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Standardised models for cracking due to restraint of imposed strain the state of the art

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1 Notation

2	a_s	distance from tensioned edge of concrete section to centre of gravity of reinforcement
3	c	concrete cover
4	f_{ct}	tensile strength of concrete
5	$f_{ctk,0.05}$	5%-quantile characteristic tensile strength of concrete
6	f_{ctm}	mean tensile strength of concrete
7	$h_{c,eff}$	depth of the effective tensile area of concrete
8	l_e	transfer length
9	s_0	slip length in crack formation stage / crack spacing in stabilised cracking stage
10	s_r	crack spacing
11	s_{rm}	mean crack spacing
12	$s_{r,max}$	maximum crack spacing
13	w	crack width
14	w_k	characteristic crack width
15	w_{mean}	mean (average) crack width
16	w_{max}	maximum crack width
17	$A_{c,eff}$	effective tensile area of concrete
18	A_{ct}	tensile area of concrete
19	A_s	area of reinforcement
20	E_c	modulus of elasticity of concrete
21	E_{cm}	mean modulus of elasticity of concrete
22	$E_{c,eff}$	effective modulus of elasticity of concrete
23	E_s	modulus of elasticity of steel
24	R	restraint factor (from linear analysis)
25	R_{ax}	restraint factor accounting for creep
26	α_e	ratio of moduli of elasticity of steel and concrete
27	ρ	reinforcement ratio
28	ρ_{eff}	effective reinforcement ratio
29	σ_s	stress in reinforcement
30	ε_{ca}	autogenous shrinkage strain
31	ε_{cd}	drying shrinkage strain
32	ε_{cm}	mean strain in concrete
33	ε_{ctu}	tensile capacity of concrete
34	ε_{free}	free (unrestrained) imposed strain
35	ε_{sm}	mean strain in steel reinforcement
36	ε_T	thermal strain
37	ϕ	diameter of reinforcement

1 **1 Introduction**

2 Crack control due to restrained volumetric changes as a serviceability limit state in reinforced concrete
3 has always been of interest to designers and researchers especially when associated with liquid-
4 retaining, air-tight but also monumental structures. Volumetric changes in concrete may arise from
5 thermal expansion (and subsequent contraction) due to hydration and ambient temperature variations,
6 drying and autogenous shrinkage. If these changes in volume are restrained, which is always the case
7 to a certain degree in all structural concrete applications, then tensile stresses are induced in concrete
8 which may result in cracking [1].

9 The type and significance of volume change restraint can vary depending on the structural arrangement.
10 Nevertheless, the restraint type is usually differentiated in three categories: (a) **end restraint**, where an
11 element is externally restrained at its extremities, (b) **continuous edge restraint**, where an element is
12 externally restrained along its edge, usually the base, and (c) **internal restraint**, occurring due to
13 temperature and moisture variations in the cross-section. The case of internal restraint is mostly relevant
14 for massive concrete structures, in which self-equilibrated temperature gradients prevail, but also in thin
15 sections, in which drying shrinkage is internally restrained by reinforcement. In typical cross-sections
16 variations of temperature and/or shrinkage are sufficiently insignificant that the practical analysis of
17 these elements can be performed assuming uniform temperature/shrinkage in the cross-section with
18 satisfactory accuracy, or the effect of self-equilibrating stresses resulting from these differences can be
19 accounted for implicitly in the analysis. Scenarios of (a) and (b), in turn, may be more commonly dealt
20 with in practice, e.g., ground slabs, suspended slabs, pile caps, tunnel walls, tanks and generally
21 containment structures. A combination of the two is also not uncommon, such as the restraint of the
22 base and one side edge (sequential casting of segments) or restraint of the base and both side edges
23 (alternate-bay casting of segments). Nevertheless, it has been observed that although the arrangement
24 of the restraint influences the amount and range of cracks, i.e. the area of the element which requires
25 additional reinforcement, it does not have direct effect on the width of the cracks [2]. Hence, crack
26 width control of the elements with mixed restraint conditions can be performed using the approach for

1 either case (a) or (b), depending on the geometrical factors of the element (see Fig. 1). Therefore, the
2 cases of end- and edge-restrained element are considered as representative.

3 The aim of the serviceability limit state design of cracking of reinforced concrete structures is
4 predominately to ensure durability of the structure, but also its aesthetically pleasant appearance. It is
5 crucial in structures with increased tightness requirements (water tightness or limited permeability). For
6 this reason, several analytical models have been developed to constitute a standardised approach in
7 different regions of the world. Experience of the authors has shown that crack width estimates may vary
8 significantly depending on the method used, which results not only from the basic assumptions of the
9 model with respect to the specifics of the analysed structure, but also from the way in which the values
10 of relevant input parameters are determined in the method. This paper gives an extensive review of the
11 codified methods currently used around the world for the restraint-induced crack width calculation with
12 their historical background. These methods are implemented in the documents of different legal
13 importance which include European standards: current version of EN 1992-1-1 [4] and EN 1992-3 [5]
14 and new final draft FprEN 1992-1-1 [6], as well as guideline CIRIA C766 [7] and National Annexes to
15 EN 1992-1-1 in Germany [8] and Austria [9]; Australian standard AS 3600 [10] and recommended
16 practice document CIA Z7/06 [11]; Japanese AIJ-SRC recommendations for practice [12] and JCI
17 guideline [13]; and American ACI reports 207.2R-07 [14] and 224R-01 [15]. In the paper these
18 documents will be referred to as “*reviewed documents*”.

19 **2 Philosophy of the models**

20 The design criterion for serviceability limit state of cracking is a target crack width estimated under
21 decisive load combination, both in case of the cracking caused by direct loads and by restraint of
22 imposed strains. The width of the crack is defined as a product of the slip length $2l_e$ and mean strain
23 difference between steel and concrete ($\varepsilon_{sm} - \varepsilon_{cm}$) over this length:

$$24 \quad w = 2l_e \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \quad (1)$$

25 Two stages are usually distinguished in the phenomenon of cracking: the crack formation stage and the
26 stabilised cracking stage [16]. The crack formation stage is a stage at rather small load levels in which

1 stand-alone single cracks exist, whereby concrete and steel strains reach compatibility between cracks.
2 The stabilised cracking stage is only reached at higher load levels when concrete and steel strains are
3 not compatible over the entire length of the element. Figure 2 illustrates the strain conditions of both
4 stages. Depending on the reinforcement ratio, the tensile strength of the concrete and the concrete cover,
5 the boundary between the two cracking stages ranges for practical cases between $\sigma_s = 150 \text{ N/mm}^2$
6 and 300 N/mm^2 . In any case, both cracking stages differ significantly in the respective underlying slip
7 length. For single cracks, the slip length is the full transfer length to both sides of the crack, $s_0 = 2l_e$;
8 for stabilised cracking, the slip length correlates with the crack spacing, $s_0 = s_r \leq 2l_e$ [17].

9 Many codes neglect the single crack (crack formation) stage and treat the crack width independently of
10 the load level for the conditions of a stabilised cracking stage by defining the slip length solely via the
11 crack spacing. By doing so, the crack width is generally defined as a product of crack spacing s_r and
12 mean strain difference:

$$13 \quad w = s_r \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \quad (2)$$

14 Such an approach has been adopted in European standards on design of concrete structures: Model Code
15 2010 [18] and EN 1992-1-1 [4]. The standards provide a method for crack width calculation based on
16 the mechanical behaviour of a reinforced concrete tie subjected to uniaxial tension. It is assumed that
17 once the acting tensile force reaches cracking force, the stabilised cracking stage is reached. The mean
18 crack width is then calculated as the product of mean (average) crack spacing s_{rm} and difference
19 between mean strain of steel and concrete (averaged along the transfer length). It must be noted that
20 determination of the mean crack spacing in such an approach is usually the result of calibration on the
21 experiments with specimens in the stabilised cracking stage. However, since the cracks are not
22 distributed uniformly along these specimens, the conversion between the mean crack width w_{mean} and
23 the design crack width w_k is done implicitly with the conversion factor $\beta > 1$ expressed with the use
24 of the maximum crack spacing $s_{r,\text{max}} = \beta \cdot s_{rm}$. Factor β is hereby again the result of calibration on
25 experiments with specimens in the stabilised cracking stage and refers usually to a characteristic crack
26 spacing. The crack width obtained with such an approach should be understood as the characteristic crack

1 width w_k , a statistical value defined as the width with a 5% probability of being exceeded (a 95%
2 quantile) [19].

3 This approach, originally developed for the elements under direct loads, has been adopted based on the
4 proposal of *Beeby* [20] for design of the elements subjected to the restraint of imposed strains. It has
5 been implemented in EN 1992-3 [5] dedicated to the design of water-tight structures, which serves as
6 an extension to the main part EN 1992-1-1 regarding e.g. serviceability limit state of cracking due to
7 imposed deformations. This philosophy has been also adopted in the guidelines CIRIA C660 [21] and
8 its successor C766 [7], which have been developed complimentary to EN 1992-3, as well as continued
9 in the new draft FprEN 1992-1-1 [6]. On the contrary, the German and Austrian National Annexes to
10 EN 1992-1-1 do not follow such strategy in the firm belief that calibration of experiments in the
11 stabilised cracking stage does not represent the actual behaviour of the element, which is often
12 characterised by stand-alone single cracks, especially in case of imposed and restrained strains.
13 Therefore, explicit distinction between single crack stage and stabilised cracking stage is provided there.
14 In addition, the calculation of slip length is not the result of calibration to experiments but based on
15 mechanical considerations with the condition that the tensile strength of concrete cannot be exceeded
16 at any point in the element. As also in the current EC2 and FprEN 1992-1-1, the simplification of a
17 (slip-independent) constant mean bond stress along the slip length of $\tau_{sm} = 1.8f_{ct,eff}$ is assumed,
18 whose effect on the results in the application of the mechanical based approach has been discussed in
19 e.g. [22]. The result is not a characteristic but theoretically maximum slip length and maximum crack
20 width w_{max} , respectively. Further details are given in [17] and [22].

21 As for the second component of the crack width formula, namely the strain difference ($\varepsilon_{sm} - \varepsilon_{cm}$), it
22 depends on the restraining condition (either end- or edge-restraint). In an element which is restrained at
23 the ends, cracking changes internal forces globally and crack widths depend on the degree of
24 reinforcement and tensile strength of concrete, but not on the applied strain. It means that a stabilised
25 crack spacing is assumed to be formed just after the cracking stresses are reached. For the other case,
26 an element restrained along one edge, as cracking changes forces only locally, the mean strain difference

1 between steel and concrete depends on the magnitude of the restrained part of the strain of uncracked
2 section.

3 If the reduction of the restraint forces due to cracking is taken into account, it is rather unlikely that a
4 stabilised cracking stage will be reached for typical magnitudes of imposed strains, element's
5 dimensions and reinforcement ratios [17]. In this sense, the German and Austrian Annexes to EN 1992-
6 1-1 control the crack width for imposed deformations on the basis of the crack formation stage. There
7 exist also other methods which acknowledge the fact that the mechanism of restrained shrinkage
8 cracking differs from the mechanism of cracking under direct loads and they are based on the
9 assumption that the element under the action of the restrained imposed strains is in a crack formation
10 stage. These include the method developed by *Base and Murray* [23], whose modified version has been
11 adopted in Japanese recommendations AIJ-SRC [12], and the method developed by *Gilbert* [24] and
12 *Nejadi and Gilbert* [25], adopted in the Australian recommendations CIA Z7 [11]. In both methods the
13 transfer length is calculated, defined after *Favre* [26] as dependent on the diameter and reinforcement
14 ratio, but independent of the concrete cover, and compared with the crack spacing. The computation
15 procedure follows development of stresses in steel and concrete and the formation of subsequent cracks
16 until (if) the stabilised cracking stage is established. It must be noted that the formula for crack width
17 is derived based on the mean crack spacing. It should be also mentioned here that the Base–Murray
18 method implemented in AIJ-SRC recommendations [12] has been modified to account for the actual
19 degree of restraint and restraining condition, and hence can be used for both end-restrained elements
20 and edge-restrained (wall) elements, as well as to account for the effect of creep.

21 Finally, a method for calculation of the width of the restraint-induced cracks is provided by the ACI
22 reports. In the ACI reports family the design for crack control of reinforced concrete structures subjected
23 to restrained imposed strains was provided by the ACI 207.2R-95 [27], later replaced with a newer
24 version ACI 207.2R-07 [14]. The provisions of the current version of the report ACI 207.2R-07 [14]
25 are limited to cracking risk assessment and for the crack width calculations refer to ACI 224R-01 [15]
26 – the report dedicated to design of elements under direct loads. The previous version of the report ACI
27 207.2R-95 [27] referred to ACI 224.2R-92 [28] from which it adopted the formulas for crack width

1 under direct loads for calculations of the elements under imposed strains. The same formulas are still
2 recommended by an updated version of the report ACI 224R-01 [15]. It was, therefore, concluded by
3 the authors that the combined provisions of the current versions of the ACI 207.2 [14] and 224 [15]
4 report can be used for the design of the elements in questions following the procedure proposed in [27].
5 This approach allows to check whether stabilised cracking stage has been reached: the spacing between
6 the cracks can be re-calculated from the obtained crack width, and this value can be compared with the
7 minimum crack spacing in a tie for a stabilised crack spacing.

8 It should be mentioned that the methods implemented in the current standards for determination of
9 restraint stresses in edge-restraint elements are all based on the so-called Compensation Plane Method
10 developed in Japan by the JCI Technical Committee on Thermal stress of massive concrete structures
11 in 1985 [29]. A companion method for crack width calculation with the use of the CPM has been also
12 proposed [30]. This approach was advocated by the Japan Concrete Institute until the 2016 version of
13 their report [13] after which it was replaced with a fully three-dimensional FEM-based method for
14 determination of stresses caused by volumetric changes. Alongside, a concept of cracking index was
15 introduced which is used for the assessment of cracking risk and crack width calculation [31]. The
16 formula for the maximum crack width calculation according to the cracking index and reinforcement
17 ratio is an empirical formula derived with a statistical analysis of a large database of the results obtained
18 in numerical simulations of wall- and non-wall structures in Japan. The formula has been calibrated so
19 its physical meaning is implicit.

20 **3 End-restrained elements**

21 In general, in the end-restrained elements the crack spacing is determined as mean crack spacing in a
22 tie and a multiplication factor $\beta > 1$ is used to convert mean crack width w_{mean} into the characteristic
23 crack width w_k . It is agreed that the crack spacing depends on the diameter and ratio of reinforcement,
24 and some of the methods acknowledge also the influence of the concrete cover. The magnitude of the
25 restrained imposed strain affects the number of cracks but not the crack width, which is characteristic
26 for the crack formation stage. Table 1 summarises the formulas in the crack width calculations for end-
27 restrained elements with different standardised methods. It is worth noting that in the new proposal of

1 the draft FprEN 1992-1-1 a new coefficient $k_{1/r}$ has been introduced which accounts for the effect of
2 curvature on the actual width of the crack in case of the elements under flexure / eccentric tension. In
3 case of pure tension, however, there is no curvature and the coefficient $k_{1/r} = 1$.

4 **3.1 Methods based on stabilised cracking stage assumption**

5 The European standards and recommendations [4][5][6][7] as well as the Australian standard AS 3600
6 [10] are based on the model in which the crack spacing is derived from the model of an end-restrained
7 element for a stabilised cracking stage (see Fig. M.2 in [5]). The mean crack spacing s_{rm} obtained with
8 such a derivation is then converted into the maximum spacing $s_{r,max}$ by multiplying it by the
9 conversion factor of 1.7 (which is done explicitly in FprEN 1992-1-1 [6] and implicitly in the remaining
10 documents). EN 1992-1-1 [4] and CIRIA C766 [7] (following CIRIA C660 [21]) adopted the same
11 formula for the crack spacing with a difference in the way how the bond properties are considered –
12 CIRIA C766 [7] recommends to assume that at the stage of hardening of concrete the quality of bond
13 between steel and concrete is poor. To account for the poor bond quality at early age the coefficient k_1
14 should be increased by $1/0.7$ (for sections > 300 mm thick with cover < 50 mm), which gives the value
15 of 1.14 in comparison to 0.8 suggested by EN 1992-1-1 for good bond condition. In the authors opinion,
16 it may be further deduced that if cracking occurs at early-age, poor bond assumption should be
17 maintained for the long-term analysis. In the opposite situation, i.e. if cracking is not predicted in the
18 early ages of concrete hardening, for the purpose of the long-term analysis the quality of bond can be
19 considered as good, and thus $k_1 = 0.8$.

20 For the crack spacing, the draft FprEN 1992-1-1 [6] proposes an evolution of the relationship given in
21 the current EN 1992-1-1. To facilitate comparison, the following considerations are related in both cases
22 to the mean crack spacing, which in case of FprEN 1992-1-1 is given explicitly, while in case of the
23 current EN 1992-1-1 is expressed as $s_{r,max}/1.7$. Comparing the two equations, changes in k_i
24 coefficients can be noticed. Coefficient k_3 has been reduced from 2.0 to 1.5, which results in smaller
25 influence of the concrete cover on the crack width. A new coefficient k_b has been introduced which
26 accounts for the bond properties depending on the position of the reinforcing bar during concreting.

1 This new coefficient replaces coefficient k_1 which related crack spacing to the bond properties of
2 reinforcing steel attributing it to the surface characteristics of the bars (ribbed vs. plain). Coefficient k_2 ,
3 dependent on the distribution of strain in cross-section, has been replaced with a new coefficient k_{fl} . In
4 case of pure tension, the value of $k_{fl} = 1$ as it was for the coefficient k_2 , but in case of the elements
5 under pure or eccentric flexure it claims a more precise determination of strains in extreme fibres; new
6 formulation of this coefficient is presented in [32]. However, the mechanical consistency of such
7 formulation can be seen as contradictory, as shown in [22]. The value $1/7.2$ results from the assumptions
8 of the model for crack spacing in stabilised cracking stage in which the transfer length is derived based
9 on the slip-independent mean bond stress with a value of $\tau_{sm} = 1.8 \cdot f_{ct,eff}$ [32]. This leads to the
10 formula for the transfer length $l_e = A_{c,eff}/(1.8 \cdot u_s)$, where the ratio between the effective tensile area
11 of concrete and the bar perimeter $A_{c,eff}/u_s$ can be written in relation to the bar diameter and
12 reinforcement ratio as $\phi/(4 \cdot \rho_{eff})$. The value $1/4$ represented as k_4 in EN 1992-1-1 [4] is described in
13 the model of FprEN 1992-1-1 [6] as an empirical coefficient, while it results from equating $A_s/u_s =$
14 $\phi/4$ which is not empirical. Accounting for the assumption of the constant bond law it gives the value
15 of $1/4 \cdot 1/1.8 = 1/7.2$ in FprEN 1992-1-1.

16 Even though in both cases the crack-inducing strain is related to the degree of reinforcement and tensile
17 strength of concrete, in the current version of EN 1992-1-1 [4] and CIRIA C766 [7] different formula
18 has been proposed for the elements subjected to imposed strains as opposed to the situation of the
19 elements under direct tension. The reinforcement ratio based on the full cross section, ρ , is used for
20 calculation of the strain difference in the elements under imposed strains while the effective one ρ_{eff} is
21 used in case of direct tension. Moreover, CIRIA C766 states that cracking may occur at stresses lower
22 than the mean tensile strength of concrete, as proposed by EN 1992-1-1, and recommends using the
23 0.05% characteristic tensile strength instead of the mean one, which is 30% lower.

24 The draft FprEN 1992-1-1 [6] recommends using the same formula for the end-restrained elements
25 regardless of the source of tension (direct loads or imposed strains). The crack-inducing strain depends
26 on the actual stress in reinforcing steel induced by the imposed strain. The same recommendation is
27 given by AS 3600 [10]. This way the method for determination of the crack-inducing strain in end-

1 restrained elements subjected to imposed deformation currently given by EN 1992-3 [5] has been
2 removed from the recommendations of FprEN 1992-1-1 [6]. It is the authors understanding comparing
3 the two methods that the FprEN 1992-1-1 [6] formula allows for superposition of stresses induced by
4 the restraint of imposed strains and additional mechanical load because it explicitly incorporates the
5 stress in steel σ_s which value can be calculated for any load acting on the analysed element. On the
6 other hand, coefficient k in EN 1992-1-1 [4], which accounts for the effect of self-equilibrating stresses
7 on the reduction of the force resultant caused by imposed strains, was neglected. Coefficient k_c in EN
8 1992-1-1 [4] approach has also been removed, however, its effect is implicitly taken into account by
9 the choice of location of the reinforcement for which the stress σ_s is determined (for pure tension, pure
10 flexure or eccentric tension appropriate stress block is considered for determination of the value of
11 stress in the analysed layer of reinforcement).

12 **3.2 Methods based on crack formation stage assumption**

13 The procedure for crack width calculation proposed in EN 1992-1-1 [4] has not been adopted for
14 national application in Germany [8] and Austria [9]. Instead of the calibrated model from EN 1992-1-
15 1, which is based upon an experimentally observed mean crack spacing in the stabilised cracking stage,
16 the German and Austrian Annexes propose an apparatus for crack width calculation on the basis of a
17 mechanical model. This mechanical model is derived from the bond stress–slip relationship at the
18 reinforcement–concrete interface in the slip length with particular regard to the cracking stage, namely
19 single crack stage or stabilised cracking stage. The model allows to calculate the maximum possible
20 slip length and, consequently, the maximum crack width, w_{\max} . The slip length / crack spacing in this
21 approach depends on the diameter and ratio of reinforcement (with coefficient k_4 set as
22 $1/(3.6 \cdot k_1 \cdot k_2)$), but not on the concrete cover (with k_3 set to 0).

23 Also the CIA Z7/06 [11], AIJ-SRC [12] and ACI 207.2R-95 [27] methods do not assume that the
24 stabilised cracking stage has been reached. In the approach of CIA Z7 and AIJ-SRC recommendations
25 for the crack width in end-restrained elements, the crack width is determined for the transfer length
26 $s_0 = 2l_e$ which depends on the diameter and ratio of reinforcement. In parallel, the expected number

1 of cracks is calculated under the assumption that the number of cracks must be large enough to
2 compensate the imposed strains so that the tensile stress in concrete does not reach its tensile strength
3 again. By knowing the number of cracks, the mean crack spacing s_{rm} can be calculated. As long as the
4 crack spacing is larger than the transfer length ($s_{rm} > s_0$), the element is in crack formation stage.
5 According to these approaches the crack spacing – and consequently the width – are independent of the
6 concrete cover. Furthermore, in the methods of CIA Z7 and AIJ-SRC the width of the crack depends
7 on the magnitude of the imposed deformation, in contrary to the methods based on the stabilised
8 cracking stage assumption. It must be, however, emphasised that the methods were derived for and are
9 applicable to lightly reinforced structures (with a reinforcement ratio $< 0.8\%$); by exceeding the limits
10 of their applicability the results of the calculations may become unreliable.

11 It must be also mentioned that the AIJ-SRC recommendation has been developed specifically for the
12 design of wall (edge-restrained) elements. It is, however, based on the method developed originally by
13 *Base and Murray* [23] for end-restrained elements, where full restraint at extremities ($R = 1$) was
14 assumed. Therefore, even though the guideline does not allow for it explicitly, it seems reasonable to
15 the authors to use this method in its original form for the design of end-restrained elements.

16 In the ACI report 207.2R-95 [27] the cracking model for end-restrained members was based on the
17 assumption that additional cracking of the element must be provided by enough reinforcing steel at a
18 controlled stress level to equal the total restraint force induced at the member ends. The report adopts
19 the formulas for the elements under direct loads to be used for calculation of the elements under imposed
20 strains. The ACI report 224.2R-92 [28] proposed two formulas for calculation of the width of the crack
21 in the elements under flexure and tension, which differed only in the value of the α coefficient. The
22 original formula developed by *Gergely and Lutz* [33] for flexure in which $\alpha = 0.011$ was modified in
23 ACI 224 with $\alpha = 0.0145$ for tension as it is acknowledged that the maximum crack width in tension
24 elements is larger than it would appear from the formula for flexural elements. Nevertheless, ACI
25 207.2R-95 [27] recommended to use the expression for flexure ($\alpha = 0.011$) for restraint-induced crack
26 width calculations as it provides sufficient limitations in determining crack reinforcement without
27 additional conservatism. It was added, though, that the designer is always at liberty to choose a more

1 conservative expression for direct tension ($\alpha = 0.0145$). The width of the crack depends on the
2 concrete cover, expressed with the cover-to-bar centre distance, as well as reinforcement ratio and
3 tensile strength, enclosed in the formula for the stress in steel. The spacing between the cracks can be
4 re-calculated from the obtained crack width – this value depends on the magnitude of the imposed strain.
5 ACI 224 states that the maximum crack spacing in a tie is equal to about four times concrete cover
6 thickness. Although the report does not propose it explicitly, in the authors opinion the calculated crack
7 spacing can be compared with the maximum crack spacing in the stabilised crack spacing $s_{r,max} = 4 \cdot$
8 c_{eff} to check whether stabilised cracking stage has been reached in the analysed element.

9 **4 Edge-restrained elements**

10 The crack width in an edge-restrained element is, in general, a product of crack spacing, s_r , and crack-
11 inducing strain, ε_{cr} , which depends on the restrained part of the imposed strain $\varepsilon_r = R \cdot \varepsilon_{free}$ and
12 extensibility of concrete (ability of concrete to extend between the neighbouring crack), which is related
13 to its tensile capacity ε_{ctu} :

$$14 \quad w = s_r \cdot \varepsilon_{cr} = s_r \cdot (\varepsilon_r - k_t \cdot \varepsilon_{ctu}) \quad (3)$$

15 Table 2 summarises the formulas in the crack width calculations for edge-restrained elements with
16 different standardised methods. A note about the influence of curvature on the crack width considered
17 in FprEN 1992-1-1 should also be mentioned (see Sec. 3).

18 **4.1 Methods based on stabilised cracking stage assumption**

19 According to the European standards and recommendations [4][5][6][7] as well as the Australian
20 recommendation CIA Z7 [11], which adopts here the provisions of CIRIA, the crack spacing in an edge-
21 restrained element is also derived from the model of a tie in which stabilised cracking stage is assumed.
22 The same formulas and recommendations are used as for the end-restrained element (see Sec. 3.1).

23 **4.2 Methods based on crack formation stage assumption**

24 In case of the approaches of AIJ-SRC recommendation [13] and ACI 207.2R-95 report [27], the same
25 methods developed for the end-restrained elements are used for the edge-restrained ones. The idea

1 behind the methods is that an edge-restrained element can be divided into virtual horizontal strips
2 (referred to as lifts), and each strip can be treated as an end-restrained element in which the actual level
3 of restraint is adjusted with the use of the restraint factor derived for a given geometry of the wall. In
4 the AIJ-SRC approach, the reinforcement is calculated per 1 m of the wall's height and by default
5 uniformly distributed reinforcement is applied. Nevertheless, it appears to the authors that such an
6 approach allows to differentiate the degree of restraint in each 1-m strip and, consequently, differentiate
7 the reinforcement ratio at the height of the wall.

8 In the ACI approach it is explicitly recommended to divide the wall into virtual lifts and perform
9 calculations for each lift separately. That is because the report acknowledges the fact that developing
10 cracks reach different heights, and thus at different levels of the wall the crack spacing varies. This
11 allows to account for the positive effect of geometry of the wall on the restraining moment of the internal
12 stress block (corresponding to the maximum base restraint) required to be transferred by the
13 reinforcement at each level of the wall, which guarantees a sufficiently small crack spacing at this level
14 to reduce the width of the crack.

15 **4.3 Crack-inducing strain and effect of creep**

16 In the edge-restrained element, the mean strain difference depends on the magnitude of the restrained
17 part of the strain of uncracked section which is calculated as a product of the unrestrained imposed
18 strain $\varepsilon_{\text{free}}$ and restraint factor R , which follows from the Compensation Plane Method. The restraint
19 factor, which represents the degree of external restraint, accounts for both the axial restraint (restraint
20 of axial force) and rotational restraint (restraint of bending moment). The degree of restraint varies
21 throughout the volume of the element. It can be determined from numerical analysis of the structure
22 (e.g. 3D FEM-based calculations) for any point in the structure. Standards and recommendations,
23 however, usually provide the recommended values of the restraint factors only in critical points or
24 sections, where the cracks are expected. All of the reviewed standardised methods in their simplified
25 recommendations assume full rotational restraint and relate the restraint only to the axial restraint.
26 Therefore, whenever "full restraint" is referred to, one should understand fixed support conditions

1 (idealistic case), while “partial restraint” relates to the actual restraining conditions (0-100%) exerted
2 on the analysed structure.

3 The current EN 1992-3 [6] does not provide any methods for determination of the imposed strains but
4 gives recommendations on the degree of external restraint for different restraint configurations.
5 However, since it is known that the first crack initiates in the central part of the base-restrained element
6 and this crack usually has the greatest width [34], it seems reasonable to the authors in determination
7 of the cracking strain to assume the highest recommended value of the restraint factor at the base, which
8 according to EN 1992-3 is equal to 0.5. Provisions of EN 1992-3 were elaborated in CIRIA C660 [21]
9 and later in CIRIA C766 [7]. First of all, CIRIA C660 introduced guidelines to determine the magnitude
10 of imposed strains caused by various phenomena, including cement hydration, shrinkage and impact of
11 ambient temperature. Further on, CIRIA C766 accounted for different scales and contribution of
12 individual components of strain on the restrained strain, both at early age and in long term. Finally, the
13 CIRIA guidelines acknowledged that the restraint factor R_{ax} proposed in EN 1992-3 already accounts
14 for the effect of creep, and the guidelines use explicitly additional coefficients K_c to represent the effect
15 of creep on reduction of the restrained part of strain. Therefore, the restraint factor R of CIRIA
16 guidelines should be interpreted as $R_{ax} = K_c \cdot R$ [21]. The maximum value of the restraint factor for an
17 edge-restrained element is given as equal to 0.8 (hence at early age $R_{ax} = 0.65 \cdot 0.8 = 0.52$ which
18 matches approximately the provisions of EN 1992-3). Nevertheless, even though the CIRIA C766
19 guidelines recommend separate consideration of the phenomena involved, no detailed provisions are
20 given on the variation of the degree of restraint. The philosophy of CIRIA C766 has been followed in
21 the draft FprEN 1992-1-1 [6], although the effect of creep on reduction of stresses is still enclosed in
22 the restrained factor as in EN 1992-3. Another difference between the approach of CIRIA C766 [7] and
23 FprEN 1992-1-1 [6] is in determination of the critical values of strains induced by individual
24 phenomena. More importantly, however, recommendations on the degree of restraint are very limited
25 in comparison to both CIRIA C766 [7] and current version of EN 1992-3 [5]. Except for giving a general
26 definition of the restraint factor, the draft FprEN 1992-1-1 only recommends that unless more detailed
27 values are available, the value of the restraint factor R_{ax} for the edge-restrained element can be taken

1 as 0.5. In the background document to Annex D [35], it is further explained that three different values
2 of the restraint factor are related to the fact that the degree of restraint changes with age of concrete and
3 with time if the structural system (and consequently restraining conditions) change. It is recommended
4 that the precise values of the restraint factor can be derived from other relevant guidelines (with no
5 reference to specific works) or determined from the FEM-based analysis of the structural system with
6 or without creep. In the latter case, however, there are no guidelines how this effect should be accounted
7 for.

8 Recommendations for determination of the imposed strain are also given by AIJ-SCR [12], however,
9 they are limited only to drying and autogenous shrinkage (thermal effects are neglected in the
10 recommendations but they are not explicitly excluded). It must be emphasised that the original Base–
11 Murray method [23] was derived for a generic source of imposed strain so in the authors opinion the
12 modifications introduced in its version implemented in AIJ-SRC recommendations should not affect its
13 range of applicability regarding the source of the load. The guideline does not provide specific
14 recommendations for determination of the restraint factor.

15 Early-age thermal strains can be assessed with the help of ACI 207.2 report [14]. The report provides
16 also guidance for determination of the restraint factor without creep. The restraint factor in the centre
17 section of the edge-restrained element, so in the section where the first and widest crack is likely to
18 occur, depends on the L/H ratio and relative axial stiffness of the element $(A_c E_c)/(A_F E_F)$:

$$19 \quad R = K_R \cdot \frac{1}{1 + \frac{A_c E_c}{A_F E_F}} \quad (4)$$

20 where K_R is a degree of structural geometry restraint; its distribution along the centre section of the
21 analysed element is given in a graph depending on the L/H ratio of the element [14]. In this case the
22 restraint factor depends only on the structural system. According to ACI 207.2 [14], this approach
23 applies when creep plays a lesser role, i.e. in thinner concrete sections where temperature changes occur
24 over a relatively short period of time. This approach is evidenced by the ACI 207.2 [14] expression for
25 crack spacing that does not include concrete shrinkage. In ACI 207.2 [14] it was stated that creep can
26 be expected to reduce significantly the stresses induced by drying shrinkage because of the long period

1 required for full drying shrinkage to develop. Due to the fact that simplified approach does not include
2 the shrinkage of concrete, according to the authors, this leads to a certain extent to the fact that the
3 positive effect of creep and the negative effect of concrete shrinkage cancel each other out.

4 It should be noted that CIRIA C766 [7] refers also to the method of ACI 207.2 [14] for determination
5 of the restraint factor, pointing out that the diagrams for K_R proposed by ACI 207.2 revised by *Emborg*
6 [36] should be used. Moreover, CIRIA C766 [7] takes into account the positive effect of creep also in
7 the concrete hardening period, which due to the coefficient $K_{c1} = 0.65$ is significantly lower than in
8 long term ($K_{c2} = 0.5$). In addition, it should be emphasised that in case of imposed deformations which
9 result, e.g., from the operational specifics of the structure, when imposed deformations take place in a
10 relatively short period of time, the creep effect may not be taken into account.

11 A critical review of Eq. (4) can also be found in [37], where an underestimation of restraint stresses by
12 Eq. (4) is reported.

13 A fully mechanical procedure for determination of the restrained strain was proposed by *Gilbert* [38]
14 whose method has been adopted in the CIA Z7 recommendations [11]. The restrained part of the strain
15 is determined from the tensile stress induced at the bottom of the restrained element by restraining force
16 caused by the imposed strain $\varepsilon_r = \sigma_{cs}/E_{c,eff}$, where $E_{c,eff}$ is an effective age-adjusted modulus of
17 elasticity which accounts for creep (if it is appropriate/justified). The restraining factor can be then
18 derived as a ratio between the restrained part of strain and free imposed strain $R = \varepsilon_r/\varepsilon_{free}$. A detailed
19 step-by-step procedure for determination of the restrained tensile stress σ_{cs} based on a new and old
20 concrete relative geometry can be found in [38]. It is worth mentioning that in CIA Z7, ε_{free} is the
21 difference of the free contraction strains between old and new concrete, as recommended also by CIRIA
22 766 for long-term analysis, while in FprEN 1992-1-1 no explicit comment is made in this regard.

23 All the methods, except for the current EN 1992-3 [5], account for the positive effect of concrete
24 extensibility on the width of the crack. Not the entire restrained part of the imposed strain is a crack-
25 inducing strain because after formation of the crack concrete has some potential to extend between the
26 cracks, this way reducing their width. The standards relate the extensibility of concrete to its tensile

1 capacity, expressed as the ratio between tensile strength and modulus of elasticity. However, each
2 method has different approach to determine this tensile capacity as well as suggests to consider its
3 different portions. Tensile capacity according to FprEN1992-1-1 considers mean tensile strength of
4 concrete and neglects creep (it is assumed that the tensile capacity is obtained under short-term loading).
5 CIRIA C766, in turn, acknowledges that the imposed strains have sustained character and consequently
6 accounts for the effect of creep. However, as it also considers a reduced value of tensile strength
7 (characteristic), the predicted strain relieve of both methods is eventually of comparable magnitude. The
8 CIA Z7 and AIJ-SRC recommendations relate strain relieve to full tensile capacity which is calculated
9 neglecting creep (short-term loading). Finally, ACI 207.2-95 gives fixed recommendations for tensile
10 capacity of concrete, which are 0.0001 for early-age cracking and 0.00015 for seasonal cracking.
11 Tensile capacity of 0.0001 is also recommended by AIJ-SRC. In typical cases these values are higher
12 than the actually calculated f_{ct}/E_c ratio. It should be noted that the effect of creep is not taken into
13 consideration in the provisions of the ACI 207.2 reports [14][27] accounting for the fact that in non-
14 massive elements temperature drop occurs relative quickly, thus creep modifies stresses only slightly.
15 Therefore, it is only noted that the main effect of creep is realised through partially relieving drying
16 shrinkage and thus recommends to reduce the value of drying shrinkage strain in calculations.

17 **5 Tensile strength and stiffness**

18 Another challenge in SLS design of reinforced concrete structures subjected to imposed strains is caused
19 by the fact that an important portion of these strains (mainly thermal strains due to hydration and
20 autogenous shrinkage) develops during the period of hardening of concrete. The critical moment for the
21 early-age crack control is when the imposed strains caused by cement hydration are high enough to
22 induce stresses which cannot be withstood by the element. In case of typical thin or semi-massive
23 elements (with the thickness of several cm such as slabs of walls) this happens within the first few days,
24 when stiffness is well developed while tensile strength is far from its reference value at 28 days.
25 Guidelines, such as CIRIA C766, recommend to perform the analysis at 3 days after casting. It is
26 believed by the authors that this is the case in C766 due to the guidance document focusing on liquid
27 retaining containers and structures, with element geometries relevant to basement walls and slabs, tank

1 walls and other non-massive elements. However, with an increasing massivity of the element, which
2 detailed definition can be found in **Erreur ! Source du renvoi introuvable.**, the critical time will be
3 extended to even up to couple of months in extreme cases of e.g. dams.

4 This requires that the values of mechanical properties (tensile strength, modulus of elasticity and tensile
5 capacity) at this age are known. That is why the guidelines provide recommendations to predict not
6 only the final values of mechanical properties but also their development in time. Table 3 presents a
7 summary on the approaches to determine relevant mechanical properties of concrete proposed by the
8 reviewed documents. For each reviewed document this summary includes definition of the effective
9 tensile strength used for calculation of the crack width, definition of the modulus of elasticity and their
10 time-development.

11 It should be mentioned here that these basic formulas were derived for concretes cured in isothermal
12 conditions under constant ambient temperature of 20°C. Nevertheless, in several cases, due to heat of
13 hydration and ambient conditions the temperature within the element is different. Since development
14 of mechanical properties depends on the temperature-time history within the element, in the above
15 mentioned basic formulas this can be expressed by using an equivalent age of concrete t_{eq} instead of
16 the real time t .

17 European standards and recommendations [4][6][7] give relationships for tensile strength and modulus
18 of elasticity as functions of compressive strength, which is useful at the design phase when usually only
19 a class of concrete is known. EN 1992-1-1 and FprEN 1992-1-1 relate to the mean tensile strength for
20 the crack width prediction while CIRIA C766 relates to the characteristic tensile strength, which is 30%
21 lower, to account for the fact that loads induced by imposed deformations are sustained for long periods
22 of time and the cracks are most likely to form at the weakest locations [7]. It is also noted that the crack
23 width calculated with this formula gives the upper bound value as it does not take into account the effect
24 of stress relief due to tension stiffening [7]. Regarding the modulus of elasticity, CIRIA C766 and
25 FprEN 1992-1-1 account for the influence of the aggregate type. All these three documents propose an
26 exponential function for time-development of mechanical properties in which the development rate
27 depends on the type of cement. The formula for time-development in FprEN 1992-1-1 has been adjusted

1 for currently used binders: the values of shape coefficient s have been updated and new time parameters
2 were introduced: reference time for which the final value of a mechanical property has been determined
3 t_{ref} , which allows to predict development of strength and stiffness beyond 28th day of curing, and the
4 time for which strength and stiffness gain begin (dormant time) t_{dor} . It should be noted that the draft
5 FprEN 1992-1-1 predicts faster development of tensile strength (n exponent of 0.6 instead of 1.0) and
6 slightly slower development of the modulus of elasticity (n exponent of 1/3 instead of 0.3) in
7 comparison to the current EN 1992-1-1.

8 The time development function proposed by EN 1992-1-1 has been also adopted in AIJ-SRC
9 recommendation [12], however, the values of s coefficients have been adjusted to reflect the behavior
10 of cements used in Japan. Since the tensile strength is not used explicitly in calculations, there are no
11 recommendations given to calculate its value. As for the modulus of elasticity, the document provides
12 a formula also based on the compressive strength which additionally accounts for the type of cement
13 and aggregate used in the concrete mix.

14 The CIA Z7 recommendation [11] refers to AS 3600 standard [10] regarding mechanical properties of
15 concrete. The standard also relates tensile strength and modulus of elasticity to compressive strength of
16 concrete. There is no time-development function provided, only specific recommendations for early-
17 age and long-term values of tensile strength and modulus of elasticity.

18 In the ACI reports family mechanical properties and their time-development are defined in the ACI
19 209.2R-08 [40] after ACI 209R-92 [41]. The time-development function has a form of a rational
20 function and the rate of this development depends on the type of cement. Nevertheless, ACI 207.2R-07
21 gives different recommendations for the final values of the mechanical properties used for serviceability
22 limit state design of restraint-induced cracking which are lower than the generally recommended values.

23 **6 Minimum reinforcement for control of crack width due to imposed strains**

24 The reason for defining the minimum reinforcement requirement can be twofold. Firstly, minimum
25 reinforcement must be provided to avoid yielding of reinforcement at first crack (i.e. the crack
26 localisation condition in which the full tension cross-section is considered). Secondly, it can be a

1 simplified method to control the width of cracks due to restraint stresses based on the limitation of steel
2 stresses to σ_{lim} (depending on the allowable crack width). The minimum reinforcement requirements
3 given by the reviewed documents are enlisted in Table 4.

4 It is generally agreed that the minimum reinforcement ratio is a ratio of the effective tensile strength of
5 concrete (specified in each method) and stress in steel. The yield strength of steel is used to ensure that
6 reinforcing steel does not yield, but whenever the width of the crack must be limited, the actual stress
7 in reinforcement should be explicitly determined. EN 1992-1-1 [4] and the recommendations which
8 follow its approach introduced additional parameters to account for some of the relevant phenomena.

9 EN 1992-1-1 [4] introduces additional coefficients: k_c to account for the stress distribution in section
10 and k for the effect of non-uniform self-equilibrating stresses which lead to reduction of the cracking
11 force. The latter coefficient allows to implicitly account for the effect of internal restraint. National
12 Annexes to EN 1992-1-1 in Germany [8] and Austria [9] allow to reduce the value of k to 0.8 in case
13 of restraint caused by temperature variation due to heat of hydration and by shrinkage. This formula has
14 been adopted in CIA Z7 recommendations [11]. CIRIA C766 [7], which also adopted the formula of
15 EN 1992-1-1 for the minimum reinforcement, in addition to limiting concrete strength by using its
16 characteristic value, introduces coefficient k_{Edge} to account for different restraining conditions of the
17 analysed element. In case of the end-restrained element $R_{Edge} = 0$, while for the edge-restrained
18 element the guideline accounts for the confining effect of the base restraint and thus $R_{Edge} > 0$ thanks
19 to which the minimum reinforcement ratio can be reduced.

20 The approach for the minimum reinforcement has been modified in FprEN 1992-1-1 [6] with respect
21 to the current version of EN 1992-1-1 [4]. The main text of the draft provides the requirement for the
22 minimum reinforcement to avoid steel yielding. Following the recommendations implemented in the
23 German and Austrian National Annexes to EN 1992-1-1 [8][9], Informative Annex S to FprEN 1992-
24 1-1 proposes in addition a requirement of the minimum reinforcement for crack width control under the
25 assumption that the crack width must only be limited for the cracking force of the effective tensile area

1 of concrete. However, a yet another definition of the effective tensile area has been proposed in FprEN
2 1992-1-1 to limit this area for thick members.

3 **7 Interpretation of crack width**

4 The aim of the serviceability limit state of cracking is to limit the width of the cracks to a target
5 allowable width under the decisive combination of loads. The interpretation of this crack width is,
6 however, a highly disputable issue. To begin with, the standardised models are design models which do
7 not intend to compare the design and actual crack widths. The draft FprEN 1992-1-1 [6] states explicitly
8 that crack width calculated with the method proposed by the standard should only be considered as a
9 nominal value for the crack width at the surface of the element, to be compared with the limit values
10 provided in the standard, and not as values actually measured on site. However, from the authors
11 experience, it is a common practice to make such comparison in order to evaluate the structure's
12 compliance with the design. Therefore, there is a need for coherence between the calculated and
13 measured crack widths.

14 The so-called calibrated models, as opposed to the mechanical models, are themselves calibrated on the
15 results of experimental tests. The crack widths and spacings in experiments are not exactly predictable
16 – at best the range of their expected sizes can be narrowed down. Within this range of expected sizes,
17 the real values are a random product from the interplay of the scatter of tensile strength along the
18 member, scatter of bond properties along the slip lengths and load level (in terms of associated cracking
19 stage). Furthermore, the actual details of test specimens and test procedure and crack measurements
20 may introduce additional scatter of observed crack widths. Finally, it must be remembered that the
21 models were developed for some model types of structures, so the reliability of design may decrease as
22 the target designed structure differs from the model structure. This means that the obtained crack
23 spacings and crack widths in experiments are not free of doubt.

24 Secondly, the design models predict either a characteristic crack width w_k or a maximum possible crack
25 width w_{max} (the difference between both design philosophies is explained in [16]). In any case, the
26 comparison of measurements with code predictions must strongly regard the model philosophy. For
27 example, the quality of the model for characteristic crack width is strongly based on the crack spacing

1 in the stabilised cracking stage – and usually evaluated with regard to the scatter of crack spacing
2 prediction. In design, however, only the maximum crack width is of importance so these models should
3 also be evaluated against the maximum crack width of experiments. On the contrary, the model for the
4 maximum crack width is often poorly represented as it has a large scatter in case of crack spacing
5 prediction for experiments. But this is no surprise since this model assumes the maximum possible slip
6 length according to the cracking stage in order to make a safe side prediction for the maximum crack
7 width.

8 Furthermore, in the reviewed documents the crack width is calculated for the theoretical situation of
9 reaching the cracking force. In experiments, however, the steel stress is somewhat relieved after each
10 crack formation, which is discussed – but not considered in the design – in CIRIA C766 [7]. At lower
11 degree of reinforcement and lower classes of concrete, this “stress reduction” after cracking can reach
12 significant magnitude of even up to 50%. That is why the time instance at which the measurements are
13 made is of importance as the crack width at the moment of measurement may not necessarily reflect the
14 maximum-in-the-history width of the crack.

15 Finally, although the calculated maximum crack width is determined by the analysis, it is generally
16 accepted that the cracking process is highly probabilistic and some of the observed cracks in the
17 structure may be larger than the calculated ones and should not be directly compared. Considering the
18 uncertainties of both calculation and measurements, a relative difference of 20% between the calculated
19 and the actual maximum crack width is assumed not to be an abnormal situation. The 20% margin
20 comes from the distribution of the ratio between the mean and maximum crack width, which based on
21 the results of *Beeby* [42], varies between 1.5 and 2. The conversion factor of 1.7 corresponds to the
22 mean value of the distribution (with $1.7 + 20\% \text{ of } 1.7 = 2$).

23 Last but not least, the crack formation in thick concrete members is strongly affected by primary and
24 secondary cracking [43]. The crack width in the primary crack can hereby increase significantly in
25 comparison to the secondary ones. The mechanics behind were intensively elaborated in [44] and from
26 the result it can be concluded, that conventional tie models are not appropriate for crack width
27 calculation of thick concrete members.

1 **8 Summary, conclusions and future work**

2 An extensive review of the available standardised methods for calculation of restraint-induced cracking
3 was summarised briefly and allowed to formulate several conclusions to be forwarded in the next
4 paragraphs.

5 Cracking induced by the restraint of imposed strains is governed by the single crack (crack formation)
6 stage which is acknowledged by the methods implemented in CIA Z7/06, AIJ-SRC, ACI Report 207.2
7 as well as German and Austrian National Annexes to EN 1992-1-1. Other methods, namely those
8 following the philosophy of the Model Code 2010 (including EN 1992-1-1 and CIRIA C766), assume
9 that stabilised crack spacing is reached instantaneously; the same models for crack spacing as for the
10 elements in direct tension are recommended for the design of the elements subjected to the restraint of
11 imposed strains. For the acceptance of the design models in practice, it is of importance that the
12 underlying design philosophy reflects the real behaviour of the element on the whole. On one hand, the
13 leading models in Europe are those based on the stabilised cracking stage, and hence seem to fail this
14 aim. On the other hand, however, it should be emphasised that these models attempt to determine the
15 crack width for extreme situations which may occur at a given point in time, which in some cases maybe
16 more useful and definitely simpler for the designer who may not necessarily be aware of the complex
17 behaviour of the structure in question.

18 Two representative restraint conditions are distinguished in the structures subjected to imposed strains:
19 end restraint and edge restraint. The formulas for the end-restrained elements are derived from the
20 model of the tie, for the assumed cracking stage. In the methods for the edge-restrained elements, crack
21 spacing is also derived from the model of the tie, because the edge-restrained element can be represented
22 as a tie in which the actual degree of restraint is adjusted with the restraint factor. One of the biggest
23 differences among the reviewed methods is in the approach to determining the difference of strains
24 between cracks (a crack-inducing strain). In case of the AIJ-SRC method, this value is not given
25 explicitly, and strains are limited only to shrinkage-induced strains. In the approaches of EN 1992-3
26 and CIA Z7/06, this value is defined, but in a general way without specifying the extent and type of

1 deformation in detail. Finally, CIRIA C766 and FprEN 1992-1-1 provide a precise method for
2 determination of the crack-inducing strain for a specific range and type of imposed strains.

3 The differences in the methods result not only from the model assumptions but also from the way in
4 which the values of relevant input parameters are determined in the method. This relates to the:

- 5 • Magnitude of the free imposed strain $\varepsilon_{\text{free}}$. There are non-negligible differences among the
6 reviewed documents regarding determination of the decisive values of the imposed strains (thermal
7 strain – early-age and long-term, autogenous and drying shrinkage strain) which influence the
8 magnitude of the crack-inducing strain.
- 9 • Effective tensile strength $f_{ct,eff}$. There is no agreement whether the mean tensile strength or the
10 characteristic tensile strength (weakest-link theory proposed in CIRIA C766) should be considered.
11 Furthermore, the reviewed documents differ in the estimation of the final value of the tensile
12 strength as well as in the prediction of its time-development during hardening of concrete.
13 Consequently, for the same class of concrete different standards would predict different values of
14 the effective tensile strength for early-age and long-term crack width predictions.
- 15 • Degree of restraint which has a decisive influence on the magnitude of the restrained part of strain.
16 The effect of the degree of restraint on the crack width is visible in case of the edge-restrained
17 elements designed according to the approach proposed in EN 1992-3 (followed by CIRIA C766
18 and CIA Z7) in which the stabilised cracking stage is assumed and the width of the crack depends
19 directly on the value of the restrained part of strain. The effect is, on the other hand, negligible in
20 case of the methods in which crack formation stage, such as AIJ-SRC. In this case the increase in
21 the restrained strain would lead to formation of new cracks rather than increase of the width of the
22 existing ones.
- 23 • Effect of creep. Taking into account the restraint factor in engineering calculations requires its
24 interpretation in terms of the size/value of the creep effect or its absence. Thus, the positive effect
25 of this factor should be defined individually for a given structural element depending on the age of
26 the concrete and the nature of the imposed deformation (i.e. at early age or in long-term, the type
27 of deformation which takes place in relatively short or long period of time).

1 Finally, interpretation of the calculated crack width is a disputable issue. Apart from the mechanical
2 model implemented in the German / Austrian NAs to EC2 which provides the maximum possible crack
3 width to be expected in the structure, the methods provide either mean (CIA Z7/07, AIJ-SRC) or
4 characteristic (EN 1992-1-1, ACI Report 224) width of the crack which should not be directly compared
5 with the actual crack width on the site. The authors discuss in the paper matters which influence the
6 measured crack width but which are usually not taken into account by the design models.

7 Despite extensive research related to the modelling of serviceability behaviour of reinforced concrete
8 elements subjected to imposed strains, there is no agreement on the formulation for the crack width
9 calculation or on the modelling approach. The construction practice demands, however, that coherence
10 is ensured between the calculated and measured crack width which would allow to check the compliance
11 of the structure with the design. This requires integrity with the behaviour of reinforced concrete and
12 mechanical consistency of the models which can be achieved by properly designed experiments and
13 refined numerical modelling. Realisation of these tasks is being covered by the RILEM TC 287-CCS
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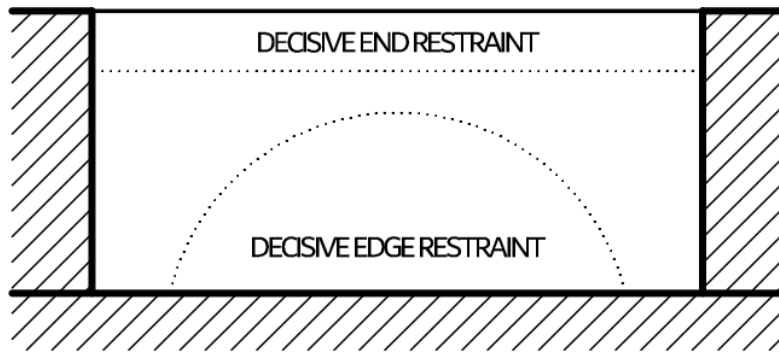
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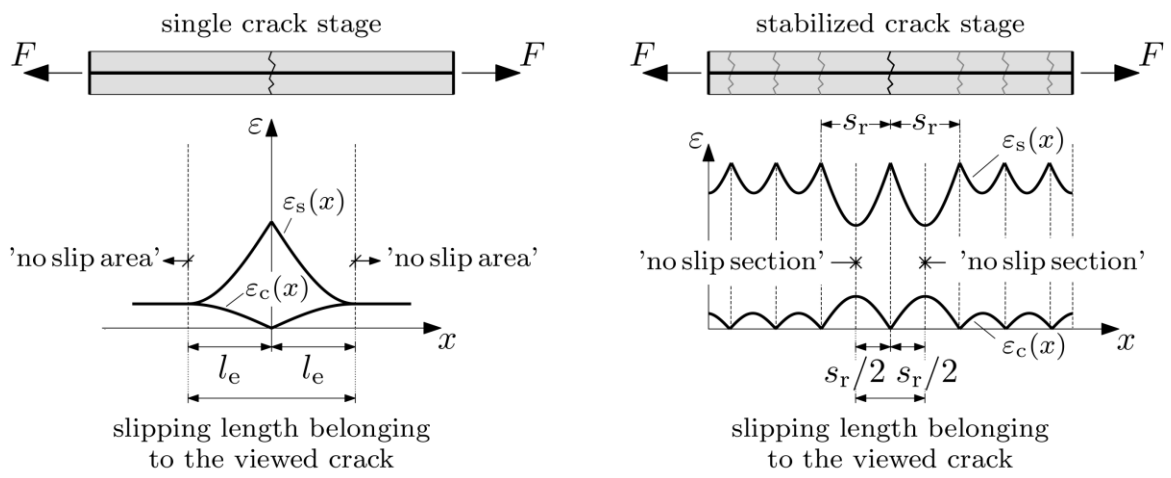


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Figure 1. Local restraining conditions in an element with mixed base/side restraint [3].

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Figure 2. Strain conditions and slip lengths according to cracking stage.

Table 1. Summary of the methods for crack width calculation in end-restrained elements from reviewed documents.

Code	Crack spacing s_r	Strain difference $\varepsilon_{sm} - \varepsilon_{cm}$	Crack width
EN 1992-1-1 [4] & 1992-3 [5]	$s_{r,max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_{eff}}$ $k_1 = 0.8 \text{ for good bond}$	for imposed deformations: $\frac{0.5 \alpha_e \cdot k_c \cdot k \cdot f_{ctm}}{E_s} \left(1 + \frac{1}{\alpha_e \cdot \rho}\right)$ for direct loads: $\frac{\sigma_s - k_t \frac{f_{ctm}(1 + \alpha_e \cdot \rho_{eff})}{\rho_{eff}}}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$ $k_t = 0.4$	$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$
CIRIA C766 [7]	$s_{r,max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_{eff}}$ $k_1 = 1.14 \text{ for good bond}$	$\frac{0.5 \cdot \alpha_e \cdot k_c \cdot k \cdot f_{ctk,0.05}}{E_s} \left(1 + \frac{1}{\alpha_e \cdot \rho}\right)$	$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$
FprEN 1992-1-1 [6]	$s_{rm} = 1.5 \cdot c + \frac{k_{fl} \cdot k_b}{7.2} \cdot \frac{\phi}{\rho_{eff}} \leq \frac{1.3}{k_w} (h - x)$ $k_w = 1.7$	$\frac{\sigma_s - k_t \frac{f_{ctm}(1 + \alpha_e \cdot \rho_{eff})}{\rho_{eff}}}{E_s} \geq (1 - k_t) \frac{\sigma_s}{E_s}$ $k_t = 0.4; \text{ where } \sigma_s = f_{ctm} \cdot A_s$	$w_k = k_w \cdot k_{\frac{1}{r}} \cdot s_{rm} \cdot (\varepsilon_{cm} - \varepsilon_{sm}) = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$ $k_{\frac{1}{r}} = 1 \text{ for pure tension}$
AS 3600 [10]	$s_{r,max} = 3.4 \cdot c + 0.3 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_{eff}}$ $k_1 = 0.8 \text{ for good bond}$	$\frac{\sigma_s - k_t \frac{0.9 f_{ct,split}}{\rho_{eff}} (1 + \alpha_e \cdot \rho_{eff})}{E_s} + \varepsilon_{free} \geq 0.6 \frac{\sigma_s}{E_s}$ $k_t = 0.6 \text{ and } \varepsilon_{free} = \varepsilon_{ca} + \varepsilon_{cd}$	$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$
NA to EN 1992-1-1 [8][9]	$s_{r,max} = \frac{\phi}{3.6 \cdot \rho_{eff}} \leq \frac{\phi}{3.6} \cdot \frac{\sigma_s}{f_{ctm}}$	for direct loads: $\frac{\sigma_s - k_t \frac{f_{ctm}(1 + \alpha_e \cdot \rho_{eff})}{\rho_{eff}}}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$	$w_{max} = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$
CIA Z7 [11]	transfer length: $s_0 = 0.1 \frac{\phi}{\rho}$ crack spacing: $s_{rm} = \frac{2 \cdot s_0 \cdot (1 + \xi)}{3 \cdot \xi} \geq s_0$ where $\xi = f(\alpha_e; \rho; \varepsilon_{free})$	not determined explicitly	$w_{mean} = - \left[\frac{\sigma_{c1}^*}{E_{c,eff}} \cdot \left(s_{rm} - \frac{2}{3} s_0 \right) + \varepsilon_{free} \cdot s_{rm} \right]$
AIJ-SRC [12]	transfer length: $2l_e = 0.1 \frac{\phi}{\rho}$ crack spacing: $s_{rm} = \frac{l}{m} \geq 2l_e$ where $m = 1 + \frac{L \cdot \alpha_e \cdot \rho}{2 \cdot l_e} \cdot \left(\frac{R \cdot \varepsilon_{free} - \varepsilon_{ctu}}{b \cdot \varepsilon_{ctu}} \right)$	not determined explicitly ε_{free} is limited to drying and autogenous shrinkage	$w_{mean} = 2l_e \left(\frac{\sigma_s}{E_s} + \frac{R \cdot \varepsilon_{free}}{b} \right)$ with $R = 1$ for full restraint at extremities
ACI 207.2 [14] & 224 [15]	$s_{r,max} = \frac{w}{1.5 \cdot (R \cdot \varepsilon_{free} - \varepsilon_{ctu})}$ based on the known crack width	not determined explicitly	$w_k = 0.011 \cdot \sigma_s \cdot \sqrt[3]{a_s \cdot A_{c,eff}} \cdot 10^{-3}$ where $\sigma_s = \frac{f_{ct}}{\rho}$

Table 2. Summary of the methods for crack width calculation in edge-restrained elements from reviewed documents.

Code	Crack spacing s_r	Strain difference $\varepsilon_{sm} - \varepsilon_{cm} = \varepsilon_{cr}$	Crack width
EN 1992-1-1 [4] & EN 1992-3 [5]	$s_{r,max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_{eff}}$ <p>$k_1 = 0.8$ for good bond</p>	$\varepsilon_{cr} = \varepsilon_r = R_{ax} \cdot \varepsilon_{free}$	$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$
CIRIA C766 [7]	$s_{r,max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_{eff}}$ <p>$k_1 = 1.14$ for poor bond (at early age)</p>	$\varepsilon_{cr} = \varepsilon_r - 0.5\varepsilon_{ctu} = R \cdot \varepsilon_{free} - 0.5\varepsilon_{ctu}$ $= K_{c1} \cdot R_1 \cdot [\varepsilon_{T1} + \varepsilon_{ca}(3)] + K_{c2} \cdot R_2$ $\cdot [\varepsilon_{T2} + \varepsilon_{ca}(28) - \varepsilon_{ca}(3)] + K_{c2} \cdot R_3 \cdot \varepsilon_{cd}$ $- 0.5\varepsilon_{ctu}$ <p>where $\varepsilon_{ctu}(t) = \frac{f_{ct,0.05}(t)}{E_{cm}(t) \cdot K_c}$</p>	$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$
FprEN 1992-1-1 [6]	$s_{rm} = 1.5 \cdot c + \frac{k_{fl} \cdot k_b}{7.2} \cdot \frac{\phi}{\rho_{eff}}$ $\leq \frac{1.3}{k_w} (h - x)$ <p>$k_w = 1.7$</p>	$\varepsilon_{cr} = \varepsilon_r - k_t \cdot \varepsilon_{ctu} = R_{ax} \cdot \varepsilon_{free} - k_t \cdot \varepsilon_{ctu}$ $= R_{ax,1} \cdot [\varepsilon_{T1} + \Delta\varepsilon_{ca}(t)] + R_{ax,2} \cdot \varepsilon_{T2}$ $+ R_{ax,3} \cdot \Delta\varepsilon_{cd}(t) - k_t \varepsilon_{ctu}$ <p>where $\varepsilon_{ctu}(t) = \frac{f_{ctm}(t)}{E_{cm}(t)}$ and $k_t = 0.4$</p>	$w_k = k_w \cdot k_{\frac{1}{r}} \cdot s_{rm} \cdot (\varepsilon_{cm} - \varepsilon_{sm})$ $= s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$ <p>$k_{\frac{1}{r}} = 1$ for pure tension</p>
CIA Z7 [11]	$s_{r,max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_{eff}}$ <p>$k_1 = 1.14$ for good bond</p>	$\varepsilon_{cr} = \varepsilon_r - \varepsilon_{ctu} = R \cdot \varepsilon_{free} - \varepsilon_{ctu}$ <p>where $\varepsilon_r = \sigma_{cs}/E_{c,eff}$ and $R = \varepsilon_r/\varepsilon_{free}$</p>	$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$
AIJ-SRC [12]	<p>transfer length: $2l_e = 0.1 \frac{\phi}{\rho}$</p> <p>crack spacing: $s_{rm} = \frac{l}{m} \geq 2l_e$</p> <p>where $m = 1 + \frac{L \cdot \alpha_e \cdot \rho}{2 \cdot l_e} \cdot \left(\frac{R \cdot \varepsilon_{free} - \varepsilon_{ctu}}{b \cdot \varepsilon_{ctu}} \right)$</p>	<p>not determined explicitly</p> <p>ε_{free} is limited to drying and autogenous shrinkage</p>	$w_{mean} = 2l_e \left(\frac{\sigma_s}{E_s} + \frac{R \cdot \varepsilon_{free}}{b} \right)$ <p>R is adjusted to represent the actual degree of restraint in a wall</p>
ACI 207.2 [14] & 224 [15]	$s_{r,max} = \frac{w}{1.5 \cdot (R \cdot \varepsilon_{free} - \varepsilon_{ctu})}$ <p>based on the known crack width</p>	$\varepsilon_{cr} = \varepsilon_r - \varepsilon_{ctu} = R \cdot \varepsilon_{free} - \varepsilon_{ctu}$	$w_k = 0.0145 \cdot \sigma_s \cdot \sqrt[3]{a_s \cdot A_{c,eff}} \cdot 10^{-3}$

Table 3. Summary of the functions for mechanical properties of concrete from reviewed documents.

Code	Tensile strength	Modulus of elasticity	Time-development function
EN 1992-1-1 [4]	$f_{ct} = f_{ctm} = 0.3 \cdot f_{ck}^{2/3}$ $f_{ct}(t) = \beta_{cc}(t) \cdot f_{ctm}$	$E_c = E_{cm} = 22[0.1(f_{ck} + 8)]^{0.3}$ $E_c(t) = \beta_{cc}(t)^{0.3} \cdot E_{cm}$	$\beta_{cc}(t) = \exp \left[s \left(1 - \sqrt{\frac{28}{t}} \right) \right]$ <i>s</i> – depends on type of cement
CIRIA C766 [7]	$f_{ct} = f_{ctk0.05} = 0.7 \cdot f_{ctm,28}$ $f_{ct}(t) = \beta_{cc}(t) \cdot f_{ctk0.05}$	$E_c = \alpha \cdot E_{cm}$, α depends on aggregate type $E_c(t)$ following EN 1992-1-1	Following EN 1992-1-1
FprEN 1992-1-1 [6]	$f_{ct} = f_{ctm} = 0.3 \cdot f_{ck}^{2/3}$ $f_{ct}(t) = \beta_{cc}(t)^{0.6} \cdot f_{ctm}$	$E_c = E_{cm} = k_E \cdot (f_{ck} + 8)^{1/3}$, k_E depends on aggregate type $E_c(t) = \beta_{cc}(t)^{1/3} \cdot E_{cm}$	$\beta_{cc}(t) = \exp \left[s \cdot \left(1 - \sqrt{\frac{t_{ref} - t_{dor}}{t - t_{dor}}} \right) \cdot \sqrt{\frac{28 - t_{dor}}{t_{ref} - t_{dor}}} \right]$ <i>s</i> , t_{ref} and t_{dor} depend on type of cement
CIA Z7 [11]	$f_{ct} = 0.32 \cdot \sqrt{f_{cm}}$ $f_{ct}(t = 3) = 0.18 \cdot \sqrt{f_{cm}}$	E_c tabulated for concrete class $E_c(t = 3) = 0.68 \cdot E_c$ for OPC	n/a
AIJ-SRC [12]	Not used explicitly in the model	$E_c = 3.35 \cdot 10^4 \cdot \gamma_1 \cdot \gamma_2 \cdot \left(\frac{\rho_c}{2.4} \right)^2 \cdot \left(\frac{f_{ck}}{60} \right)^{\frac{1}{3}}$ $f_{ck}(t) = \beta_{cc}(t) \cdot f_{ck}$	$\beta_{cc}(t)$ following EN 1992-1-1, but different values of <i>s</i> coefficient
ACI 207.2 [14] & 209.2 [40]	$f_{ct} = 0.5 \cdot \sqrt{f_{ck}}$ $f_{ct}(t) = \beta_{cc}(t) \cdot f_{ct}$	$E_c = 4,700 \cdot \sqrt{f_{ck}}$ $E_c(t) = \beta_{cc}(t)^{1/2} \cdot E_c$	$\beta_{cc}(t) = \frac{t}{a + b \cdot t}$ <i>a</i> and <i>b</i> – depend on type of cement

Table 4. Summary of the minimum reinforcement requirements for crack width control in RC elements under pure tension from reviewed documents.

Code	Minimum reinforcement ratio	Comments
EN 1992-1-1 [4]	$\rho_{\min} = \frac{A_{s,\min}}{A_{ct}} = k_c \cdot k \cdot \frac{f_{ctm}}{\sigma_{s,\lim}}$	$\sigma_{s,\lim}$ can be taken as equal to f_{yk} , but to limit the crack width it might be necessary to assume smaller value of the allowable stress $h_{c,\text{eff}} = \min(2.5a_s; 0.5h)$
NA to EN 1992-1-1 [8][9]	$\rho_{\min} = \frac{A_{s,\min}}{A_{ct}} = k_c \cdot k \cdot \frac{f_{ctm}}{\sigma_{s,\lim}}$ Additional regulations for thick concrete members under centric restraint at each surface of the element: $A_{s,\min} = \max\left(\frac{f_{ct,\text{eff}} A_{c,\text{eff}}}{\sigma_s}; \frac{k \cdot f_{ct,\text{eff}} \cdot 0.5A_c}{f_{yk}}\right)$ where $f_{ct,\text{eff}} = f_{ctm}$ in general or $f_{ct,\text{eff}} = 0.5f_{ctm}$ if cracking is expected within first 3 days	$\sigma_s = \sqrt{w_k \cdot \frac{3.48 \cdot 10^6}{\phi_s^*}}$ k according to EN 1992-1-1 [4]; for externally-caused imposed strains (e.g. settlement) $k = 1$ while for restraint due to internally-caused imposed strains (hydration heat, shrinkage) $k = 0.8$ can be reduced $h_{c,\text{eff}}$ must be increased according to the member's thickness: $\frac{h_{c,\text{eff}}}{a_s} = \begin{cases} \frac{h}{2a_s} & \text{for } \frac{h}{a_s} \leq 5 \\ \frac{h}{10a_s} + 2 & \text{for } 5 < \frac{h}{a_s} < 30 \\ 5 & \text{for } \frac{h}{a_s} \geq 30 \end{cases}$
CIRIA C766 [7]	$\rho_{\min} = \frac{A_{s,\min}}{A_{ct}} = k_{\text{Edge}} \cdot k_c \cdot k \cdot \frac{f_{ctk,0.05}}{f_{yk}}$ with $k_{\text{Edge}} = 1 - 0.5 \cdot R_{\text{Edge}}$	end-restrained element $R_{\text{Edge}} = 0$ edge-restraint element R_{Edge} – the edge restraint factor at the location of the maximum crack width
FprEN 1992-1-1 [6]	$\rho_{\min} = \frac{A_{s,\min}}{A_{ct}} = k_h \cdot \frac{f_{ctm}}{f_{yk}}$	Annex S (informative) – minimum reinforcement for crack width control at each surface of the element: $A_{s,\min} = \max\left(\frac{f_{ctm} \cdot A_{c,\text{eff}}}{\sigma_{s,\lim}}; \frac{k_h \cdot f_{ctm} \cdot 0.5A_c}{f_{yk}}\right)$ with $A_{c,\text{eff}} = h_{c,\text{eff}} \cdot b_{c,\text{eff}}$ where $h_{c,\text{eff}} = \min[a_{s,y} + 5\phi; 10\phi; 3.5a_{s,y}; 0.5h]$ and $b_{c,\text{eff}} = \min[a_{s,x} + 5\phi; 10\phi; 3.5a_{s,x}; 0.5b]$ Limit stress in steel can be taken as: $\sigma_{s,\lim} = \frac{6 \cdot f_{ctm}}{\phi} \cdot \left(-c + \sqrt{c^2 + 0.3 \cdot \frac{E_s \cdot w_{\lim} \cdot \phi}{k_w \cdot k_1 \cdot f_{ctm}}} \right)$

$$k_{\frac{1}{r}} = 1 \text{ for tension}$$

CIA Z7 [11]

$$\rho_{\min} = \frac{A_{s,\min}}{A_{ct}} = k_c \cdot k \cdot \frac{f_{ctm}}{f_{yk}}$$

AIJ-SRC [12]

$$\rho_{\min} = \frac{A_{s,\min}}{A_{ct}} = \frac{f_{ct}}{f_{yk}}$$

ACI 207.2 [14] & ACI

$$\rho_{\min} = \frac{f_{ct}}{\sigma_s}$$

224 [15]

$$\sigma_s = \frac{w_{\lim}}{0.11 \cdot \sqrt[3]{a_s \cdot A_{c,\text{eff}}}} \text{ to limit the width of crack of } \sigma_s = f_{yk}; \text{ Reference to recommendations of } Gilbert [23]$$