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**Age-Related Differences in Strategic Monitoring During
Arithmetic Problem Solving**

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Age-Related Differences in Strategic Monitoring During Arithmetic Problem Solving

1. Introduction

Multiple-strategy use is one of the ubiquitous features of human cognition at all ages. Indeed, several decades of research in children (see Siegler, 1996, 2007, for overviews) and during adulthood (see Lemaire, 2016, for an overview) have shown that participants use a variety of strategies to accomplish cognitive tasks. Participants' performance and age-related changes in cognitive performance depend on strategies. One important issue of research on strategies is how participants choose among strategies on a given item. The present study contributes to this issue by investigating strategic monitoring and control during arithmetic problem solving. Specifically, this study examines how participants judge whether a selected strategy for a problem is the better or the poorer strategy.

Previous empirical works on strategies showed that strategy selection and strategy execution are influenced by participants, stimulus, and situation characteristics (Siegler, 2007). These factors act individually and in interaction with each other. For example, Lemaire, Arnaud, and Lecacheur (2004) asked young and older adults to provide estimates of two-digit multiplication problems (e.g., 43×38) with a rounding-down strategy (doing $40 \times 30 = 1200$) or a rounding-up strategy (doing $50 \times 40 = 2000$), under different levels of speed/accuracy pressures. The authors found that older adults selected and executed the rounding-down strategy more slowly under accuracy-pressure conditions than under no-pressure conditions, especially when they solved easy problems. Young adults, however, were less influenced by the time pressure condition. Such findings show that young and older adults are differently affected by problem and task characteristics.

Computational models of strategy selection proposed several mechanisms to account for how people choose and execute strategies on each problem: Lovett and Anderson's (1996) Adaptive Control of Thought-Rational (ACT-R), Siegler and Shipley's (1995) Adaptive Strategy Choices Model (ASCM), Lovett and Schunn's (1999) Represent, Construct, Choose, Learn (RCCL) model, Neches' (1987) Heuristic Procedure Modification (HPM) model, Rieskamp and Otto's (2006) Strategy Selection Learning (SSL) model, and Siegler and Araya's (2005) Strategy, Choice, and Discovery Simulation* (SCADS*). All these models proposed that choosing among multiple strategies crucially involves associative mechanisms such as activating the relative costs/benefits of each strategy and selecting the strategy that works best for a given problem on the basis of problem and strategy characteristics. All models also assume that strategies including fewer and/or simpler procedures (e.g., retrieving the correct solution of arithmetic problems like $12 = 3 \times 4$ directly from memory) are easier to execute than strategies including more and/or more complex procedures (e.g., adding 3 four times). Finally, these models assume that based on past experience, children and adults select more and more frequently the better strategy on each problem. So, when participants have to solve a new problem, they assess problem features, they activate strategies available to solve the present problem, select the most strongly associated strategy with the problem to be solved or with a related problem, execute the selected strategy, and store strategy performance relative to the problem features. Associative mechanisms are a key component of these models and have proven sufficient to account for most findings on strategy choices and execution such as the effects of problem difficulty or strategy characteristics.

In addition to associative mechanisms, two of the existing computational models, namely Lovett and Schunn's (1999) RCCL and Siegler and Araya's (2005) SCADS*, assume that

strategy choices involve metacognitive mechanisms. In RCCL, the metacognitive system enables participants to interrupt a strategy mid-execution if participants estimate that the current strategy is not the best strategy or if it is an inappropriate strategy. In SCADS*, the metacognitive system which is key to create or discover new strategies, includes the attentional spotlight (devoted to allocate attentional resources to strategy execution, especially when strategies are not automatized), strategy change heuristics (devoted to evaluating efficiency of current strategy and if a recombination of strategy components is possible to create a new, more efficient strategy), and goal-sketch filters (which ensures that the newly created strategies are valid for solving a given category of problems). In sum, models of strategies include metacognitive processes to evaluate strategies once selected and, possibly to interrupt strategies mid-execution to switch for a better strategy (RCCL) or to create and discover new legitimate strategies (SCADS*). Here, we test the possibility that metacognitive processes are also involved in strategy selection decisions.

Previous empirical works suggest a role for metacognitive processes (Ardiale & Lemaire, 2012, 2013; see also Luwel, Torbeyns, & Verschaffel, 2003) both when a strategy has been selected and is under current execution, as assumed by RCCL, and before a strategy is selected. For example, Ardiale and Lemaire (2012, 2013) asked young and older adults to estimate products of two-digit multiplication problems like 37×64 . Problems were displayed with a cue indicating which of two rounding strategies to use. After executing this cued strategy for one second, participants could choose to change (or not) strategy if they judged that the cued strategy was not the best strategy for this item (i.e., the strategy that yields the closest estimate to correct product). The authors found that both young and older adults were able to interrupt execution of strategy and switch strategy when the cued strategy was not the best one. Such results suggest that participants are able both to assess current strategic operations to judge whether the selected

strategy is the best (meta-strategic monitoring) and to use the outputs of this assessment to regulate their strategic behaviors (meta-strategic control). However, unknown is how young and older adults performed these better strategy judgments, an issue that we pursued in the present experiment. Ardiale and Lemaire also found that older adults revised initial strategy selections less often than young adults. Unknown is whether age-related changes in meta-strategic monitoring processes (accomplishing better strategy judgment) or in executive control processes (responsible for switching strategies) are responsible for older adults' being less able to revise initial strategy selections once engaged in strategy execution.

In the metacognitive literature (see Hertzog, 2015; Castel, Middlebrooks, & McGillivray, 2015, for reviews), results on aging effects are conflicting. Some findings suggest that older adults may suffer from metacognitive monitoring impairments; namely, they are less able than young adults to evaluate their own cognitive performance (e.g., Jacoby & Rhodes, 2006). In contrast, other studies do not show any age-related differences in metacognitive monitoring processes (e.g., Price & Murray, 2012). However, even when monitoring processes are spared, studies indicate that their outputs are not necessarily used by older adults to regulate or control their performance (e.g., Hertzog & Hultsch, 2000). For instance, Souchay and Isingrini (2004) found that older adults did not allocate their study time as efficiently as young adults during a self-paced learning task. As most previous studies on aging and metacognition have been carried out in the memory domain, unknown are whether strategic monitoring and control change with age while young and older adults accomplish problem solving tasks. We address this issue in the context of arithmetic problem solving tasks.

1.1. Overview of the present study

As a first step to examine the role of metacognitive processes in strategic behaviors during arithmetic problem solving, we asked young and older participants to accomplish a better strategy judgment task. On each trial, participants were given arithmetic problems and a strategy. They had to decide as quickly as possible if the cued strategy was the better or the poorer of two available strategies (i.e., which strategy yielded the answer that was the closest to the correct product) for each problem. These strategies consist in rounding both operands up or rounding both operands down to their closest decades. Asking participants to choose among a pre-defined set of strategies enables to reduce the variability due to the fact that some people know more strategies than others. Also, previous works have shown that when participants are left free to choose whichever strategies they want to accomplish these tasks, they do use the above mentioned strategies spontaneously. Designs restricting the number of available strategies to choose among do not yield different findings regarding age-related changes in rates of better strategy selection and in strategy performance (e.g., LeFevre, Greenham, & Waheed, 1993; Lemaire, Lecacheur, & Farioli, 2000). Our hypothesis is that to be able to make accurate strategy selection judgments on each problem, participants will have to detect and weigh information enabling them to evaluate which strategy is the better strategy to solve the problem (meta-strategic monitoring), then to decide whether the cued strategy is actually the better (meta-strategic control).

Two types of information can possibly be used by participants to make accurate strategy selection judgments. Indeed, participants (a) select a strategy based on the problem characteristics without considering the cued strategy and/or (b) execute the cued strategy to see whether it was truly the better strategy to provide the closest estimate of the correct product. In this context, to understand processes underlying better strategy judgment in young and older

adults, we gave participants a better strategy selection task (i.e., participants were given arithmetic problems and had to select the better strategy among two available strategies) and a strategy execution task (participants were given arithmetic problems and a cue strategy that they had to execute). As these three tasks were given to the same individuals on the same problems, we were able to test whether participants' better strategy judgments were related to strategy selection and strategy execution. If participants try to select a strategy based on problems' characteristics and use the result of this selection when making their strategy judgments, we expect to find a positive relation between strategy judgment and strategy selection performance. However, if participants execute the cued strategy and use the result of this strategy execution as a cue to guide their strategy judgments, we expect to find a positive relation between strategy judgment and strategy execution performance.

The set of problems was carefully selected so as to examine whether problems' structural features that are known to influence strategy selection and strategy execution influence (a) better strategy judgment and (b) the relations between strategy judgment, selection, and execution. The two main problem characteristics that influence strategy selection and execution are first which strategy is better for a given problem (half the problems were best estimated with one strategy, the other problems with another strategy) and the size of unit digits (i.e., problems with both unit digits vs. one unit digit smaller or larger than 5) (see Lemaire, 2016, for an overview). Moreover, manipulating these variables enabled us to determine whether these two problem characteristics interacted with the strategy cued on each problem as participants accomplished the better strategy judgment task.

From a developmental perspective, this study was expected to determine whether there are age-related differences in (a) participants' better strategy judgments, (b) the relations

between strategy judgment, selection, and execution and (c) which problems' structural features influence these age differences. Specifically, if meta-strategic monitoring and control processes are impaired in aging, we expect to observe a decrease in the accuracy of older adults' strategy judgments as compared to young adults. In that context, the manipulation of problems' features should inform us on factors that affect these age-related changes in meta-strategic skills. In contrast, if meta-strategic processes are spared in aging, we should observe similar patterns of strategy judgments in young and older participants.

2. Method

2.1. Participants

The final sample included 73 French-speaking participants divided into two age groups: 44 healthy young adults (25 females; mean age = 24.45 years; $SD = 2.83$; age range = 19–30) and 29 healthy older adults (19 females; mean age = 72.86 years; $SD = 4.81$; age range = 64–85) years. Young adults were undergraduates from Aix-Marseille University. Older adults were recruited from the community of Marseille, and all had scores larger than 27 ($M = 29.6$) in the Mini Mental State Examination (MMSE), a screening measure of cognitive functioning (Folstein, Folstein, & McHugh, 1975). Two additional participants (one in each age group) were tested but were not included in the final analyses because they did not understand the test instructions.

Information about participants' sex, age, verbal fluency, and arithmetic fluency were collected at the end of the experiment. The latter two variables were assessed using the French version of the Mill-Hill Vocabulary Scale (Deltour, 1993) and the addition and subtraction-multiplication subtests of the French Kit (French, Ekstrom, & Price, 1963), respectively. As often found, older participants' verbal fluency ($M = 27.90$ vs. 22.05; $F(1,71) = 43.68$, $p < .001$, η^2_p

= .38) and arithmetic fluency ($M = 86.58$ vs. 35.34 ; $F(1,71) = 142.22$, $p < .001$, $\eta^2_p = .67$) were larger than those of young adults.

2.2. Stimuli

The stimuli were 64 two-digit multiplication problems presented in a standard form (e.g., 32×67). Based on previous findings in arithmetic (see Cohen Kadosh & Dowker, 2015, for an overview), the following factors were controlled: (a) no operand had a 0 or a 5 unit digit, (b) digits were not repeated within operands (e.g., 33×42), (c) digits were not repeated in the same unit or decade positions across operands (e.g., 62×67), (d) no reverse order of operands was used (e.g., 56×23 vs. 23×56), (e) the first operand was larger than the second operand in half the problems, and (f) the operand with the smallest unit digits was in the left position in half the problems (e.g., 42×36) and in the right position in the other problems (e.g., 23×41).

In this study, half the problems ($N = 32$) had homogeneous unit digits and half had heterogeneous unit digits. Problems with homogeneous unit digits had the unit digit of both operands smaller (e.g., 32×63) or larger (e.g., 38×69) than 5. In problems with heterogeneous unit digits, the unit digit of one operand was smaller than 5 while that of the other operand was larger than 5 (e.g., 42×69). These two types of problems were included because previous studies had found that all participants – whatever their age – select the best rounding strategy more easily when the size of the unit digits was homogenous than when it was heterogeneous (e.g., LeFevre, Greenham, & Waheed, 1993; Lemaire et al., 2004). Homogeneous and heterogeneous problems had comparable exact products when solved with each rounding strategy (*Means* = 2426 and 2408 for homogeneous and heterogeneous problems, respectively).

Finally, half the problems were so-called rounding-down problems because they were best estimated (i.e., closest products from the correct products) with the rounding-down strategy

(e.g., 86×21) and half rounding-up problems because they were best estimated with the rounding-up strategy (e.g., 74×39).

2.3. Procedure

Participants were tested individually in a quiet room using a laptop computer equipped with E-prime software (Schneider, Eschman, & Zuccolotto, 2002). They underwent an approximately 45-minute session including three main phases. These experimental phases were administered in the following order: (a) a judgment phase, (b) a selection phase, and (c) an execution phase. Before the test, participants were told that their task was to give an approximate answer to each arithmetic problem that is as close as possible to the correct answer without actually calculating the correct answer. To this end, they had to select between rounding both operands down (rounding-down strategy) and rounding both operands up (rounding-up strategy) on each problem. The better strategy for a given problem was the strategy that yielded the answer that was closest to the correct product for this problem. All participants were presented the 64 problems in random order. However, a different random order was used in each of the three phases. Moreover, the test was preceded by an initial practice trial including six arithmetic problems so that participants could get familiarized with the apparatus and the general procedure used in each of the three experimental phases. The stimuli were presented in 60-point Courier black font in the center of the computer screen. Each problem was preceded by a blank screen for 500 ms that was followed by a warning signal (“##”) displayed for 400 ms.

2.3.1. Judgment phase

The arithmetic problems were presented one by one in the center of the computer screen for an unlimited time. Below each problem, one of the two rounding strategies was provided (e.g., “ 23×56 , RD?”). Participants were instructed to judge whether the given strategy was the

better of the two available rounding strategies to find the closest estimate from the correct product. Specifically, half the participants were asked to press the “L” key on a AZERTY keyboard when they thought that the cued strategy was the better strategy (“yes” response) and to press the “S” key when they thought that the cued strategy was not the better strategy (“no” response), and vice versa. The better strategy was presented for half the problems and the poorer strategy for the other problems. We collected participants’ rates of better strategy judgments and mean judgment times.

2.3.2. Selection phase

During this phase, no strategy was cued with each problem. Participants were asked to select which one of the two available rounding strategies (i.e., rounding-down or rounding-up) was the better strategy to estimate the correct product of each problem. Half the participants were instructed to press the “L” key when they thought that the rounding-up strategy was the better strategy and to press the “S” key when they thought that the rounding-down strategy was the better one, and vice versa. We recorded the number of better strategy selections and mean selection times.

2.3.3. Execution phase

In this third phase, participants were asked to execute aurally (e.g., “ $20 \times 40 = 800$ ”) the strategy that was displayed above each problem (e.g., RD, 26×42). The better strategy was provided for half the problems, and the poorer strategy for the other problems. Once the given strategy was executed, the experimenter pressed a response key to move on to the next problem as soon as possible after participant’s response. We collected the number of strategies correctly executed and mean execution times.

3. Results

3.1. Age-Related Differences in Strategy Judgment, Selection, and Execution

Unless otherwise noted, differences are significant to at least $p < .05$. Given that older adults had better arithmetic fluency than young adults, this variable was included as a covariate in all the analyses. The aim of these analyses was to examine whether there were age-related differences in young and older adults' better strategy judgment, selection and execution, and whether these differences interacted with different problem features. Globally, if meta-strategic monitoring and control processes are impaired during aging, we expected older adults to show poorer strategy judgments relative to young adults. This age effect was expected to increase with problem difficulty (e.g., on problems with heterogeneous unit digits as compared to problems with homogeneous unit digit) and by whether the better strategy was cued on each problem (e.g., a rounding-up strategy cued on rounding-down problems).

3.1.1. Strategy judgment

3.1.1.1. Rates of correct better strategy judgment

Mean rates of correct better strategy judgment (coded 1 if participants said "yes" when the better strategy was cued or if participants said "no" when the poorer strategy was cued, and 0 otherwise) were analyzed with a mixed-design, 2 (Age: young, older adults) x 2 (Unit digits: homogeneous, heterogeneous) x 2 (Cued strategy: rounding-down, rounding-up) x 2 (Problem type: rounding-down, rounding-up problems) ANOVA, with age as the only between-participants factor. Older adults (.78) made poorer strategy judgment than young adults (.83), $F(1,69) = 4.68$, $p = .03$, $\eta^2_p = .06$. Moreover, participants made better strategy judgments on problems with homogeneous unit digits (.92) than on problems with heterogeneous unit digits (.78), $F(1,69) = 30.38$, $p < .001$, $\eta^2_p = .31$. Two interactions came out significant: Cued Strategy x

Problem Type, $F(1,69) = 14.04$, $p < .001$, $\eta^2_p = .17$, and Unit Digits x Cued Strategy x Problem Type, $F(1,69) = 4.00$, $p = .04$, $\eta^2_p = .05$. On problems with heterogeneous unit digits, difference in rates of correct better strategy judgment between rounding-up problems and rounding-down problems was larger when the cued strategy was rounding-down (.77 vs. .60) than when rounding-up was cued (.79 vs. .66), $F(1,69) = 17.90$, $p < .001$, $\eta^2_p = .25$. On problems with homogeneous unit digits, difference in rates of correct better strategy judgment between rounding-up problems and rounding-down problems was larger when the cued strategy was rounding-down (.92 vs. .89) than when rounding-up was cued (.92 vs. .90) $F(1,69) = 5.13$, $p = .02$, $\eta^2_p = .05$. Finally, the Age x Cued Strategy x Problem Type interaction, $F(1,69) = 5.70$, $p = .02$, $\eta^2_p = .08$, was significant. In older adults, differences in rates of better strategy judgment between rounding-up problems and rounding-down problems were smaller when the rounding-down strategy was cued (.81 vs. .70) relative to when the rounding-up strategy was cued (.88 vs. .72), $F(1,69) = 21.0$, $p < .001$. Young adults had larger rates of better strategy judgment on rounding-up problems than on rounding-down problems when the rounding-down strategy was cued (.87 vs. .78), but not when the rounding-up strategy was cued (.84 vs. .82); $F(1,69) = 8.63$, $p = 0.05$. No other main or interaction effects reached significance, all $ps > .17$.

In sum, when the cued strategy was rounding-up, young adults had equal rates of better strategy judgment on rounding-up and rounding-down problems. However, when the rounding-down strategy was cued, they had higher better strategy judgments on rounding-up than on rounding-down problems. Older adults had higher rates of better strategy judgment on rounding-up problems than on rounding-down problems when the rounding-down strategy was cued, and on rounding-up problems than on rounding-down problems when the rounding-up strategy was cued.

Insert Table 1 About Here

3.1.1.2. Better strategy judgment times

Mean judgment times for correct better strategy judgments were analyzed with a mixed-design ANOVA, 2 (Age: young, older adults) x 2 (Unit digits: homogeneous, heterogeneous) x 2 (Cued strategy: rounding-down, rounding-up) x 2 (Problem Type: rounding-down, rounding-up problems), with age as the only between-participants factor. Participants were faster to make better strategy judgments on problems with homogeneous unit digits (3043 ms) than on problems with heterogeneous unit digits (4551 ms), $F(1,69) = 5.39, p = .02, \eta^2_p = .08$. The Unit Digits x Cued Strategy x Problem Type interaction was marginally significant, $F(1,69) = 3.20, p = .07, \eta^2_p = .05$. Participants made faster better strategy judgments on rounding-up problems with heterogeneous unit digits than on rounding-down problems with heterogeneous unit digits when rounding-down was cued (4603 vs. 4947 ms); this problem-related difference was smaller when rounding-up was cued (4236 vs. 4836 ms), $F(1,69) = 2.74, p = .04$. No differences were found on problems with homogeneous unit digits, $F < 1$. No other effects reached significance, all $ps > .09$.

Overall, these results showed that both young and older participants were influenced in better strategy judgments and in how long it took them to make those judgments by the type of the unit digits on each problem and by whether the better or poorer strategy was cued. Regarding age effects, our results showed different levels of sensitivity to the cued strategy and to the type of problem on young and older adults' correct judgment rates. Young adults were sensitive to whether the cued strategy was the better or poorer strategy only when this cued strategy was rounding-down (they made highly accurate strategy judgments when the cued strategy was

rounding-up, whether it was cued on rounding-up or and rounding-down problems). Older adults were sensitive to whether the cued strategy was the better or poorer strategy on both rounding-down and rounding-up problems.

3.1.2. Strategy selection

3.1.2.1. Better strategy selection rates

Mean rates of better strategy selection (see means in Figure 1) were analyzed with a mixed-design ANOVA, 2 (Age: young, older adults) x 2 (Unit digits: homogeneous, heterogeneous) x 2 (Problem type: rounding-down, rounding-up), with age as the only between-participants factor. Participants selected the better strategy more often on problems with homogeneous unit digits (.98) than on problems with heterogeneous unit digits (.80), $F(1,69) = 30.69, p < .001, \eta^2_p = .31$. Participants also selected the better strategy more often on rounding-up problems (.94) than on rounding-down problems (.85), $F(1,69) = 6.94, p = .01, \eta^2_p = .09$. Finally, the Unit Digits x Problem Type interaction came significant, $F(1,69) = 6.07, p = .02, \eta^2_p = .08$. Specifically, participants selected the better strategy on rounding-up problems with heterogeneous unit digits more often than on rounding-down problems with heterogeneous unit digits (.89 vs. .72; $F(1,69) = 34.04, p < .001$). No differences were found on homogeneous problems, $F < 1$. No other main or interaction effects were significant, all $ps > .07$.

Insert Figure 1 About here

3.1.2.2. Better strategy selection latencies

Mean selection latencies for providing estimates with the better strategy were analyzed with a mixed-design ANOVA, 2 (Age: young, older adults) x 2 (Unit digits: homogeneous,

heterogeneous) x 2 (Problem Type: rounding-down, rounding-up), with age as the only between-participants factor. Participants were faster to select the better strategy on problems with homogeneous unit digits (1356 ms) than on problems with heterogeneous unit digits (2253 ms), $F(1,69) = 6.14, p = .02, \eta^2_p = .08$. No other effects were significant, all $ps > .08$.

These results indicate that how often young and older participants selected the better strategy was influenced by the unit digits of the problems and by whether the better strategy on each problem was rounding-up or rounding-down (i.e. better strategy selection was harder on rounding-down problems than on rounding-up problems, but only when problems had heterogeneous unit digits). How long it took participants to select the better strategy was only influenced by the unit digits of the problems. Overall, young and older participants showed similar patterns of results for strategy selection.

3.1.3. Strategy Execution

3.1.3.1. Rates of correct strategy execution

Mean rates of correct strategy execution (see means in Table 2) were analyzed with mixed design ANOVA, 2 (Age: young, older adults) x 2 (Unit Digits: homogeneous, heterogeneous) x 2 (Cued strategy: rounding-down, rounding-up) x 2 (Problem type: rounding-down, rounding-up problems), with age as the only between-participants factor. Participants executed strategies correctly more often when the rounding-down strategy (.95) was cued than when the rounding-up strategy was cued (.92), $F(1,69) = 6.58, p = .01, \eta^2_p = .09$. No other significant effects were found, all $ps > .08$.

 Insert Table 2 About here

3.1.3.2. Strategy execution times

Mean execution times were analyzed with a mixed-design ANOVA, 2 (Age: young, older adults) x 2 (Unit digits: homogeneous, heterogeneous) x 2 (Cued strategy: rounding-down, rounding-up) x 2 (Problem type: rounding-down, rounding-up problems), with age as the only between-participants factor. Participants were faster to execute strategies on problems with homogeneous unit digits (4291 ms) than on problem with heterogeneous unit digits (4774 ms), $F(1,69) = 9.67, p = .003, \eta^2_p = .12$. Participants were also faster with the rounding-down strategy (4337 ms) than with the rounding-up strategy (4787 ms), $F(1,69) = 17.85, p < .001, \eta^2_p = .21$, and when the better strategy was cued (4445 ms) than when the poorer strategy was cued (4679 ms), $F(1,69) = 9.58, p = .003, \eta^2_p = .12$. The Age x Unit Digits interaction came out significant, $F(1,69) = 4.18, p = .04, \eta^2_p = .06$. Older adults were faster to execute strategies on problems with homogeneous unit digits (4029 ms) than on problem with heterogeneous unit digits (4230 ms), $F(1,69) = 2.21, p = .14$; this strategy difference was smaller in young adults (4838 vs. 4952 ms), $F(1,69) = 0.04, p = .82$. Finally, a Unit Digit x Cued Strategy x Problem Type interaction was significant, $F(1,69) = 10.06, p = .002, \eta^2_p = .13$. On problems with homogeneous unit digits, difference in execution times was larger on rounding-up problems (4396 ms) than on rounding-down problems (5048 ms) when the rounding-up strategy was cued than when the rounding-down strategy was cued (4185 vs. 4104 ms), $F(1,69) = 5.48, p = .01$. Conversely, on problems with heterogeneous unit digits, difference in execution times was larger on rounding-down (4745 ms) than on rounding-up problems (4313 ms) when the rounding-up strategy was cued than when the rounding-down strategy was cued (4739 vs. 4967 ms), $F(1,69) = 11.30, p < .001$. No other effect was significant, all $ps > .08$.

Overall, these results showed that participants were influenced in how often they correctly executed the cued strategy and how long it took them to execute this strategy by the unit digits in a problem, by the type of problems, and by whether the better or poorer strategy was cued on each problem.

3.2. Relations between Strategy Judgment, Selection, and Execution.

The goal of the following analyses was to examine whether participants' strategy judgments were related to strategy selection and strategy execution, and to investigate whether these relationships changed with participants' age and/or with problems' features. Specifically, correlations between rates and latencies of better strategy judgments, of better strategy selection, and of strategy execution were carried out for each age group and homogeneous and heterogeneous problems, separately. These correlations are presented in Table 3.

Insert Table 3 About Here

Young and older participants' rates of better strategy judgment correlated with both rates of better strategy selection ($r_s = .69$ and $.84$ in young and older adults, respectively) and better strategy selection times ($r_s = -.65$ and $-.68$) when all problems were examined together. Similarly, young and older participants' better strategy judgment times correlated with rates of better strategy selection ($r_s = -.55$ and $-.68$) and with better strategy selection times ($r_s = .59$ and $.53$). None of the execution scores correlated with better strategy judgment or selection scores ($r_s < -.20$, *ns*).

When homogeneous and heterogeneous problems were analyzed separately, young and older participants' rates of better strategy judgment on homogeneous problems correlated

significantly with better strategy selection times ($r_s = -.37$ and $-.42$). No significant correlations were found between rates or latencies of better strategy selection and better strategy judgment times on homogeneous problems ($r_s < -.18$, *ns*). On heterogeneous problems, young participants' rates of better strategy judgment correlated with rates of better strategy selection ($r = .47$), but not with better strategy selection times ($r = -.07$, *ns*). Better strategy judgment times did not significantly correlate with rates of better strategy selection ($r = -.16$, *ns*), or with latencies of better strategy selection ($r = -.14$, *ns*). Older participants' rates of better strategy judgment on heterogeneous problems correlated with both rates of better strategy selection ($r = .80$) and better strategy selection times ($r = -.42$). Better strategy judgment times correlated with rates of better strategy selection ($r = -.44$), but not with better strategy selection times ($r = .13$, *ns*). No correlations were found between rates of better strategy judgment, judgments times, and execution scores ($r_s < -.24$, *ns*).

The relationships between (a) rates of better strategy judgment and better strategy selection times, (b) rates of better strategy judgment and of better selection, and (c) judgment latencies and rates of better strategy selection differed between the two age groups for heterogeneous problems. We tested differences between these correlations with *R*-to-*Z* Fisher tests. Correlations between rates of better strategy judgment and better strategy selection times ($r_s = .42$ and $-.07$, $p = .07$) and between rates of better strategy judgment and of better strategy selection ($r_s = .80$ and $.47$, $p = .01$) were larger for older than for young participants. However, difference between young and older adults in correlation latencies of better strategy judgment and rates of better strategy selection were not significant ($r_s = -.44$ and $-.16$, $p = .12$).

Finally, on the basis of these correlations, path analyses were conducted to more precisely test the relationships between better strategy judgment scores and better strategy

selection scores. As no relationships were found with the execution scores, these two variables were not included in the model. Maximum likelihood estimates of the parameters were obtained using structural equation modeling techniques implemented in Lisrel 9.2 (Jöreskog & Sörbom, 2015). Several goodness-of-fit indices were used to evaluate the model's acceptability: the chi-square divided by degrees of freedom (χ^2/df), the root mean square error of approximation (RMSEA), the comparative fit index (CFI), and the standardized root mean square residual (SRMR). To indicate an adequate fit, the χ^2/df ratio had to be less than 2, the RMSEA .08 or lower, the CFI higher than .95, and the SRMR not above .10 (Brown, 2006). The path diagrams and fit indexes for the four models are displayed in Figure 2.

Insert Figure 2 About here

Overall, these results indicated that better strategy judgment scores were predicted by better strategy selection scores in both young and older adults and for both homogeneous and heterogeneous problems. However, although the influence of better strategy selection scores on better strategy judgment scores were similar in both age group for homogeneous problems, the model showed a better fit in older adults ($\chi^2/df = 2.40$; $RMSEA = .07$; $CFI = .99$; $SRMR = .01$) than in young adults ($\chi^2/df = 11.35$; $RMSEA = .34$; $CFI = .99$; $SRMR = .02$) for heterogeneous problems. To confirm these results we also conducted these path analyses with age (in years) as a manifest variable. The influence of age on better strategy selection and better strategy judgment was larger for heterogeneous problems ($\chi^2/df = 1.82$; $RMSEA = .04$; $CFI = .99$; $SRMR = .03$) than for homogeneous problems ($\chi^2/df = 2.83$; $RMSEA = .10$; $CFI = .97$; $SRMR = .06$). The path diagrams and fit indexes for the two models are provided in the Appendix.

4. General Discussion

The main goal of the present experiment was to examine how participants make better strategy judgments, a process assumed to be involved while they select and execute strategies to accomplish cognitive tasks. By documenting effects of problem features and participant characteristics on how young and older adults judge whether a cued strategy is the better strategy and by investigating relations among strategy judgment, strategy selection, and strategy execution as a function of different types of problems, we aimed at determining whether strategic behaviors involve metacognitive processes. The present results replicate previous findings on strategic behaviors and age-related differences in how participants select and execute strategies. They also document new findings that shed important lights on metacognitive processes involved while participants judge whether a cued strategy is the better strategy on a given problem. We discuss the implications of the present findings to further our understanding of strategic behaviors in young and older adults.

The present results replicated previous findings on how young and older adults select and execute strategies. Thus, like in many previous studies (see Lemaire, 2016; Uittenhove & Lemaire, 2015, for overviews), participants were crucially influenced by problem features. They selected the better strategy on each problem when this better strategy was the rounding-up strategy and the problems were homogeneous problems. They executed the rounding-down strategy more quickly when it was the better strategy and it was cued on homogeneous problems. Moreover, both young and older adults were influenced by problem features (e.g., both were better at selecting and executing the better strategy on homogeneous problems than on heterogeneous problems). Intriguingly, older participants also appeared to be better than young participants at executing a cued strategy. Paradoxically, this finding could be explained by the

fact that older adults have previously been shown to be less accurate than young adults to check whether a cued strategy was the better (e.g., Ardiale & Lemaire, 2012, 2013). The fact that older participants checked less often or less effectively the relevance of a cued strategy to a specific problem might have conferred them some advantages in the execution phase. Indeed, this could have enabled them to execute the cued strategy every time as if it was the better, without giving it a second thought, which was not the case of young adults.

The most original findings in the present study concerns strategy judgment. We found that participants were influenced in their strategy judgments by problems features. Indeed, participants made better strategy judgments on each problem when problems were homogeneous problems and when the better strategy was cued on rounding-up problems. Moreover, above and beyond global age-related differences in strategy judgment (young adults being better than older adults), aging effects differed across problem types. Specifically, older adults made more accurate judgments on rounding-up problems than on rounding-down problems when the cued strategy was rounding-up, while young adults did not show such problem-related differences. Regarding relationships between better strategy judgment, strategy selection, and strategy execution, our data revealed that strategy selection, but not strategy execution, correlated with strategy judgment. Moreover, these relationships were larger for older adults than for young adults. The present findings have important implications for further understanding strategic behaviors and strategic variations with age, especially for understanding the possible influence of metacognitive processes on these variations. To discuss these implications, we propose a theoretical framework of how strategy judgments occur in young and older adults and discuss how this framework accounts for our data and yields new hypotheses and predictions regarding relationships between metacognitive monitoring and strategic behaviors.

4.1. A Theoretical Framework for Strategy Judgment

To account for our results, we propose a theoretical framework that is based on previous findings and proposals both in the metacognitive and strategy selection literatures. First, our framework is in part based on Thompson and colleagues' works on the role of metacognitive processes during reasoning tasks. According to Thompson and colleagues (Thompson, Evans, & Campbell, 2013; Thompson & Johnson, 2014; Thompson, Prowse Turner, & Pennycook, 2011), when participants have to evaluate conclusions of selection tasks they can rely on two types of processes: automatic and fast processes that give rise to heuristic responses (System 1) or conscious and slow processes that give rise to analytic responses (System 2). In this context, the role of metacognitive processes is to determine when analytic processes have to be substituted to heuristic processes through the monitoring of current cognitive operations. Specifically, Thompson and collaborators claimed that metacognitive decisions are based on two main cues. The first cue is the speed and ease with which a conclusion is evaluated (i.e., answer fluency). Participants are less likely to allocate rethinking times to answers that are produced quickly (Thompson et al., 2011). The second cue is the occurrence of a conflict between collected information. Participants are more likely to engage in analytic rethinking on conflict relative to non-conflict problems (De Neys & Glumicic, 2008; Thompson & Johnson, 2014). In sum, metacognitive processes are supposed to lead participants to rethink about their answers when they produce these answers slowly or when they are confronted with conflicting information. Importantly, the probability of changing answers increases with the amount of rethinking times.

Our model also integrates assumptions regarding how strategies are selected made by theories of strategy choices (e.g., RCCL, SCADS*, ACT-R). These computational models assume that strategies are selected on a problem-by-problem basis. Specifically, when they are

confronted with a new problem, participants assess problem features before activating strategies available to solve the problem and selecting the most strongly associated strategy with the problem. The strength of this association determines the ease with which a strategy is selected.

Based on these assumptions, we assume that four components (see Figure 3) are involved in participants' strategy judgments: (a) an analysis and selection system, (b) a comparison system, (c) a decision system, and (d) a regulatory system.

4.1.1. Component 1: Analysis and selection system

We assume that the first component of the evaluation of a selected strategy involves an analysis of problem features (e.g., size of unit digits, odd/even status of operands) that are (or not) crucial for determining which strategy is the better one on each problem. This means that when a problem is presented, participants focus their attention on problem features and analyze these features during encoding. That is, while encoding operands, participants analyze unit digits and determine, for example, whether unit digits of both operands are smaller versus larger than 5 or if only one operand is smaller/larger than 5. Following previous models of strategy choices, we assume that such analyses result in activating a set of strategies as well as a number of strategy characteristics (e.g., relative speed, accuracy, and demands in cognitive resources of available strategies). In the present context, because problem features like size of unit digits are correlated with which strategy is the better one, the better strategy is selected on the basis of most strongly correlated problem features (e.g., small unit digits=rounding down). The output of the analysis of the problem features is used in the selection of the better strategy. Together with which strategy is supposed to be the better strategy, this first component also activates information about that pre-selected strategy (e.g., how easy it is to execute, how easy it has been to be selected). In the context of strategy judgment tasks, the output of this first system includes

information about which strategy is expected to be the better one on each problem and information about the fluency with which this strategy is selected. In addition to analyzing problems' features and selecting the better strategy, the first component also enables participants to encode the cued strategy. At this stage, however, it is unknown whether participants analyze the problem features and select the better strategy before focusing their attention on the cued strategy [Path A, in Figure 3], analyze the cued strategy first, then use it to guide their analysis of problem features [Path B1], or analyze the cued strategy first, store it in their working memory and then compare it with the better strategy once selected [Path B2].

4.1.2. Component 2: Comparison system

The second component of strategy judgment involves a system or a set of processes that compares the selected and the cued strategies. Once the first component has selected a strategy, the second component compares the selected strategy with the cued strategy. Whether this second component is activated only after the first component provided its output or while the first component is still computing and selecting which strategy is the better one is at this stage undetermined. Therefore, we make no theoretical commitments regarding the timing of these components. Comparing the selected and cued strategies yields two types of information, conflicting or non-conflicting information. Non-conflicting information is provided when the selected strategy matches the cued strategy whereas conflicting information occurs when the selected strategy differs from the cued strategy.

4.1.3. Component 3: Decision system

Based on the information provided by the second component, the system decides whether the cued strategy is the better strategy or whether it is not the better strategy. The system will

make a “yes” decision when the comparison system provides a match or non-conflicting information and will make a “no” decision when the comparison system provides a non-matching or conflicting information. Note that wrong decisions (e.g., “no” with non-conflicting, match information) can occur even when the comparison system provides accurate information.

4.1.4. Component 4: Regulatory system

The role of this system is to determine whether the output of the decision system (i.e., a yes/no answer) results from the fact that the better or the poorer strategy was actually cued or from an error of the previous two systems. In this context, this regulatory system can lead to two outputs: (a) acceptance or rejection of the output of the decision system, or (b) revision of this output. Following Thompson et al. (2011), we assume that the output of the regulatory system depends on answer fluency and on whether there is a conflict between collected information. These two cues determine the likelihood that participants engage in revising their initial strategy judgments by re-analyzing either the problem’s features or the cued strategy. Specifically, conflicting information and low response fluency are both expected to increase re-processing of available information. Note that “answer/response fluency” refers here to the speed and ease with which a strategy is pre-selected by the Analysis and Selection component, not to the speed with which a problem is solved (i.e., arithmetic fluency). The influence of the latter variable has been controlled in all our analyses.

In sum, our theoretical framework assumes that strategy judgment involves metacognitive processes that are based on four systems or components. These systems are activated more or less serially as outputs of each systems feed processes of the next system.

4.2. How does this framework account for the present findings?

Our theoretical framework made several predictions. First, participants are expected to show faster better strategy judgment times and higher rates of better strategy judgments when the better strategy was cued (non-conflicting situation) than when the poorer strategy was cued (conflicting situation). We should also observed higher rates of correct judgments on problems where the strategy selection was easy (high fluency) than when the strategy selection was hard (low fluency). These two variables are also expected to interact with each other. Moreover, this framework should also help to determine whether and which meta-strategic monitoring and control processes are impaired in aging.

4.2.1. Problem-Related Differences in Strategy Judgment

Our results showed that participants made faster strategy judgments when the better strategy was cued (non-conflicting situation) on rounding-up problems (i.e., problems on which strategy selection is easy, as revealed by higher rates of correct answers in the better strategy selection task) than when the poorer strategy was cued (conflicting situation) on rounding-down problems (low fluency). Moreover, participants made better strategy judgment when the poorer strategy was cued (conflicting situation) on rounding-down problems (low fluency) than when the better strategy was cued (non-conflicting situation) on rounding-up problems (high fluency). This pattern suggests that participants allocate more rethinking times to problems in conflicting situation or when a low feeling of fluency is experienced, resulting in slower strategy judgment, but higher rates of better strategy judgment.

4.2.2. Age-related differences in strategy judgment

Age-related differences in strategy judgment are also consistent with our framework. Within this new framework, when the rounding-up strategy was cued on rounding-up problems,

the answer fluency resulting from the analysis and selection system combined with the fact that the comparison system yielded a match between the selected and the cued strategy increased the likelihood of giving a correct “yes” response [Path A1a]. This is consistent with the fact that older adults showed higher rates of better strategy judgment when the rounding-up strategy was cued on rounding-up problems than when it was cued on rounding-down problems. Interestingly, when the rounding-up strategy was cued, young adults did not show any differences in their rates of better strategy judgments which were good whatever the type of problems. We assume that this occurs because young participants engaged in more analytic processes and allocated more rethinking times to problems than older participants when the comparison system resulted in a mismatch between the selected strategy and the cued strategy [Path A2b], increasing their chances of making better strategy judgments on conflicting problems. These age-related differences were only found on heterogeneous problems. When easier (more fluent) homogeneous problems were presented, older and young adults showed comparable better strategy judgments.

In other words, the findings of the present study suggest that young participants use the presence of less fluent and conflicting information as a signal indicating that a further analysis is required. Carrying out this further analysis led participants to increase their rates of better strategy judgment for less fluent and conflicting problems, resulting in equal rates of better strategy judgment whatever the type of problems. Conversely, older participants do not appear to use the presence of conflicting information or less fluent answers as a signal indicating that extensive re-analysis is necessary. Indeed, they showed higher rates of better strategy judgment for fluent and non-conflicting information than for less fluent and conflicting information.

4.2.3. Relationships between strategy judgment and strategy selection

Better strategy selection correlated with strategy judgment. This suggests that processes involved in strategy judgment and better strategy selection tasks are not unrelated. Correlations between strategy judgment and better strategy selection are consistent with the involvement of an analysis and selection system in strategy judgment that would capture the same mechanisms than those involved in a strategy selection task. From a developmental point of view, our results indicated weaker relationships between rates of better strategy judgment and better strategy selection latencies on heterogeneous problems in young adults than in older adults. This suggests that young adults do not rely on the speed with which they select the better strategy to make their judgments on heterogeneous problems. These findings can easily be interpreted with our new framework. In this experiment, our results consistently showed that problems with heterogeneous unit digits were less quickly (less fluently) answered than problems with homogeneous unit digits. It is possible that lack (or weaker) fluency resulting from the analysis and selection system leads young participants to allocate rethinking times to heterogeneous problems before making judgments [Paths A1b and A2b], reducing the correlation between rates of better strategy judgment and latencies of better strategy selection.

4.3. Summary and Future directions

In sum, our findings suggest that meta-strategic monitoring and control processes are involved in strategy selection, and that there are age-related differences in these processes. Specifically, within our framework, our results suggest that differences in how young and older participants judge whether a strategy is the better strategy possibly results from an alteration of the regulatory system. Older participants appear to be unable to use the presence of conflicting information or less fluent answers as a signal indicating that a re-analysis of the problem is

needed. One possible explanation for this apparent difficulty in re-examining strategy selection decisions could be found in a reduction of older adults' cognitive resources. Indeed, according to Thompson et al. (2011), "System 2" processes make high demands on executive resources that are known to be limited in older adults (see Glisky, 2007, for an overview).

One limitation of the present study is some ceiling effects. These ceiling effects appeared on all problems for rates of correct strategy execution. This could have led to the underestimation of important relations between participants' strategy execution and strategy judgment performance. Note however that participants' strategy execution times did not reveal ceiling effects and yet, no correlations were found with measures of better strategy judgment. Moreover, ceiling effects also appeared on homogeneous problems for rates of better strategy judgment and rates of better strategy selection. But, when only problems that did not show ceiling effect (i.e., heterogeneous problems) are considered, our findings and conclusions remain strictly unchanged. Nevertheless, future studies should be conducted to replicate these results by examining age-related differences in strategic judgments on heterogeneous problems only.

Furthermore, there are two other directions for future works on the role of metacognitive processes during strategy selection. First, from an empirical perspective, future studies may collect further evidence for determining how our proposed four components work together to achieve metacognitive strategy judgments and how aging influences these different components. For example, patterns of eye movements may shed important light on different issues, such as whether participants compare the selected and cued strategies only when the analysis and selection system completed its task and generated all necessary and sufficient information about what strategy is expected to be the better one on each problem or whether the comparison processes start before these analysis and selection are finished. Further investigations should also

be conducted using other types of material to examine whether our model applies only to the better strategy judgment task used in the present study or whether it can be generalized to other strategy selection tasks, in the arithmetic domain (e.g., using different computational tasks); but also in other major cognitive domains (e.g., memory, language).

From a theoretical perspective, future works could expand previous computational models to include assumptions on the role of metacognitive models during strategy selection. Two of the classical computational models of strategies (RCCL and SCADS*) assume that strategy choices involve metacognitive mechanisms. In these models, the metacognitive system enables participants to evaluate strategies once selected and to interrupt strategies mid-execution to switch for a better strategy (RCCL) or to discover new legitimate strategies (SCADS*). The present data and theoretical proposal suggest that metacognitive processes are involved earlier in the strategy selection processes than what was previously thought, when participants try to select the better strategy on each problem.

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References

- Ardiale, E., & Lemaire, P. (2012). Within-item strategy switching: An age comparative study in adults. *Psychology and Aging, 27*, 1138–1151. doi:10.1037/a0027772
- Ardiale, E., & Lemaire, P. (2013). Effects of execution duration on within-item strategy switching in young and older adults. *Journal of Cognitive Psychology, 25*, 464–472. doi:10.1080/20445911.2013.789854
- Castel, A., Middlebrooks, C., & McGillivray, S. (2015). Monitoring memory in old age: Impaired, spared, and aware. J., Dunlosky, SK Tauber, (Eds.), *The Oxford handbook of metamemory*, 463–483.
- Cohen Kadosh, R., & Dowker, A. (2015). *The Oxford Handbook of Numerical Cognition*: Oxford University Press, USA.
- De Neys, W., & Glumicic, T. (2008). Conflict monitoring in dual process theories of thinking. *Cognition, 106*, 1248–1299. doi:10.1016/j.cognition.2007.06.002
- Deltour, J. (1993). Echelle de vocabulaire de Mill Hill de JC Raven. *Adaptation française et normes européennes du Mill Hill et du Standard Progressive Matrices de Raven (PM38)*. Braine-le-Château: Editions l'application des techniques modernes.
- Folstein, M. F., Folstein, S. E., & McHugh, P. R. (1975). “Mini-mental state”: a practical method for grading the cognitive state of patients for the clinician. *Journal of psychiatric research, 12*, 189–198.
- French, J. W., Ekstrom, R. B., & Price, L. A. (1963). *Kit of reference tests for cognitive factors*: Educational Testing Service.
- Hertzog, C. (2015). Aging and metacognitive control. In J. Dunlosky & S. K. Tauber (Eds.), *Oxford Handbook of Metamemory*: Oxford University Press.

- Hertzog, C., & Hultsch, D. F. (2000). Metacognition in adulthood and old age. In F. I. M. C. T. A. Salthouse (Ed.), *The handbook of aging and cognition (2nd ed.)* (pp. 417–466). Mahwah, NJ, US: Lawrence Erlbaum Associates Publishers.
- Jacoby, L. L., & Rhodes, M. G. (2006). False Remembering in the Aged. *Current Directions in Psychological Science, 15*, 49–53. doi:10.1111/j.0963-7214.2006.00405.x
- Jöreskog, K.G. & Sörbom, D. (2015). LISREL 9.20 for Windows [Computer software]. Skokie, IL: Scientific Software International, Inc.
- LeFevre, J.-A., Greenham, S. L., & Waheed, N. (1993). The Development of Procedural and Conceptual Knowledge in Computational Estimation. *Cognition and Instruction, 11*, 95–132. doi:10.1207/s1532690xci1102_1
- Lemaire, P. (2016). *Cognitive Aging: The Role of Strategies*. London: Routledge
- Lemaire, P., Arnaud, L., & Lecacheur, M. (2004). Adults' Age-Related Differences in Adaptivity of Strategy Choices: Evidence From Computational Estimation. *Psychology and Aging, 19*, 467–481. doi:10.1037/0882-7974.19.3.467
- Lemaire, P., Lecacheur, M., & Farioli, F. (2000). Children's strategy use in computational estimation. *Canadian Journal of Experimental Psychology, 54*, 141–148. doi:10.1037/h0087336
- Lovett, M. C., & Anderson, J. R. (1996). History of Success and Current Context in Problem Solving: Combined Influences on Operator Selection. *Cognitive Psychology, 31*, 168–217. doi:10.1006/cogp.1996.0016
- Lovett, M. C., & Schunn, C. D. (1999). Task representations, strategy variability, and base-rate neglect. *Journal of Experimental Psychology: General, 128*, 107–130. doi:10.1037/0096-3445.128.2.107

- Luwel, K., Torbeyns, J., & Verschaffel, L. (2003). The relation between metastrategic knowledge, strategy use and task performance: Findings and reflections from a numerosity judgement task. *European Journal of Psychology of Education, 18*, 425–447. doi:10.1007/bf03173245
- Neches, R. (1987). Learning through incremental refinement of procedures. In D. Klahr, P. Langley, & R. Neches (Eds.), *Production system models of learning and development* (pp. 163–219). Cambridge, MA, US: The MIT Press.
- Price, J., & Murray, R. G. (2012). The region of proximal learning heuristic and adult age differences in self-regulated learning. *Psychology and Aging, 27*, 1120–1129. doi:10.1037/a0029860
- Rieskamp, J., & Otto, P. E. (2006). SSL: A Theory of How People Learn to Select Strategies. *Journal of Experimental Psychology: General, 135*, 207–236. doi:10.1037/0096-3445.135.2.207
- Schneider, W., Eschman, A., & Zuccolotto, A. (2002). *E-Prime: User's guide*: Psychology Software Incorporated.
- Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*: Oxford University Press.
- Siegler, R. S. (2007). Cognitive variability. *Developmental Science, 10*(1), 104–109. doi:10.1111/j.1467-7687.2007.00571.x
- Siegler, R. S., & Araya, R. (2005). A computational model of conscious and unconscious strategy discovery. *Advances in child development and behaviour, 33*, 1–44.

- Siegler, R. S., & Shipley, C. (1995). Variation, selection, and cognitive change. In T. J. S. G. S. Halford (Ed.), *Developing cognitive competence: New approaches to process modeling* (pp. 31–76). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.
- Souchay, C., & Isingrini, M. (2004). Age related differences in metacognitive control: Role of executive functioning. *Brain and Cognition, 56*, 89–99. doi:10.1016/j.bandc.2004.06.002
- Thompson, V. A., Evans, J. S. B. T., & Campbell, J. I. D. (2013). Matching bias on the selection task: It's fast and feels good. *Thinking & Reasoning, 19*, 431–452.
doi:10.1080/13546783.2013.820220
- Thompson, V. A., & Johnson, S. C. (2014). Conflict, metacognition, and analytic thinking. *Thinking & Reasoning, 20*, 215–244. doi:10.1080/13546783.2013.869763
- Thompson, V. A., Prowse Turner, J. A., & Pennycook, G. (2011). Intuition, reason, and metacognition. *Cognitive Psychology, 63*, 107–140. doi:10.1016/j.cogpsych.2011.06.001
- Uittenhove, K., & Lemaire, P. (2015). The Effects of Aging on Numerical Cognition. In R. C. Kadosh & A. Dowker (Eds.), *The Oxford Handbook of Numerical Cognition* (pp. 345–366): Oxford University Press, USA.

Appendix

Insert Figure 4 About here

Fig 4. Path analyses examining the relations between age (in years), better strategy selection scores, and better strategy judgment scores. Asterisks indicate that the test of the unstandardized path coefficients is statistically significant. R^2 indicate the proportions of variance associated with the predictors used in this model.

Table 1

Mean Correct Judgment Rates and Mean Judgment Times (in ms) in Young and Older Adults on Rounding-Down or Rounding-up Problems with Homogenous and Heterogeneous Unit Digits Cued with the Rounding-Down Strategy or the Rounding-Up Strategy.

Problem Type	Cued Strategy	Young Adults			Older Adults			Total
		Homogeneous Problems	Heterogeneous Problems	<u>Means</u>	Homogeneous Problems	Heterogeneous Problems	<u>Means</u>	<u>Means</u>
<i>Correct Judgment Rates</i>								
Rounding-Down	Rounding-down	0.95	0.62	0.78	0.83	0.57	0.70	0.74
	Rounding-up	0.92	0.71	0.82	0.84	0.61	0.72	0.80
	<u>Means</u>	0.94	0.67	0.80	0.84	0.59	0.71	0.76
Rounding-Up	Rounding-down	0.94	0.80	0.87	0.87	0.75	0.81	0.84
	Rounding-up	0.91	0.76	0.84	0.94	0.83	0.88	0.86
	<u>Means</u>	0.93	0.78	0.86	0.91	0.79	0.85	0.85
Total	<u>Means</u>	0.93	0.72	0.83	0.87	0.69	0.81	0.81
<i>Judgment Times (in ms)</i>								
Rounding-Down	Rounding-down	2528	4593	3560	3526	5301	4413	3986
	Rounding-up	2837	4129	3744	3526	5022	4334	4039
	<u>Means</u>	2683	4361	3652	3526	5162	4374	4013
Rounding-Up	Rounding-down	2824	4443	3634	3481	4764	4122	3878
	Rounding-up	2539	4651	3334	3327	4344	3836	3585
	<u>Means</u>	2682	4547	3484	3404	4554	3979	4231
Total	<u>Means</u>	2682	4454	3568	3465	4858	4176	4135

Note. Standard Errors ranged from .02 to .05 for the mean correct judgment rates and from 162 to 594 for the mean judgment times.

Table 2

Mean Correct Execution Rates and Mean Execution Times (in ms) in Young and Older Adults on Rounding-Down or Rounding-up Problems with Homogenous and Heterogeneous Unit Digits, Cued with the Rounding-Down Strategy or the Rounding-Up Strategy.

Problem Type	Cued Strategy	Young Adults			Older Adults			Total <u>Means</u>
		Homogeneous Problems	Heterogeneous Problems	<u>Means</u>	Homogeneous Problems	Heterogeneous Problems	<u>Means</u>	
<i>Correct Execution Rates</i>								
Rounding-Down	Rounding-down	0.93	0.92	0.93	0.98	0.97	0.98	0.96
	Rounding-up	0.91	0.88	0.90	0.95	0.94	0.94	0.92
	<u>Means</u>	0.92	0.90	0.92	0.96	0.95	0.96	0.94
Rounding-Up	Rounding-down	0.95	0.92	0.93	0.97	0.96	0.96	0.95
	Rounding-up	0.89	0.89	0.89	0.95	0.94	0.95	0.92
	<u>Means</u>	0.92	0.90	0.91	0.96	0.95	0.96	0.93
Total	<u>Means</u>	0.92	0.90	0.91	0.96	0.95	0.96	0.94
<i>Execution Times (in ms)</i>								
Rounding-Down	Rounding-down	4510	4868	4689	3698	3759	3729	4209
	Rounding-up	5447	5048	5247	4649	4380	4539	4893
	<u>Means</u>	4978	4958	4968	4173	4069	4134	4551
Rounding-Up	Rounding-down	4405	5138	4772	3966	4351	4158	4465
	Rounding-up	4990	5553	5272	3801	4055	4091	4681
	<u>Means</u>	4697	5345	5022	3883	4203	4124	4573
Total	<u>Means</u>	4838	5151	4995	4028	4136	4129	4562

Note. Standard Errors ranged from .02 to .02 for the mean correct execution rates and from 182 to 333 for the mean execution times.

Table 3

Correlation Matrix in Young (N = 44) and Older Adults (N = 29)

	Young Adults					Older Adults				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
<i>All Problems</i>										
1.%Strat Judgment	1.00					1.00				
2. Judgment Times	-0.72*	1.00				-0.69*	1.00			
3. %Strat Selection	0.69*	-0.55*	1.00			0.84*	-0.68*	1.00		
4. Selection Latencies	-0.65*	0.59*	-0.64*	1.00		-0.68*	0.53*	-0.72*	1.00	
5. %Strat Execution	0.17	-0.06	0.03	-0.01	1.00	0.05	0.08	0.14	-0.20	1.00
6. Execution Times	0.23	0.23	-0.04	0.15	-0.53*	0.01	0.22	-0.07	0.01	-0.17
<i>Homogeneous-Unit Problems</i>										
1.%Strat Judgment	1.00					1.00				
2. Judgment Times	-0.17	1.00				-0.40*	1.00			
3. %Strat Selection	0.24	0.02	1.00			0.31	-0.18	1.00		
4. Selection Latencies	-0.37*	0.17	-0.47*	1.00		-0.42*	0.06	-0.16	1.00	
5. %Strat Execution	0.19	0.03	-0.12	-0.19	1.00	0.10	0.14	0.25	-0.10	1.00
6. Execution Times	0.15	0.11	0.00	-0.02	-0.58*	0.08	0.18	0.00	-0.13	-0.25
<i>Heterogeneous-Unit Problems</i>										
1.%Strat Judgment	1.00					1.00				
2. Judgment Times	-0.34	1.00				-0.51*	1.00			
3. %Strat Selection	0.47*	-0.16	1.00			-0.80*	-0.44*	1.00		
4. Selection Latencies	-0.07	-0.14	-0.34	1.00		-0.42*	0.13	-0.41	1.00	
5. %Strat Execution	-0.12	0.07	-0.05	0.12	1.00	-0.10	0.25	0.07	0.24	1.00
6. Execution Times	-0.08	0.09	0.17	-0.03	-0.48*	0.20	0.14	0.13	-0.21	-0.07

*p<.05

Figure Captions

Figure 1. Rates of better strategy selection on homogeneous and heterogeneous problems for which the better strategy was rounding-down or rounding-up.

Figure 2. Path analyses for the model examining the relationships between better strategy selection scores and better strategy judgment scores. Asterisks indicated that the test of the unstandardized path coefficients was statistically significant. R^2 indicated the proportion of variability associated with the predictors used in this model.

Figure 3. Description of the framework of strategy judgment.