Human Torque Estimation for an LMI-based Convex Control Rehabilitation Strategy using Assistive Robots

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***Abstract*— The number of people affected by motor impairment has increased considerably in recent years, raising, in consequence, the interest in the use of assistive robotic devices for additional motricity and rehabilitation purposes. The main challenge for this kind of devices is dealing with the interaction between the human and the robot for a cooperative movement, especially in active rehabilitation schemes, where patients are incited to participate in the movement tasks, in order to increase efficiency of the rehabilitation protocol. A question that arises in this specific context is how to estimate human contribution in order to personalize the rehabilitation tasks. Current solutions being based on cumbersome measurement devices, this paper suggest an observer-based solution, allowing to estimate the human torque based on few dynamical measurements, in addition to an LMI-based computed-torque controller. This suggested scheme has been tested and validated using OpenSim software, which is widely used for biomechanical modeling, simulation and analysis.**

I. INTRODUCTION

Recent studies by the World Stroke Organization [1], show that there is an increase in the number of people with motor impairment. An approach to aid patients affected by this consist in robot-assisted rehabilitation, which has shown long-lasting effects and can be more efficient than conventional therapy [2]. However, the interaction between human and assistive robots comes with its own set of challenges, such as providing patients with personalized assistance to reduce the intervention of robotic actuation and, therefore, improve their condition [3], [4].

In order to customize an exoskeleton aid for patients, some studies adopt an "Assist-as-needed" control strategy which aims to adapt the actuation given by the robot to increase or decrease difficulty for a patient trying to follow a certain trajectory. Most of these works require either measuring torques or forces, such as in [5], using expensive sensors or estimating the torque via electromyography (EMG), for example in [6]. However, EMG is known to have a low signal-to-noise ratio, providing an inaccurate torque estimation, in addition to be cumbersome for the patients. Availability of the contribution of the patient allows to design an adaptive control of the robotic assistance.

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In the context of robot manipulators, computed torque control (CTC) is a well-known strategy for trajectory tracking, which makes use of a nonlinear dynamic model of the system obtained from Euler-Lagrange equations for feedback linearization [7]. This technique has a wide range of applications, such as serial [8] and parallel [9] manipulators, and has also been used for exoskeleton control problems [10]. In the field of assistive rehabilitation where a trajectory should be followed, this methodology seems particularly suitable.

Furthermore, classical CTC is usually associated with linear controllers, such as PD controller, and is based on eliminating non-linear terms by means of feedback linearization. In this paper, a variation of the computed-torque approach with a nonlinear controller is introduced, based on convex control using linear matrix inequalities (LMIs). This approach is known for being a systematic procedure allowing to deal with a large class of non-linearities. Originally developed for linear parameter varying (LPV) systems and Takagi-Sugeno (TS) models, it consist in a convex rewriting of a nonlinear system [11] and a control law constructed from a convex sum of gains called parallel distributed compensation (PDC). These gains can be obtained via LMI conditions by means of the direct Lyapunov method for asymptotic stability of the origin.

The main originality of this work is the estimation of the human contribution in terms of torque using only few measurements, namely the joint angle and velocity, allowing to overcome the need of expensive and cumbersome measurement devices. The estimated human torque is used to guarantee the stability of the whole closed loop system, ensured by the separation principle.

Moreover, the availability of open-source software for musculoskeletal models simulation, such as OpenSim [12], makes the testing and validation of estimation and control schemes more tractable under realistic conditions. This allowed us to test the proposed scheme on a realistic arm model and to validate the estimation and the control performances.

First, a brief explanation of the classical LMI-based convex control procedure is presented, followed by some information about mechanical modeling of human-robot interactions. As for main results, a methodology for stabilizing dynamics of tracking error by means of nonlinear computed torque and convex optimization techniques; then a procedure to design an observer to obtain a human torque estimation is addressed, validated by the widely known separation principle.

II. PRELIMINARIES

In this section, some essential knowledge will be presented: first, the classical procedure for convex rewriting and LMI-based convex control; second, the modeling of human-robot dynamics based on Euler-Lagrange equations of motion is shown.

A. LMI-based Convex Control

Consider a nonlinear system that can be written in a quasi-LPV form

$$\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t), \tag{1}$$

for sake of simplicity, dependence on time t will be omitted in the sequel. Variables $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^r$ are the state vector and input vector, respectively. Provided that the system (1) has non-constant terms in $A(x)$ and $B(x)$ that are bounded for any $x \in \Omega$, $0 \in \Omega$, namely, $z_i(x) \in [z_i^0, z_i^1]$, $i \in \{1, 2, \ldots, p\}$, then convex functions:

$$w_0^i(z) = \frac{z_i^1 - z_i(x)}{z_i^1 - z_i^0},\ w_1^i(z) = 1 - w_0^i(z),\ i \in \{1, 2, \ldots, p\} \tag{2}$$

can be defined, which allows rewriting of any non-constant term in $A(x)$ and $B(x)$ as

$$z_i(x) = w_0^i(z)z_i^0 + w_1^i(z)z_i^1 = \sum_{j=0}^{1} w_j^i(z)z_i^j,\ i \in \{1, 2, \ldots, p\}. \tag{3}$$

This meaning that nonlinear system (1) can be rewritten as:

$$\dot{x} = \sum_{\mathbf{i} \in \mathbb{B}^p} \mathbf{w_i}(z) \left(A_\mathbf{i} x + B_\mathbf{i} u\right), \tag{4}$$

where convexity of functions (2) is employed to guarantee $\mathbf{w_i}(z) \in [0, 1]$, $\forall x \in \Omega$ and $\sum_{\mathbf{i} \in \mathbb{B}^p} \mathbf{w_i}(z) = 1$, $\forall x \in \mathbb{R}^n$; $\mathbb{B} = \{0, 1\}$, $\mathbf{i} = (i_1, i_2, \ldots, i_p)$, $\mathbf{w_i}(z) = w_{i_1}^1(z)w_{i_2}^2(z) \cdots w_{i_p}^p(z)$, such that the linear TS dynamical model is a convex combination of local linear models in the region of interest, this is $A_\mathbf{i} = A(z)|_{\mathbf{w_i}(z)=1}$ and $B_\mathbf{i} = B(z)|_{\mathbf{w_i}(z)=1}$ [13].

Based on the convex structure (4), a parallel distributed compensation (PDC) control law can be applied:

$$u = \sum_{\mathbf{k} \in \mathbb{B}^p} \mathbf{w_k}(z) F_\mathbf{k} x, \tag{5}$$

where $F_\mathbf{k}$, with $\mathbf{k} \in \mathbb{B}^p$, are gains to be found by LMI computation:

$$X = X^T > 0, \tag{6}$$

$$A_\mathbf{i} X + B_\mathbf{i} M_\mathbf{k} + X A_\mathbf{i}^T + M_\mathbf{k}^T B_\mathbf{i}^T < 0, \tag{7}$$

as $F_\mathbf{k} = M_\mathbf{k} X^{-1}$. Thus, any trajectory within the outermost Lyapunov level $\{x : x^T X^{-1} x \leq c\} \subset \Omega$ goes asymptotically to 0 [13]. Grouping similar terms that share the same factor in convex product $\mathbf{w_i}(z)\mathbf{w_k}(z)$ can lead to less conservative LMI conditions, as can be seen in [14], as well by applying Tanaka's relaxation [15]:

$$\begin{aligned} &X = X^T > 0, \\ &G_\mathbf{ii}^T P + P G_\mathbf{ii} < 0, \\ &\left(\frac{G_\mathbf{ik} + G_\mathbf{ki}}{2}\right)^T P + P \left(\frac{G_\mathbf{ik} + G_\mathbf{ki}}{2}\right) \leq 0, \end{aligned}$$

where $G_\mathbf{ik} = A_\mathbf{i} - B_\mathbf{i} F_\mathbf{k}$.

B. Modeling Human-Robot Dynamics

There are many works in literature where the human body is described as a series of rigid bodies, for example, [16] and [17]. This consideration allows to simplify the study and analysis of human motion. Therefore, a dynamical model of its motions can be obtained by expressing the accelerations in terms of forces and inertia by Euler-Lagrange formulation:

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q), \tag{8}$$

where $q \in \mathcal{R}^n$ is the position vector of the joints, $\dot{q}$ and $\ddot{q}$ are the velocity and acceleration vectors, $\tau \in \mathcal{R}^n$ is the vector of torques applied on each individual joint, $M(q) \in \mathcal{R}^{n \times n}$ is an inertia matrix, $C(q, \dot{q}) \in \mathcal{R}^{n \times n}$ the Coriolis matrix and $G(q) \in \mathcal{R}^n$ represents gravitational forces. At joint level, τ is the sum of torque provided by the exoskeleton τ_r and the one applied by the human τ_h. Considering that τ_r is used for control, system (8) can be rewritten as follows:

$$\tau_r = M(q)\ddot{q} + N(q, \dot{q}) - \tau_h, \tag{9}$$

where $N(q, \dot{q}) = C(q, \dot{q})\dot{q} + G(q)$ will be used for simplicity. Note that the physical parameters of the body are needed for the Euler-Lagrange equations, in addition to the position, the velocity and the torque applied by the human. In this work, only the position and the velocity will be required to design a nonlinear observer.

III. PROPOSED METHODOLOGY

A. LMI-based Computed-Torque Control

Since the control objective is to follow a trajectory, we consider a variation of the computed-torque law, similar to [18]:

$$\tau_r = M(q)(\ddot{q}_d - M^{-1}(q)u) + N(q_d, \dot{q}_d) - \hat{\tau}_h, \tag{10}$$

where $q_d \in \mathcal{R}^n$ is the vector of desired positions, $u \in \mathbb{R}^m$ $\hat{\tau}_h$ is an estimation of the human torque. By equivalence of (9) and (10), the following can be written:

$$M(q)(\ddot{q}_d - M^{-1}(q)u) + N(q_d, \dot{q}_d) - \hat{\tau}_h = M(q)\ddot{q} + N(q, \dot{q}) - \tau_h,$$

which leads to:

$$u = M(q)(\ddot{q}_d - \ddot{q}) + N(q_d, \dot{q}_d) - N(q, \dot{q}) + \tau_h - \hat{\tau}_h.$$

By including $\hat{\tau}_h$ in the computed torque law (10) the estimation error of human torque will appear in the dynamics of the tracking error, which will provide a better insight of what, as will be presented afterward.

In [19], expressions with a convergent Taylor series can be found to be written as $f(q_d) - f(q) = F(q_d, q)e$ with

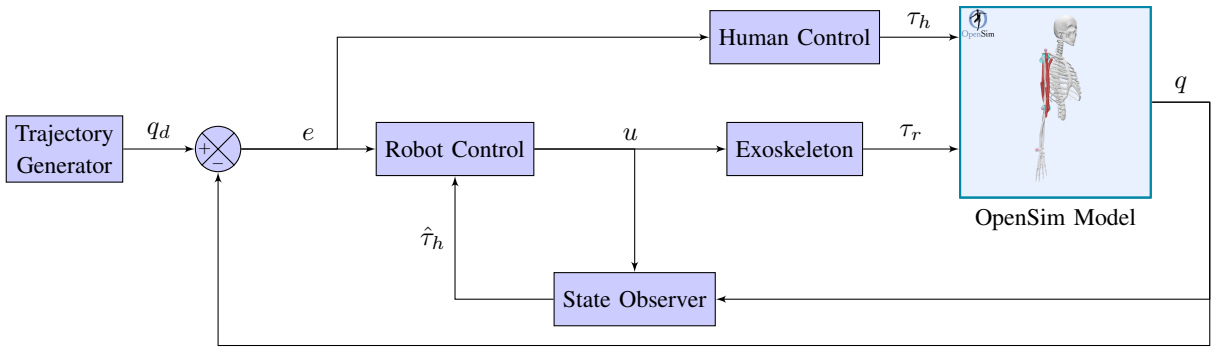

Fig. 1. Control block diagram of the system

$e = q_d - q$ being the tracking error. By utilizing this method, it is possible to write

$$\begin{aligned}\ddot{e} =& M^{-1}(q)(u - N(q_d, \dot{q}_d) + N(q, \dot{q}) - \tau_h + \hat{\tau}_h) \\ =& M^{-1}(q)(u - N_1(x)e - N_2(x)\dot{e} - \tau_h + \hat{\tau}_h)\end{aligned}$$

where $x = [q_d^T \ \dot{q}_d^T \ q^T \ \dot{q}^T]^T$ and $N(q_d, \dot{q}_d) - N(q, \dot{q}) = N_1(x)e + N_2(x)\dot{e}$. This allows obtaining a dynamic system of the tracking error:

$$\dot{\bar{e}} = E(q)\left(\begin{bmatrix} 0 & I \\ -N_1(x) & -N_2(x)\end{bmatrix}\bar{e} + \begin{bmatrix}0\\ I\end{bmatrix}u + \begin{bmatrix}0\\ I\end{bmatrix}(\tau_h - \hat{\tau}_h)\right),$$

with $E(q) = diag\{(I, M^{-1}(q))\}$ and $\bar{e} = [e^T \ \dot{e}^T]^T$. Notice that previous system is in the form

$$\dot{\bar{e}} = A(x)e + B(x)(u + \tau_h - \hat{\tau}_h), \tag{11}$$

where

$$A(x) = \begin{bmatrix} 0 & I \\ -M^{-1}(q)N_1(x) & -M^{-1}(q)N_2(x)\end{bmatrix},$$
$$B(x) = \begin{bmatrix}0\\ M^{-1}(q)\end{bmatrix}.$$

To achieve correct tracking of a desired trajectory, it is necessary to stabilize system (11). This can be done by rewriting it as a convex model by following steps described in section II-A, defining non-constant terms $z(x) \in [z^0, z^1]$ to construct convex functions $\mathbf{w_i}(z)$, which results in[1]:

$$\dot{\bar{e}} = \sum_{\mathbf{i}\in\mathbb{B}^p} \mathbf{w_i}(z)\left(A_\mathbf{i}\bar{e} + B_\mathbf{i}(u + \tau_h - \hat{\tau}_h)\right), \tag{12}$$

where convex functions $\mathbf{w_i}(z) \in [0,1]$, $\forall x \in \Omega$ and $\sum_{\mathbf{i}\in\mathbb{B}^p}\mathbf{w_i}(z) = 1$, $\forall x \in \mathbb{R}^{4n}$; $\mathbb{B} = \{0,1\}$, $\mathbf{i} = (i_1, i_2, \ldots, i_p)$, $\mathbf{w_i}(z) = w^1_{i_1}(z)w^2_{i_2}(z)\cdots w^p_{i_p}(z)$.

Thus, by applying a PDC control law

$$u = \sum_{\mathbf{k}\in\mathbb{B}^p} \mathbf{w_k}(z)F_\mathbf{k}\bar{e},$$

[1]Notice that since $M(q)$ is positive-definite, as it is an inertia matrix, which implies that $E(q) > 0$; this allows to extract said matrix from (11) in order for less non-constant terms $z_i(t)$ to appear on the convex model (12), as necessary conditions only require stability for $E(q)^{-1}\dot{\bar{e}}(t)$.

with $\mathbf{k} = (k_1, k_2, \ldots, k_p)$ and $F_\mathbf{k}$ gains to be obtained by computing LMI conditions (6)-(7). Leading to a closed-loop system:

$$\dot{\bar{e}} = \sum_{\mathbf{i},\mathbf{k}\in\mathbb{B}^p} \mathbf{w_i}(z)\left((A_\mathbf{i} + B_\mathbf{i}F_\mathbf{k})\bar{e} + B_\mathbf{i}(\tau_h - \hat{\tau}_h)\right). \tag{13}$$

Note that it is needed that the torque estimation error converges to zero, that is $\lim_{t\to\infty}\tau_h - \hat{\tau}_h = 0$, in order to ensure the stability of (12). However, since it is practically challenging to have an accurate torque measurement, in the following section a nonlinear observer is proposed inorder to estimate the human torque.

B. Human Torque Observer

In order to estimate the human torque, consider that $\tau_h = l_h f_h$, where $f_h \in \mathbb{R}^n$ are linear forces applied by the muscles and $l_h \in \mathbb{R}^{n\times n}$ correspond to their moment arms for each joint, which are considered constant due to the small distance between the muscle and the joint of interest. Thus, its derivative $\dot{\tau}_h = l_h\dot{f}_h$ and double derivative $\ddot{\tau}_h = l_h\ddot{f}_h$ are taken into consideration to propose the assumption $\dddot{\tau}_h = 0$. This statement would imply that $\ddot{\tau}_h$ is constant, while still considering the rate of change of torque τ_h to be variable. With this assumption and by obtaining the acceleration dynamics of the joints from (9):

$$\ddot{q} = M^{-1}(q)(\tau_r + \tau_h - N(q, \dot{q})), \tag{14}$$

which can be written, along with the assumption $\dddot{\tau}_h = 0$, as

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \\ \tau_h \\ \dot{\tau}_h \\ \ddot{\tau}_h\end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ -M^{-1}(q)N_q(q,\dot{q}) & -M^{-1}(q)N_{\dot{q}}(q,\dot{q}) & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix} q \\ \dot{q} \\ \tau_h \\ \dot{\tau}_h \\ \ddot{\tau}_h\end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}(q) \\ 0 \\ 0 \\ 0\end{bmatrix}\tau_r, \tag{15}$$

$$y = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0\end{bmatrix}\begin{bmatrix} q \\ \dot{q} \\ \tau_h \\ \dot{\tau}_h \\ \ddot{\tau}_h\end{bmatrix}, \tag{16}$$

where $N(q,\dot{q}) = N_q(q,\dot{q})q + N_{\dot{q}}(q,\dot{q})\dot{q}$. Since the model (15)-(16) is in the form

$$\begin{aligned}\dot{x} =& A_o(y)x_o + B_o(y)u\\ y =& C_o x_o\end{aligned}$$

with $x_o = [q^T \ \dot{q}^T \ \tau_h^T \ \dot{\tau}_h^T \ \ddot{\tau}_h^T]^T$; similarly to the tracking error (12), it can be rewritten by defining p non-constant terms, that depend uniquely on the output, $z_i(y)$ in $A_o(y)$ and $B_o(y)$, as in (3), to construct convex functions $w_{\mathbf{i}}(z)$, as in (2), and get a convex model of the form:

$$\begin{aligned}\dot{x}_o =& \sum_{\mathbf{i}\in\mathbb{B}^p} \mathbf{w_i}(z)\left(A_{o\mathbf{i}}x_o + B_{o\mathbf{i}}u\right),\\ y =& C_o x_o,\end{aligned}$$

where $\mathbf{w_i}(z) \in [0,1]$, $\forall y \in \Omega$ and $\sum_{\mathbf{i}\in\mathbb{B}^p}\mathbf{w_i}(z) = 1$, $\forall x_o \in \mathbb{R}^{5n}$; $\mathbb{B} = \{0,1\}$, $\mathbf{i} = (i_1, i_2, \ldots, i_p)$, $\mathbf{w_i}(z) = w^1_{i_1}(z)w^2_{i_2}(z)\cdots w^p_{i_p}(z)$, $A_{o\mathbf{i}} = A_o(y)|_{\mathbf{w_i}(z)=1}$, $B_{o\mathbf{i}} = B_o(y)|_{\mathbf{w_i}(z)=1}$ and $C_{o\mathbf{i}} = C_o$. Premise variables $z_i(y)$ depend on measurable states, both position of the joint q and its velocity $\dot{q}$. A nonlinear observer in a convex form can be applied:

$$\dot{\hat{x}}_o = \sum_{i\in\mathbb{B}^p} w_{\mathbf{i}}(z)\left(A_{o\mathbf{i}}\hat{x}_o + B_{o\mathbf{i}}u + L_{\mathbf{i}}(y-\hat{y})\right), \tag{17}$$

$$\hat{y} = C_o\hat{x}_o, \tag{18}$$

where $\hat{x}_o$ and $\hat{y}$ represent estimations of states and output, respectively, while $L_{\mathbf{i}}$ are gains to be obtained via LMI conditions:

$$PA_{o\mathbf{i}} - N_{\mathbf{i}}C_o + A_{o\mathbf{i}}^T P - C_o^T N_{\mathbf{i}}^T < 0, \tag{19}$$

with $L_i = P^{-1}N_{\mathbf{i}}$, guaranteeing that the observation error $e_o = x_o - \hat{x}_o$ goes asymptotically to $e_o = 0$ [13]. These conditions come from dynamics of observation error e_o, which can be written as

$$\dot{e}_o = \sum_{\mathbf{i}\in\mathbb{B}^p} \mathbf{w_i}(z)\left(A_{o\mathbf{i}}x_o + B_{o\mathbf{i}} - A_{o\mathbf{i}}\hat{x}_o - B_{o\mathbf{i}} - L_{\mathbf{i}}(y-\hat{y})\right) \tag{20}$$

$$= \sum_{\mathbf{i}\in\mathbb{B}^p} \mathbf{w_i}(z)\left(A_{o\mathbf{i}} - L_{\mathbf{i}}C_o\right)e_o, \tag{21}$$

which is the source of LMI conditions (19) and will be used to determine stability of the controller-observer system in the next section.

C. Controller-observer stability

Since the goal is stabilizing the whole system, which is:

$$\begin{aligned}\dot{\bar{e}} =& \sum_{\mathbf{i},\mathbf{k}\in\mathbb{B}^p} \mathbf{w_i}(z)\left((A_{\mathbf{i}} + B_{\mathbf{i}}F_{\mathbf{k}})\bar{e} + B_{\mathbf{i}}(\tau_h - \hat{\tau}_h)\right)\\ \dot{e}_o =& \sum_{\mathbf{i}\in\mathbb{B}^p} \mathbf{w_i}(z)\left(A_{o\mathbf{i}} - L_{\mathbf{i}}C_o\right)e_o,\end{aligned}$$

provided that the third derivative of the human torque is equal to zero ($\dddot{\tau}_h = 0$) , the extended system can be written in the following form:

$$\begin{bmatrix}\dot{\bar{e}}\\ \dot{e}_o\end{bmatrix} = \sum_{\mathbf{i},\mathbf{k}\in\mathbb{B}^p} \mathbf{w_i}(z)\begin{bmatrix}A_{\mathbf{i}} + B_{\mathbf{i}}F_{\mathbf{k}} & B_{\mathbf{i}}Q\\ 0 & A_{o\mathbf{i}} - L_{\mathbf{i}}C_o\end{bmatrix}\begin{bmatrix}\bar{e}\\ e_o\end{bmatrix} \tag{22}$$

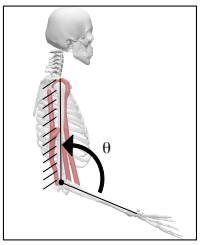

Fig. 2. OpenSim arm26 model

where $Q = diag\{0_{2n}, I_n, 0_{2n}\}$ is a matrix used to extract the observation error of human torque $\tau_h - \hat{\tau}_h$. Note that assumption on the human torque, $\dddot{\tau}_h = 0$, is not restrictive and can be easily satisfied, as what has been discussed in section III-B. Since the extended system (22) is in a triangular form, the separation principle holds, [20][2]. A control diagram of the system with the controller and observer can be seen in Fig. 1.

IV. SIMULATION

Tests were carried out using the model "arm26" [21] provided by OpenSim®, to which was added a torque actuator on the elbow, representing the assistance of an exoskeleton or an orthosis. The controller and observer were implemented is the MATLAB®/Simulink® environment, and a co-simulation framework has been established in order to communicate with OpenSim and to recover the dynamics as well as the real human torque. The parameters of the "arm26" model are presented in [21]. In this section, we will consider the rehabilitation of an elbow, so there is an assumption that the upper arm will not be moving for experimentation, which leads to a situation as shown in Fig. 2. By Euler-Lagrange formulation, it is possible to write

$$\tau_r = ml^2\ddot{\theta} + mgl\sin(\theta) - \tau_h, \tag{23}$$

where $\tau_r \in \mathbb{R}$ refers to torque applied by the robot, $\tau_h \in \mathbb{R}$ to the one applied by the human, $\theta \in \mathbb{R}$ represents the elevation angle of the elbow, m, l, $g \in \mathbb{R}$ correspond to the mass of the arm, its length and force of gravity, respectively. Note that system (23) is in the same form as (9). In order to implement a computed-torque controller, the physical parameters m, l need to be known or identified using movement data. By applying a computed-torque law of the form (10):

$$\tau_r = ml^2\ddot{\theta}_d - u + mgl\sin(\theta_d) - \hat{\tau}_h, \tag{24}$$

which depends on desired trajectory $\theta_d \in \mathbb{R}$ and the human torque estimation $\hat{\tau}_h \in \mathbb{R}$. By equating (23) and (24):

$$\begin{aligned}&ml^2\ddot{\theta} + mgl\sin(\theta) - \tau_h = ml^2\ddot{\theta}_d + mgl\sin(\theta_d) - \hat{\tau}_h\\ &ml^2(\ddot{\theta}_d - \ddot{\theta}) = -mgl\sin(\theta_d) + mgl\sin(\theta) + \hat{\tau}_h - \tau_h\\ &\ddot{e} = \frac{1}{ml^2}(-mgl\sin(\theta_d) + mgl\sin(\theta) + \hat{\tau}_h - \tau_h),\end{aligned}$$

[2]Note that for the proposed controller, estimation error of human torque appears similarly to an external disturbance. It is only necessary to ensure stability of the observer and design the controller by considering $\tau_h - \hat{\tau}_h = 0$, but it is also possible to reduce its effect on the controller by disturbance attenuation techniques, such as $\mathcal{H}_\infty$.

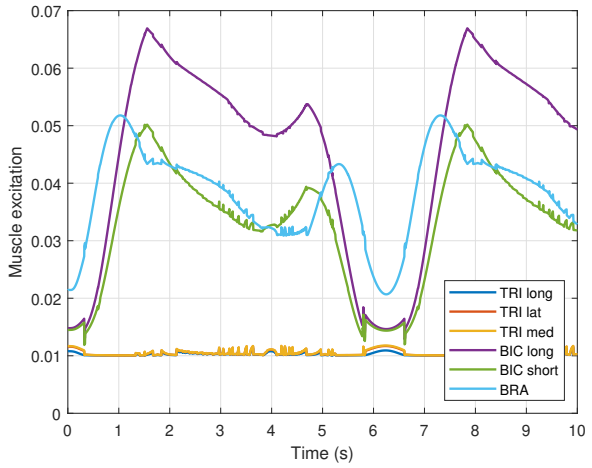

Fig. 3. Muscle excitation profiles.

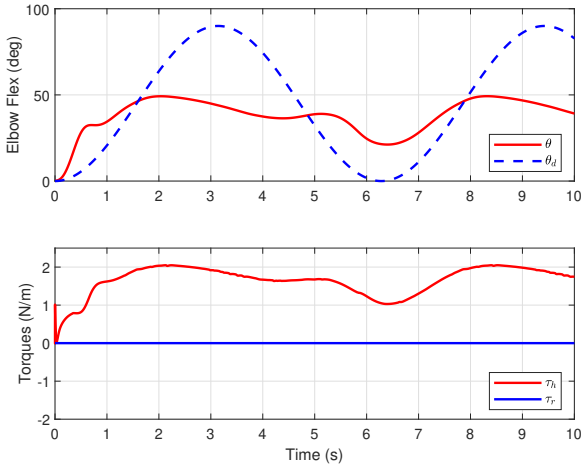

Fig. 4. Human movement simulation (without robot assistance)

where $e = \theta_d - \theta$. Then, it is possible to write:

$$\begin{bmatrix}\dot{e}\\ \ddot{e}\end{bmatrix} = \begin{bmatrix}0 & I\\ -\frac{g}{l}\frac{\sin(e-\theta_d)}{e-\theta_d} & 0\end{bmatrix}\begin{bmatrix}e\\ \dot{e}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ml^2}\end{bmatrix}(\tau_r + \tau_h - \hat{\tau}_h),$$

where non-constant term $z_1(\theta, \theta_d) = \frac{\sin(e-\theta_d)}{e-\theta_d} \in [-0.217,\ 1]$ is selected to construct the convex function:

$$w_0^1(\theta,\theta_d) = \frac{z_1^1 - z_1(\theta,\theta_d)}{z_1^1 - z_1^0},\ w_1^1(\theta,\theta_d) = 1 - w_0^1(\theta,\theta_d),$$

which allows writing

$$\begin{bmatrix}\dot{e}\\ \ddot{e}\end{bmatrix} = \sum_{i\in\mathbb{B}} w_i(\theta,\theta_d)\left(\begin{bmatrix}0 & 1\\ -\frac{g}{l}z_1^i(\theta,\theta_d) & 0\end{bmatrix}\begin{bmatrix}e\\ \dot{e}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ml^2}\end{bmatrix}(\tau_r + \tau_h - \hat{\tau}_h)\right),$$

which can be submitted to a PDC control law (5), leading to a closed-loop system of the form:

$$\begin{bmatrix}\dot{e}\\ \ddot{e}\end{bmatrix} = \sum_{i\in\mathbb{B}} w_i(\theta,\theta_d)\left(\left(\begin{bmatrix}0 & 1\\ -\frac{g}{l}z_1^{\mathbf{i}}(\theta,\theta_d) & 0\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ml^2}\end{bmatrix}F_{\mathbf{i}}\right)\begin{bmatrix}e\\ \dot{e}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ml^2}\end{bmatrix}(\tau_h - \hat{\tau}_h)\right),$$

which gains $F_{\mathbf{i}}$ are computed by means of LMI conditions (6)-(7) via the LMI Toolbox in MATLAB®. A solution is obtained with the following gains:

$$F_0 = [-358.19 \quad -13.40],$$
$$F_1 = [-399.66 \quad -13.87].$$

The next step is to design a human torque observer. For this, consider the following system

$$\ddot{\theta} = \frac{1}{ml^2}(\tau_r + \tau_h - mgl\sin(\theta)),$$

and by assuming $\dddot{\tau}_h = 0$, it is possible to write:

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ \tau_h\\ \dot{\tau}_h\\ \ddot{\tau}_h\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0 & 0\\ -\frac{g}{l}\frac{\sin(\theta)}{\theta} & 0 & \frac{1}{ml^2} & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ \tau_h\\ \dot{\tau}_h\\ \ddot{\tau}_h\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ml^2}\\ 0\\ 0\\ 0\end{bmatrix}\tau_r,$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ \tau_h\\ \dot{\tau}_h\\ \ddot{\tau}_h\end{bmatrix}.$$

By repeating the convex modeling procedure, non-constant term $z_1(\theta) = \frac{\sin(\theta)}{\theta} \in [-0.217\ 1]$ is selected to construct a required convex function for a nonlinear observer of the form (17)-(18).

$$\frac{d}{dt}\begin{bmatrix}\hat{\theta}\\ \dot{\hat{\theta}}\\ \hat{\tau}_h\\ \dot{\hat{\tau}}_h\\ \ddot{\hat{\tau}}_h\end{bmatrix} = \sum_{i\in\mathbb{B}} w_{\mathbf{i}}(\theta)\left(\begin{bmatrix}0 & 1 & 0 & 0 & 0\\ -\frac{g}{l}z_1^{\mathbf{i}}(\theta) & 0 & \frac{g}{l} & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{\theta}\\ \dot{\hat{\theta}}\\ \hat{\tau}_h\\ \dot{\hat{\tau}}_h\\ \ddot{\hat{\tau}}_h\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ml^2}\\ 0\\ 0\\ 0\end{bmatrix}\tau_r + L_{\mathbf{i}}(y - \hat{y})\right)$$

$$\hat{y} = \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{\theta}\\ \dot{\hat{\theta}}\\ \hat{\tau}_h\\ \dot{\hat{\tau}}_h\\ \ddot{\hat{\tau}}_h\end{bmatrix},$$

which gains $L_{\mathbf{i}}$ are computed by means of LMI conditions (19) with a decay rate $\alpha = 15$, resulting in:

$$L_0 = \begin{bmatrix}15.50 & 14.52\\ -23.89 & 175.71\\ -165.98 & 942.79\\ -4540.56 & 25778.87\\ -41922.10 & 237940.50\end{bmatrix} \quad L_1 = \begin{bmatrix}15.49 & -64.53\\ 117.67 & 175.71\\ 804.13 & 942.79\\ 21997.70 & 25778.87\\ 203100.27 & 237940.50\end{bmatrix}$$

The simulations were carried out using the following parameters: $m = 1.53\ kg$, $g = 9.81\ N/m$ and $l = 0.21\ m$. The movement done only by the human is in Fig. 4 with the muscle excitation in Fig. 3, showing that the human is not capable of following the desired trajectory $\theta_d = 1.57\sin(t + 1.5\pi) + 1.5$ (in radians). Fig. 5 shows that when robot assistance is added, the elbow angle follows the desired trajectory. Furthermore, Fig. 6 shows that the relative error for the torque estimation with respect to the real torque, $e_{or} = \frac{\tau_h - \hat{\tau}_h}{\tau_h} \times 100\%$, is less than 4% after the settling time, which is acceptable for this application.

V. CONCLUSIONS

In this paper, a nonlinear observer for human torque estimation was proposed, providing an alternative to the use of torque measurement or estimation using expensive or cumbersome sensors. The proposed variation of the computed torque control presents an improvement with respect to the classical one in terms of robustness considerations. A future

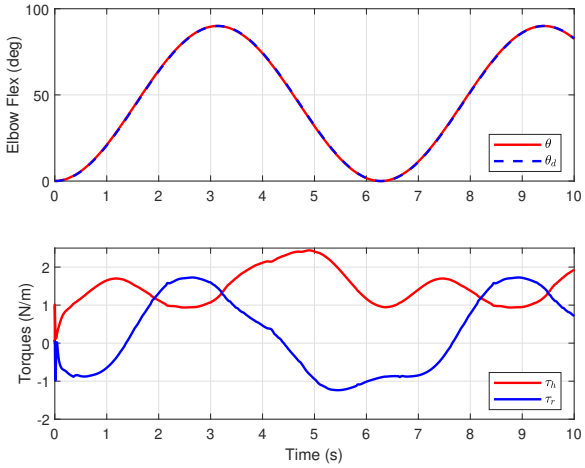

Fig. 5. Human-robot simulation

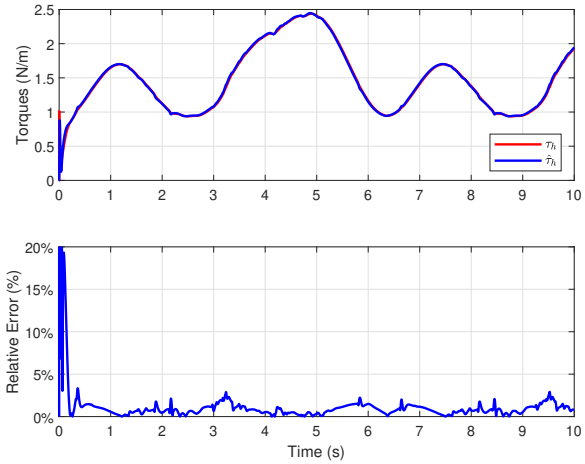

Fig. 6. Relative error of human torque observer

improvement could be making use of an LMI-based convex control approach in order to deal with uncertainties, disturbance rejection or to consider performance specifications. Future work may also consider Assist-as-Needed approaches, allowing to modify the rehabilitation trajectories based on the patient capacities. Furthermore, a possible improvement for the observer would be to consider only a knowledge of the position, instead of both the position and the velocity.

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