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On the design of optimal parametric insurance

Alexis Louaas* and Pierre Picard†

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Abstract

Under parametric insurance, the indemnity is a function of a publicly-observable parameter vector correlated with the loss incurred by the policyholder. The parameter vector yields a loss index, which is the best estimate of the loss, the basis risk being the random difference between the actual loss and the loss index. We show that the design of optimal parametric insurance depends on whether the parameter vector and the basis risk are independently distributed or not, and we analyze how it is affected by the attitude toward risk of the policyholder.

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1 Introduction

Parametric insurance consists in conditioning the indemnity paid to the policyholder not on the financial value of the losses incurred, but on publicly observable information correlated with these losses. This information may be parametric *stricto sensu*, as for instance in crop insurance when the payment to the farmer depends on average rainfall in a specific area during a given period. It may also take the form of a modeled-loss index reflecting a specific risk exposure. This is the case in property insurance when the payout depends on the policyholder's expected loss calculated on the basis of the wind speed of a hurricane measured at various points along its path, or according to the magnitude and epicenter of an earthquake.

The main advantage of parametric insurance is to eliminate the moral hazard issue and to avoid the claim-handling costs associated with the assessment of policyholders' actual losses. The primary concern is the basis risk retained by the policyholder, i.e. the fact that the parametric insurance trigger does not exactly match his actual losses. Parametric insurance covers are offered by direct insurers and they are also widely used as triggers in alternative risk transfer mechanisms, particularly catbonds. They now play an important role in the coverage of agriculture climate-related risks (particularly in developing countries) and of property catastrophic risks, and they tend to spread over a larger range of risk lines. Given this growing role, understanding the drivers of optimal parametric insurance is an objective of prime importance in the theory of insurance markets.

In what follows, we will analyze parametric insurance as the optimal solution to a risk-sharing problem with public and private information. Public information takes the form of a multi-dimensional parameter vector, and the issue then is how it should be used to define the indemnity paid to the policyholder. The basis risk is the difference between the loss incurred (which is private information of the policyholder) and the conditional expected loss based on this public information, which will be called the

loss index.

While the context of imperfect information on incurred losses is often implicit in the approach to parametric insurance, several important issues emerge when the problem is explicitly stated in that way. Firstly, should the parametric insurance payout be a function of the loss index (i.e., the best estimate of the loss based on the parameter vector), or should it depend on the parameter vector itself? Secondly, how can we characterize the optimal parametric indemnity schedule? In particular, does it look like commonly observed policies, such as index-based deductible contracts, or is it different? Thirdly, under which conditions does a change in public information affecting the basis risk improve the efficiency of the risk-sharing mechanism? Fourthly, does the usual relationship between insurance demand and risk aversion extend to the case of parametric insurance? As we will see, the answers to these questions depend heavily on whether the parameter vector and the basis risk are independently distributed or not.

It is in the area of agriculture risk management that parametric insurance has been most widely studied. Without being exhaustive, this includes the analysis of area-yield crop insurance by Miranda (1991) and Barnett and al. (2005), the interaction with the poverty issue in lower-income countries by Barnett and Mahul (2007), Chantarat et al. (2007), Chantarat et al. (2013) and Skees (2008), the effect of insurance on the adoption of new technologies by Mobarak and Rosenzweig (2013), Carter et al. (2016), Biffis and Chavez (2017) and Biffis et al. (2022), and the statistical analysis of the basis risk by Carter et al. (2017), Kusuma et al. (2018), and Clement et al. (2018). The design of optimal parametric insurance in a microeconomic setting hitherto has received much less attention. As we will see below, this is an issue logically related to the insurance demand problem with background risk, as studied by Gollier (1996). Clarke (2016) analyses parametric insurance in an expected utility setting, with the main conclusion that the basis risk may make it unattractive for strongly risk-averse policyholders. Bryan (2019) considers the case where the policyholder may

be ambiguity averse. Teh and Woolnough (2019) analyze how parametric triggers can be compared, and they determine a partial order ranking for any risk averse individual. Hott and Regner (2023) study how a risk-averse farmer optimally combines parametric insurance, indemnity insurance and savings in a two-period model.

The rest of the paper is organized as follows. Section 2 presents our general setting: the risk-averse policyholder has private information on his actual loss and public information takes the form of a parameter vector that defines the set of feasible parametric insurance contracts. We define the loss index and the basis risk, and we highlight the specificity of index-based insurance. Section 3 and 4 characterize the optimal parametric insurance, depending on whether the basis risk and the parameter vector are independent random variables or not, respectively, and we show that conclusions strongly differ in both cases. We particularly focus attention on the shape of the indemnity schedule, and on how it is affected by the policyholder's attitude toward risk, characterized by risk aversion and prudence. Section 5 analyzes how a more accurate information provided by the parameter vector reduces the basis risk and improves the effectiveness of parametric insurance. Section 6 concludes, and Section 7 includes complementary results and the proofs.

2 Setting

2.1 Model

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, with states of the world $\omega \in \Omega$. We consider a risk-averse individual who incurs a state-contingent loss : his risk exposure is defined by random variable $X(\cdot) : \Omega \rightarrow [0, \bar{x}]$ with loss $X(\omega)$ in state ω , where \bar{x} is the maximum possible loss (with possibly $\bar{x} = +\infty$). The state of the world - and thus the loss $X(\omega)$ - is privately observed by the individual. However, in each state ω , a signal $Y(\omega) \in \mathcal{S}$ is publicly observed, where $\mathcal{S} \subset \mathbb{R}^n$ is a measurable vector space.

The multivariate random variable $Y(\cdot) : \Omega \rightarrow \mathcal{S}$ defines a multi-dimensional state-dependent public information $Y(\omega)$, that will be called the *parameter vector* in what follows. Parametric insurance consists in conditioning the insurance payout on the parameter vector $Y(\omega)$ rather than on the loss $X(\omega)$.

Hence, a *parametric insurance contract* is defined by an indemnity function $I(\cdot) : \mathcal{S} \rightarrow \mathbb{R}_+$ that specifies the insurance payout $I(y)$ as a function of parameter vector y . When the individual purchases such a parametric cover at price P (the insurance premium), his random final wealth is

$$W_f = w_0 - X + I(Y) - P,$$

where w_0 is his initial wealth. The individual's attitude toward risk is characterized by a twice-differentiable von Neumann-Morgenstern utility function u , such that $u' > 0$, $u'' < 0$, and his expected utility is written as

$$\mathbb{E}u(W_f) = \mathbb{E}u(w_0 - X + I(Y) - P). \quad (1)$$

We will follow the usual approach to the analysis of insurance demand, by assuming that the insurance premium is proportional to the expected indemnity.¹ This is written as

$$P = (1 + m)\mathbb{E}I(Y), \quad (2)$$

where m is the loading factor, with $m \geq 0$ reflecting all costs incurred by insurers for assuming the risk $I(Y)$.² An optimal parametric insurance contract maximizes $\mathbb{E}u(W_f)$ with respect to P and $I(\cdot)$ subject to the pricing rule (2).

¹See Schlesinger (2013).

²In practice, parametric insurance is often subsidized by governments or international institutions, and the insurance premium may even be lower than the expected indemnity (i.e., m may be negative). The price of parametric insurance is indeed often used as a policy instrument, for instance when smallholder farmers face severe liquidity constraints that prevent them from purchasing crop insurance. We here restrict attention to the case of competitive insurance markets where insurers's costs are reflected in loaded premiums.

Let $Y(\Omega) \subset \mathcal{S}$ be the set of possible parameter vectors. Let us define $Z(\cdot) : Y(\Omega) \rightarrow \mathbb{R}_+$ by

$$Z(y) = \mathbb{E}[X(\omega) \mid Y(\omega) = y] \text{ for all } y. \quad (3)$$

In what follows, $Z(y)$ is called the *loss index* associated with parameter vector y . Hence, the loss index $Z(y)$ is the conditional expected value of the loss when parameter vector y is observed, and we denote $Z = Z(Y)$ the corresponding random variable, with $Z \in [\underline{z}, \bar{z}] \subset [0, \bar{x}]$.

We also define $\tilde{\varepsilon}(\cdot) : \Omega \rightarrow \mathbb{R}$ by

$$\tilde{\varepsilon}(\omega) = X(\omega) - Z(Y(\omega)) \text{ for all } \omega, \quad (4)$$

with

$$\mathbb{E}[\tilde{\varepsilon}(\omega) \mid Y(\omega) = y] = 0 \text{ for all } y.$$

Thus, $\tilde{\varepsilon}(\omega)$ is the difference between the true loss $X(\omega)$ and the loss index $Z(Y(\omega))$ in state ω , and it is called the *basis risk*. In other words, the basis risk $\tilde{\varepsilon}$ is a zero-mean random variable corresponding to the difference between the loss X and its conditional expected value $Z = Z(Y)$.

Frequently, under parametric insurance, the indemnity paid to the policyholder is a function of the loss index $Z(y)$ induced by parameter vector y , and not a function of the parameter vector y itself. We will refer to such a case as *index-based insurance*. Hence, a parametric insurance contract defined by $\{P, I(\cdot), \mathcal{S}\}$ is index-based when there exists $J(\cdot) : [\underline{z}, \bar{z}] \rightarrow \mathbb{R}_+$, such that $I(y) = J(Z(y))$ for all $y \in \mathcal{S}$.

We have $X = Z + \tilde{\varepsilon}$ and an optimal index-based insurance contract maximizes

$$\mathbb{E}u(W_f) = \mathbb{E}u(w_0 - Z - \tilde{\varepsilon} + J(Z) - P), \quad (5)$$

with respect to P and $J(\cdot)$, subject to

$$P = (1 + m)\mathbb{E}J(Z). \quad (6)$$

Interestingly, this is formally equivalent to searching for the optimal cover of an individual with insurable risk exposure Z and non-insurable background risk $\tilde{\varepsilon}$ as studied by Gollier (1996).

As will be set out in detail below, the characterization of an optimal parametric insurance contract strongly depends on whether or not the parameter vector Y and the basis risk $\tilde{\varepsilon}$ are stochastically independent. The assumptions underlying these two cases become particularly clear when Ω is a finite-dimensional vector space and the parameter vector $Y(\omega)$ observed in state ω is a publicly observable subvector of ω . Consider this case and assume that $\Omega = \Omega_1 \times \Omega_2$, with $\omega = (\omega_1, \omega_2)$, $\omega_1 \in \Omega_1$, $\omega_2 \in \Omega_2$ and let $(\Omega_1, \mathcal{F}_1, \mathbb{P}_1)$ and $(\Omega_2, \mathcal{F}_2, \mathbb{P}_2)$ two probability spaces, with $\mathcal{F} = \mathcal{F}_1 \otimes \mathcal{F}_2$. Assume further that $\mathcal{S} = \Omega_1$ and $Y(\omega) = \omega_1$, meaning that the parameter vector coincides with component ω_1 of state vector ω .

When $X(\omega)$ depends additively on ω_1 and ω_2 , i.e., $X(\omega) = X_1(\omega_1) + X_2(\omega_2)$, and $\mathbb{P} = \mathbb{P}_1 \times \mathbb{P}_2$, i.e., $(\Omega, \mathcal{F}, \mathbb{P})$ is a product probability space combining $(\Omega_1, \mathcal{F}_1, \mathbb{P}_1)$ and $(\Omega_2, \mathcal{F}_2, \mathbb{P}_2)$, we have

$$Z(Y(\omega)) = X_1(\omega_1) + \mathbb{E}X_2,$$

and

$$\tilde{\varepsilon}(\omega) = X(\omega) - Z(Y(\omega)) = X_2(\omega_2) - \mathbb{E}X_2,$$

which does not depend on the observable parameter vector Y that only reveals ω_1 . In that case, the parameter vector Y and the basis risk $\tilde{\varepsilon}$ are independently distributed. Conversely, if the observable and non-observable components of the state vector $\omega = (\omega_1, \omega_2)$ affect the loss $X(\omega)$ either non-additively or in a non-independent way, then generically Y and $\tilde{\varepsilon}$ are not independent.

2.2 Applications

We may illustrate the above through the cases of crop insurance and hurricane insurance. In each case, we assume $\Omega = \Omega_1 \times \Omega_2$, with $\omega = (\omega_1, \omega_2)$, $\omega_1 \in \Omega_1$, $\omega_2 \in \Omega_2$, with

$\mathcal{S} = \Omega_1$ and $Y(\omega_1) = \omega_1$.

2.2.1 Crop insurance

Consider the case of a farmer facing uncertain meteorological circumstances and other hazards affecting his crop. Here, w_0 is the harvested crop value under optimal conditions, and loss X is the decrease in this value due to adverse events. Assume that the parameter vector corresponds to the intensity of daily precipitations from planting to harvesting, publicly observed through satellite data. Hence, ω_1 and ω_2 correspond to information about raining and to other farm-specific random events (e.g., pest attack, local flood or hail storm), respectively. Assuming that there are 180 days from plantation to harvest gives $\Omega_1 = \mathcal{S} = \mathbb{R}_+^{180}$, and $\omega_1 = (\omega_1^1, \dots, \omega_1^{180})$ where ω_1^i is the precipitation intensity on day $i = 1, \dots, 180$, and $Z(y)$ is the expected decrease in crop yield under w_0 when the raining trajectory $y = (y^1, \dots, y^{180})$ has been observed. An index-based crop insurance contract would specify the indemnity paid to the farmer, as a function of $Z(y)$.³ In this example, Y and $\tilde{\varepsilon}$ are independent random variables if the rainfall trajectory and other hazards affect the crop yield independently and additively, and generically they are not otherwise.

2.2.2 Hurricane insurance

Consider an individual located in an area subject to hurricanes, with property at risk w_0 . The track of a hurricane is characterized by the longitude and latitude coordinates of its center, and by the speed and direction of wind. Assume that satellite imagery provides this four-dimension information m times along the path of the hurricane. We have $\omega_1 = (\omega_1^1, \dots, \omega_1^m) \in \Omega_1 = \mathcal{S} = \mathbb{R}_+^{4m}$ where $\omega_1^i \in \mathbb{R}_+^4$ is the information provided

³As a practical illustration of the construction of a yield index using rainfall data, see for instance Omondi et al. (2021): they analyze how expected crop growth in Kenya depends on satellite weather data, including onset days, rainfall depths, dry spells, and rainfall occurrence for four crop growth stages.

by satellite data, with $i = 1, \dots, m$. Furthermore, ω_2 corresponds to local factors that may affect damages from a hurricane, e.g., seasonal tidal range favoring storm surge or torrential rains triggering landslides. A vulnerability model then relates the data on the hurricane track (i.e., ω_1) and the potential damages incurred in a given territory, thereby leading to the loss index $Z(y)$.⁴ Here, if the hurricane track and seasonal or local factors affect damages to property independently and additively, then Y and $\tilde{\varepsilon}$ are independently distributed, and this is not the case otherwise.

3 Independent basis risk

This section focuses attention on the case where parameter vector Y and basis risk $\tilde{\varepsilon}$ are independent random variables (in short, the basis risk is independent), which implies that the loss index Z and the basis risk $\tilde{\varepsilon}$ are also independent.⁵ Let us define indirect utility $v(w)$ by

$$v(w) \equiv \mathbb{E}_{\tilde{\varepsilon}} u(w - \tilde{\varepsilon}),$$

with $v' > 0$, $v'' < 0$. When Y and $\tilde{\varepsilon}$ are independent, we may write

$$\begin{aligned} \mathbb{E}u(W_f) &= \mathbb{E}_Y[\mathbb{E}_{\tilde{\varepsilon}} u(w_0 - Z(Y) + I(Y) - P - \tilde{\varepsilon})] \\ &= \mathbb{E}_Y[v(w_0 - Z(Y) + I(Y) - P)], \end{aligned}$$

This is analogous to the standard approach to risk analysis under independent background risk: when facing a zero-mean independent background risk $\tilde{\varepsilon}$, the individual's attitude toward the risk affecting his insurable wealth is the same as if there were no background risk and his utility function were $v(\cdot)$ instead of $u(\cdot)$. In other words, in that case, the optimal parametric insurance contract maximises $\mathbb{E}_Y[v(w_0 - Z(Y) + I(Y) - P)]$

⁴Typically, hurricane vulnerability models show that wind produces damages when their speed exceeds a threshold that depends on the property at risk, and over this threshold property damages may increase rapidly with wind speed. See Katz (2002), Pielke (2007), Nordhaus (2010) and Emanuel (2011).

⁵Note that the inverse property does not hold unless $Z^{-1}(\cdot)$ is single-valued.

with respect to $I(\cdot)$ and P , subject to (2). This is very similar to a standard optimal insurance problem, with random loss $Z(Y)$ and utility function $v(w)$, the only difference being that the insurance payout depends on the determinants of the loss $Y \in \mathcal{S}$ rather than on the loss itself $Z(Y) \in \mathbb{R}_+$. It is very intuitive and confirmed by the proof of the following proposition that two parameter vectors $y_1, y_2 \in Y$ such that $Z(y_1) = Z(y_2)$ should lead to the same indemnity. Thus, the optimal contract is index-based and it maximizes

$$\mathbb{E}u(W_f) = \mathbb{E}v(w_0 - Z + J(Z) - P),$$

with respect to P and $J(\cdot)$, subject to (3). This corresponds to a standard insurance demand problem with utility function $v(\cdot)$, in which the loss would be $Z \in [\underline{z}, \bar{z}]$ and not $X \in [0, \bar{x}]$. We know that the optimal solution to such a problem is a straight deductible contract, unless there is no loading, in which case full insurance would be optimal. When Z is the indemnity trigger with deductible z_0 , full insurance and partial insurance correspond to $z_0 = \underline{z}$ and $z_0 > \underline{z}$, respectively.⁶ This is summarized in Proposition 1.

Proposition 1 *If Y and $\tilde{\varepsilon}$ are independently distributed, then the optimal parametric insurance contract is index-based. The insurance payout $J(Z)$ is equal to the conditional expected loss Z if $m = 0$, and it provides full coverage of conditional expected losses above a deductible $z_0 > \underline{z}$ if $m > 0$. In other words, $J(Z) = \max\{Z - z_0, 0\}$, with $z_0 = \underline{z}$ if $m = 0$ and $z_0 > \underline{z}$ if $m > 0$.*

The analogy with the optimal insurance problem under an independent background risk allows us to answer the simple but controversial following question: considering two individuals with the same risk exposure X and the same publicly observable parameter vector Y , does the more risk averse one purchase more parametric insurance? Put differently, does the standard result according to which more risk aversion means more insurance demand (when $m > 0$), also applies in the case of parametric insurance?

⁶Since $X = Z + \tilde{\varepsilon} \geq 0$ and $\mathbb{E}\tilde{\varepsilon} = 0$, we necessarily have $\underline{z} > 0$ when the basis risk is independent.

To answer this question, consider two individuals indexed by $h = 1, 2$, with utility functions $u_1(w)$ and $u_2(w)$, indirect utility functions $v_1(w)$ and $v_2(w)$, and optimal deductibles z_{01} and z_{02} , respectively. According to standard results in the theory of insurance demand, the larger the index of absolute risk aversion, the larger the demand for insurance, and thus, under constant loading, the lower the deductible. Let us denote $A_u(w) = -u''(w)/u'(w)$ and $A_v(w) = -v''(w)/v'(w)$ the Arrow-Pratt index of absolute risk aversion, for direct and indirect utility function $u(\cdot)$ and $v(\cdot)$, respectively. The question we are asking is whether $A_{u_2}(w) > A_{u_1}(w)$ for all w implies $z_{02} < z_{01}$. Since the optimal deductible maximizes the policyholder's expected indirect utility, we know that $z_{02} < z_{01}$ if $A_{v_2}(w) > A_{v_1}(w)$ for all w . Consequently, the larger the degree of risk aversion (for utility function u), the larger the demand for parametric insurance if $A_{u_2}(w) > A_{u_1}(w)$ implies $A_{v_2}(w) > A_{v_1}(w)$.

When this last property holds, we say that the background risk preserves comparative risk aversion in the sense of Arrow-Pratt. It has been shown in the literature on background risks that additional assumptions are required for this to be true. This is the case, in particular, when $h = 1$ and/or $h = 2$ displays nonincreasing risk aversion. This is also true if one reinforces the comparison of risk aversion by following the approach of Ross (1981).⁷ Hence, either by postulating decreasing absolute risk aversion, or by comparing risk aversion in the manner of Ross, we may conclude that the existence of an independent background risk preserves comparative risk aversion. When at least one of these two assumptions hold, we say that risk aversion is strongly comparable. The following Proposition states that, in such a setting, the more risk averse the individual, the larger his demand for parametric (index-based) insurance.

⁷ $h = 2$ is said to be more risk averse than $h = 1$ in the sense of Ross (1981), if there exists a positive scalar λ and a decreasing and concave function g such that $u_2(w) = \lambda u_1(w) + g(w)$ for all w . It can be shown that comparative risk aversion in the sense of Ross (1981) implies comparative risk aversion in the sense of Arrow-Pratt, i.e., $A_{u_2}(w) > A_{u_1}(w)$, but the reverse is not true. When $h = 2$ is more risk averse than $h = 1$ in the sense of Ross, then $A_{v_2}(w) > A_{v_1}(w)$. See Propositions 24 and 25 in Gollier (2004).

Proposition 2 *When risk aversion is strongly comparable, the optimal index-based insurance coverage $J(Z) = \max\{Z - z_0, 0\}$ is increasing in risk aversion (i.e., the larger the risk aversion, the lower the deductible z_0) if $m > 0$, and it is equal to the conditional expected loss $J(Z) = Z$ independently from risk aversion when $m = 0$.*

It is also worth emphasizing the ambiguous effect of a greater or lesser basis risk on the demand for parametric insurance, when the parameter vector and the basis risk are independently distributed. The analogy with insurance demand under independent background risk shows that, for a given random loss index Z , a larger independent basis risk $\tilde{\varepsilon}$ in the sense of Rothschild and Stiglitz (1970)⁸ leads the risk-vulnerable individual to purchase more parametric insurance, i.e., to choose a lower deductible z_0 .⁹ However, this is no more the case if the increase in the size of the background risk $\tilde{\varepsilon}$ corresponds to a less accurate loss index Z , the risk exposure $X = Z + \tilde{\varepsilon}$ being unchanged. In that case, as illustrated in the simulations below, the increase in the basis risk reflects a decrease in the quality of the parametric insurance cover, and this may lead the individual to purchase less parametric insurance, i.e., to choose a larger deductible z_0 .¹⁰

These conclusions are illustrated below by simulations where the individual displays

⁸In what follows, when lotteries are compared, the increasing-risk criterion is always in the sense of Rothschild and Stiglitz (1970).

⁹Risk vulnerability corresponds to the case where an exogenous zero-mean background risk raises the aversion to another independent risk. As shown by Eeckhoudt et al. (1996), risk vulnerability occurs when absolute risk aversion is decreasing and convex with wealth. There is also risk vulnerability when absolute risk aversion and absolute prudence are both decreasing, a case called standard risk aversion by Kimball (1993).

¹⁰The accuracy of the loss index refers to the informational content of the underlying parameter vector. Under the comparative accuracy criterion defined in Section 5, the more accurate this information, the lower the basis risk and the larger the expected utility associated with optimal parametric insurance, these conclusions being true whether the parameter vector and the basis risk are independent or not. However, as will be shown, a lower basis risk *per se* is not the guarantee of a better index-based parametric insurance.

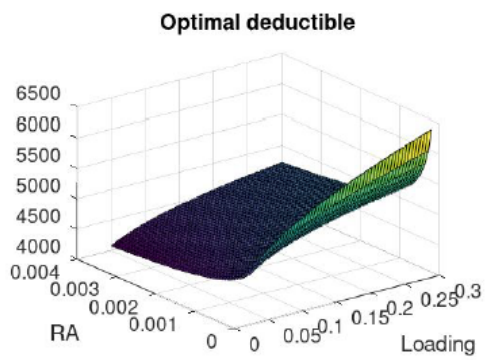
CARA preferences, i.e. $u(w) = -\exp(-\rho w)$ with $\rho > 0$ the index of absolute risk aversion, and where Z and $\tilde{\varepsilon}$ are independently and normally distributed, with $\tilde{\varepsilon} \sim \mathcal{N}(0, \sigma_{\tilde{\varepsilon}}^2)$ and $Z \sim \mathcal{N}(\mu, \sigma_z^2)$, and thus $X \sim \mathcal{N}(\mu, \sigma_z^2 + \sigma_{\tilde{\varepsilon}}^2)$.¹¹ We calibrate the loss index and the basis risk by assuming $\sigma^2 \equiv \sigma_z^2 + \sigma_{\tilde{\varepsilon}}^2 = 800^2$ with $R^2 = \sigma_z^2/\sigma^2$ and $1 - R^2 = \sigma_{\tilde{\varepsilon}}^2/\sigma^2$ measuring the accuracy of the index and the size of the basis risk, respectively. Other calibration parameters include initial wealth $w_0 = 50,000$ and expected loss $\mu = 5,000$. The gains from parametric insurance are measured as $(ce - ce_n)/ce_n$, where ce and ce_n denote the certainty equivalent of final wealth, with and without parametric insurance, respectively.

In Figures 1a and 1b, the accuracy of the loss index is fixed, and the size of the basis risk is $1 - R^2 = 0.3$. The index of absolute risk aversion ρ varies from 0.0002 to 0.004 and the loading factor m varies from 0.05 to 0.30. In accordance with Proposition 2, Figure 1a shows that the optimal deductible z_0 is decreasing with risk aversion, and as expected it is also increasing with the loading factor. Figure 1b shows that the certainty equivalent gains range from 0 to 14%: the larger the loading, the lower the gains from insurance. Higher levels of risk aversion also increase gains since coverage is more valuable for more risk averse individuals.

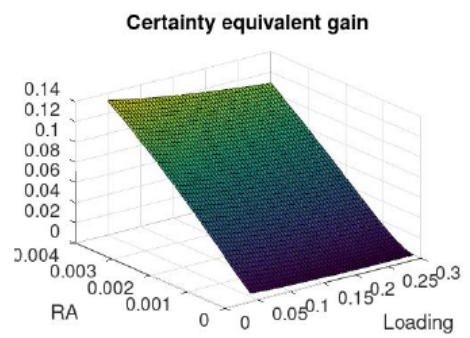
In Figures 2a and 2b, the size of the basis risk $1 - R^2$ varies from 0 to 0.8 and m varies from 0.05 to 0.30, while the index of absolute risk aversion is now fixed and equal to $\rho = 0.0021$, the middle of the previous interval. Figure 2a shows that the optimal deductible increases with basis risk, indicating that, for a given loss exposure X , individuals purchase less parametric insurance when the loss index is less accurate and basis risk increases. As expected and highlighted in Figure 2b, the gain from parametric insurance is decreasing in basis risk and loading.¹²

¹¹Gaussian distributions are used for the sake of computational simplicity, although they have infinite support.

¹²For the sake of readability, the axis in Figure 2b have been reversed.

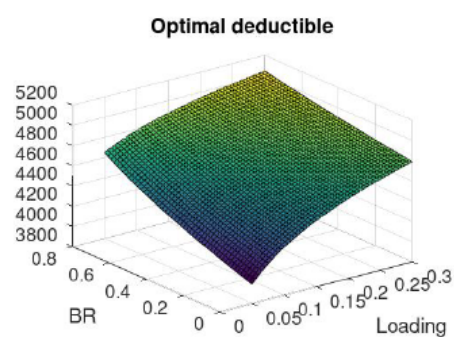


(a) Deductible as a function of loading and risk aversion (RA).

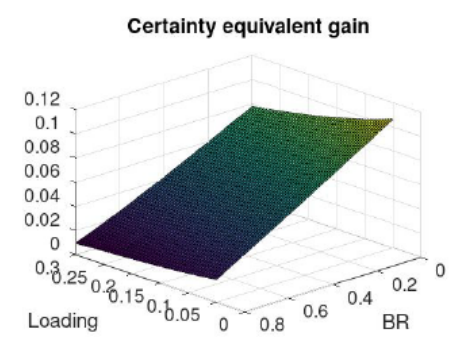


(b) Certainty equivalent gain as a function of loading and risk aversion (RA).

Figure 1



(a) Deductible as a function of basis risk (BR) and loading.



(b) Certainty equivalent gain as a function of basis risk (BR) and loading.

Figure 2

4 Dependent basis risk

4.1 Optimal indemnity schedule

When Y and $\tilde{\varepsilon}$ are dependent random variables (in short, the basis risk is dependent), the optimal parametric insurance contract maximizes

$$\mathbb{E}u = \mathbb{E}_Y\{\mathbb{E}_{\tilde{\varepsilon}}[u(w_0 - Z(y) - \tilde{\varepsilon} + I(y) - P) \mid Y = y]\},$$

with respect to $I(\cdot)$ and P , subject to (2). Proposition 3 characterizes the optimal solution to this problem when $u''' > 0$, i.e. the individual is prudent.

Proposition 3 *When $u''' > 0$, the optimal parametric-insurance indemnity schedule is written as $I(Y) = \max\{\widehat{Z}(Y) - \widehat{z}_0, 0\}$, where the trigger is the adjusted risk $\widehat{Z}(Y)$ such that $\widehat{Z}(Y) > Z(Y)$. For any $y_1, y_2 \in \mathcal{S}$ such that $I(y_1), I(y_2) > 0$, if the conditional probability distribution of $\tilde{\varepsilon}$ corresponds to a larger basis risk when $Y = y_2$ than when $Y = y_1$, then $\widehat{Z}(y_2) - Z(y_2) > \widehat{Z}(y_1) - Z(y_1)$.*

In the case of a dependent basis risk, the optimal parametric insurance contract is not index-based, except in the particular case considered in Corollary 1 below. The first part of Proposition 3 states that the optimal parametric cover of the prudent policyholder is written as a straight deductible contract, in which the trigger is an adjusted random loss $\widehat{Z}(Y)$ larger than the expected loss $Z(Y)$ and the deductible is \widehat{z}_0 , hence with indemnity $I(Y) = \max\{\widehat{Z}(Y) - \widehat{z}_0, 0\}$.¹³ Further characterizing the indemnity schedule $I(Y)$ requires to be more specific about the relationship between the loss adjustment $\widehat{Z}(Y) - Z(Y)$ and the parameter vector Y . The second part of

¹³The proof of Proposition 3 shows that the optimal indemnity schedule $I(Y) = \max\{\widehat{Z}(Y) - \widehat{z}_0\}$ maximizes the policyholder's expected utility in a problem where the risk exposure is $\widehat{Z}(Y)$ without basis risk. Note however that the increase of risk exposure from $Z(Y)$ to $\widehat{Z}(Y)$ affects the optimal deductible \widehat{z}_0 , hence an ambiguous effect of the basis risk on the optimal indemnity $I(Y)$. See Eeckhoudt et al. (1991) on the effect of an increase in risk exposure on optimal insurance with deductible.

the proposition shows that $\widehat{Z}(Y) - Z(Y)$ depends on the relation between Y and the size of the basis risk. Considering two parameter vectors y_1 and y_2 in S , if the conditional distribution of the basis risk is more risky when $Y = y_2$ than when $Y = y_1$, then the loss adjustment is larger in the first case than in the second. In that sense, and perhaps paradoxically, a larger conditional basis risk stimulates the demand for parametric insurance.¹⁴

Corollary 1 *Assume that the conditional distribution of $\tilde{\varepsilon}$ given $Y = y$ only depends on $Z(y)$. Then, when $u''' > 0$ the optimal parametric insurance is index-based and it is written as $I(Y) = \max\{\xi(Z(Y)) - \widehat{z}_0, 0\}$, with $\xi' > 1$ (respect. $\xi' < 1$) if an increase in $Z(Y)$ makes the conditional distribution of $\tilde{\varepsilon}$ more risky (respect. less risky).*

Corollary 1 states a direct consequence of the second part of Proposition 3. If the conditional distribution of the basis risk is more risky when the expected loss is larger, then the increase in the insurance payout is larger than the increase in expected loss, which corresponds to a vanishing deductible, as established by Gollier (1996) in his study of optimal indemnity insurance with basis risk. In the opposite case, the indemnity schedule entails an increasing deductible.¹⁵

¹⁴We may consider a simple version of our model, in which $X \in \{0, L\}$ and $S = \{0, 1\}$, with L a single-valued possible loss. X and Y are assumed to be positively correlated, and an indemnity I is paid when $Y = 1$. One may check that Y and $\tilde{\varepsilon}$ are **not** independently distributed in this case (see subsection 7.1 of the appendix for details). Clarke (2016) considers this binary model in the case of constant absolute or relative risk aversion, and he simulates the variations of the optimal indemnity I when the risk aversion parameter changes. He shows that I may not be uniformly increasing with risk aversion, which establishes by a counterexample that Proposition 2 is no more valid when the parameter vector and the basis risk are not independently distributed.

¹⁵Proposition 3 and Corollary 1 may be illustrated by considering the case of constant absolute risk aversion $u(w) = -\exp(-\rho w)$, with a Gaussian basis risk $\tilde{\varepsilon}(y) \mapsto \mathcal{N}(0, \sigma_\varepsilon(y)^2)$ for all $y \in \mathcal{S}$, where $\tilde{\varepsilon}(y) \equiv \tilde{\varepsilon}|_{Y=y}$. In that case, simple calculations yield $\widehat{Z}(y) = Z(y) + \rho\sigma_\varepsilon(y)^2/2$ for all $y \in \mathcal{S}$. When Y and $\tilde{\varepsilon}$ are independent, we have $\sigma_\varepsilon(y) = \sigma_\varepsilon$ for all y , and Proposition 3 is equivalent to Proposition 1 with $\widehat{z}_0 = z_0 + \rho\sigma_\varepsilon^2/2$. When Y and $\tilde{\varepsilon}$ are not independent, the risk adjustment $\widehat{Z}(y) - Z(y)$ is

Assuming that the basis risk only depends on expected loss is very restrictive and, in general, the optimal parametric indemnity schedule $I(Y)$ cannot be written as a function of $Z(Y)$. In other words, in general the optimal parametric insurance contract is not index-based and, in that case, characterizing this indemnity schedule requires additional assumptions about the informational content of the parameter vector. To do so, we may consider the case where parameter vector Y is splitted in two components, one affecting the expected loss and the other being related with the basis risk. Let us write $Y = (Y_a, Y_b)$, with $Y_a \in \mathcal{S}_a \subset \mathbb{R}^{n_a}, Y_b \in \mathcal{S}_b \subset \mathbb{R}^{n_b}, n_a + n_b = n$ and $\mathcal{S} = \mathcal{S}_a \times \mathcal{S}_b$. We assume that component Y_a is a sufficient statistic for the expected loss $Z(Y)$, while only component Y_b may be correlated with the basis risk $\tilde{\varepsilon}$. Proposition 4 considers this case and shows how the two components of the parameter vector should be combined in order to provide the optimal coverage.

Proposition 4 *If $Z(y_a) = \mathbb{E}[X \mid Y = (y_a, y_b)]$ for all $y_a \in \mathcal{S}_a, y_b \in \mathcal{S}_b$, and Y_a and $\tilde{\varepsilon}$ are independently distributed, then the optimal parametric insurance is written as $I(Y) = \max\{Z(Y_a) - z_0(Y_b), 0\}$, where payout $I(Y)$ is equal to the excess of the expected loss $Z(Y_a)$ above a deductible $z_0(Y_b)$ that depends on component Y_b of the parameter vector. Furthermore, when $u''' > 0$, if the conditional probability distribution of $\tilde{\varepsilon}$ corresponds to a larger basis risk when $Y_b = y_{b2}$ than when $Y_b = y_{b1}$, then $z_0(y_{b2}) < z_0(y_{b1})$.*

Proposition 4 provides conditions under which the optimal parametric insurance contract entails full coverage of expected loss $Z(Y_a)$ above a deductible $z_0(Y_b)$. Furthermore, the larger the basis risk conditionally on Y_b , the lower the deductible. The proportional to the conditional variance $\sigma_\varepsilon(y)^2$, which illustrates the second part of Proposition 3 in the case of a Gaussian distribution. When the distribution of $\tilde{\varepsilon}(y)$ only depends on $Z(y)$, there exists a function $\bar{\sigma}_\varepsilon(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $\sigma_\varepsilon(y) = \bar{\sigma}_\varepsilon(Z(y))$ for all y . In this case, the optimal contract is index-based and, with the notation of Corollary 1, we may write $\xi(Z(y)) = Z(y) + \rho \bar{\sigma}_\varepsilon(Z(y))^2 / 2$, with $\xi' > 1$ if $\bar{\sigma}'_\varepsilon > 0$ and $\xi' < 1$ if $\bar{\sigma}'_\varepsilon < 0$: there is a vanishing deductible in the first case, and an increasing deductible in the second.

intuition is simple. Proposition 4 assumes that $Z(Y)$ only depends on subvector Y_a and that the conditional probability distribution of $\tilde{\varepsilon}$ may only depend on the other subvector Y_b . Hence, conditionally on Y_b , the basis risk $\tilde{\varepsilon}$ and the expected loss $Z(Y_a)$ are independently distributed. Consequently, as in Proposition 1, conditionally on Y_b , the optimal insurance entails full coverage of the expected loss $Z(Y_a)$ above a deductible that depends on Y_b . Under prudence, the larger the conditional basis risk $\tilde{\varepsilon}_{|Y_b}$, the lower the deductible $z_0(Y_b)$, and thus the larger the coverage of expected losses $Z(Y_a)$.¹⁶

4.2 An illustrative example

For illustrative purposes, consider the case of a risk-averse firm facing a double risk of property loss and price uncertainty. To be concrete, assume that the firm is an electrical energy supplier with normal output q in kWh per year, sold at unit price p , as specified in long-term contracts with customers.¹⁷ Accidents due to meteorological uncertainty may induce repair costs, as for example when electricity pylons are blown over or offshore windmills are damaged when a hurricane hits power plants. For simplicity, it is assumed that these property damages do not affect the firm's yearly output (i.e., repair does not entail significant production delay) and we denote $\tilde{\ell}$ the repair costs, with $\tilde{\ell} = \bar{\ell}(Y_a) + \tilde{\eta}_a$ where $Y_a \in \mathcal{S}_a$ is a random vector of publicly observable meteorological data and $\tilde{\eta}_a$ is a zero-mean random variable, Y_a and $\tilde{\eta}_a$ being independently distributed. The actual output is $q(1 + \tilde{\eta}_b)$ where $\tilde{\eta}_b$ is a zero-mean random variable, pairwise independent from Y_a and $\tilde{\eta}_a$. The difference $q\tilde{\eta}_b$ between actual and normal outputs results from all factors that may affect electricity production for

¹⁶Interestingly, this is reminiscent of the precautionary motive of the prudent insured highlighted by Schlesinger (2013), whose intuition was provided by Eeckhoudt & Schlesinger (2006), and which states that uncertainty about uninsurable losses exacerbates insurance demand.

¹⁷This presumably reflects purchasers' risk-aversion. Equivalently, we may assume that electricity output is sold at spot price, the electricity supplier being able to hedge its price risk at actuarial price through forward exchange contracts.

privately observed reasons, such as technological failures, delivery delays by subcontractors or wind speed outside accident risk, in the case of a wind farm. It is sold or purchased in a centralized spot market, at publicly observable price $Y_b \in \mathcal{S}_b = \mathbb{R}_+$, Y_b and $\tilde{\eta}_b$ being also independent, with zero-mean net proceeds $q\tilde{\eta}_b Y_b$. Production costs (apart from repair costs) are fixed and denoted C .

The firm's profit is written as $q(p + \tilde{\eta}_b Y_b) - \bar{\ell}(Y_a) - \tilde{\eta}_a - C$, which corresponds to previous notations with $w_0 = qp - C$ and $X = \bar{\ell}(Y_a) + \tilde{\eta}_a - q\tilde{\eta}_b Y_b$. In other words, initial wealth w_0 is the difference between normal turnover and fixed cost, while loss X is the sum of repair cost and net purchases in the spot market. We have $\mathbb{E}[\tilde{\eta}_b Y_b] = 0$ because $\tilde{\eta}_b$ and Y_b are independently distributed with $\mathbb{E}\tilde{\eta}_b = 0$, which gives $\mathbb{E}[X | Y_a, Y_b] = \bar{\ell}(Y_a) = Z(Y_a)$ and $\tilde{\varepsilon} = X - Z(Y_a) = \tilde{\eta}_a + q\tilde{\eta}_b Y_b$.

We deduce from Proposition 4 that the optimal parametric cover is a straight deductible contract, where the trigger is the expected repair cost $\bar{\ell}(Y_a)$ under meteorological data Y_a and the deductible $z_0(Y_b)$ depends on the electricity spot price Y_b . Furthermore, $\tilde{\varepsilon}_{|Y_b=y_{b2}}$ is more risky than $\tilde{\varepsilon}_{|Y_b=y_{b1}}$ if y_{b2} is larger than y_{b1} . Hence, if the electricity supplier is prudent, the larger the spot price, the lower the deductible.¹⁸

5 Comparing information structures

The accuracy of the loss index reflects the informational content of the parameter vector, and many practical debates about parametric insurance are related to this accuracy and to its effect on the basis risk and on the efficiency of parametric insurance. Indeed, (Y, \mathcal{S}) defines an *information structure* because observing signal $y \in \mathcal{S}$ implies $\omega \in \mathcal{O}(y) = Y^{-1}(y)$, with $\mathcal{P} = \{\mathcal{O}(y), y \in \mathcal{S}\}$ a partition of Ω . In other words, one knows that ω is in $\mathcal{O}(y)$ when parameter vector y is observed.

As explicitly formulated in Definition 1 below, an information structure is *more accurate* than another one when it corresponds to a finer partition \mathcal{P} of Ω , which

¹⁸See the appendix (subsection 7.2) for an illustrative simulation of this example.

corresponds to a partial order among information structures. In particular, when Ω is a finite-dimensional vector space and $Y(\omega)$ is a publicly observable subvector of ω , extending this subvector (i.e., observing more components of ω) leads to a finer partition of Ω , and thus to a more accurate information structure.¹⁹

Definition 1 *Information structure (\mathcal{S}_1, Y_1) is more accurate than information structure (\mathcal{S}_2, Y_2) , when, for all $y_2 \in \mathcal{S}_2$, there exists a set $\mathcal{K}(y_2) \subset \mathcal{S}_1$ such that*

$$\mathcal{O}_2(y_2) = \cup_{y_1 \in \mathcal{K}(y_2)} \mathcal{O}_1(y_1),$$

with $\{\mathcal{K}(y_2), y_2 \in \mathcal{S}_2\}$ a partition of \mathcal{S}_1 .

In Definition 1, partition $\mathcal{P}_1 = \{\mathcal{O}_1(y_1), y_1 \in \mathcal{S}_1\}$ induced by (\mathcal{S}_1, Y_1) is finer than partition $\mathcal{P}_2 = \{\mathcal{O}_2(y_2), y_2 \in \mathcal{S}_2\}$ induced by (\mathcal{S}_2, Y_2) : each set $\mathcal{O}_2(y_2)$ belonging to \mathcal{P}_2 is the union of a number of sets $\{\mathcal{O}_1(y_1), y_1 \in \mathcal{K}(y_2)\}$ belonging to \mathcal{P}_1 . Equivalently, as shown in Lemma 1, we may express y_2 as a function of y_1 through a function $y_2 = \Phi(y_1)$ such that $\mathcal{K}(y_2) = \Phi^{-1}(y_2)$.

Lemma 1 *Information structure (\mathcal{S}_1, Y_1) is more accurate than information structure (\mathcal{S}_2, Y_2) if and only if there exists a function $\Phi(\cdot) : \mathcal{S}_1 \rightarrow \mathcal{S}_2$, such that*

$$Y_2(\omega) = \Phi(Y_1(\omega)) \text{ for all } \omega \in \Omega.$$

Information structure (\mathcal{S}_1, Y_1) weakly dominates information structure (\mathcal{S}_2, Y_2) if the optimal parametric-insurance contract based on (\mathcal{S}_1, Y_1) is weakly preferred to the optimal contract based on (\mathcal{S}_2, Y_2) , for any increasing concave utility function. Dominance is strong if, in addition, optimal expected utility is strictly larger for at least one utility function.

¹⁹For instance, in parametric crop insurance, information yielded by ground-based sensors may be added to weather satellite data. However, ground-based remote sensing is costly, and thus there is a trade-off between the costs and benefits of using such an additional information.

Proposition 5 *Assume that (\mathcal{S}_1, Y_1) is more accurate than (\mathcal{S}_2, Y_2) . In that case, (\mathcal{S}_1, Y_1) weakly dominates (\mathcal{S}_2, Y_2) . Furthermore, assume that there exist positive-probability sets $\mathcal{A}_2 \subset \mathcal{S}_2$, $\mathcal{A}_1^1(y_2), \mathcal{A}_1^2(y_2) \subset \mathcal{K}(y_2) \subset \mathcal{S}_1$ with $\mathcal{A}_1^1(y_2) \cap \mathcal{A}_1^2(y_2) = \emptyset$ for all $y_2 \in \mathcal{A}_2$, such that (i): $X(\omega^1) > X(\omega^2)$ if $Y^1(\omega^1) \in \mathcal{A}_1^1(y_2)$ and $Y^1(\omega^2) \in \mathcal{A}_1^2(y_2)$ for $y_2 \in \mathcal{A}_2$, and (ii): For some concave utility function $u(\cdot)$, we have $I_2^*(y_2) > 0$ when $y_2 \in \mathcal{A}_2$, where $I_2^*(\cdot) : \mathcal{S}_2 \rightarrow \mathbb{R}_+$ is the optimal indemnity schedule under information structure (\mathcal{S}_2, Y_2) . Then (\mathcal{S}_1, Y_1) strongly dominates (\mathcal{S}_2, Y_2) .*

Proposition 5 is cumbersome, but its intuition is simple. Obviously, when (\mathcal{S}_1, Y_1) is more accurate than (\mathcal{S}_2, Y_2) , then any indemnity schedule $I_2(Y_2)$ based (\mathcal{S}_2, Y_2) can be replicated by another indemnity schedule $I_1(Y_1) = I_2(\Phi(Y_1))$ based on (\mathcal{S}_1, Y_1) , hence the weak dominance property. More specifically, in its second part, Proposition 5 postulates that there exists a positive-probability set $\mathcal{A}_2 \subset \mathcal{S}_2$ such that any parameter vector $y_2 \in \mathcal{A}_2$ is the image of subsets $\mathcal{A}_1^1(y_2)$ and $\mathcal{A}_1^2(y_2) \subset \mathcal{S}_1$ by function $\Phi(\cdot)$. Hence, information structure (\mathcal{S}_1, Y_1) separates the states ω leading to $\mathcal{A}_1^1(y_2)$ from those leading to $\mathcal{A}_1^2(y_2)$, which cannot be done through (\mathcal{S}_2, Y_2) . Assume that the policyholder's loss is larger in the first case than in the second one, and start from the optimal parametric insurance contract based on Y_2 . Increasing the insurance payout when $y_1 \in \mathcal{A}_1^1(y_2)$ and decreasing it when $y_1 \in \mathcal{A}_1^2(y_2)$, while keeping the expected payment unchanged, increases the risk-averse policyholder's expected utility for an unchanged insurance premium. This is possible when the utility function is such that the optimal parametric insurance contract based on (\mathcal{S}_2, Y_2) provides positive coverage when $y_2 \in \mathcal{A}_2$.²⁰

In Proposition 6, we consider two information structures (\mathcal{S}_1, Y_1) and (\mathcal{S}_2, Y_2) , with their loss index and basis risk $Z_1, \tilde{\varepsilon}_1$ and $Z_2, \tilde{\varepsilon}_2$, respectively. Since they correspond to

²⁰Note that the optimal parametric insurance contract may be index-based under (\mathcal{S}_2, Y_2) while being dominated by a non-index based contract under (\mathcal{S}_1, Y_1) . In particular, increasing the insurance payout when $y_1 \in \mathcal{A}_1^1(y_2)$ and decreasing it when $y_1 \in \mathcal{A}_1^2(y_2)$ may transform an index-based indemnity function $I_2^*(\cdot)$ into another one that is no more index-based.

the same risk exposure X , we have

$$Z_1 + \tilde{\varepsilon}_1 = Z_2 + \tilde{\varepsilon}_2 = X. \quad (7)$$

Proposition 6 *Assume that (\mathcal{S}_1, Y_1) is a more accurate information structure than (\mathcal{S}_2, Y_2) . Then $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$, and Z_1 is more risky than Z_2 .*

Proposition 6 shows that a more accurate information takes the form of smaller basis risk and more risky loss index, the increasing-risk criterion being in the sense of Rothschild-Stiglitz (1970), in both cases. A completely uninformative parameter vector would lead to a constant loss index $Z = \mathbb{E}X$ equal to the unconditional expected loss, while the variations of an informative loss index reproduce the change in incurred losses more or less precisely. When the information structure is more accurate, the parameter vector provides a more precise information on the state, and the index reproduces more closely the changes in the loss, with less residual uncertainty, hence a more variable (more risky) loss index and a lower basis risk.

Propositions 5 and 6 show that a more accurate information structure simultaneously provides a better optimal parametric insurance coverage and a smaller basis risk. Therefore, the question naturally arises as to whether a decrease in the basis risk (reflecting a change in the underlying information structure) is a sufficient condition for the dominance of the new information structure over the previous one. The answer to this question is in fact negative, even in the case of independent basis risk. This is illustrated in the following example, presented in more detail in the appendix (subsection 7.3).²¹ Consider Z_1 and Z_2 two loss indices distributed in the same support $[\underline{z}, \bar{z}]$, with $0 < \underline{z} < \bar{z} < \bar{x}$, with independent basis risks $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ distributed in $[-\underline{z}, \bar{x} - \bar{z}]$, with densities $f_{\tilde{\varepsilon}_1}(\varepsilon)$ and $f_{\tilde{\varepsilon}_2}(\varepsilon)$, respectively.²² We know from Proposition 1 that an

²¹This ties in with the conclusion of Teh and Woolnough (2019) that higher correlation between loss and index does not necessarily equate to a better index.

²²It is shown in the appendix that, when Z_1 and Z_2 have a common support $[\underline{z}, \bar{z}]$, their underlying information structures (\mathcal{S}_1, Y_1) and (\mathcal{S}_2, Y_2) cannot be compared by the accuracy criterion.

index-based straight deductible contract is optimal in each case, and we denote $z_{0i}^*(m)$ the optimal deductible as a function of the loading factor m when parametric insurance is based on loss index Z_i . Let \bar{m}_i be the upper bound of the loading factor under which the policyholder purchases insurance, thus with $z_{0i}^*(m) < \bar{z}$ if $m < \bar{m}_i$, and $z_{0i}^*(\bar{m}_i) = \bar{z}$. It can be shown that \bar{m}_i satisfies

$$\int_{-\bar{z}}^{\bar{x}-\bar{z}} u'(w_0 - \bar{z} - \varepsilon) f_{\tilde{\varepsilon}_i}(\varepsilon) d\varepsilon - (1 + \bar{m}_i) \int_0^{\bar{x}} u'(w_0 - x) f_X(x) dx, \quad (8)$$

which may be interpreted, the absence of insurance being a statu quo, as an equality between cost and benefit of providing a small positive coverage when $Z_i = \bar{z}$. Assume that the policyholder is prudent, and that $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$ with $\mathbb{E}\tilde{\varepsilon}_1 = \mathbb{E}\tilde{\varepsilon}_2 = 0$. Equation (8) then gives $\bar{m}_1 < \bar{m}_2$. When $m = \bar{m}_1$, the policyholder is willing to purchase parametric insurance based on Z_2 while he would prefer to remain uninsured if Z_1 were used. When m is slightly lower than \bar{m}_1 , he is willing to purchase insurance whatever the index and, by a continuity argument, when m is close to \bar{m}_1 , his optimal expected utility is higher with Z_2 than with Z_1 , although the basis risk is larger.

6 Conclusion

Reframing the parametric insurance problem in an imperfect information setting brings about new insights into the design of optimal coverage. The most important conclusion that emerges in this context is the fact that optimal parametric insurance depends on the stochastic relationship between the parameter vector and the basis risk. If they are independently distributed, then, with some caveats such as the strong comparability criterion used in Proposition 2, important results of insurance demand theory extend to the parametric insurance setting. As we have seen, this follows from the similarity with the insurance demand problem under independent background risk. Under constant loading, a straight deductible contract triggered by the loss index is optimal. Furthermore, the amount of insurance demand of two individuals who face the same risk exposure depend on their respective degrees of risk aversion.

Conclusions are far less simple when the parameter vector and the basis risk are not independently distributed, a case that may be more relevant in many concrete situations. The reason is simple: if two parameter vectors leading to the same loss index correspond to different distributions of the basis risk, then they provide different information on the loss, and this should be reflected in the optimal insurance coverage. In that case, the optimal parametric insurance is generally not index-based. In other words, the insurance payout should depend on the parameter vector itself, and not only on the best estimate of the loss that can be inferred from this information.

Once said that, this raises questions about, at least, two issues: the structure of the optimal indemnity schedule and the relationship between the attitude toward risk and the demand for insurance. With respect to the first question, we have shown that the optimal indemnity schedule corresponds to a straight deductible contract applied to an adjusted expected loss exposure. When the policyholder is prudent, the larger the basis risk conditionally on the parameter vector, the larger the risk adjustment. In other words, if the conditional basis risk increases when we move from a parameter vector to another one, then the risk adjustment should be larger in the second case than in the first one. This adjustment takes a more simple form when the parameter vector can be splitted in two independently distributed subvectors, affecting the expected loss and the conditional basis risk, respectively. In that case, the optimal indemnity schedule takes the form of a conditional deductible, and the larger the conditional basis risk, the lower the conditional deductible. Concerning the relationship between the attitude toward risk and the demand for parametric insurance, risk aversion and prudence codetermine the demand for parametric insurance, and, contrary to the case where the basis risk and the loss index are independently distributed, a lower degree of risk aversion does not necessarily means a lower demand for insurance with a lower premium, if this risk aversion effect is more than compensated by a larger degree of prudence. In other words, risk aversion and the downside risk aversion that characterizes prudence may go in opposite direction, which invalidates the usual comparative

static analysis of insurance choices.

Finally, parametric insurance should also be appraised from a risk-sharing efficiency standpoint, and this depends on the accuracy of the parameter vector and the induced basis risk. Increasing this accuracy usually means measuring additional parameters, but this may be costly. Trading-off the benefits from a more accurate information with the costs of collecting this information is part and parcel of the design of optimal parametric insurance.

7 Appendix

7.1 Binary model

As Clarke (2016), we may consider the case where the possible loss is single-valued $X \in \{0, L\}$ with $\mathcal{S} = \{0, 1\}$, and joint probabilities as follows:

$$\begin{array}{rcc} & Y = 0 & Y = 1 \\ X = 0 & \pi_{00} & \pi_{01} \\ X = L & \pi_{10} & \pi_{11} \end{array}$$

with

$$\begin{aligned} \mathbb{P}[X = L | Y = 1] &= \frac{\pi_{11}}{\pi_{01} + \pi_{11}}, \\ \mathbb{P}[X = L | Y = 0] &= \frac{\pi_{10}}{\pi_{00} + \pi_{10}}. \end{aligned}$$

We assume $\mathbb{P}[X = L | Y = 1] > \mathbb{P}[X = L | Y = 0]$, which holds if

$$\frac{\pi_{11}}{\pi_{01}} > \frac{\pi_{10}}{\pi_{00}}. \quad (9)$$

Parametric insurance provides a payout I when $Y = 1$. Clarke (2016) considers the CARA and CRRA classes of utility functions, and he shows that the optimal indemnity I^* may be non-monotonic with respect to the coefficient of absolute or relative risk aversion γ . More precisely, he shows that either $I^*(\gamma) = 0$ for all $\gamma \in (0, \infty)$, or

$I^*(\gamma) = 0$ for all $\gamma < \gamma_1$, $I^*(\gamma)$ is strictly increasing for all $\gamma_1 < \gamma < \gamma_2$ and $I^*(\gamma)$ and strictly decreasing for all $\gamma_2 < \gamma < \infty$ for some $\gamma_1 < \gamma_2 < \infty$. In words, the optimal coverage is increasing and then decreasing with risk aversion, so the most risk averse individual does not necessarily purchase more insurance.

We may write

$$Z(0) = L \frac{\pi_{10}}{\pi_{10} + \pi_{00}} \quad Z(1) = L \frac{\pi_{11}}{\pi_{11} + \pi_{01}},$$

with $Z(1) > Z(0)$ from (9), and

$$\tilde{\varepsilon}|_{Y=0} = \begin{cases} -L \frac{\pi_{10}}{\pi_{10} + \pi_{00}} & \text{with probability } \frac{\pi_{00}}{\pi_{10} + \pi_{00}} \\ L \frac{\pi_{00}}{\pi_{10} + \pi_{00}} & \text{with probability } \frac{\pi_{10}}{\pi_{10} + \pi_{00}} \end{cases},$$

and

$$\tilde{\varepsilon}|_{Y=1} = \begin{cases} -L \frac{\pi_{11}}{\pi_{11} + \pi_{01}} & \text{with probability } \frac{\pi_{01}}{\pi_{11} + \pi_{01}} \\ L \frac{\pi_{01}}{\pi_{11} + \pi_{01}} & \text{with probability } \frac{\pi_{11}}{\pi_{11} + \pi_{01}} \end{cases}.$$

which shows that the distribution of $\tilde{\varepsilon}$ differs according to whether $Y = 0$ or $Y = 1$: hence Y and $\tilde{\varepsilon}$ are not independent. Because of that, Proposition 2 is not valid, and the optimal insurance payout may not be increasing with respect to risk aversion. In fact, when the loss index and the basis risk are not independently distributed, the optimal parametric covers results from the interaction between risk aversion and prudence, as shown in Proposition 3.

Some intuition of this interaction may be obtained as follows. Conditionally on $X = 0$, the net expected transfer from the insurer to the policyholder is

$$\bar{T}_0 = \frac{\pi_{01}}{\pi_{00} + \pi_{01}} I - P,$$

and the actual net transfer is

$$\tilde{T}_0 = \begin{cases} I - P & \text{with prob. } \frac{\pi_{01}}{\pi_{00} + \pi_{01}} \\ -P & \text{with prob. } \frac{\pi_{00}}{\pi_{00} + \pi_{01}} \end{cases},$$

with $P = (1 + m)(\pi_{01} + \pi_{11})I$. Similarly, in state $X = L$, the net expected and actual transfers to the policyholder are

$$\bar{T}_L = \frac{\pi_{11}}{\pi_{10} + \pi_{11}} I - P,$$

and

$$\tilde{T}_L = \begin{cases} I - P & \text{with prob. } \frac{\pi_{11}}{\pi_{10} + \pi_{11}} \\ -P & \text{with prob. } \frac{\pi_{10}}{\pi_{10} + \pi_{11}} \end{cases},$$

respectively. We have $\bar{T}_L > \bar{T}_0$ because of (9), meaning that, on average, the payment received by the policyholder is larger when $X = L$ than when $X = 0$. Because of this first effect, as in a standard insurance demand problem, the larger the degree of absolute risk-aversion, the larger the optimal average indemnity, obtained through an increase in I . However, the actual transfer \tilde{T}_0 or \tilde{T}_L (conditionally on $X = 0$ or L , respectively) is uncertain, and we may have $\text{Var}(\tilde{T}_L) > \text{Var}(\tilde{T}_0)$. In that case, because this uncertainty on the conditional payment (as measured by its variance) is larger in the loss state than in the no-loss state, the downward risk aversion inherent in prudence creates a countervailing effect that reduces insurance demand. Under CARA and CRRA preferences, when parameter γ increases, the coefficient of absolute prudence also increases. The countervailing effect reflecting prudence becomes stronger and it may dominate the risk aversion effect, hence a possible decrease in insurance demand.

7.2 Optimal conditional deductible: simulation

For the sake of illustration, Table 1 displays the results of simulations of the optimal conditional-deductible contract, in the case of the electricity supplier considered in sub-section 4.2. We postulate CARA preferences and normal distributions for $\tilde{\eta}_a$ and $\tilde{\eta}_b$, and we assume that electricity spot price Y_b takes only two values y_{b2} (high) and y_{b1} (low) with equal probability, with $z_0(y_{b1})$ (left) and $z_0(y_{b2})$ (right) in each cell. We assume $y_{b2} = \delta p_{b2}$ and $y_{b1} = \delta p_{b1}$, with $p_{b2} > p_{b1}$, where parameter δ calibrates the variability of Y_b .²³ When $\delta = 0$ (first column), we have $y_{b1} = y_{b2} = 0$, and

²³The calibration of parameters is as follows: $q = 1, p = 0.1, p_{b1} = 0.096, p_{b2} = 0.104$. We also assume $C = 0.6pq = 0.06$ and $\mathbb{E}X = 0.4, w = 0.016, \sigma_a = 100\mathbb{E}X/q = 1.6$ and $\sigma_b = 0.3\sigma_a = 0.48$, where σ_a and σ_b are the standard deviation of $\tilde{\eta}_a$ and $\tilde{\eta}_b$, respectively.

δ	0	0.25	0.5	0.75	1
R^2					
m	0.5	0.39	0.24	0.14	0.09
0.1	13,935-13,935	14,182-13,642	14,636-12,476	14,744-9,884	14,744-6,104
0.2	14,624-14,624	14,885-14,345	15,545-13,385	15,966-11,106	15,976-7,336
0.3	15,122-15,122	15,394-14,854	16,224-14,064	17,272-12,412	17,411-8,771
0.4	15,556-15,556	15,836-15,296	16,769-14,609	18,106-13,246	18,478-9,838
0.5	15,976-15,976	16,260-15,720	17,217-15,057	18,638-13,778	19,317-10,677

Table 1: Example 1 - CARA utility function. Each cell reports $z_0(y_{b1})$ (left) and $z_0(y_{b2})$ (right) for various levels of loading factor m and price gap δ .

the two deductibles are therefore identical. When $\delta > 0$, the deductible depends on the observed value of Y_b , with a higher degree of basis risk, captured by the higher (conditional) variance $\mathbb{V}(\tilde{\varepsilon}|Y_b = y_{bi}) = \sigma_a^2 + \delta^2 q^2 p_{bi}^2 \sigma_b^2$ when $i = 2$ than when $i = 1$. In accordance with Proposition 4, we have $z_0(y_{b1}) > z_0(y_{b2})$ when $\delta > 0$.²⁴

7.3 Better insurance with higher basis risk: an example

Assume that X is distributed in an interval $[0, \bar{x}]$ with density $f_X(x)$. Consider two loss indices Z_1 and Z_2 with density function $f_{Z_i}(z)$ and c.d.f. $F_{Z_i}(z)$ for loss index Z_i , with $i = 1$ or 2 . These loss indices are distributed in the same support $[\underline{z}, \bar{z}]$, with $0 < \underline{z} < \bar{z} < \bar{x}$, with independent basis risks $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$, respectively, also distributed in the same support $[-\underline{z}, \bar{x} - \bar{z}]$. We assume that $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$ in the sense of

²⁴The increase in δ increases both the lower price y_{b1} and the upper price y_{b2} , but it also accentuates the difference between the two prices, with two consequences. First, the variance of the basis risk variable $\tilde{\varepsilon}$ increases, as reflected by the decreasing values of the R^2 . In the case of a prudent policyholder, this should result in a lower deductibles. We do observe this effect for $z_0(y_{b1})$ but not for $z_0(y_{b2})$ that increases with δ . This is because the difference between the two states y_{b1} and y_{b2} is also accentuated, with more additional variance in the state where the electricity price is high than in the state where it is low. As a consequence, the prudent policyholder requires a higher deductible in the less risky state in order to afford a significantly lower deductible in the riskier state.

Rothschild and Stiglitz (1970). We also assume $f_{Z_1}(\bar{z}) = f_{Z_2}(\bar{z}) > 0$ and we denote \bar{f} this common value of the loss-index densities at the top of the support of Z_1 and Z_2 .

From Proposition 1, in both cases the optimal parametric insurance contract is an index-based straight deductible policy. Let $U_i(z_0, m)$ be the policyholder's expected utility when parametric insurance is based on loss index Z_i with deductible $z_0 \in [\underline{z}, \bar{z}]$ and loading m . We have

$$U_i(z_0, m) = \int_{\underline{z}}^{z_0} \left[\int_0^{\bar{x}} u(w_0 - x - \hat{P}_i(z_0, m)) f_X^i(x | z) dx \right] f_{Z_i}(z) dz \\ + \int_{z_0}^{\bar{z}} \left[\int_0^{\bar{x}} u(w_0 - x - \hat{P}_i(z_0, m) + z - z_0) f_X^i(x | z) dx \right] f_{Z_i}(z) dz,$$

for $i = 1$ or 2 , where $f_X^i(x | z)$ is the conditional density function of X when $Z_i = z$, and

$$\hat{P}_i(z_0, m) = (1 + m) \int_{z_0}^{\bar{z}} (z - z_0) f_{Z_i}(z) dz$$

is the insurance premium when index Z_i is used.

Let $z_{0i}^*(m) = \arg \max\{U_i(z_0, m), z_0 \in [\underline{z}, \bar{z}]\}$ be the optimal deductible under loss index Z_i as a function of the loading factor, for $i = 1, 2$, and let $\bar{m}_1 = \sup\{m > 0 \mid z_{01}^*(m) < \bar{z}\}$. We assume that $z_0 \rightarrow U_1(z_0, m)$ is unimodal in the neighbourhood of \bar{z} , and thus $z_{01}^*(m)$ is uniquely defined and continuous when m is close to \bar{m}_1 , with $z_{01}^*(\bar{m}_1) = \bar{z}$ and $z_{01}^*(m) < \bar{z}$ when $m < \bar{m}_1$.

Let $\Delta U(z_0, m) = U_2(z_0, m) - U_1(z_0, m)$, with $\Delta U(\underline{z}, m) < 0$ and $\Delta U(\bar{z}, m) = 0$ for all m . A simple calculation yields

$$\frac{\partial U_i(z_0, m)}{\partial z_0} = - \int_{z_0}^{\bar{z}} \left[\int_0^{\bar{x}} u'(w_0 - x - \hat{P}_i(z_0, m) + z - z_0) f_X^i(x | z) dx \right] f_{Z_i}(z) dz \\ - \hat{P}'_i(z_0, m) \mathbb{E} u'_i(z_0, m),$$

where

$$\hat{P}'_i(z_0, m) = -(1 + m)[1 - F_{Z_i}(z_0)],$$

and

$$\begin{aligned}\mathbb{E}u'_i(z_0, m) &= \int_{\underline{z}}^{z_0} \left[\int_0^{\bar{x}} u'(w_0 - x - \widehat{P}_i(z_0, m)) f_X^i(x | z) dx \right] f_{Z_i}(z) dz \\ &+ \int_{z_0}^{\bar{z}} \left[\int_0^{\bar{x}} u'(w_0 - x - \widehat{P}_i(z_0, m) + z - z_0) f_X^i(x | z) dx \right] f_{Z_i}(z) dz.\end{aligned}$$

This gives

$$\frac{\partial U_i(z_0, m)}{\partial z_0} \Big|_{z_0=\bar{z}} = 0 \text{ for all } m, \quad (10)$$

for $i = 1$ and 2 , and thus

$$\frac{\partial [\Delta U(z_0, m)]}{\partial z_0} \Big|_{z_0=\bar{z}} = 0 \text{ for all } m. \quad (11)$$

We have $z_{01}^*(m) < \bar{z}$ when $m < \bar{m}_1$ and thus the unimodal function $z_0 \longrightarrow U_1(z_0, m)$ is decreasing in the neighbourhood of \bar{z} in that case. Using (10) then yields $\partial^2 U_1(z_0, m) / \partial z_0^2 > 0$ when z_0 is close to \bar{z} and $m < \bar{m}_1$. Symmetrically, we have $\partial^2 U_1(z_0, m) / \partial z_0^2 < 0$ when z_0 is close to \bar{z} and $m > \bar{m}_1$, and by continuity we deduce

$$\frac{\partial^2 U_1(z_0, \bar{m}_1)}{\partial z_0^2} \Big|_{z_0=\bar{z}} = 0. \quad (12)$$

Since X and $\tilde{\varepsilon}_i$ are independent, and X conditionally on $Z_i = \bar{z}$ is distributed in $[\bar{z} - \underline{z}, \bar{x}]$, we may write

$$\begin{aligned}\frac{\partial^2 U_i(z_0, m)}{\partial z_0^2} \Big|_{z_0=\bar{z}} &= \bar{f} \left[\int_{\bar{z}-\underline{z}}^{\bar{x}} u'(w_0 - x) f_X^i(x | \bar{z}) dx - (1 + m) \mathbb{E}u'_i(\bar{z}) \right] \\ &= \bar{f} A_i(m) \text{ for all } m,\end{aligned}$$

where[

$$A_i(m) = \int_{-\underline{z}}^{\bar{x}-\bar{z}} u'(w_0 - \bar{z} - \varepsilon) f_{\tilde{\varepsilon}_i}(\varepsilon) d\varepsilon - (1 + m) \int_0^{\bar{x}} u'(w_0 - x) f_X(x) dx,$$

and where $f_{\tilde{\varepsilon}_i}(\varepsilon)$ denotes the density function of $\tilde{\varepsilon}_i$. Assume that the policyholder is prudent (i.e. $u''' > 0$). We then have $A_2(m) > A_1(m)$ because $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$, with $A_1(\bar{m}_1) = 0$, and $A_2(m) > A_1(m) > 0$ when $m < \bar{m}_1$. Hence, we have

$$\frac{\partial^2[\Delta U(z_0, m)]}{\partial z_0^2} \Big|_{z_0=\bar{z}} = \bar{f}[A_2(m) - A_1(m)] > 0, \quad (13)$$

when $m < \bar{m}_1$. We deduce from equations (10) and (12) that, when $m < \bar{m}_1$, function $z_0 \rightarrow \Delta U(z_0, m)$ has a local minimum in $[\underline{z}, \bar{z}]$ at $z_0 = \bar{z}$ with $\Delta U(z_0, m) > 0$ when z_0 is smaller than (and close to) \bar{z} . When m is close to \bar{m}_1 , $z_{01}^*(m)$ is close to \bar{z} . The definition of $z_{02}^*(m)$ implies $U_2(z_{02}^*(m), m) \geq U_2(z_{01}^*(m), m)$, and thus we may write

$$\begin{aligned} U_2(z_{02}^*(m), m) - U_1(z_{01}^*(m), m) &\geq U_2(z_{01}^*(m), m) - U_1(z_{01}^*(m), m) \\ &= \Delta U(z_{01}^*(m), m) \\ &> 0, \end{aligned}$$

when m is smaller than (and close to) \bar{m}_1 . In that case, the optimal expected utility is higher when parametric insurance is based on Z_2 rather than on Z_1 , although $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$.

Remark 1 *It has been assumed that Z_1 and Z_2 are distributed in the same support $[\underline{z}, \bar{z}]$, and for this reason the underlying information structures (\mathcal{S}_1, Y_1) and (\mathcal{S}_2, Y_2) cannot be compared by the accuracy criterion. To show this, assume a contrario that (\mathcal{S}_1, Y_1) is more accurate than (\mathcal{S}_2, Y_2) and denote $[\underline{z}_i, \bar{z}_i] \subset [0, \bar{x}]$ the support of Z_i . With the notation of Definition 1, we may write:*

$$\begin{aligned} \underline{z}_2 &= \inf\{\mathbb{E}[X(\omega) \mid \omega \in \mathcal{O}_2(y_2)], y_2 \in \mathcal{S}_2\} \\ &= \inf\{\mathbb{E}[X(\omega) \mid \omega \in \cup_{y_1 \in \mathcal{K}(y_2)} \mathcal{O}_1(y_1)], y_2 \in \mathcal{S}_2\} \\ &> \inf\{\inf\{\mathbb{E}[X(\omega) \mid \omega \in \mathcal{O}_1(y_1)], y_1 \in \mathcal{K}(y_2)\}, y_2 \in \mathcal{S}_2\} \\ &= \inf\{\mathbb{E}[X(\omega) \mid \omega \in \mathcal{O}_1(y_1)], y_1 \in \mathcal{S}_1\} = \underline{z}_1. \end{aligned}$$

Similarly, we have $\bar{z}_2 < \bar{z}_1$. Hence the supports of Z_1 and Z_2 differ, the one corresponding to the less accurate information structure being included in the other.

7.4 Proof of Proposition 1

Let $I^*(\cdot) : \mathcal{S} \rightarrow \mathbb{R}_+$, P^* the optimal indemnity schedule and premium, with optimal expected utility $\bar{u}^* = \mathbb{E}v(w_0 - Z + I^*(Y) - P^*)$. Define $\bar{J}(\cdot) : Z(Y(\Omega)) \rightarrow \mathbb{R}_+$ by $\bar{J}(z) = \mathbb{E}_Y[I^*(Y) | Z(Y) = z]$. Using $u'' < 0$ allows us to write

$$\begin{aligned} \bar{u}^* &= \mathbb{E}_Z[\mathbb{E}_Y[v(w_0 - z + I^*(Y) - P^*) | Z = z]] \\ &\leq \mathbb{E}_Z[v(w_0 - z + \mathbb{E}_Y[I^*(Y) | Z = z] - P^*) | Z = z] \\ &= \mathbb{E}v(w_0 - Z + \bar{J}(Z) - P^*) = \bar{u}, \end{aligned}$$

with strict inequality if $I^*(\cdot)$ is not index-based in a positive-probability event. Furthermore, we have

$$\begin{aligned} \mathbb{E}\bar{J}(Z) &= \mathbb{E}_Z[\mathbb{E}_Y[I^*(Y) | Z = z]] = \mathbb{E}I^*(Y), \\ P^* &= (1 + m)\mathbb{E}I^*(Y) = (1 + m)\mathbb{E}\bar{J}(Z). \end{aligned}$$

Thus, the index-based contract $\bar{J}(\cdot), P^*$ is feasible, with higher expected utility than $I^*(\cdot), P^*$, hence a contradiction. The rest of the proof results from the optimality of a straight deductible contract with loss Z , under constant loading and utility function $v(\cdot)$.

7.5 Proof of Proposition 2

The Proposition directly follows from the analysis of comparative risk aversion when there is an independent background risk: see Proposition 24 and 25 in Gollier (2004).

7.6 Proof of Proposition 3

Let λ be a Lagrange multiplier associated with constraint (2). The first-order optimality conditions are written as

$$\begin{aligned} \mathbb{E}[u'(w_0 - Z(y) - \tilde{\varepsilon} + I(y) - P) | Y = y] - \lambda(1 + m) &\begin{cases} \leq 0 \text{ for all } y \in \mathcal{S} \\ = 0 \text{ if } I(y) > 0 \end{cases}, \\ \mathbb{E}[u'(w_0 - Z(Y) - \tilde{\varepsilon} + I(Y) - P)] &= \lambda. \end{aligned}$$

Using $\mathbb{E}[\tilde{\varepsilon} | Y = y] = 0$ for all y and $u''' > 0$ yields

$$\mathbb{E}[u'(w_0 - Z(y) - \tilde{\varepsilon} + I(y) - P) | Y = y] > u'(w_0 - Z(y) + I(y) - P).$$

Let $\widehat{Z}(y)$ be defined for all y by

$$\mathbb{E}[u'(w_0 - Z(y) - \tilde{\varepsilon} + I(y) - P) | Y = y] = u'(w_0 - \widehat{Z}(y) + I(y) - P)$$

with $\widehat{Z}(y) > Z(y)$ from $u'' < 0, u''' > 0$. Hence, the optimality conditions may be rewritten as

$$u'(w_0 - \widehat{Z}(y) + I(y) - P) - \lambda(1 + m) \begin{cases} \leq 0 \text{ for all } y \in \mathcal{S} \\ = 0 \text{ if } I(y) > 0 \end{cases},$$

$$\mathbb{E}[u'(w_0 - \widehat{Z}(Y) + I(Y) - P)] = \lambda.$$

Let \widehat{z}_0 defined by $u'(w_0 - \widehat{z}_0 - P) = \lambda(1 + m)$. We have $I(y) = \widehat{Z}(y) - \widehat{z}_0$ if $I(y) > 0$, which implies $\widehat{Z}(y) > \widehat{z}_0$ and $\widehat{Z}(y) \leq \widehat{z}_0$ if $I(y) = 0$, and thus we may write

$$I(y) = \max\{\widehat{Z}(y) - \widehat{z}_0, 0\} \text{ for all } y \in \mathcal{S}.$$

Let $y_1, y_2 \in \mathcal{S}$ with $I(y_1), I(y_2) > 0$. Denote $\Delta Z_1 = \widehat{Z}(y_1) - Z(y_1) > 0$ and $\Delta Z_2 = \widehat{Z}(y_2) - Z(y_2) > 0$. Assume that the conditional distribution of $\tilde{\varepsilon}$ is more risky when $Y = y_2$ than when $Y = y_1$, and suppose $\Delta Z_2 \leq \Delta Z_1$. The optimality conditions give

$$u'(w_0 - \widehat{Z}(y_1) + I(y_1) - P) = u'(w_0 - \widehat{Z}(y_2) + I(y_2) - P) = \lambda(1 + m),$$

or, equivalently

$$\mathbb{E}[u'(w_0 - Z(y_1) - \tilde{\varepsilon} + I(y_1) - P) | Y = y_1] = \mathbb{E}[u'(w_0 - Z(y_2) - \tilde{\varepsilon} + I(y_2) - P) | Y = y_2].$$

We have $I(y_1) = \widehat{Z}(y_1) - \widehat{z}_0 = Z(y_1) - \widehat{z}_0 + \Delta Z_1$ and $I(y_2) = \widehat{Z}(y_2) - \widehat{z}_0 = Z(y_2) - \widehat{z}_0 + \Delta Z_2$. Hence, the last equation may be rewritten as

$$\mathbb{E}[u'(w_0 + \Delta Z_1 - \widehat{z}_0 - \tilde{\varepsilon} - P) | Y = y_1] = \mathbb{E}[u'(w_0 + \Delta Z_2 - \widehat{z}_0 - \tilde{\varepsilon} - P) | Y = y_2],$$

or

$$\mathbb{E}[u'(w + \Delta Z_1 - \Delta Z_2 - \tilde{\varepsilon}) \mid Y = y_1] = \mathbb{E}[u'(w - \tilde{\varepsilon}) \mid Y = y_2],$$

where $w = w_0 + \Delta Z_2 - \hat{z}_0 - P$. However, we have

$$\begin{aligned} \mathbb{E}[u'(w + \Delta Z_1 - \Delta Z_2 - \tilde{\varepsilon}) \mid Y = y_1] &\leq \mathbb{E}[u'(w - \tilde{\varepsilon}) \mid Y = y_1] \\ &< \mathbb{E}[u'(w - \tilde{\varepsilon}) \mid Y = y_2], \end{aligned}$$

where the first inequality comes from $\Delta Z_1 \geq \Delta Z_2$ and $u'' < 0$, and the second from the fact that $\tilde{\varepsilon}_{|Y=y_2}$ is more risky than $\tilde{\varepsilon}_{|Y=y_1}$ and $u''' > 0$. This is a contradiction.

7.7 Proof of Corollary 1

The Corollary directly follows from the second part of Proposition 3.

7.8 Proof of Proposition 4

The first-order optimality conditions are

$$\begin{aligned} \mathbb{E}[u'(w_0 - Z(y_a) - \tilde{\varepsilon} + I(y) - P) \mid Y_b = y_b] - \lambda(1 + m) &\begin{cases} \leq 0 \text{ for all } y \in \mathcal{S}, \\ = 0 \text{ if } I(y) > 0, \end{cases} \\ \mathbb{E}[u'(w_0 - Z(Y_a) - \tilde{\varepsilon} + I(Y) - P)] &= \lambda. \end{aligned}$$

where $y = (y_a, y_b)$. Let us define $\mathcal{U}(w, y_b) \equiv \mathbb{E}_{\tilde{\varepsilon}}[u(w - \tilde{\varepsilon}) \mid Y_b = y_b]$, with $\mathcal{U}'_w > 0, \mathcal{U}''_{w^2} < 0$. This allows us to rewrite the optimality conditions as

$$\begin{aligned} \mathcal{U}'_w(w_0 - Z(y_a) + I(y) - P, y_b) - \lambda(1 + m) &\begin{cases} \leq 0 \text{ for all } y \in \mathcal{S} \\ = 0 \text{ if } I(y) > 0 \end{cases} \\ \mathbb{E}[\mathcal{U}'_w(w_0 - Z(Y_a) + I(Y) - P), Y_b] &= \lambda. \end{aligned}$$

When $I(y) > 0$, we have $I(y) = Z(y_a) - z_0(y_b)$, where $z_0(y_b)$ is defined by $\mathcal{U}'_w(w_0 - z_0(y_b) - P, y_b) = \lambda(1 + m)$. Furthermore, when $I(y) = 0$ we have $Z(y_a) < z_0(y_b)$. Patching up these two cases yields

$$I(y) = \max\{0, Z(y_a) - z_0(y_b)\} \text{ for all } y = (y_a, y_b).$$

Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be random variables distributed as $\tilde{\varepsilon}$ given $\tilde{y}_b = y_{b1}$ and y_{b2} , respectively, and assume that $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$. We have

$$\mathcal{U}'_w(w_0 - z_0(y_{bh}) - P, y_{bh}) = \lambda(1 + m) \text{ for } h = 1, 2.$$

When $h = 2$, this may be rewritten as

$$\mathbb{E}u'(w_0 - z_0(y_{b2}) - \tilde{\varepsilon}_2 - P) = \lambda(1 + m).$$

Since $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$, we may write $\tilde{\varepsilon}_2 \equiv \tilde{\varepsilon}_1 + \tilde{\eta}$, where random variable $\tilde{\eta}$ is such that $\mathbb{E}[\tilde{\eta} \mid \tilde{\varepsilon}_1 = \varepsilon_1] = 0$ for all ε_1 . When $u''' > 0$, we have

$$\begin{aligned} \mathbb{E}u'(w_0 - z_0(y_{b2}) - \tilde{\varepsilon}_2 - P) &= \mathbb{E}_{\tilde{\varepsilon}_1}[\mathbb{E}_{\tilde{\eta}}[u'(w_0 - z_0(y_{b2}) - \varepsilon_1 - \tilde{\eta} - P) \mid \tilde{\varepsilon}_1 = \varepsilon_1]] \\ &> \mathbb{E}_{\tilde{\varepsilon}_1}u'(w_0 - z_0(y_{b2}) - \tilde{\varepsilon}_1 - P) \\ &= \mathcal{U}'_w(w_0 - z_0(y_{b2}) - P, y_{b1}). \end{aligned}$$

We deduce

$$\mathcal{U}'_w(w_0 - z_0(y_{b2}) - P, y_{b1}) < \mathcal{U}'_w(w_0 - z_0(y_{b1}) - P, y_{b1}),$$

and using $\mathcal{U}''_{w^2} < 0$ gives $z_0(y_{b2}) < z_0(y_{b1})$.

7.9 Proof of Lemma 1

Assume $Y_2(\omega) = \Phi(Y_1(\omega))$ for all ω . Let $y_2 \in \mathcal{S}_2$. We have

$$\begin{aligned} \mathcal{O}_2(y_2) &= \{\omega \in \Omega \text{ s.t. } Y_2(\omega) = y_2\} \\ &= \{\omega \in \Omega \text{ s.t. } \Phi(Y_1(\omega)) = y_2\} \\ &= \{\omega \in \Omega \text{ s.t. } Y_1(\omega) \in \Phi^{-1}(y_2)\} \\ &= \bigcup_{y_1 \in \mathcal{K}(y_2)} \mathcal{O}_1(y_1), \end{aligned}$$

with $\mathcal{K}(y_2) = \Phi^{-1}(y_2)$. Hence $\{\mathcal{K}(y_2), y_2 \in \mathcal{S}_2\}$ is a partition of \mathcal{S}_1 , and thus (\mathcal{S}_1, Y_1) is more accurate than (\mathcal{S}_2, Y_2) .

Conversely, assume that (\mathcal{S}_1, Y_1) is more accurate than (\mathcal{S}_2, Y_2) , i.e.

$$\mathcal{O}_2(y_2) = \bigcup_{y_1 \in \mathcal{K}(y_2)} \mathcal{O}_1(y_1),$$

with $\{\mathcal{K}(y_2), y_2 \in Y_2\}$ a partition of Y_1 . Let $\Phi(\cdot) : \mathcal{S}_1 \rightarrow \mathcal{S}_2$ defined by $\Phi(y_1) = y_2$ if $y_1 \in \mathcal{K}(y_2)$ for all $y_2 \in Y_2$. For all ω there exists $y_1 \in Y_1$ such that $Y_1(\omega) = y_1$, i.e. $\omega \in \mathcal{O}_1(y_1)$. We have $\mathcal{O}_1(y_1) \subset \mathcal{O}_2(y_2)$ with $y_1 \in \mathcal{K}(y_2)$, and thus $\mathcal{O}_1(y_1) \subset \mathcal{O}_2(\Phi(y_1))$ from the definition of $\Phi(\cdot)$, which implies $Y_2(\omega) = \Phi(y_1) = \Phi(Y_1(\omega))$.

7.10 Proof of Proposition 5

Under information structure (\mathcal{S}_i, Y_i) , for $i = 1$ or 2 , the optimal contract $\{P_i^*, I_i^*(\cdot)\}$ maximizes

$$\mathbb{E}u(w_0 - X + I_i(Y_i) - P_i),$$

with respect to P_i and $I_i(\cdot) : \mathcal{S}_i \rightarrow \mathbb{R}_+$ subject to

$$P_i = (1 + m)\mathbb{E}I_i(Y_i),$$

with optimal expected utility

$$\bar{u}_i^* = \mathbb{E}u(w_0 - X + I_i^*(Y_i) - P_i^*).$$

Consider the indemnity schedule $I_1(\cdot) : \mathcal{S}_1 \rightarrow \mathbb{R}_+$ defined by $I_1(y_1) = I_2^*(\Phi(y_1))$ for all $y_1 \in \mathcal{S}_1$. In any state of nature ω , the insurance payout is the same for $I_1(\cdot)$ and $I_2^*(\cdot)$. Hence, the contract $\{P_2^*, I_1(\cdot)\}$ is feasible under information structure (\mathcal{S}_1, Y_1) with expected utility $\bar{u}_1 = \bar{u}_2^*$, and thus we have $\bar{u}_1^* \geq \bar{u}_2^*$, hence the weak dominance of (\mathcal{S}_1, Y_1) over (\mathcal{S}_2, Y_2) .

Furthermore, replacing (\mathcal{S}_2, Y_2) by (\mathcal{S}_1, Y_1) allows us to increase the insurance indemnity $I_1(y_1)$ above $I_2^*(\Phi(y_1))$ when $y_1 \in \mathcal{A}_1^1(y_2)$ and simultaneously to decrease $I_1(y_1)$ under $I_2^*(\Phi(y_1))$ when $y_1 \in \mathcal{A}_1^2(y_2)$. This can be done for all $y_2 \in \mathcal{A}_2$ in such a way that the expected insurance payout is unchanged when $Y^1(\omega) \in \mathcal{A}_1^1(y_2) \cup \mathcal{A}_1^2(y_2)$.

Furthermore $I_1(y_1)$ is kept equal to $I_2^*(\Phi(y_1))$ if $y_1 \notin \{\mathcal{A}_1^1(y_2) \cup \mathcal{A}_1^1(y_2), y_2 \in \mathcal{A}_2\}$. This change increases the insurance payout in states with higher losses and reduces this payout in states with lower losses, starting from an initial solution $I_1(\cdot)$ where these payouts are equal, and the insurance premium is unchanged. Because of the concavity of the utility function, this induces an increase in expected utility, with $\bar{u}_1^* > \bar{u}_2^*$, hence the strong dominance of (\mathcal{S}_1, Y_1) over (\mathcal{S}_2, Y_2) .

7.11 Proof of Proposition 6

Assume that (\mathcal{S}_1, Y_1) is more accurate than (\mathcal{S}_2, Y_2) . Using (7) gives $\tilde{\varepsilon}_2 = \tilde{\varepsilon}_1 + \tilde{\eta}$ where $\tilde{\eta} = Z_1(Y_1) - Z_2(Y_2)$. We may write

$$Z_2(Y_2) = \mathbb{E}[Z_1(Y_1) \mid Y_1 \in \mathcal{K}(Y_2)]$$

where $\mathcal{K}(\cdot)$ is defined as in Definition 1. Hence, we have

$$\mathbb{E}[\tilde{\eta} \mid \tilde{\varepsilon}_1 = \varepsilon_1] = \mathbb{E}[Z_1(Y_1) \mid \tilde{\varepsilon}_1 = \varepsilon_1] - \mathbb{E}[Z_1(Y_1) \mid Y_1 \in \mathcal{K}(Y_2), \tilde{\varepsilon}_1 = \varepsilon_1]. \quad (14)$$

Let $F_1(y_1 \mid \varepsilon_1)$ and $F_2(y_2 \mid \varepsilon_1)$ be the c.d.f. of Y_1 and Y_2 , respectively, conditionally on $\tilde{\varepsilon}_1 = \varepsilon_1$. Let also $F_1(y_1 \mid y_2, \varepsilon_1)$ be the c.d.f. of Y_1 conditionally on $Y_2 = y_2$ and $\tilde{\varepsilon}_1 = \varepsilon_1$. We have

$$\begin{aligned} \mathbb{E}[Z_1(Y_1) \mid \tilde{\varepsilon}_1 = \varepsilon_1] &= \int_{y_1 \in \mathcal{S}_1} Z_1(y_1) dF_1(y_1 \mid \varepsilon_1) \\ &= \int_{y_2 \in \mathcal{S}_2} \left\{ \int_{y_1 \in \mathcal{K}(y_2)} Z_1(y_1) dF_1(y_1 \mid y_2, \varepsilon_1) \right\} dF_2(y_2 \mid \varepsilon_1) \\ &= \mathbb{E}[Z_1(Y_1) \mid Y_1 \in \mathcal{K}(Y_2), \tilde{\varepsilon}_1 = \varepsilon_1], \end{aligned}$$

for all ε_1 . Using (14) then gives $\mathbb{E}[\tilde{\eta} \mid \tilde{\varepsilon}_1 = \varepsilon_1] = 0$ for all ε_1 , which shows that $\tilde{\varepsilon}_2$ is more risky than $\tilde{\varepsilon}_1$. Furthermore, we have

$$\begin{aligned} Z_2(y_2) &= \mathbb{E}[X \mid Y_2 = y_2] \\ &= \mathbb{E}[X \mid Y_1 \in \mathcal{K}(y_2)] \\ &= \mathbb{E}[Z_1(Y_1) \mid Y_1 \in \mathcal{K}(y_2)], \end{aligned}$$

for all y_2 . Consequently, we may write

$$z_2 = \int_{y_1 \in \mathcal{K}(Z_2^{-1}(z_2))} Z_1(y_1) dF_1(y_1 | Y_1 \in \mathcal{K}(Z_2^{-1}(z_2))), \quad (15)$$

for all $z_2 \in \text{im}(Z_2)$, where $F_1(y_1 | Y_1 \in \mathcal{K}(Z_2^{-1}(z_2)))$ is the distribution function of Y_1 conditionally on $Y_1 \in \mathcal{K}(Z_2^{-1}(z_2))$. We have

$$Y_1 \in \mathcal{K}(Z_2^{-1}(z_2)) \Leftrightarrow Z_2(\Phi(Y_1)) = z_2,$$

for all $z_2 \in \text{im}(Z_2)$. Using $Y_2 = \Phi(Y_1)$ and (15) yields

$$\mathbb{E}[Z_1 | Z_2(\Phi(Y_1)) = z_2] = z_2 \text{ for all } z_2 \in \text{im}(Z_2),$$

or equivalently $Z_1 = Z_2 + \tilde{\eta}_Z$ with

$$\mathbb{E}[\tilde{\eta}_Z | Z_2 = z_2] = 0 \text{ for all } z_2 \in \text{im}(Z_2)$$

which shows that Z_1 is more risky than Z_2 .

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