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# A mathematical tool to generate complex whole body motor tasks and test hypotheses on underlying motor planning

Michele Tagliabue · Alessandra Pedrocchi ·  
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**Abstract** In spite of the complexity of human motor behavior, difficulties in mathematical modeling have restricted to rather simple movements attempts to identify the motor planning criterion used by the central nervous system. This paper presents a novel-simulation technique able to predict the “desired trajectory” corresponding to a wide range of kinematic and kinetic optimality criteria for tasks involving many degrees of freedom and the coordination between goal achievement and balance maintenance. Employment of proper time discretization, inverse dynamic methods and constrained optimization technique are combined. The application of this simulator to a planar whole body pointing movement shows its effectiveness in managing system nonlinearities and instability as well as in ensuring the anatomic-physiological feasibility of predicted motor plans. In addition, the simulator’s capability to simultaneously optimize competing movement aspects represents an interesting opportunity for the motor control

community, in which the coexistence of several controlled variables has been hypothesized.

**Keywords** Motor control · Optimization · Simulation · Posture · Pointing

## 1 Introduction

In the field of neuroscience, movement simulation is becoming increasingly important to test the reliability of neurophysiological theories proposed to explain efficient human motor behavior. The computational approach seems especially useful to study the neural mechanisms underlying the definition of the motor plan, often called the “desired trajectory”, to accomplish a given task. Indeed, it is still matter of debate the nature of the processes occurring prior to movement execution that are aimed at solving the three indeterminacy problems involved in goal-oriented movement planning [24]: (1) determination of the endpoint trajectory in extrinsic coordinates; (2) selection of the best joint angle combinations able to produce the selected endpoint trajectory; (3) determination of muscle activations that will produce the planned kinematics.

Stereotyped features of human arm movements [7, 29, 41] suggest that the central nervous system (CNS) must use an optimality criterion to solve these indeterminacy problems, but its unequivocal identification is still an open point [1, 12, 17, 30, 42, 48, 51]. Although the proposed theories and the corresponding computational models have shed light on the neural processes underlying human motor planning, they show some important limitations. Indeed, since most of them focus on movement characterized by an equal number of degrees of freedoms (DoFs) in the joint and hand spaces, they entail a one-to-one correspondence between hand and

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60 joint angle trajectories. The resulting theories, therefore,  
61 cannot be easily generalized to more complex movements  
62 that require a solution to the second problem of indeter-  
63 minacy. To this end, a breakthrough was achieved in the  
64 investigation and modeling of the final arm posture in  
65 movements where task achievement leaves one uncon-  
66 strained DoF [1, 42].

67 The present work aims at broadening previous theories  
68 by considering tasks characterized by several DoFs, which  
69 better represent our everyday motor behavior. These  
70 movements require a simultaneous solution to the first- and  
71 second-order problems of indeterminacy. We address this  
72 goal by investigating and modeling transitive movements  
73 involving the whole body. By transitive motor act we mean  
74 the execution of a task with a defined visual goal, not  
75 inwardly perceived, and requiring the sensory–motor  
76 transformation, from visual to body space, which charac-  
77 terizes classical pointing movements. Moreover, by  
78 studying whole body transitive tasks performed from a  
79 standing position, a pivotal question in motor control, i.e.,  
80 the coordination between hand trajectory formation and  
81 equilibrium maintenance [13, 36], could be explored.  
82 Previous studies, performed with intransitive task, like  
83 trunk bending [2] or arm raising [38], represent important  
84 steps in the understanding of the very general question of  
85 posture and movement coordination [27], therefore, they  
86 represent a natural starting point for the investigation of  
87 the coordination between balance and transitive task  
88 performance.

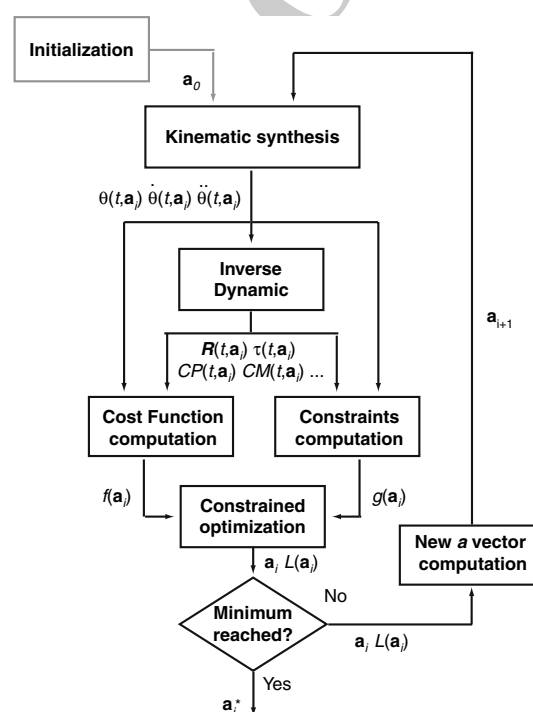
89 In the light of these considerations, it seems profitable to  
90 extend the modeling studies and related theories that have  
91 focused on both intransitive postural perturbation tasks and  
92 hand movements, to transitive whole body movements.  
93 However, this entails a remarkable increase in the complex-  
94 ity of the motor-control problem, due to the increase of  
95 the DoFs and to the intrinsic human posture instability.  
96 Therefore, previously developed simulation techniques  
97 would not be suitable. For this reason, the present paper  
98 aims at providing a tool allowing an easy application of  
99 classical planning criteria focused on movement kinetics,  
100 kinematics, balance or task accomplishment to whole body  
101 goal-oriented movements.

102 In the first part of the paper, the main structure of the  
103 simulator will be presented. Afterwards, as example of its  
104 possibilities, the simulator will be applied to a planar whole  
105 body pointing (WBP) task from standing, where the task  
106 accomplishment involves all body segments. This protocol  
107 was selected because it may be considered as an example  
108 of transitive complex movement, involving many DoFs and  
109 requiring a strong coordination between an explicit task  
110 fulfillment and equilibrium control. Furthermore, such a  
111 paradigm has led to several experimental investigations  
112 and provided a considerable amount of data [10, 19, 21, 36,

37, 43–46, 49]. In the subsequent part, an effective tech-  
nique to combine several motor planning criteria is  
discussed. Indeed, since during WBP the CNS must solve  
several problems (i.e., spatial and postural), the possibility  
of simultaneously optimizing various aspects of the motor  
plan seems particularly desirable.

## 2 Methods

The simulator structure is reported in Fig. 1. The approach  
used is based on an iterative optimization algorithm that  
searches for the movement kinematics which minimize a  
cost function explicitly defined to describe a specific motor



**Fig. 1** Block diagram representing the simulator iterative structure. At the  $i$ th iteration of the optimization process,  $\mathbf{a}_i$  is a matrix of coefficients used to synthesize body segments' angular position velocity and acceleration,  $\theta$ ,  $\dot{\theta}$  and  $\ddot{\theta}$  (kinematic synthesis block). The obtained kinematics is fed to the "Inverse dynamic" block that computes joint reaction forces and torques,  $\mathbf{R}$  and  $\boldsymbol{\tau}$ , as well as other parameters as the center of mass (CM) and the center of pressure (CP). Both kinematics and kinetics go through the "Cost function computation" and the "Constraints computation" blocks, which respectively evaluate the cost figure to be minimized,  $f(\mathbf{a}_i)$ , and the constraint fulfillment,  $g(\mathbf{a}_i)$ . Then, the "Constrained optimization" block combines these two functions in the Lagrangian function,  $L(\mathbf{a}_i)$ , to evaluate whether or not the algorithm reached the minimum of the cost function. If not, a new vector  $\mathbf{a}_{i+1}$  corresponding to a lower value of the Lagrangian function is computed and a new iteration starts, otherwise the actual set of coefficients,  $\mathbf{a}_i^*$ , corresponding to the optimal movement execution, is returned. The initialization is carried out by the "initialization" block that produces a starting matrix of coefficients,  $\mathbf{a}_0$

124 strategy. A more detailed functional description of the  
 125 blocks of the diagrams in Fig. 1 is reported in the following  
 126 paragraphs.

127 2.1 Synthesis of the movement kinematics

128 To reduce the dimensionality of the optimization problem,  
 129 the trajectories of the angular DoFs of the system,  $\theta(t)$ , (see  
 130 Fig. 2) are computed as weighted sums of *B*-spline functions [52], as reported in (1). In this way, the dimension of  
 131 the angular trajectory space basis corresponds to the  
 132 number of coefficients  $a_{j,k}$ .  
 133

$$\theta_k(t, \mathbf{a}) = \sum_{j=1}^N B_{jW}(t) \cdot a_{j,k} \quad k = 1, \dots, s \quad (1)$$

135 where  $B_{jW} = B(.|t_j, \dots, t_{j+W})$  is the *j*th *B*-spline of order *W*,  
 136 *N* is the number of coefficients (14 in the implementation)  
 137 in each column of **a** matrix (with elements  $a_{j,k}$ ) and *s* is the  
 138 number of model segments and the size of rows of the  
 139 matrix **a** (5, as shown in Fig. 2). Therefore, the total  
 140 number of coefficients  $a_{j,k}$  is  $N \cdot s = 70$  and  $\mathcal{R}^{N \cdot s}$  is the  
 141 optimization solution space basis. The selected spline

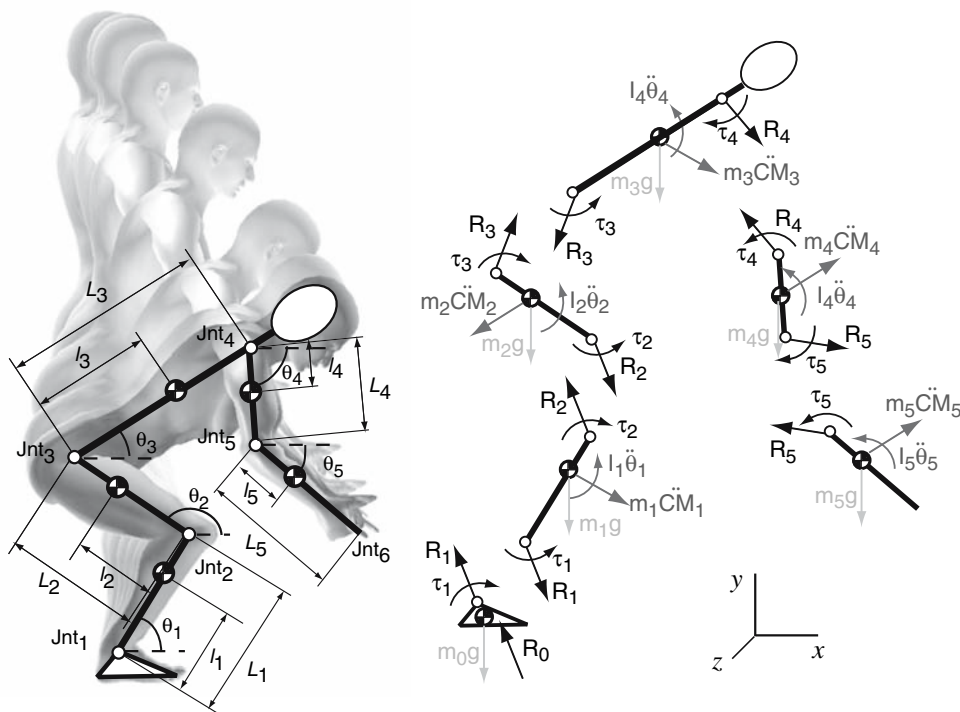
order, *W*, is 8. Such values for *N* and *W* ensure a sufficient  
 spline flexibility, even if the body segments orientation,  
 velocity and acceleration at the first frame are constrained.  
 Moreover, such a high order does not automatically force  
 to zero jerk angle profiles as do third order piecewise  
 polynomials.

The kinematic synthesis block also computes segment  
 angular velocity and acceleration  $(\dot{\theta}, \ddot{\theta})$ .

2.2 Initialization

Movement kinematics of six voluntary subjects performing  
 WBP tasks were recorded by a motion capture system (for  
 experimental protocol details, see [37]). The symmetry of  
 the task allows it to be modeled in the sagittal plane alone,  
 without loss of significant information. Therefore, only  
 joint angle projections onto the sagittal plane were  
 considered.

Angular trajectories of the segments are computed by  
 fitting *B*-splines to the average of the experimentally  
 measured behavior. Then, the initial **a** matrix is defined by  
 adding random noise to the coefficients of the interpolating  
*B*-splines. The noise maximum amplitude is set to 5% of



**Fig. 2** On the left, a schematic representation of the whole body pointing (WBP) protocol is shown. For the final position (*darker cartoon*) the biomechanical model used to describe the human body is superimposed. Jnt<sub>1-6</sub> are ankle, knee, hip, shoulder, elbow and index finger apex position.  $\theta_{1-5}$  are the shank, leg, trunk, arm and forearm angular position.  $L_{1-5}$  are the body segments' length.  $l_{1-5}$  are the distance between the segments' CM, represented by *crossed circles*,

and the distal joint. To improve figure legibility, all trimming parameters  $\Delta l$  and  $\Delta tr$  are assumed to be zero. On the right, a representation of the inverse dynamic solution is reported.  $m_k$  and  $I_k$  are the mass and inertial moment of the *k*th segment.  $\ddot{\theta}_k$  and  $\ddot{C}M_k$  are the segment angular and linear acceleration.  $\mathbf{R}_k$  and  $\tau_k$  are the reaction force and torque at the *k*th joint

Author Proof

163 the  $\mathbf{a}$  elements mean value. Greater noise tends to prevent  
164 the convergence of the algorithm because it locates the  
165 starting point of the minimum search far from the subspace  
166 corresponding to the physiological solutions.

### 167 2.3 Inverse dynamic method

168 The synthesized kinematic time course is applied to the  
169 biomechanical model of the human body reported in Fig. 2.  
170 The position ( $x(t)$ ,  $y(t)$ ,  $z(t)$ ) of each joint (Jnt) is computed  
171 as follows:

$$\mathbf{Jnt}_k(t, \mathbf{a}) = \mathbf{Jnt}_{k-1}(t, \mathbf{a}) + L_{k-1} \cdot \mathbf{v}_{k-1}(t, \mathbf{a}) \quad k = 2, \dots, N$$

with  $\mathbf{Jnt}_1(t, \mathbf{a}) = (0, 0, 0) \quad \forall t$  (2)

173 where  $\mathbf{v}_k(t, \mathbf{a})$  is a unit vector, defined as

$$\mathbf{v}_k(t, \mathbf{a}) = [\cos(\theta_k(t, \mathbf{a})) \quad \sin(\theta_k(t, \mathbf{a})) \quad 0]$$

175 describing the  $k$ th body segment orientation and  $L_k$  is its  
176 length. The constant zero value of the third component  
177 of the unit vector  $\mathbf{v}_k$  is due to the choice of applying  
178 the simulator to a planar movement. Also  $k$ th body  
179 segments center of mass (CM) position is evaluated using  
180 equation 3:

$$\mathbf{CM}_k(t, \mathbf{a}) = \mathbf{Jnt}_k(t, \mathbf{a}) + (l_k + \Delta l_k) \cdot \mathbf{v}_k(t, \mathbf{a}) + \Delta \text{tr}_k \cdot \mathbf{n}_k(t, \mathbf{a})$$

(3)

182 where  $l_k$  is the nominal distance of the segment CM from  
183 the distal joint as defined by anthropometrical tables,  $\mathbf{n}_k$  is a  
184 unit vector normal to  $\mathbf{v}_k$  defined as

$$\mathbf{n}_k(t, \mathbf{a}) = [-\sin(\theta_k(t, \mathbf{a})) \quad \cos(\theta_k(t, \mathbf{a})) \quad 0]$$

186 and  $\Delta l_k$  and  $\Delta \text{tr}_k$  are CM position corrections along  
187 longitudinal and transversal directions respectively, which  
188 are defined during the biomechanical model validation.  
189 These trimming parameters minimize the differences  
190 between the ground reaction forces and the CP position,  
191 estimated by using the inverse dynamics method (IDM) on  
192 motion capture system data and the force platform data,  
193 recorded during subjects' WBP task executions [23].  
194 Acceleration of CMs is evaluated using equation 4:

$$\begin{aligned} \ddot{\mathbf{CM}}_k(t, \mathbf{a}) = & \sum_{i=1}^{k-1} L_i \left( \ddot{\theta}_i(t, \mathbf{a}) \cdot \mathbf{n}'_i(t, \mathbf{a}) - \dot{\theta}_i^2(t, \mathbf{a}) \cdot \mathbf{v}'_i(t, \mathbf{a}) \right) \\ & + l'_k \left( \ddot{\theta}_k(t, \mathbf{a}) \cdot \mathbf{n}'_k(t, \mathbf{a}) - \dot{\theta}_k^2(t, \mathbf{a}) \cdot \mathbf{v}'_k(t, \mathbf{a}) \right) \end{aligned}$$

(4)

196 where  $\mathbf{v}'_k$ ,  $\mathbf{n}'_k$ ,  $l'_k$  are modifications of the corresponding  
197 parameters due to the corrections of the CM position,  $\Delta l_k$   
198 and  $\Delta \text{tr}_k$ , analytically defined as:

$$\begin{aligned} \mathbf{v}'_k(t, \mathbf{a}) &= [\cos(\theta'_k(t, \mathbf{a})) \quad \sin(\theta'_k(t, \mathbf{a})) \quad 0] \\ \mathbf{n}'_k(t, \mathbf{a}) &= [-\sin(\theta'_k(t, \mathbf{a})) \quad \cos(\theta'_k(t, \mathbf{a})) \quad 0] \\ l'_k &= \sqrt{(l_k + \Delta l_k)^2 + (\Delta \text{tr}_k)^2} \end{aligned}$$

with

$$\theta'_k(t, \mathbf{a}) = \theta_k(t, \mathbf{a}) + \arctan\left(\frac{\Delta \text{tr}_k}{l_k + \Delta l_k}\right)$$

Movement kinetics are estimated using the IDM. The IDM  
consists of the sequential solution of the equilibrium  
equations (5) and (6), from the endpoint to the ankle joint  
( $k = 5, \dots, 1$ ):

$$\mathbf{R}_k(t, \mathbf{a}) = m_k \cdot \ddot{\mathbf{CM}}_k(t, \mathbf{a}) + m_k \cdot \mathbf{g} + \mathbf{R}_{k+1}(t, \mathbf{a})$$

(5)

$$\begin{aligned} \boldsymbol{\tau}_k(t, \mathbf{a}) &= I_k \cdot \ddot{\theta}_k(t, \mathbf{a}) + \boldsymbol{\tau}_{k+1}(t, \mathbf{a}) \\ &+ [\mathbf{Jnt}_{k+1}(t, \mathbf{a}) - \mathbf{CM}_k(t, \mathbf{a})] \wedge \mathbf{R}_{k+1}(t, \mathbf{a}) \\ &- [\mathbf{Jnt}_k(t, \mathbf{a}) - \mathbf{CM}_k(t, \mathbf{a})] \wedge \mathbf{R}_k(t, \mathbf{a}) \end{aligned}$$

(6)

with

$$\mathbf{R}_6(t, \mathbf{a}) = \boldsymbol{\tau}_6(t, \mathbf{a}) = (0, 0, 0) \quad \forall t$$

where  $\mathbf{R}_k$  and  $\boldsymbol{\tau}_k$  are the joint reaction force and torque at  
the  $k$ th joint,  $m_k$  and  $I_k$  are the  $k$ th body segment mass and  
inertial moment (effects of the trimming parameters,  $\Delta l_k$   
and  $\Delta \text{tr}_k$ , on  $I_k$  were assumed to be negligible) and  $\mathbf{g}$  is the  
gravity acceleration vector. Moreover, solving (5) and (6)  
also for the feet ( $k = 0$ ), ground reaction forces and the  
antero-posterior position of the CP are evaluated. The CP  
position computation is possible because torque exchanged  
between feet and floor in medio-lateral direction is zero and  
the vertical coordinate of the CP always corresponds to the  
floor level. Moreover, to represent the experimental proto-  
col characteristics the feet are assumed to be still  
( $\ddot{\mathbf{CM}}_0 = 0$  and  $\dot{\theta}_0 = 0$ ).

Biomechanical model parameters,  $L$ ,  $l'$ ,  $m$  and  $I$  are  
defined as the average values obtained for the six voluntary  
subjects using anthropometrical tables [53] and the trim-  
ming procedure [23].

### 2.4 Computation of the constraints

As shown in Fig. 1, both kinematics and kinetics of the  
current solution go through the constraint block (CB). The  
solution has to fulfill two constraints: task constraints and  
anatomo-physiological constraints.

#### 2.4.1 Task constraints

Task constraints concern the kinematic variables and are  
represented by (7)–(12). Equation (7), where  $\bar{\theta}_k$  are the



236 initial segment angular positions, defines the starting posture, while (8) and 9 impose its steadiness.

$$\theta_k(0, \mathbf{a}) = \bar{\theta}_k \quad k = 1, \dots, 5 \quad (7)$$

$$\dot{\theta}_k(0, \mathbf{a}) = 0 \quad k = 1, \dots, 5 \quad (8)$$

$$\ddot{\theta}_k(0, \mathbf{a}) = 0 \quad k = 1, \dots, 5 \quad (9)$$

243 Task accomplishment constraint is defined by imposing  
244 the endpoint position at the final instant,  $t_f$ , as shown in  
245 equation 10. Moreover, to guarantee a final steady body  
246 configuration, the angular segments velocity and acceleration  
247 must be zero (equations 11 and 12).

$$\mathbf{Jnt}_6(t_f, \mathbf{a}) = \bar{\mathbf{Jnt}}_6 \quad (10)$$

$$\dot{\theta}_k(t_f, \mathbf{a}) = 0 \quad k = 1, \dots, 5 \quad (11)$$

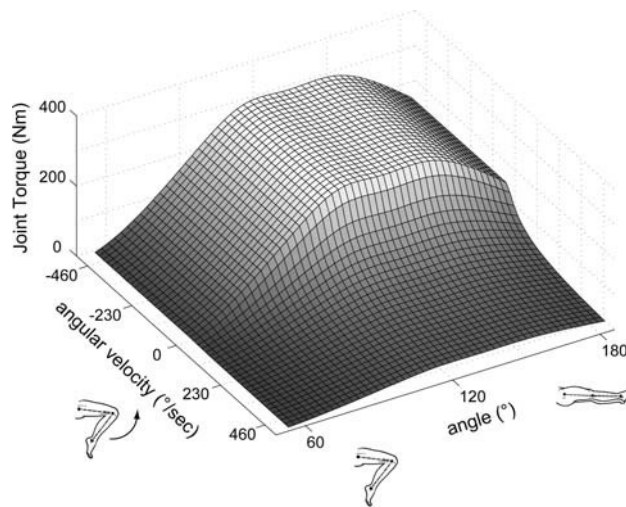
$$\ddot{\theta}_k(t_f, \mathbf{a}) = 0 \quad k = 1, \dots, 5 \quad (12)$$

253

254 *2.4.2 Anatomico-physiological constraints*

255 Anatomico-physiological constraints are aimed at representing the motor system characteristics. First, the task execution must be compatible with the range of motion of human joints. Second, joint torques,  $\tau_k(t, \mathbf{a})$ , evaluated by using IDM, must be smaller than the maximum torque exertable by the muscles acting on the  $k$ th joint,  $T_k(\theta, \dot{\theta})$ . For each muscle group, a surface representing the maximal exertable torque, depending on joint angle position and velocity, was evaluated. In order to accurately represent the actual physiological muscle characteristics, in vivo measurements reported in the literature were used [18, 26, 31, 55]. Published data were also integrated by specific experimental acquisition using a Cybex isokinetic device. Obtained data were used to define the parameters of a simplified version of the Hill equation [35], representing the relationship between the maximal force and the muscle lengthening speed. An example of resulting surfaces is shown in Fig. 3. These constraints are important because the optimum search must take into account that joints develop different torques and that even the same joint cannot produce the same maximal torque in all conditions.

277 A further imposed constraint reflects the muscles' incapability of producing excessively abrupt joint torque changes, due to excitation and activation dynamics. In the model, torques evaluated applying the IDM must not require muscular neural inputs to be less than zero or greater than one. This would indicate that the required muscle activation level is prevented due to muscular



**Fig. 3** Surface representing the maximal extensor torque as a function of angular velocity and angular position for the knee joint

dynamics [16]. Therefore, the constraint block must check that the muscle neural inputs,  $U$ , remain between 0 and 1. To estimate  $U$ , firstly, active joint torque components,  $A\tau_k$ , are evaluated as differences between the net joint torques, obtained by the IDM, and the passive components calculated by using mathematical models representing joint stiffness,  $\tau_{k,stiff}$ , and viscosity,  $\tau_{k,visc}$  [30, 39, 40, 54];

$$A\tau_k(t) = \tau_k(t, \mathbf{a}) - \tau_{k,stiff}(\theta(t, \mathbf{a})) - \tau_{k,visc}(\dot{\theta}(t, \mathbf{a})) \quad (13)$$

For each joint, active torque components are separated in flexor and extensor muscles contributions ( $A\tau_{k,1}$  and  $A\tau_{k,2}$ , respectively) taking into account the minimal cocontractions necessary to physiological transitions from a muscle group to its antagonist. In particular, the same amount of active torque is added to agonist and antagonist muscles to produce a gradual transition between active and inactive muscle states (and vice versa). Transitions are represented by a fifth order polynomial that guarantees the continuity up to the second derivative. Afterward, an approximation of the contraction level,  $A$ , is computed as the ratio between the active torque components and their corresponding maximal value.

$$A_{k,i}(t, \mathbf{a}) = \frac{A\tau_{k,i}(t, \mathbf{a})}{T_{k,i}(\theta(t, \mathbf{a}), \dot{\theta}(t, \mathbf{a}))} \quad k = 1, \dots, 5 \quad i = 1, 2 \quad (14)$$

Finally, a rough estimation of the muscle motor commands is evaluated by inverting two first order differential equations representing muscle activation and excitation dynamics [16] (equations 15 and 16, respectively).

$$E_{k,i,f-1} = \frac{A_{k,i,f} - A_{k,i,f-1} \cdot e^{-\frac{\Delta t}{T_a}}}{1 - e^{-\frac{\Delta t}{T_a}}} \quad (15)$$

$$U_{k,i,f-1} = \frac{E_{k,i,f} - E_{k,i,f-1} \cdot e^{-\frac{\Delta t}{T_e}}}{1 - e^{-\frac{\Delta t}{T_e}}} \quad (16)$$

312

314  $E_{k,i,f}$ ,  $U_{k,i,f}$  are the excitation and the corresponding neural  
 315 input of muscle groups acting on the  $k$ th joint at frame  $f$ .  
 316  $\Delta t = 0.1$  ms is the sampling period used to allow the  
 317 effective numerical solution of the differential equations  
 318 describing muscular dynamic.  $T_a$  and  $T_e$  are muscle acti-  
 319 vation and excitation time constants:  $T_a = 10$  ms and  
 320  $T_e = 50$  ms during muscle activation and deactivation  
 321 respectively, and  $T_e$  value is = 40 ms [16]. The employed  
 322 muscle model does not certainly intend to accurately  
 323 describe the single muscle dynamics, but it aims, by  
 324 imposing the neural input to range between 0 and 1, only at  
 325 roughly representing the impossibility of muscles groups to  
 326 produce abrupt changes in their contraction level.

327 The last constraint represents the need to keep the bal-  
 328 ance ensuring that the CP always lies within the supporting  
 329 base.

330 If one of the constraints is not fulfilled, CB output  
 331  $\mathbf{g}(\mathbf{a})$  ( $\mathbf{g}(\mathbf{a}): \mathcal{R}^{N \times s} \rightarrow \mathcal{R}^m$ , where  $m$  is the constraints' number)  
 332 has positive elements that penalize the current solution, as  
 333 described in Sect. 2.6.

### 334 2.5 Motor planning criteria

335 Kinematics and kinetics, corresponding to the current  
 336 combination of elements of matrix  $\mathbf{a}$ , are used in the cost  
 337 function block (CFB) to evaluate the cost figure,  $f(\mathbf{a})$ ,  
 338 representing the motor planning criteria under evaluation.  
 339 In the literature, several mathematical formulations, such  
 340 as the minimum endpoint jerk model [12], the minimum  
 341 torque change model [51] and the minimum commanded  
 342 torque change models [30], were proposed.

343 As an example, two of the expressions, which will be  
 344 used later on, are reported in (17) and (18). The first one,  
 345 where  $\text{Jnt}_6$  are the endpoint coordinates, represents the  
 346 minimum Jerk model; while the second equation, where  
 347  $x_{\text{CM}}$  is the antero-posterior position of the whole body  
 348 center of mass and  $\bar{x}_{\text{CM}}$  is its mean value, is an imple-  
 349 mentation of the CM stabilization criterion,  $\sigma_{\text{CM}}^2$  [28].

$$\text{Jerk} = \frac{1}{2} \int_0^{t_f} \left[ \left( \frac{d^3 \text{Jnt}_{6,x}}{dt^3} \right)^2 + \left( \frac{d^3 \text{Jnt}_{6,y}}{dt^3} \right)^2 \right] \cdot dt \quad (17)$$

$$\sigma_{\text{CM}}^2 = \frac{1}{t_f} \int_0^{t_f} (x_{\text{CM}}(t) - \bar{x}_{\text{CM}})^2 dt \quad (18)$$

351

353

### 2.6 Optimization algorithm 354

The simulator solves the following minimization problem: 355

$$\underset{\mathbf{a} \in \mathcal{R}^{N \times s}}{\text{minimize}} \quad f(\mathbf{a}) \quad (19)$$

$$\text{subject to} \quad g_i(\mathbf{a}) = 0 \quad i = 1, \dots, m_e \quad (20) \quad 357$$

$$g_i(\mathbf{a}) \leq 0, \quad i = m_e + 1, \dots, m \quad (21) \quad 359$$

where  $m$  is the total number of imposed constraints and  $m_e$   
 361 is the number of these constraints represented by equality. 362

Therefore, at each iteration, the optimization algorithm  
 363 block modifies the  $\mathbf{a}$  matrix to reduce the cost function  
 364 value fulfilling the imposed constraints. 365

The constrained optimization problem reported in  
 366 equations (19)–(21) can be solved as an unconstrained one  
 367 using the Lagrange function defined in (22). 368

$$L(\mathbf{a}, \boldsymbol{\lambda}) = f(\mathbf{a}) + \sum_{i=1}^m \lambda_i \cdot g_i(\mathbf{a}) \quad (22)$$

where  $\boldsymbol{\lambda}$  is the vector of Lagrange multipliers. For a more  
 370 detailed description of the optimization algorithm see [9]. 371

To minimize the local minima problem, the simulator  
 372 carries out several minimum searches starting from dif-  
 373 ferent points of the solution hyperspace and selecting the  
 374 minimum among all the final solutions. 375

### 2.7 Combination of cost figures 376

To efficiently build a multiple cost function it is necessary  
 377 to use a proper procedure to normalize the single cost  
 378 functions that have to be combined. The normalization  
 379 procedure must solve two problems: dimensional incon-  
 380 sistencies and differences in magnitude. The formula used  
 381 is reported in (23), 382

$$\text{NCF}_i = \frac{(\text{CF}_i - \text{CF}_{i,\text{min}})}{(\text{CF}_{i,\text{real}} - \text{CF}_{i,\text{min}})} \quad (23)$$

where  $\text{CF}_i$  is the cost function to be normalized.  $\text{CF}_{i,\text{min}}$  is  
 384 the minimum value obtained by minimizing it individually. 385  
 $\text{CF}_{i,\text{real}}$  is the value of the CF corresponding to the average  
 386 experimental behaviour. By using this equation, all the  
 387 normalized CFs (NCFs) are non-dimensional, have their  
 388 minimum at zero and their values tend to vary in compa-  
 389 rable ranges. NCFs can then be combined together to  
 390 obtain the multiple cost function (MCF) of equation 24. 391

In order to allow weighting differently the single  $\text{NCF}_i$  a  
 392 set of weights ( $W_{\text{NCF}_i}$ ) can be defined according to  
 393 the constraints shown in (25), (26). Therefore,  $W_{\text{NCF}_i}$  394

**Table 1** The values and results of biomechanical model and Inverse dynamic method validation

	Segment										RMSE					
	1		2		3		4		5		CP		$F_x$		$F_y$	
	$\Delta l$	$\Delta tr$	$\Delta l$	$\Delta tr$	$\Delta l$	$\Delta tr$	$\Delta l$	$\Delta tr$	$\Delta l$	$\Delta tr$	B	A	B	A	B	A
S1	21	-53	11	-48	-33	-32	-17	10	-36	10	28	9	12	11	3	3
S2	21	24	-24	31	-35	-26	17	-10	40	-10	26	12	15	13	5	5
S3	19	-27	22	0	-5	-2	-16	-10	-36	-10	16	6	10	8	3	3
S4	18	-48	21	-20	-27	12	15	10	-1	10	18	9	12	11	5	5
S5	18	18	26	-26	-18	51	-5	-5	-17	-13	13	5	12	8	9	2
S6	-18	18	27	27	6	60	-5	-5	-59	-52	27	9	14	8	7	5
Med	18	-4	22	-10	-23	5	-5	-5	-27	-10	22	9	13	10	5	4
Quart	1	31	6	22	12	31	12	8	16	8	5	1	1	1	1	1

In the first part of the table the values of the biomechanical model trimming parameters ( $\Delta l$  and  $\Delta tr$ ) are reported, in mm, for each body segment (1: shank, 2: leg, 3: Trunk & Head, 4: arm, 5: forearm&hand) and for each subject (S1–6). The second part of the table represents the results of the biomechanical model and Inverse dynamic method validation. For each subject, the table reports the percentage root mean squared error (RMSE%) between the antero-posterior centre of pressure positions (CP), the antero-posterior and vertical ground reaction forces ( $F_x$ ,  $F_y$ ) evaluated using the inverse dynamic method and the corresponding data recorded by using a force platform during the motor task execution. For each of these comparison the results obtained before, B, and after, A, the use of the above cited trimming parameters are reported. In the last two rows, median (Med) and the quartile (Quart) evaluated on six subjects are reported for all results

396 represents the fraction of the MCF due to the  $NCF_i$ .

$$MCF = 100 \cdot \sum_{i=1}^V W_{NCF_i} \cdot NCF_i \quad (24)$$

398 subject to 
$$\sum_{i=1}^V W_{NCF_i} = 1 \quad (25)$$

400 
$$W_{NCF_i} \geq 0 \quad (26)$$

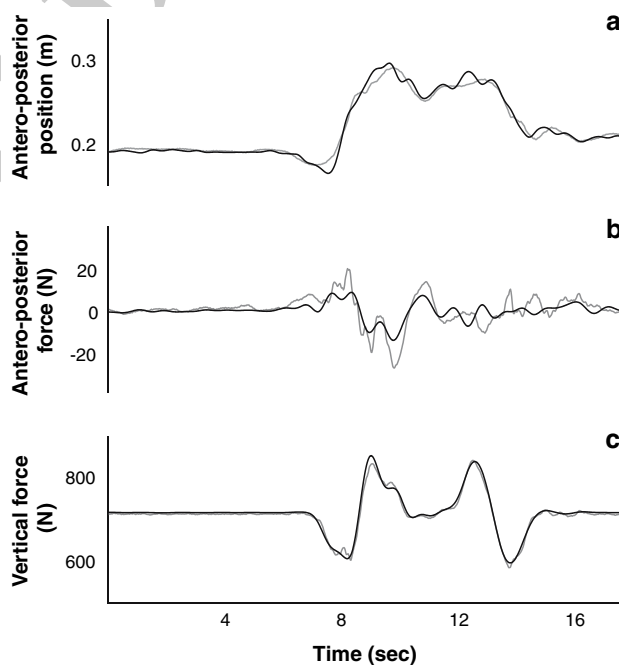
402 A multiplying global factor (i.e., 100) is also used to have  
 403 an MCF order of magnitude able to maximize the optimi-  
 404 zation algorithm efficiency. The number of cost figures ( $V$ )  
 405 that can be combined is theoretically unlimited.

### 406 3 Results

#### 407 3.1 Biomechanical model and inverse dynamic method 408 validation

409 Table 1 shows the set of trimming parameters,  $\Delta l_k$  and  
 410  $\Delta tr_k$ , and the corresponding validation improvements for  
 411 each subject. Table 1 also reports the median values of the  
 412 trimming parameters, which are applied to the biome-  
 413 chanical model used by the simulator.

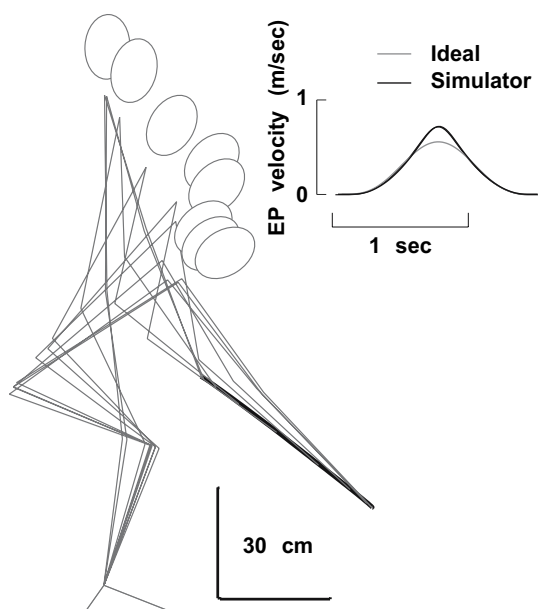
414 A representative example of the model validation is  
 415 shown in Fig. 4. A remarkable match between the IDM  
 416 results and force platform data can be observed for the  
 417 antero-posterior CP position and the vertical component of



**Fig. 4** Example of the validation of the biomechanical model and inverse dynamic methods. The data acquired by the force platform on which the subjects stood (gray lines) are compared with the corresponding data calculated by means of the inverse dynamics (black lines). In **a**, the antero-posterior position of the center of pressure is reported. Panels **b** and **c** show the antero-posterior and the vertical components of the ground reaction forces, respectively. The comparison has been made over the whole acquisition, including the period when the subject came back to the initial position

Author Proof





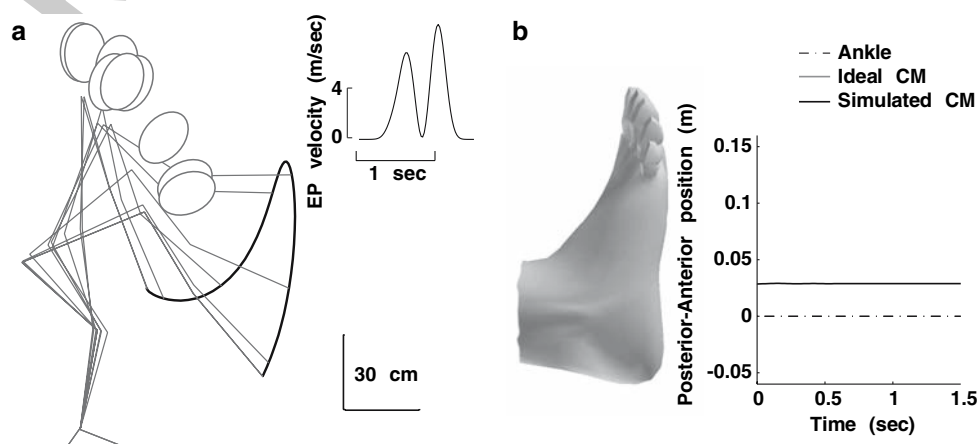
**Fig. 5** Minimum endpoint jerk model prediction for WBP. The stick diagram represents the movement execution corresponding the minimization of the third derivative of the endpoint coordinated in a Cartesian reference frame. The resulting endpoint trajectory and velocity profiles (*thick lines*) are compared with the ideal minimum endpoint jerk trajectory (*gray lines*)

418 the ground reaction forces. Considering the necessary  
419 simplifications introduced by the biomechanical model,  
420 such results can be considered fully satisfactory.

421 **3.2 Whole body movement simulation**

422 Because of the instability and the relevant nonlinearities of  
423 the system and the high number of controlled DoFs, the  
424 simulator's effectiveness in minimizing a given cost figure  
425 could, in theory, not be optimal for WBP. In order to verify  
426 whether and how the increased problem complexity would  
427 significantly affect performance, the simulator was tested

**Fig. 6** Minimum center of mass displacement model prediction for WBP. In **a** the stick diagram, representing the movement execution corresponding to this motor planning model, is reported together with the end point trajectory (*thick line*) and the endpoint velocity profile (*upper-right corner*). Panel **b** shows the anterior-posterior position of the center of mass (CM) during movement execution



428 on three different motor planning models: minimum jerk,  
429 CM stabilization and CP stabilization. Their ideal solution  
430 is well-known, therefore, the discrepancies between the  
431 obtained results and the corresponding ideal solution can  
432 be used to evaluate the simulator's performance.

433 For instance, for simple and unconstrained hand move-  
434 ments starting from the  $(x_0, y_0)$  position, with null velocity  
435 and acceleration, and arriving after  $t_f$  seconds at the  
436  $(x_{t_f}, y_{t_f})$  position, the minimum jerk model, Jerk, repre-  
437 sented by (17), produces an ideal finger trajectory  
438 described by (27) and (28) [12].

$$x(t) = x_0 + (x_0 - x_{t_f}) \cdot (15\tau^4 - 6\tau^5 - 10\tau^3) \quad (27)$$

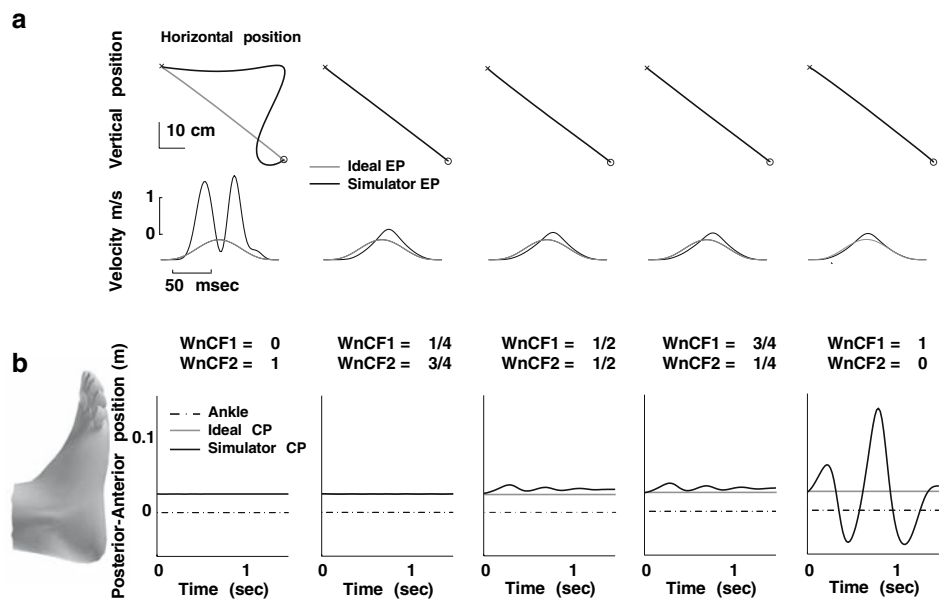
$$y(t) = y_0 + (y_0 - y_{t_f}) \cdot (15\tau^4 - 6\tau^5 - 10\tau^3) \quad (28)$$

442 Figure 5 shows a qualitative comparison between this ideal  
443 hand path and simulation results for the Jerk minimization.  
444 No differences can be noticed for the endpoint trajectory  
445 stick morphology and only slight differences can be observed  
446 between velocity profiles. In fact, evaluation of the dif-  
447 ferences between the time courses of the endpoint  
448 coordinates obtained by simulation and by using (27) and  
449 (28) gives a root mean squared error below 1 cm, corre-  
450 sponding to about 1% of the whole distance covered by the  
451 finger.

452 For CM variance minimization (equation 18), the full  
453 antero-posterior stabilization  $\sigma_{CM}^2 = 0$ , was considered as  
454 ideal solution [27, 28]. Simulation results for  $\sigma_{CM}^2$  are  
455 reported in Fig. 6. Panel B clearly shows that the simulator  
456 is perfectly able to stabilize the whole body CM ( $\sigma_{CM} =$   
457 0.06 mm) respecting all task and anatomo-physiological  
458 constraints.

459 As we did for the CM (equation 18), the cost function  
460 representing the CP stabilization model consists of its an-  
461 tero-posterior variance,  $\sigma_{CP}^2$ ; therefore its full stabilization,  
462  $\sigma_{CP}^2 = 0$ , was considered as the ideal model prediction.  
463 Despite the fact that the  $\sigma_{CP}^2$  model introduces relevant  
464 nonlinear kinetics components with respect to  $\sigma_{CM}^2$ , even

**Fig. 7** Combination of two cost functions. Predictions corresponding to five combinations of the minimum jerk (Jerk) and the minimum center of pressure displacement ( $\sigma_{CP}^2$ ) models are reported. In **a** the Jerk theoretical endpoint trajectory and velocity profiles (gray lines) are compared with the simulator predictions (black lines); **b** represents the comparison between the fully stabilized CP antero-posterior position (gray lines) and the simulation results (black lines).  $W_{NCF1}$  and  $W_{NCF2}$  are the weights given to the Jerk and  $\sigma_{CP}^2$  models, respectively



465 more accurate results ( $\sigma_{CP} = 0.03$  mm) were obtained.  
 466 Besides showing the simulator reliability in searching  
 467 optimal strategies, WBP execution predicted by this model  
 468 seems realistic, because resulting large and complicate  
 469 upper limbs movements, shown in the first column of  
 470 Fig. 7, resemble equilibrist walking on the tightrope, i.e.,  
 471 keeping the CP stable. In both simulator and funambulist  
 472 behaviours inertial effects of wide arm movements aim at  
 473 compensating unbalancing due to the rest of the body.

474 Besides being effective, the simulator proves reliable.  
 475 Having performed ten simulations for each of the three  
 476 tested CFs, the average deviation from the best solution is  
 477 below 5%.

### 478 3.3 Combination of motor planning criteria

479 In order to test the efficiency of the simulator with MCFs,  
 480 different combinations of Jerk and  $\sigma_{CP}^2$  models were  
 481 implemented. These two models were selected because  
 482 they focus on different aspects of the movements (focal and  
 483 postural) and together they involve both movement kinematics  
 484 and kinetics, therefore, their combination represents a  
 485 relevant test of the simulator's capabilities.

486 As expected, results, reported in Table 2 and in Fig. 7,  
 487 show that the lower the weight of the NCFs the bigger  
 488 the discrepancies between the MCF model predictions  
 489 and the movement characteristics theoretically predicted  
 490 by the corresponding motor planning model. Nevertheless,  
 491 a  $W_{NCF}$  value of just 1/4 still achieves a reasonable  
 492 match with the theoretical result, indicating that even  
 493 with MCFs the minimum search algorithm is working  
 494 effectively.

## 4 Discussion

495  
 496 In the following, simulator performances in predicting  
 497 WBP executions will be discussed and compared to anal-  
 498 ogous works previously proposed in literature. Moreover,  
 499 some interesting neurophysiological implications of the  
 500 new possibilities given by the simulator characteristics will  
 501 be considered.

502 Reported results clearly show a remarkable accuracy of  
 503 the proposed simulation technique. In fact, although it uses  
 504 a biomechanical model with 5 DoFs, entailing a minimum  
 505 search in a 70 (5 DoFs · 14 Coefficients of B-splines)  
 506 dimension hyperspace, the simulator achieves consistent  
 507 full minimizations, compatible with task fulfilment. Evi-  
 508 dence of the simulator's reliability are the complete  
 509 stabilization of CM and CP achieved when respective  
 510 variances ( $\sigma_{CM}^2$  and  $\sigma_{CP}^2$ ) are used as cost figures. In par-  
 511 ticular, the result related to  $\sigma_{CP}^2$ , which includes nonlinear  
 512 components related to kinetics, clearly demonstrates that  
 513 not only the simulator can efficiently handle the high

**Table 2** The discrepancies between the theoretical results and the movements

$W_{NCF1}-W_{NCF2}$	0-1	1/4-3/4	1/2-1/2	3/4-1/4	1-0
RMSE Jerk (mm)	81.10	31.50	20.80	20.30	9.50
RMSE $\sigma_{CP}^2$ (mm)	0.03	0.13	2.80	3.14	47.30

The theoretical results predicted by minimum endpoint jerk (Jerk) and minimum center of pressure displacement ( $\sigma_{CP}^2$ ) models and the movements predicted by the simulator for 5 different combinations of the weights  $W_{NCFi}$  are reported here. The discrepancy are evaluated as root mean squared errors (RMSE)

514 nonlinearities of the human motor system, but can exploit  
515 them to carry out the movement optimization.

516 Another useful characteristic of the proposed simulation  
517 technique is the possibility of imposing nonlinear con-  
518 straints on the optimization problem. Indeed, this feature  
519 allows a simple, but effective, representation of human  
520 body anatomic-physiological characteristics. For instance,  
521 in addition to what has been done for many of the models  
522 proposed in literature [12, 22, 25, 30, 44, 51], the simulator  
523 takes into account the maximal torque that different mus-  
524 cular groups can exert at each joint angular position and  
525 velocity. Moreover, having been already highlighted the  
526 importance of using mathematical models that account for  
527 muscle dynamics [15], motor command-contraction  
528 dynamics is also implemented in the simulation method  
529 proposed here. The representation of the motor system  
530 intrinsic constraints seems especially important because it  
531 allows a more effective induction of the causes of the  
532 human motor behavior. Indeed, it gives the possibility to  
533 understand whether observed movement features are direct  
534 consequences of motor system characteristics, or whether  
535 they result from a specific strategy. For instance, it was  
536 demonstrated that human movement smoothness is  
537 strongly related to the nervous and muscular system fea-  
538 tures and not to an explicit CNS goal [17]. In the proposed  
539 simulator this specific consideration seems to be well  
540 represented by the imposed limits on the muscular inputs.  
541 Indeed, all simulator solutions, even when they aimed at  
542 optimizing critical movement aspects that do not explicitly  
543 include gracefulness (e.g., CM or CP stabilization), were  
544 never characterized by an unnatural jerkiness.

545 A further important aspect differentiating the proposed  
546 simulator from most similar works in the literature [17, 22,  
547 25, 30, 32, 44, 51] is the validation of the biomechanical  
548 model representing human body segments and of the  
549 inverse/forward dynamic method (FDM). This procedure  
550 appears to be crucial, because inaccuracy in the estimation  
551 of joint torques would greatly affect the predictions  
552 obtained by implementing any motor planning model-  
553 based on movement kinetics [17, 25, 30, 48, 51] or energy  
554 [1, 42]. A further remark must be given on this topic.  
555 Although experimental evidences show that gravity deter-  
556 mines relevant differences between upward and downward  
557 arm movements [34], most of the motor planning model  
558 used in literature, even those focusing on movement  
559 kinetics or energy [1, 30, 42, 51], do not consider the  
560 gravitational component of the joint torque or the varia-  
561 tions in system potential energy. If this approximation was  
562 thought to be acceptable for arm movements, this is cer-  
563 tainly not true for WBP. Therefore, the simulator's  
564 capacity to include gravitational effects seems indispens-  
565 able to the study and modeling of motor-planning criteria  
566 underlying the execution of this category of movement.

567 Overall, the use of IDM to evaluate movement kinetics,  
568 instead of more classical FDMs, proves to be effective for  
569 the purposes of the present work. Indeed, the large number  
570 of DoFs of the multi-segment inverted pendulum repre-  
571 senting the human body, as well as its intrinsic instability  
572 and nonlinearities, would make the use of the FDM  
573 inconvenient. Moreover, although it was already demon-  
574 strated that IDM and FDM agree if applied to the same  
575 model [5], the latest would require a huge computational  
576 power and the integration of feedback control techniques.  
577 Hence, it would restrain the simulator applicability because  
578 of the need for supercomputers with parallel processors  
579 [6, 32]. On the contrary, the proposed simulator can be run  
580 even on simple PCs: the results reported in this paper were  
581 obtained using a laptop with a Pentium 4 processor  
582 (2 GHz) and they took in average  $24 \pm 7$  min.

583 The use of FDM would be necessary if the simulations  
584 were focused on the execution of movements in perturbed  
585 environments. In fact, in this case it would be necessary to  
586 include specific muscle models allowing modulations of  
587 the joint impedance through cocontraction and the stretch-  
588 reflex. Therefore, the implementation of motor control  
589 techniques such as the equilibrium point theory would be  
590 possible. However, since the aim of the present work is the  
591 prediction of the "desired trajectories" corresponding to  
592 optimization criteria in unperturbed conditions, the inclu-  
593 sion of these models does not seem of primary importance.  
594 Indeed, it was shown that joint impedance is preponderant  
595 in perturbed movements and it is minimized for unper-  
596 turbed ones [8]. In the literature, this idea was even used to  
597 solve the third indeterminacy problem in simulation using  
598 FDM [3, 4, 32, 33] and, in general, to estimate muscle  
599 contributions to net joint torques [5, 11, 20, 33, 35, 50]. In  
600 particular, the third indeterminacy problem was solved by  
601 minimizing the global effort or the metabolic cost of  
602 movement. Since most of the planning criteria presented in  
603 the literature, for which the present simulator has been  
604 conceived, aim at solving just the two first indeterminacy  
605 problems [1, 12, 14, 17, 30, 42, 51], detailed models of  
606 each muscle, including uni- or bi-articular ones, and opti-  
607 mal force sharing criteria have not been integrated, though it  
608 would be theoretically possible.

609 In general, flexibility is a very important characteristics  
610 of the proposed simulation method. Although for simplicity  
611 and clarity reasons it is here applied to a bi-dimensional  
612 and symmetric movement, the simulator could also be  
613 employed for three-dimensional and asymmetric move-  
614 ments. In this case, the motion of the segments should be  
615 described also in terms of roll and yaw. Therefore, a line of  
616 the matrix  $\mathbf{a}$  should be dedicated to the description of each  
617 of these angle trajectories. Moreover, in order to apply the  
618 IDM, the model segments should be characterized by their  
619 inertial moment in the frontal and horizontal planes. Also

620 joint stiffness and viscosity in these two planes should be  
621 modelled. Accordingly, specific constraints on the joint  
622 range of motion and maximal torques should be included.

623 All results obtained for single motor planning criteria  
624 clearly show that the simulator integrates goal-oriented and  
625 postural components of the movement subordinating the  
626 equilibrium control to the endpoint trajectory generation  
627 (Jerk) or vice versa ( $\sigma_{CM}^2$ ,  $\sigma_{CP}^2$ ). For instance, wide and  
628 dangerous CP oscillations can be observed in simulations  
629 minimizing Jerk and large upper limb movement are pre-  
630 dicted when the CM or CP stabilization is achieved. This  
631 could suggest that the criterion actually used by CNS to  
632 plan a whole body transitive movement should include  
633 both its goal-oriented and postural components. Further-  
634 more, this hypothesis is in line with the idea proposed in  
635 literature that the true optimality criterion is likely to  
636 involve a mix of cost terms and that weights defining a  
637 multiattribute cost could be used as command signals by  
638 higher-level centers [47].

639 From this point of view, the demonstrated capability to  
640 combine different cost figures, using MCFs, seems to be  
641 particularly interesting, because it gives the possibility to  
642 test the validity of these theories also in the case of complex  
643 WBP task. In particular, results related to the simultaneous  
644 minimization of the hand trajectory jerkiness and the CP  
645 variance, although only illustrative and without any par-  
646 ticular neurophysiological relevance, show the simulator's  
647 effectiveness in optimizing multiple cost functions. Indeed,  
648 the proposed combination technique succeeds in balancing  
649 the two MCF components and in producing a satisfactory  
650 gradual transition from one criterion to the other. However,  
651 it is important to notice that different characteristics of each  
652 of the motor planning criteria, such as specific nonlinearities  
653 of the cost function, could affect the results.

## 654 5 Conclusion

655 The proven efficacy and reliability of the proposed human  
656 movement simulator in identifying the "desired trajecto-  
657 ries" corresponding to the optimization of various cost  
658 functions, strongly suggest that it could be effectively used  
659 to verify whether classical theories on motor planning  
660 could be extended to whole body movements involving  
661 several DoFs and requiring the coordination between task  
662 accomplishment and maintenance of balance. In addition,  
663 its capability of combining several optimality criteria  
664 seems especially interesting because it allows one to  
665 explore new motor control theories.

666 Moreover, the simulator's flexibility would allow its  
667 extension to further applications. For instance, it could be  
668 used to simulate various task. In particular, if the simula-  
669 tion technique was applied to a proper three-dimensional

670 biomechanical model, the features of movements with  
671 relevant medio-lateral components could be predicted too.  
672 Also different force fields acting on the subject (e.g., hyper  
673 or micro-gravity) or subject motor deficits (e.g., weakening  
674 of specific muscle groups or reduction of joint mobility)  
675 could be easily implemented.

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