ . Importantly, this also shows that  $G_{\text{bath}}$  (see the previous Section II) satisfies the thermal identity (27) as well as  $G$  when  $\rho_R$  is a thermal state.

### IV. APPARENT TEMPERATURE OF THE PHASEONIUM

In Eq. (5) of [18], the average photon number of the cavity  $n_\phi$  is given to follow the dynamics

$$\dot{n}_\phi = \alpha [2\rho_{aa}(n_\phi + 1) - (\rho_{bb} + \rho_{cc} + \rho_{bc} + \rho_{cb})n_\phi], \quad (34)$$

where  $\rho_{xy} := \langle x | \rho_R | y \rangle$ ,  $x, y = a, b, c$ , and  $\alpha$  is a simple rate factor. The steady state average photon number is obtained by taking  $\dot{n}_\phi = 0$  in the previous equation, leading to

$$\frac{n_\phi + 1}{n_\phi} = \frac{\rho_{bb} + \rho_{cc} + \rho_{bc} + \rho_{cb}}{2\rho_{aa}}. \quad (35)$$

The steady state temperature  $T_\phi$  of the cavity is related to the average photon number by  $n_\phi = (e^{\Omega/k_B T_\phi} - 1)^{-1}$ , where  $\Omega$  is the frequency of the cavity. Substituting in (35) one obtains

$$k_B T_\phi = \Omega \left( \ln \frac{\rho_{bb} + \rho_{cc} + \rho_{bc} + \rho_{cb}}{2\rho_{aa}} \right)^{-1}, \quad (36)$$

which is precisely the expression obtained in Eq. (10) of the main text (substituting  $\Omega$  by  $\omega$ ). One should note that in order to serve better the purpose of the article the expression of  $T_\phi$  used in [18] is an approximated version of the above expression.

### V. INCREASE AND DECREASE OF THE APPARENT TEMPERATURE BY COHERENCE

From Eq. (5) of the main text, the apparent temperature is increased by the coherence if and only if

$$\frac{\sum_{n=1}^N l_n (\rho_{n-1} + c_{n-1})}{\sum_{n=1}^N l_{n-1} (\rho_n + c_n)} \leq \frac{\sum_{n=1}^N l_n \rho_{n-1}}{\sum_{n=1}^N l_{n-1} \rho_n}, \quad (37)$$

which is equivalent to the condition

$$\langle A_S A_S^\dagger \rangle_0 \sum_{n=1}^N l_{n-1} c_n - \langle A_S^\dagger A_S \rangle_0 \sum_{n=0}^{N-1} l_{n+1} c_n \geq 0, \quad (38)$$

where the subscript 0 indicates that the expectation values are taken in the decohered state, namely,  $\langle A_S A_S^\dagger \rangle_0 := \sum_{n=1}^N l_n \rho_{n-1}$ , and  $\langle A_S^\dagger A_S \rangle_0 := \sum_{n=1}^N l_{n-1} \rho_n$ . For the three-level system in the  $\Lambda$ -configuration considered in the main text we have that the coherence increases the apparent temperature if and only if  $c_0$  is negative, which can also be verified directly from Eq. (8) of the main text.

It is worth noting that we use the expression ‘‘increase the apparent temperature’’ in a broad sense, meaning ‘‘hotter’’: if the temperature is negative (like inverted-population states), an increase of the temperature corresponds in fact to an increase of its absolute value, so that the resulting state is ‘‘hotter’’.

## VI. INCREASE AND DECREASE OF THE APPARENT TEMPERATURE BY CORRELATIONS

From Eq. (6) of the main text, the apparent temperature is increased by the correlations if and only if

$$\frac{\sum_{i=1}^m \langle A_i A_i^\dagger \rangle_{\rho_R} + c}{\sum_{i=1}^m \langle A_i^\dagger A_i \rangle_{\rho_R} + c} \leq \frac{\sum_{i=1}^m \langle A_i A_i^\dagger \rangle_{\rho_R}}{\sum_{i=1}^m \langle A_i^\dagger A_i \rangle_{\rho_R}}, \quad (39)$$

which is equivalent to the condition

$$c \sum_{i=1}^m (\langle A_i A_i^\dagger \rangle_{\rho_R} - \langle A_i^\dagger A_i \rangle_{\rho_R}) \geq 0. \quad (40)$$

If  $\langle A_i A_i^\dagger \rangle_{\rho_R} \geq \langle A_i^\dagger A_i \rangle_{\rho_R}$  (meaning that the individual subsystems are not in inverted-population states), the correlations increase (reduce) the apparent temperature if and only if  $c$  is positive (negative). If the individual subsystems are in inverted-population states,  $\langle A_i A_i^\dagger \rangle_{\rho_R} \leq \langle A_i^\dagger A_i \rangle_{\rho_R}$ , then it is the contrary: the correlations increase (reduce) the apparent temperature if and only if  $c$  is negative (positive).

As in the previous Section, it is worth noting that we use the expression ‘‘increase the apparent temperature’’ in a broad sense, meaning ‘‘hotter’’: if the temperature is negative (like inverted-population states), an increase of the temperature corresponds in fact to an increase of its absolute value, so that the resulting state is ‘‘hotter’’.

## VII. REDUCTION OF TEMPERATURE BY 50% DUE TO THERMAL CORRELATIONS

As an illustration of the power of correlations we give a simple example of a pair of two-level atoms interacting

via the XY-coupling, also used in [22] and discussed in the main text,

$$H_{int} = -g(aS^\dagger + a^\dagger S^-), \quad (41)$$

with coupling constant  $g$ . The collective ladder operator  $S^-$  is given by  $S^- = |g\rangle_1 \langle e| + |g\rangle_2 \langle e|$ , where  $|e\rangle_i$  and  $|g\rangle_i$  are the excited and ground states of the atom  $i = 1, 2$ , respectively, and  $S^\dagger$  is given by the complex conjugated of  $S^-$ . We assume that the pair of atoms is prepared in an thermal entangled state at temperature  $T$ ,  $\rho = Z^{-1} e^{-\beta H}$  [55, 56], where  $H = \omega S_1^z + \omega S_2^z + H_{int}$ ,  $S_i^z = \frac{1}{2}(|e\rangle_1 \langle e| - |g\rangle_1 \langle g|)$ ,  $\beta = 1/T$ , and  $Z = \text{Tr}(e^{-\beta H}) = 2[\cosh \beta\omega + \cosh \beta\lambda]$  is the partition function. Using the parametrisation of [22] we can re-write the thermally entangled state as

$$\rho = Z^{-1} \left[ e^{\beta\omega} |gg\rangle \langle gg| + e^{-\beta\omega} |ee\rangle \langle ee| + e^{\beta\lambda} |\Psi_-\rangle \langle \Psi_-| + e^{-\beta\lambda} |\Psi_+\rangle \langle \Psi_+| \right], \quad (42)$$

with the maximally entangled Bell states  $|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(\pm|ge\rangle + |eg\rangle)$ . From the expression (42) one can obtain easily the following quantities

$$\begin{aligned} \langle gg|\rho|gg\rangle &= Z^{-1} e^{\beta\omega} \\ \langle ee|\rho|ee\rangle &= Z^{-1} e^{-\beta\omega} \\ \langle eg|\rho|ge\rangle &= \langle ge|\rho|eg\rangle = Z^{-1} \cosh \beta\lambda \\ \langle eg|\rho|eg\rangle &= \langle ge|\rho|ge\rangle = -Z^{-1} \sinh \beta\lambda. \end{aligned} \quad (43)$$

Substituting in the expression of the apparent temperature presented in Eq. (7) of the main text, we have the following simple expression for the apparent temperature of the atomic pair,

$$e^{\omega/\mathcal{T}_{at}} = \frac{e^{\beta\omega} + e^{-\beta\lambda}}{e^{-\beta\omega} + e^{-\beta\lambda}}. \quad (44)$$

Without thermal entanglement ( $\lambda = 0$ ), we recover as expected  $\mathcal{T}_{at} = 1/\beta = T$ . However, for strong thermal entanglement (or strong coupling)  $\lambda \geq \omega$ , we obtain  $e^{\omega/\mathcal{T}_{at}} = e^{2\beta\omega}$ , or in other words,  $\mathcal{T}_{at} = T/2$ . This simple illustration shows the power of control that correlations have on the apparent temperature.

## VIII. DELOCALISED EXCITATION

An efficient way of creating correlations is by collective excitations. Let us consider for instance an ensemble of  $N$  identical two-level systems with one collective excitation (for instance a cloud of  $N$  two-level atoms initially in the ground state absorb one photon from their common bath). Then, the state of the ensemble is

$$|\phi\rangle = \frac{1}{\sqrt{N}}(|10\dots 0\rangle + |01\dots 0\rangle + \dots + |00\dots 1\rangle). \quad (45)$$

When interacting with a common bath the apparent temperature  $\mathcal{T}_{del}$  of the cloud with a delocalised excitation is

given by Eq. (6) of the main text,

$$e^{\omega/\mathcal{T}_{\text{del}}} = \frac{\sum_{i=1}^N \langle \phi | \sigma_i \sigma_i^\dagger | \phi \rangle + \sum_{i \neq j=1}^N \langle \phi | \sigma_i \sigma_j^\dagger | \phi \rangle}{\sum_{i=1}^N \langle \phi | \sigma_i^\dagger \sigma_i | \phi \rangle + \sum_{i \neq j=1}^N \langle \phi | \sigma_i \sigma_j^\dagger | \phi \rangle}, \quad (46)$$

where we introduced the ladder operators of the atoms  $\sigma_i = |g\rangle_i \langle e|$  and  $\sigma_i^\dagger = |e\rangle_i \langle g|$ , with  $|e\rangle_i$  and  $|g\rangle_i$  the excited and ground level of the atom  $i$ , respectively. From (45) one can obtain easily the following identities,

$$\begin{aligned} \langle \phi | \sigma_i \sigma_i^\dagger | \phi \rangle &= \frac{N-1}{N} \\ \langle \phi | \sigma_i^\dagger \sigma_i | \phi \rangle &= \frac{1}{N} \\ \langle \phi | \sigma_i \sigma_j^\dagger | \phi \rangle &= \frac{1}{N}. \end{aligned} \quad (47)$$

Substituting in (46) we have

$$e^{\omega/\mathcal{T}_{\text{del}}} = \frac{2(N-1)}{N}, \quad (48)$$

which has to be compared with the apparent temperature for a localised excitation (for instance the first atom is excited and the others are in the ground state) which is (can be obtained in a similar way as above)  $e^{\omega/\mathcal{T}_{\text{loc}}} = N-1$ . For  $N=2$ , the two apparent temperature are equal. However, for increasing number of atoms, the correlations grow linearly so that for large  $N$  their effect becomes dominant:  $\mathcal{T}_{\text{loc}} \simeq 0$  while  $\mathcal{T}_{\text{del}} = \frac{\omega}{\ln 2} \simeq 1.5\omega$ . For comparison, when the cloud is in a thermal state at temperature  $T = \frac{\omega}{\ln 2}$ , its internal energy is  $\omega N/3$ . Then, from an internal energy equal to  $\omega$ , the correlations (created due to the delocalised excitation) rise the apparent temperature up to a level achieved by a thermal energy of  $\omega N/3$ . This is one more striking example of the impact of correlations on apparent temperature.