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# A Survey on Fuzzy Control for Mechatronics Applications

**Abstract.** Fuzzy control has become one of the most effective tools for dealing with complex engineering processes. Over the years, research on fuzzy control systems has continuously evolved, witnessing numerous theoretical contributions and successful real-world achievements. The concept of model-free or data-driven fuzzy control was initially introduced with specific heuristics incorporated into the design. Due to the lack of a systematic framework for stability analysis in model-free fuzzy control, the significance of model-based fuzzy control has grown extensively. This approach ensures systematic design based on precise fuzzy models of the process. This survey focuses on the fundamental aspects of three prominent classes of fuzzy control. First, the paper commences with a review of Takagi-Sugeno fuzzy control systems. This includes discussions on stability analysis and controller design, exploring techniques to derive less conservative and/or complex results from a numerical burden perspective. Second, various aspects of data-driven fuzzy control are analyzed in detail including a classification of the most popular data-driven control techniques and their combination with fuzzy control; a representative Iterative Feedback Tuning-based fuzzy controller is described. Third, this survey explores the fundamental aspects of evolving fuzzy control, with a particular emphasis on the significance of stability and control laws, which are not usually the primary focus of evolving intelligent systems research. For each discussed class of fuzzy control, the paper provides a selective list of mechatronics applications to illustrate their performance effectiveness, emphasizing research papers published after 2011. Finally, drawing from recent advances in fuzzy control theory and mechatronics applications, future research directions and associated challenges are discussed.

**Keywords.** Adaptive fuzzy control; data-driven fuzzy control; evolving systems; fuzzy control; Mamdani fuzzy controllers; mechatronics; model-based fuzzy control; stability; Takagi-Sugeno-Kang fuzzy controllers.

## **1. Introduction**

Mechatronics applications include mechanical systems, electronic systems, control solutions and information technology as highlighted in (Dragos, Preitl, Precup, Cretiu, & Fodor, 2010). The incorporation of advanced hardware and software architectures in mechatronics applications is useful due to the advantages achieved, including high dynamics, steady-state and robust performance under various operating conditions, and control of processes with unmeasurable or estimated parameters.

As highlighted in (Nguyen, Dinh, Chong, Iwasaki, Precup, & Ruderman, 2023), mechatronics applications are now ubiquitous in daily life, including aircraft, spacecraft, chemical process equipment, manufacturing, homes and buildings, automobiles, ships, and trains. Along with the pressing demands for high performance, reliability and affordability, the need to make these mechatronics applications safer, greener and smarter is growing rapidly. Mechatronics applications must perform a variety of challenging tasks in dynamic working environments, with or without human intervention. To achieve this goal, new sensing, actuation and control technologies have been extensively integrated into mechatronics applications, which are becoming increasingly complex. This integrated complexity leads to great challenges in decision making, motion planning, control and automation of mechatronics applications. These challenges justify the study and research interest drawn towards fuzzy control for mechatronics applications, motivated by the intelligence features and flexibility of fuzzy controllers. If the controller design is carried out systematically, these challenges are properly solved and thus transformed into advantages of using fuzzy control for mechatronics.

As shown in (Precup, & Preitl, 1999a), next (Precup, & Hellendoorn, 2011) and (Precup, Roman, & Safaei, 2021), the “classical” engineering approaches to

characterize real-world problems are essentially qualitative and quantitative, based on more or less accurate mathematical models. In such approaches, expressions such as “average temperature”, “high humidity”, “low pressure”, “very high speed”, referring to the variables specific to the behaviour of a controlled process, are subjected to relatively difficult quantitative interpretations. This is because “classical” automation deals with variables and information that are processed with well-specified numerical values. In this respect, the design and tuning of the controller and its subsequent implementation in the control system require a quantitative modelling of the controlled process that is as accurate as possible. Advanced control strategies (e.g., adaptive, predictive, and variable-structure) even require continuous re-evaluation of the models, including the structures and/or values of the parameters that characterize these (parametric) models.

Lotfi A. Zadeh established the foundations of fuzzy set theory (Zadeh, 1965), which at first seemed to be only mathematical entertainment. The boom in computer science in the 1970s opened up the first prospects for practical applications of the theory in the field of automatic control, and these first applications belong to Ebrahim H. Mamdani and his co-author Sedrak Assilian (Mamdani, 1974; Mamdani, & Assilian, 1975). The reference application of fuzzy control involves some “special” controllers based on fuzzy set theory, called fuzzy controllers, in cement kiln control (Holmblad, & Ostergaard, 1982). In the 1980s, the so-called fuzzy boom took place in Japan, the USA and later in Europe in the field of fuzzy control applications, covering several domains ranging from the electrical household industry to the control of vehicles, transportation systems and robots. This is partly due to the spectacular development of electronic technologies and computer systems, which made it possible to manufacture circuits with very high information processing speed, dedicated (by design and use) to a specific purpose, including fuzzy information processing, and also the development of

computer-aided design programs, which allowed the control system designer to efficiently use a large amount of information about the controlled process and the control equipment.

The applications of fuzzy control reported until now point out two important aspects related to this control strategy (Precup, & Preitl, 1999a; Precup, & Hellendoorn, 2011; Precup, Roman, & Safaei, 2021):

- In certain situations (such as the control of nonlinear processes that are difficult to model mathematically, or, even more challenging, the control of ill-defined processes), fuzzy control can be a viable alternative to classical, crisp (conventional) control.
- Compared to conventional control, fuzzy control can rely heavily on and focus on the experience of a human operator, and a fuzzy controller can model this experience more accurately (in a linguistic way) than a conventional controller.

The main features of fuzzy control are discussed in (Precup, & Preitl, 1999a; Precup, & Hellendoorn, 2011; Precup, Roman, & Safaei, 2021) as follows:

- Fuzzy control uses the so-called fuzzy controllers or fuzzy logic controllers that ensure nonlinear input-output static maps that can be influenced/modified based on the designer's option.
- Fuzzy control can handle multiple variables from the controlled process. Therefore, fuzzy control structures are considered to belong to the class of Multi Input-Multi Output (MIMO) systems with interactions, and fuzzy control can be viewed as a multi-input or multi-output controller similar to state feedback controllers. In this regard, fuzzy controllers can be viewed as a family of

relatively simple and easy-to-understand nonlinear state feedback or output-feedback controllers.

- Fuzzy controllers are basically without dynamics. However, the applications and performance of fuzzy controllers and fuzzy control systems can be significantly expanded by inserting dynamics (i.e., derivative and/or integral components) into fuzzy controller structures, resulting in the so-called fuzzy controllers with dynamics or typical fuzzy controllers because of the analogy to the typical conventional controllers, namely Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) and Proportional-Derivative (PD), which are popular in industrial applications.
- Fuzzy controllers are flexible in terms of modifying the transmission characteristics (through input-output static maps), thus ensuring the possibility of developing a wide range of adaptive control system structures.

The approach based on human experience is applied in fuzzy controllers by expressing the performance specifications imposed on the control systems and then elaborating the control signal in terms of “natural” IF-THEN rules belonging to the set of rules

$$\begin{array}{l} \dots \\ \text{IF (antecedent) THEN (consequent),} \\ \dots \end{array} \quad (1)$$

where the antecedent (the premise) refers to the current situation concerning the controlled process evolution (usually compared to the desired evolution), and the consequent (the conclusion) refers to the measures that should be taken – in the form of the control signal  $u$  – in order to fulfil the desired evolution of the control system imposed by the performance specifications. The set of these rules forms the rule base of the fuzzy controller.



The block diagram of the principle (considered as classical in the literature) of a fuzzy control system considered as a single-input system with respect to the reference input  $r$  and a single-output system with respect to the controlled output  $y$  is shown in Figure 1 (a). The reference input is also referred to as the set-point or the desired output  $y_d$ . The second input fed to the controlled process / the fuzzy control system is the disturbance input  $d$ .

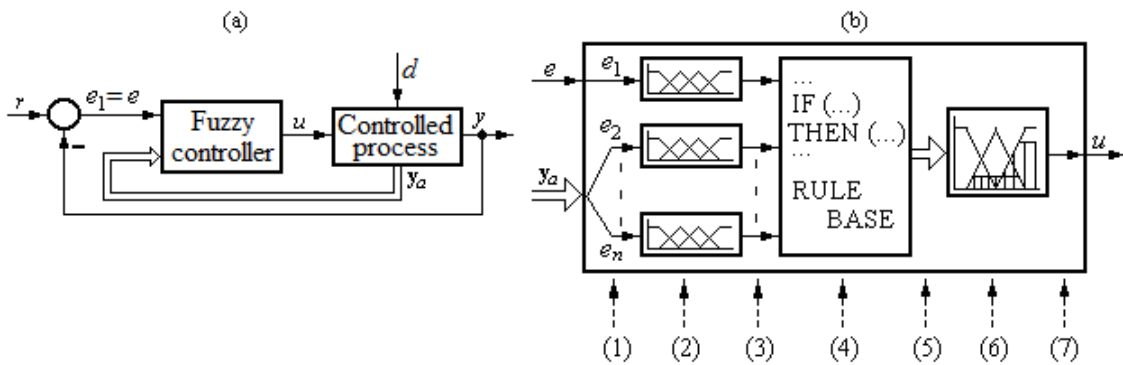


Figure 1. Fuzzy control system structure (a) and fuzzy controller structure (b) (Precup, & Preitl 1999a; Precup, & Hellendoorn, 2011; Precup, Roman, & Safaei, 2021).

Figure 1 (b) also highlights the operating principle of a fuzzy controller in its classical version, characterizing Mamdani fuzzy controllers, with the following variables and modules: (1) the crisp inputs, (2) the fuzzification module, (3) the fuzzified inputs, (4) the inference module, (5) the fuzzy conclusions, (6) the defuzzification module, and (7) the crisp output. In this regard, the operating principle of the Mamdani fuzzy controller includes the sequence of operations (a), (b) and (c) (Precup, & Preitl, 1999a; Precup, & Hellendoorn, 2011; Precup, Roman, & Safaei, 2021):

(a) The crisp input information, expressed as measured variables, reference input (or set-point) and control error  $e$ , is converted into a fuzzy representation. This operation is called fuzzification of crisp information, and it uses scheduling variables

instead of input variables in Takagi-Sugeno-Kang fuzzy controllers, also called Takagi-Sugeno (T-S) fuzzy controllers or Sugeno fuzzy controllers. In addition to the control error, the vector  $\mathbf{y}_a$  of additional variables (shown in Figure 1) can be used as crisp input information applied to the fuzzy controller; such variables, with an important effect on the control system behaviour, include the state variables.

(b) The fuzzified information is processed using the rule base, which consists of fuzzy IF-THEN rules referred to as fuzzy control rules illustrated in (1), that must be adequately defined in order to meet the performance specifications imposed on the control system. The principles for evaluating and processing the rule base represent the inference mechanism/engine, and the result is the “fuzzy” form of the fuzzy control signal  $u$  produced by the fuzzy controller.

(c) The fuzzy control signal must be converted into a crisp formulation with a well-specified physical nature, directly understandable and usable by the actuator, in order to be capable of controlling the process. This operation is called defuzzification.

These three operations characterize the three sub-systems or modules in the structure of a fuzzy controller shown in Figure 1 (b), namely the fuzzification module (2), the inference module (4) and the defuzzification module (6). All these modules are supported by an appropriate database.

The structure shown in Figure 1 (b) becomes more complex in the context of uncertainties in terms of type-2 fuzzy sets with additional parameterization of the input membership functions and insertion of a type-reducer module in conjunction with the defuzzification module. This leads to interval type-2 fuzzy controllers, with different applications besides the general ones of type-2 fuzzy logic ones reviewed in (Mittal, Jain, Vaisla, Castillo, & Kacprzyk, 2020). As outlined in (Wu, 2012; Precup, David,

Roman, Szedlak-Stinean, & Petriu, 2021), the main differences between the interval type-2 fuzzy controllers and the type-1 fuzzy controllers are:

- adaptivity, i.e., changing the embedded type-1 fuzzy sets when computing the boundaries of the type-reduced interval with respect to input changes,
- novelty, i.e., the upper and lower membership functions of the same interval type-2 fuzzy set can be used simultaneously in computing each bound of the type-reduced interval,
- flexible parameterization, i.e., the larger number of parameters of interval type-2 fuzzy controllers as compared to type-1 fuzzy controllers allows a greater possibility to modify the nonlinear input-output map of the fuzzy controllers to compensate for the unfavourable nonlinearities of the controlled process.

These differences have been exploited and transformed into the main advantage of interval type-2 fuzzy controllers pointed out in (Lam & Seneviratne, 2008), namely interval type-2 fuzzy model-based control directly handles the uncertainties in nonlinear control systems. These uncertainties include parameter uncertainties, mismeasurement uncertainties, observation uncertainties, and communication uncertainties.

As stated in (Precup, & Hellendoorn, 2011), in most applications a fuzzy controller is used for direct feedback control or at the low level in hierarchical control system structures. However, a fuzzy controller can also be used at the supervisory level, such as in adaptive control system structures. Nowadays, fuzzy control is not only used to directly express the knowledge about the controlled process, which was originally called model-free fuzzy control. A fuzzy controller can be designed from a fuzzy model obtained in terms of system identification techniques, and thus it can be considered in the framework of model-based fuzzy control. Nevertheless, as discussed in (Precup,

Roman, & Safaei, 2021), fuzzy control can also be analysed and designed in the framework of data-driven control, where no process model is involved in the fuzzy controller tuning. The most commonly used are (Precup, & Hellendoorn, 2011):

- Mamdani fuzzy controllers, also called linguistic fuzzy controllers, with either fuzzy consequents, which represent type-I fuzzy systems according to the classifications proposed in (Sugeno, 1999), or singleton consequents, which belong to type-II fuzzy systems. These fuzzy controllers are usually used as direct controllers. These notations of the fuzzy systems type concern their structure and are different from those of the fuzzy sets type, which concern the definitions of the fuzzy sets. The notations of the fuzzy sets type are next extended to be used for fuzzy controllers as well.
- Takagi-Sugeno-Kang fuzzy controllers or Takagi-Sugeno controllers or Sugeno fuzzy controllers, also known as type-III fuzzy systems in the context of (Sugeno, 1999), especially when affine consequents are employed, and typically used as both supervisory and direct closed-loop controllers.

Several surveys and position papers highlight specific topics in fuzzy control, make characterizations, and present valuable points of view, and a part of the most representative ones is briefly discussed below. An overview of fuzzy modelling for control is provided in (Babuška, & Verbruggen, 1996). The stability analysis methods for type-II fuzzy control systems are analysed in detail in (Sugeno, 1999). An overview of neuro-fuzzy rule generation in a more general soft computation setting is given in (Mitra, & Hayashi, 2000). The fusion of computationally intelligent methods, including fuzzy logic and sliding mode control, is discussed in (Kaynak, Erbatur & Ertugrul, 2001). Conclusions on the perspectives of fuzzy control systems are formulated in

(Sala, Guerra, & Babuška, 2005) and continued after ten years in (Sala, Guerra, & Tanaka, 2015) with quasi-linear parameter varying (LPV) model-based control design, major applications and improvements. A survey on the analysis and design methods of model-based fuzzy control systems using quasi-LPV model-based control design through some milestones and key applications is carried out in (Feng, 2006). Other recent surveys of application results in LPV control/estimation validated by experiments or high-fidelity simulations can be found in (Hoffmann, & Werner, 2014; Li, Nguyen, Du, Wang, & Zhang, 2021). A survey on type-I, type-II, and type-III fuzzy systems focusing on model-based approaches developed via Lyapunov stability and linear matrix inequality (LMI) formulations is offered in (Nguyen, Taniguchi, Eciolaza, Campos, Palhares, & Sugeno, 2019).

Since type-2 fuzzy controllers are characterized by essentially more parameters than type-1 fuzzy controllers, their systematic design is more difficult. Therefore, their optimal tuning is a convenient way to ensure their systematic design if appropriate optimization problems are defined. Several surveys on the use of metaheuristic algorithms in the optimal tuning of type-2 fuzzy controllers have been reported in (Castillo, Melin, Garza, Montiel, & Sepúlveda, 2011; Castillo, & Melin, 2012; Castillo, & Melin, 2014; Valdez, Castillo, Cortes-Antonio, & Melin, 2020), and this issue is also discussed in other reviews that deal with both type-1 and type-2 fuzzy controllers, including (Hagras, 2008; Precup, Angelov, Costa, & Sayed-Mouchaweh, 2015; Hamza, Yap, Choudhury, Chiroma, & Kumbasar, 2018).

This paper builds on the authors' previous surveys on industrial applications of fuzzy control reported until 2011 (Precup, & Hellendoorn, 2011), nature-inspired optimal control of industrial applications (Precup, Angelov, Costa, & Sayed-Mouchaweh, 2015), and the status of fuzzy control systems and perspectives in 2019

(Nguyen, Taniguchi, Eciolaza, Campos, Palhares, & Sugeno, 2019), and provides an overview of mechatronic applications of fuzzy control, which includes several areas. However, papers published before 2011 are considered here if they are representative of the topics discussed. Some of these applications are industrial, while others are laboratory-based. A large part of these applications includes real-time experimental results, but significant results with strong theory and rich simulation studies are also included. The authors are aware of the fact that the number of publications on the topic of fuzzy control combined with mechatronics is extremely large, so that an exhaustive list is impossible. Selected papers are listed in the References section of this paper. Unfortunately, many excellent papers have been omitted, and we hope that their authors will accept the apologies of the authors of this paper. In addition, this survey paper is not able to cover all categories of fuzzy control applications of mechatronic systems, which may include manufacturing, robotics, automotive and process industries, servo systems and actuators. The importance of this survey is twofold:

- First, a comprehensive and up-to-date treatment of fuzzy controllers is offered. This is explained in terms of theory, presented at a minimally understandable level so as to cover a wide range of applications.
- Second, the organization of the paper allows for a systematic description of different control system structures, divided into model-based fuzzy control, data-driven fuzzy control, and evolving systems. Conventional, state feedback and adaptive controllers are discussed in the remaining sections.

The paper is organized as follows. Section 2 discusses model-based fuzzy control. Next, Section 3 focuses on data-driven fuzzy control. Applications of fuzzy

control in the general evolving framework are presented in Section 4. Section 5 provides concluding remarks, perspectives, and challenges of fuzzy control.

## **2. Model-based fuzzy control**

This section first reviews some main features and research mainstreams related to fuzzy model-based approaches, recently discussed in the literature. A special focus is put on the classical T-S fuzzy control systems with linear consequents and the T-S fuzzy control systems with nonlinear consequents. Then, this section discusses some notable successful mechatronics applications of fuzzy model-based approaches, essentially developed with the Lyapunov stability method and convex optimization techniques under LMI constraints. Other complementary and related theoretical and application results of fuzzy model-based techniques with other design tools, for instance, sliding mode control, adaptive control, predictive control, etc., can be found in (Feng, 2006; Precup, & Hellendoorn, 2011).

### ***2.1. Takagi-Sugeno model-based fuzzy control***

In the first decade after Mamdani's successful application of fuzzy logic control, researchers encountered substantial criticisms from conventional control theorists (Sugeno, 1999; Sala, Guerra, & Babuška, 2005; Nguyen, Taniguchi, Eciolaza, Campos, Palhares, & Sugeno, 2019). The primary reason was the absence of stability analysis for fuzzy control during that period. In response to these critiques, Sugeno's research group at the Tokyo Institute of Technology introduced T-S fuzzy systems in 1985.

The practical background of T-S model-based fuzzy control is the special structure and operation of T-S fuzzy models described in Section 1, which make the T-S fuzzy models behave as bumpless interpolators between local linear or nonlinear models placed in the rule consequents. The models or controllers in the rule consequents can be designed separately and relatively easily, possibly using the designer's experience, to

capture or control the nonlinear mechanisms of the process in different regions of the premise or scheduling space, and appropriate stability analysis is needed to ensure the stability of the fuzzy control systems.

Generally speaking, there are two main approaches for constructing a T-S fuzzy model: i) identifying the model using measured or simulated data (Takagi, & Sugeno, 1985; Babuška & Verbruggen, 1996), ii) analytical construction of a T-S model that exactly represents or approximates a given nonlinear dynamic system (Tanaka, & Wang, 2004). The key feature of a T-S fuzzy model is to express the local dynamics of each fuzzy implication rule by a linear system model as

$$\begin{aligned}
 &\text{Model rule } i : \\
 &\text{IF } z_1(t) \text{ IS } M_{i1} \text{ AND } \dots \text{ AND } z_p(t) \text{ IS } M_{ip} \quad (2) \\
 &\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t), \quad i = 1 \dots r,
 \end{aligned}$$

where  $\mathbf{x}(t) \in \mathfrak{R}^n$  is the state vector,  $\mathbf{u}(t) \in \mathfrak{R}^m$  is the control input vector;  $\mathbf{y}(t) \in \mathfrak{R}^q$  is the output vector,  $\mathbf{z}(t) = [z_1(t) \ z_2(t) \ \dots \ z_p(t)]^T \in \mathfrak{R}^p$  is the vector of premise variables or scheduling variables, which may be functions of the state variables, external disturbances and/or time, the superscript  $T$  indicates matrix transposition,  $M_{ij}$  is the fuzzy set, specifically the linguistic term, and  $r$  is the number of fuzzy model rules. The constant subsystem matrices  $\mathbf{A}_i \in \mathfrak{R}^{n \times n}$  and  $\mathbf{B}_i \in \mathfrak{R}^{n \times m}$ , for  $i = 1 \dots r$ , are known. The overall fuzzy model of the system is obtained by fuzzy “blending” of the linear subsystem models as

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^r h_i(\mathbf{z}(t)) [\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t)], \quad (3)$$

where

$$h_i(\mathbf{z}(t)) = \frac{\omega_i(\mathbf{z}(t))}{\sum_{i=1}^r \omega_i(\mathbf{z}(t))}, \quad \omega_i(\mathbf{z}(t)) = \prod_{j=1}^p M_{ij}(z_j(t)). \quad (4)$$



The term  $M_{ij}(z_j(t))$  represents the grade of membership of  $z_j(t)$  in  $M_{ij}$ . The membership functions  $h_i(\mathbf{z}(t))$ , for  $i = 1 \dots r$ , satisfy the convex sum property

$$\begin{aligned} \sum_{i=1}^r h_i(\mathbf{z}(t)) &= 1, \\ 0 \leq h_i(\mathbf{z}(t)) &\leq 1, \quad i = 1 \dots r. \end{aligned} \tag{5}$$

The discrete-time T-S fuzzy model is constructed in the same way by replacing  $\dot{\mathbf{x}}(t)$  with  $\mathbf{x}(t+1)$  in (2) and (3), with  $t$  indicating the continuous time argument and the discrete time argument, respectively. However, for the sake of simplicity and illustration, continuous-time T-S fuzzy models are mainly discussed hereafter.

Among various analytical methods to construct a T-S fuzzy model from a given nonlinear system, the sector nonlinearity approach (Wang, Tanaka, & Griffin, 1996) has received the most attention since it provides a systematic method to derive a strictly equivalent T-S fuzzy representation within a compact set of the state space. However, for existing T-S fuzzy results based on the sector nonlinearity approach, the numerical complexity of stability analysis, observation and control design conditions exponentially grows with respect to the number of premise variables (Tanaka, & Wang, 2004). This limits the applicability of these results to systems with only few nonlinearities (Li, Xie, Zhao, Gao, Hu, & Wong, 2021). To overcome this major practical issue, several approaches have been proposed to reduce the numerical complexity of T-S fuzzy systems. Based on a singular value decomposition method, the authors in (Yam, Baranyi, & Yang, 1999) proposed an approach to reduce the fuzzy rules, yielding approximate T-S fuzzy models. In (Taniguchi, Tanaka, Ohtake, & Wang, 2001), some nonlinearities were transformed into system uncertainties to reduce the number of local linear sub-models of T-S fuzzy systems. Exploiting the linear dependencies between the T-S fuzzy local sub-models obtained with the sector nonlinearity approach, a reduced-complexity model can be directly obtained from the

original T-S fuzzy model in (Dehak, Nguyen, Dequidt, Vermeiren, & Dambrine, 2020). However, the reduction approaches in (Taniguchi, Tanaka, Ohtake, & Wang, 2001; Dehak, Nguyen, Dequidt, Vermeiren, & Dambrine, 2020) can lead to over-conservative stability analysis and control design results. To avoid this drawback, a reduced-complexity approach was proposed in (Dehak, Nguyen, Dequidt, Vermeiren, & Dambrine, 2022), which can also systematically derive an equivalent polytopic representation of a given nonlinear system within a compact set of the state space. However, in contrast to the sector nonlinearity approach, the model complexity in (Dehak, Nguyen, Dequidt, Vermeiren, & Dambrine, 2022) only grows proportionally, rather than exponentially, with the number of premise variables. In particular, this approach allows introducing some specific slack variables at the modelling step to reduce the design conservatism.

State feedback fuzzy control has been widely applied to stabilize a T-S fuzzy system. The most popular state feedback fuzzy control scheme is based on the concept of parallel distributed compensation (PDC) (Tanaka, & Sugeno, 1992; Tanaka, Ikeda, & Wang, 1996; Wang, Tanaka, & Griffin, 1996). The key idea is to construct the fuzzy controller sharing the same premise membership functions and the same number of rules as the T-S fuzzy model (Tanaka, Ikeda, & Wang, 1996). Then, the fuzzy controller is constructed via the PDC as

$$\begin{aligned}
&\text{Control rule } i : \\
&\text{IF } z_1(t) \text{ IS } M_{i1} \text{ AND } \dots \text{ AND } z_p(t) \text{ IS } M_{ip} \\
&\text{THEN } \mathbf{u}(t) = \mathbf{K}_i \mathbf{x}(t), \quad i = 1 \dots r.
\end{aligned} \tag{6}$$

The overall fuzzy controller can be inferred from (6) as follows:

$$\mathbf{u}(t) = \sum_{i=1}^r h_i(\mathbf{z}(t)) \mathbf{K}_i \mathbf{x}(t). \tag{7}$$

The goal of the fuzzy controller design is to determine the local feedback (matrix) gains  $\mathbf{K}_i$ , for  $i = 1 \dots r$ , in the linear consequent parts such that the closed-loop T-S fuzzy system with the models (3) and (7) is asymptotically stable and verifies some predefined performance specifications.

For illustrations, let us consider the stability of the closed-loop system with the models (3) and (7). Substituting (7) into (3), the state-space equation of the closed-loop T-S fuzzy system becomes

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\mathbf{z}(t)) h_j(\mathbf{z}(t)) (\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t). \quad (8)$$

The most basic stabilization result for system (8) is based on the quadratic Lyapunov candidate function of the form

$$V(\mathbf{x}) = \mathbf{x}^T \mathbf{P} \mathbf{x}, \quad \mathbf{P} \succ 0, \quad (9)$$

where  $\mathbf{P} \succ 0$  denotes a symmetric positive definite matrix. To ensure the asymptotic stability of the closed-loop system (8), the time derivative of  $V(\mathbf{x})$  along its trajectory is required to be negative, i.e.,  $\dot{V}(\mathbf{x}) = \dot{\mathbf{x}}^T \mathbf{P} \mathbf{x} + \mathbf{x}^T \mathbf{P} \dot{\mathbf{x}} < 0$ , which results in

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\mathbf{z}(t)) h_j(\mathbf{z}(t)) [(\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j)^T \mathbf{P} + \mathbf{P}(\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j)] \prec 0. \quad (10)$$

The stability condition (10) is expressed in terms of bilinear matrix inequality (BMI) due to the coupling between the Lyapunov matrix  $\mathbf{P}$  and the control gains  $\mathbf{K}_j$ , for  $j = 1 \dots r$ , which cannot be solved effectively with standard numerical solvers (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). To convexify the control design conditions, a

congruence transformation can first be performed by pre- and post-multiplying (10)

with  $\mathbf{Q} = \mathbf{P}^{-1}$ , which yields

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))[\mathbf{Q}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j)^T + (\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j)\mathbf{Q}] \prec 0. \quad (11)$$

With a simple change of variable  $\mathbf{M}_j = \mathbf{K}_j\mathbf{Q}$ , for  $j = 1 \dots r$ , the stability condition (11)

becomes

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))[(\mathbf{A}_i\mathbf{Q} + \mathbf{B}_i\mathbf{M}_j)^T + (\mathbf{A}_i\mathbf{Q} + \mathbf{B}_i\mathbf{M}_j)] \prec 0. \quad (12)$$

The condition (12) is now expressed as an LMI constraint of infinite dimension due to its direct dependency on the membership functions  $h_i(\mathbf{z}(t))$ , for  $i = 1 \dots r$ . To make the condition for checking the closed-loop stability feasible and tractable, the membership functions should be dropped out from (12) to obtain a finite set of LMI control design conditions. To this end, numerous relaxation results have been proposed to drop out the membership functions with different degrees of conservativeness and numerical complexities, which can be categorized into two classes: i) without requiring slack variables such as Tanaka's relaxation (Tanaka, & Wang, 2004), Tuan's relaxation (Tuan, Apkarian, Narikiyo, & Yamamoto, 2001); ii) involving slack variables such as Liu and Zhang's relaxation (Liu, & Zhang, 2003), Pólya-based relaxation (Sala & Ariño, 2007; Montagner, Oliveira, & Peres, 2009). In particular, Pólya-based relaxation results offer asymptotically necessary and sufficient LMI-based conditions for the stability and performance of T-S fuzzy systems (Sala, & Ariño, 2007). Nevertheless, approaches based on Pólya's theorem are more conceptual than feasible, as the computational load escalates rapidly, often leading to crashes in most numerical solvers (Nguyen, Taniguchi, Eciolaza, Campos, Palhares, & Sugeno, 2019). Although the PDC

concept provides a natural, effective and systematic framework for T-S fuzzy controller and observer designs using LMI-based techniques, the obtained results can be very conservative (Lam, 2018). Therefore, most of the research effort has been focussed on reducing the design conservatism of T-S fuzzy model-based approaches. Apart from finding an effective way to drop the membership functions from the design conditions as discussed above, there are two main directions to reduce the conservativeness of the design conditions (Nguyen, Taniguchi, Eciolaza, Campos, Palhares, & Sugeno, 2019): i) using different families of Lyapunov function candidates, ii) exploit better the information on the membership functions for stability analysis.

First, most of PDC-based results were obtained using a common quadratic Lyapunov function, where a single Lyapunov matrix must be used for stability analysis of all local sub-models of the T-S fuzzy systems. To relax this constraint, more general classes of Lyapunov candidate functions have been leveraged for T-S fuzzy model-based approaches, for instance, piecewise Lyapunov functions (Johansson, Rantzer, & Arzen, 1999; Feng, 2004), line integral Lyapunov functions (Rhee, & Won, 2006; Mozelli, Palhares, & Avellar, 2009), fuzzy Lyapunov functions depending on the membership functions (Tanaka, Hori, & Wang, 2003; Guerra, & Vermeiren, 2004; Zheng, Xie, Nguyen, & Qu, 2023), polynomial Lyapunov functions depending on the membership functions with an arbitrary degree (Zhang, & Xie, 2011), multidimensional fuzzy Lyapunov functions (Lee, Joo, & Tak, 2014), etc. In particular, together with the use of fuzzy Lyapunov functions and slack decision variables, the non-PDC control concept has been proposed to further reduce the design conservatism of T-S fuzzy approaches (Guerra, & Vermeiren, 2004; Xie, Ma, Zhao, Ding, & Wang, 2012; Zheng, Xie, Nguyen, & Qu, 2023). More insightful discussions on the advantages and

drawbacks of each type of Lyapunov candidate functions can be found in (Nguyen, Taniguchi, Eciolaza, Campos, Palhares, & Sugeno, 2019).

Second, the membership functions are used to interconnect the local sub-models of the T-S fuzzy system, which represent the nonlinearities of the original plant to be controlled. However, only their convex sum property (5) has been exploited in most of the existing works based on various types of Lyapunov candidate functions. As far as the shape characteristics of the membership functions, i.e., the intrinsic nonlinear nature of the plant, are not exploited for stability analysis, the concerns related to the conservativeness issue still remain (Lam, 2018; Sala, & Arino, 2008). To reduce the source of conservativeness, the membership-function-approximation approaches using staircase and/or piecewise linear membership functions (Lam, & Narimani, 2009; Zhang, Lam, Qiu, Liu & Chen, 2018). Moreover, membership-bound-dependent approaches have been proposed to exploit the bound information of membership functions for stability analysis (Sala, & Arino, 2008; Li, Xie, Zhao, Gao, Hu, & Wong, 2021). Further discussions on T-S fuzzy membership-function-dependent stability analysis can be found in the survey paper (Lam, 2018).

It is important to note that using nonquadratic Lyapunov functions for membership-function-dependent stability analysis usually leads to a higher number of convex stability constraints and a high number of involved decision variables, thus a high degree of computational complexity. Then, the resulting T-S fuzzy control and estimation results can be impractical, especially for complex processes with a high number of premise variables (Dehak, Nguyen, Dequidt, Vermeiren, & Dambrine, 2022). Research effort has been also devoted to reducing the numerical design complexity. Two notable approaches can be distinguished. First, the numerical complexity reduction of the T-S fuzzy results can be done by “controlling” the number of slack decision

variables introduced in the control design for relaxation purposes (Sala, & Arino, 2008; Montagner, Oliveira, & Peres, 2009; Xie, Lu, & Yue, 2022a). Second, researchers have tried to find alternative T-S fuzzy modelling methods to reduce the excessive number of fuzzy rules caused by the classical sector nonlinearity approach as discussed above. Apart from the previously mentioned modelling reduction methods, T-S fuzzy modelling with nonlinear consequents has received increasing research attention, especially N-TS fuzzy systems (Dong, Wang, & Yang, 2009) and fuzzy polynomial systems (Tanaka, Yoshida, Ohtake, & Wang, 2008) as described hereafter.

**N-TS fuzzy systems.** Using similar techniques as for the classical T-S fuzzy modelling, i.e., the sector nonlinearity approach, a N-TS fuzzy model can be derived from a nonlinear system as follows (Dong, Wang, & Yang, 2009):

$$\begin{aligned}
& \text{Model rule } i : \\
& \text{IF } z_1(t) \text{ IS } M_{i1} \text{ AND } \dots \text{ AND } z_p(t) \text{ IS } M_{ip} \\
& \text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{G}_i \boldsymbol{\varphi}(t), \quad i = 1 \dots r,
\end{aligned} \tag{13}$$

where the same system notations as in (2) are used. The vector of nonlinearities  $\boldsymbol{\varphi}(t) \in \mathfrak{R}^h$  satisfies the following sector-boundedness condition:

$$\boldsymbol{\varphi}(t)^T \boldsymbol{\Lambda} (\mathbf{E} \mathbf{x}(t) - \boldsymbol{\varphi}(t)) \geq 0, \tag{14}$$

where the matrix  $\mathbf{E} = [\mathbf{E}_1^T \dots \mathbf{E}_h^T]^T \in \mathfrak{R}^{n \times h}$  is given, and  $\boldsymbol{\Lambda} \in \mathfrak{R}^{h \times h}$  is a positive definite diagonal matrix. Note that for engineering applications, since the states of the systems are amplitude-bounded, the characterization of the boundedness condition (14) is always possible (Khalil, 2002). Since some nonlinearities, i.e., premise variables, of the original complex plant can be retained in the consequent parts with  $\boldsymbol{\varphi}(t)$  of the T-S fuzzy models, this N-TS fuzzy modelling generally requires less fuzzy rules, i.e., local sub-models, than the classical T-S fuzzy modelling with linear consequents (Takagi, &

Sugeno, 1985). Using the PDC concept, a fuzzy state feedback controller of the N-TS system (13) is given in the following form, where the nonlinearity  $\boldsymbol{\varphi}(t)$  is incorporated into the control scheme:

$$\begin{aligned}
& \text{Control rule } i : \\
& \text{IF } z_1(t) \text{ IS } M_{i1} \text{ AND } \dots \text{ AND } z_p(t) \text{ IS } M_{ip} \\
& \text{THEN } \mathbf{u}(t) = \mathbf{K}_{xi}\mathbf{x}(t) + \mathbf{K}_{\phi_i}\boldsymbol{\varphi}(t), \quad i = 1 \dots r.
\end{aligned} \tag{15}$$

where the control gains  $\mathbf{K}_{xi} \in \mathfrak{R}^{m \times n}$  and  $\mathbf{K}_{\phi_i} \in \mathfrak{R}^{m \times h}$ , for  $i = 1 \dots r$ , are to be determined.

It is clear that using the same fuzzy inference method, the N-TS fuzzy model (13) and its state feedback controller (15) can be respectively obtained as

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^r h_i(\mathbf{z}(t))[\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{G}_i\boldsymbol{\varphi}(t)] \tag{16}$$

and

$$\mathbf{u}(t) = \sum_{i=1}^r h_i(\mathbf{z}(t))[\mathbf{K}_{xi}\mathbf{x}(t) + \mathbf{K}_{\phi_i}\boldsymbol{\varphi}(t)]. \tag{17}$$

**Remark 1.** It can be seen that if  $\mathbf{G}_i = \mathbf{0}$  in (13) and  $\mathbf{K}_{\phi_i} = \mathbf{0}$  in (15), for  $i = 1 \dots r$ , then the N-TS fuzzy system (13) (respectively the fuzzy controller (15)) reduces to the classical T-S fuzzy system (2) (respectively the fuzzy controller (6)), widely studied in the fuzzy control literature. Therefore, N-TS fuzzy models encompass the classical T-S fuzzy models as a special case. Then, one can follow the same approach as Chapter 14 in (Tanaka, & Wang, 2004) to demonstrate that N-TS fuzzy models can be used as a universal approximator for smooth nonlinear control systems.

Exploiting the boundedness condition (14), and similar Lyapunov candidate functions as the case of the classical T-S fuzzy systems, N-TS fuzzy-model-based



control approaches have been actively developed for nonlinear control of complex systems (Dong, Wang, & Yang, 2010; Coutinho, Araujo, Nguyen, & Palhares, 2020; Araujo, Coutinho, Nguyen, & Palhares, 2021; Xie, Yang, Wan, Xia, & Shi, 2022). In particular, an N-TS fuzzy static output feedback controller was developed in (Nguyen, Coutinho, Guerra, Palhares, & Pan, 2021) for nonlinear systems subject to state and control input constraints, where the sector-bounded nonlinearity  $\varphi(t)$  was also incorporated in the construction of the nonquadratic Lyapunov function to further reduce the control design conservatism. As illustrated in these references, N-TS fuzzy model-based control approaches can offer less conservative design results and less computational burden than classical T-S fuzzy model-based approaches. In particular, using N-TS fuzzy modelling, an effective solution has been recently proposed for the challenging T-S fuzzy observer design with unmeasured premise variables, where all the unmeasurable premise variables of the original plant are isolated in the nonlinear consequents (Pan, Nguyen, Guerra, & Ichalal, 2020; Nguyen, Pan, Guerra, & Wang, 2020; Quadros, Leite, & Palhares, 2022; Peixoto, Nguyen, Guerra, & Palhares, 2023).

**Polynomial fuzzy systems.** As suggested in (Tanaka, Yoshida, Ohtake, & Wang, 2008), a polynomial fuzzy model can be described as

$$\begin{aligned}
& \text{Model rule } i : \\
& \text{IF } z_1(t) \text{ IS } M_{i1} \text{ AND } \dots \text{ AND } z_p(t) \text{ IS } M_{ip} \\
& \text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i(\mathbf{x}(t))\hat{\mathbf{x}}(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{u}(t), \quad i = 1 \dots r,
\end{aligned} \tag{18}$$

where  $\mathbf{A}_i(\mathbf{x}(t)) \in \mathfrak{R}^{n \times n}$  and  $\mathbf{B}_i(\mathbf{x}(t)) \in \mathfrak{R}^{n \times m}$  are polynomial matrices in  $\mathbf{x}(t)$ , and  $\hat{\mathbf{x}}(\mathbf{x}(t)) \in \mathfrak{R}^n$  is a column vector whose entries are all monomials in  $\mathbf{x}(t)$ . Following a similar defuzzification procedure as the case of the classical T-S fuzzy system, the polynomial fuzzy model (18) can be represented by

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^r h_i(\mathbf{z}(t)) [\mathbf{A}_i(\mathbf{x}(t)) \hat{\mathbf{x}}(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t)) \mathbf{u}(t)]. \quad (19)$$

Inspired by the PDC control concept, the following polynomial fuzzy controller can be constructed to stabilize the system (19):

$$\begin{aligned} &\text{Control rule } i : \\ &\text{IF } z_1(t) \text{ IS } M_{i1} \text{ AND } \dots \text{ AND } z_p(t) \text{ IS } M_{ip} \\ &\text{THEN } \mathbf{u}(t) = \mathbf{K}_i(\mathbf{x}(t)) \hat{\mathbf{x}}(\mathbf{x}(t)), \quad i = 1 \dots r, \end{aligned} \quad (20)$$

which can be represented in a compact form as

$$\mathbf{u}(t) = \sum_{i=1}^r h_i(\mathbf{z}(t)) \mathbf{K}_i(\mathbf{x}(t)) \hat{\mathbf{x}}(\mathbf{x}(t)). \quad (21)$$

The closed-loop polynomial fuzzy system can be formed from (19) and (21) as

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\mathbf{z}(t)) h_j(\mathbf{z}(t)) [\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t)) \mathbf{K}_j(\mathbf{x}(t))] \hat{\mathbf{x}}(\mathbf{x}(t)). \quad (22)$$

For stability analysis and control design of the fuzzy model (19), the following polynomial Lyapunov candidate function has been widely used:

$$V(\mathbf{x}) = \hat{\mathbf{x}}^T(\mathbf{x}) \mathbf{P}(\mathbf{x}) \hat{\mathbf{x}}(\mathbf{x}), \quad (23)$$

where  $\mathbf{P}(\mathbf{x}) \in \mathfrak{R}^{n \times n}$  is a symmetric positive definite polynomial matrix. The control problem related to the polynomial fuzzy system (22) is to determine the feedback gains  $\mathbf{K}_j(\mathbf{x}(t))$ , for  $j = 1 \dots r$ , and the polynomial Lyapunov matrix  $\mathbf{P}(\mathbf{x})$  such that  $\mathbf{x}(t) \rightarrow \mathbf{0}$ , when  $t \rightarrow \infty$ , with the standard assumption that  $\hat{\mathbf{x}}(\mathbf{x}(t)) = \mathbf{0}$  if and only if  $\mathbf{x}(t) = \mathbf{0}$ . The control design conditions are recast in the form of the sum of squares (SOS) constraints,

which can be effectively solved using the well-known Matlab SOSTOOLS toolbox (Prajna, Papachristodoulou, & Parrilo, 2002).

**Remark 2.** As in the case of N-TS fuzzy models, it can be highlighted that if  $\hat{\mathbf{x}}(\mathbf{x}(t)) = \mathbf{x}(t)$ , and  $\mathbf{A}_i(\mathbf{x}(t))$ ,  $\mathbf{B}_i(\mathbf{x}(t))$ ,  $\mathbf{K}_i(\mathbf{x}(t))$  and  $\mathbf{P}(\mathbf{x})$  are constant matrices, for  $i = 1 \dots r$ , then the polynomial fuzzy model (19) reduces to the T-S fuzzy model (3), and the polynomial Lyapunov function (23) is the same as the classical quadratic Lyapunov function. Hence, the SOS-based approaches to polynomial fuzzy models encompass the classical LMI-based approaches to T-S fuzzy models as a special case. Therefore, it is expected that polynomial fuzzy model-based approaches using SOS tools can provide significantly more relaxed stability results than the classical LMI-based T-S fuzzy approaches (Tanaka, Yoshida, Ohtake, & Wang, 2008; Sala, & Arino, 2009).

Similar to classical T-S fuzzy model-based approaches, the main research issues to be considered when investigating the stability of the polynomial fuzzy system (22) are related to the types of Lyapunov candidate functions used for stability analysis, the explicit dependency of the information of the membership functions in the stability analysis conditions, and the convexification techniques used for stability analysis and control design. These research issues are directly related to the design conservatism, the numerical computational burden, and the applicability of polynomial fuzzy model-based approaches. A deeper discussion on all these aspects concerning polynomial fuzzy systems can be found in the recent survey (Lam, 2018). It is important to note that despite their appealing theoretical interests, there are only very few successful engineering applications of polynomial fuzzy model-based approaches. Qi, Liu, Ataka, Lam, & Althoefer (2016) presented a notable kinematic polynomial fuzzy model-based control method with experimental validations for continuum manipulators, whose Jacobian is not required to be continuously updated for end-effector trajectory tracking

tasks. However, most of the papers on SOS-based polynomial fuzzy approaches found in the current literature are essentially illustrated with simple academic examples and simulation results.

## ***2.2. Model-based fuzzy control for mechatronics applications***

Due to their effectiveness and systematic designs, T-S fuzzy model-based approaches have been largely applied to the control and estimation problems of engineering applications, especially in mechatronics. A selected list of references related to the use of T-S fuzzy control techniques for aerospace and marine applications, robotics and biomechanics applications, and automotive and transportation applications, is presented hereafter.

**Aerospace and marine applications.** A T-S fuzzy integral sliding mode control method was proposed in (Hu, Wu, Hu, & Gao, 2013) for the control problem of flexible air-breathing hypersonic vehicles subject to non-symmetric dead zone nonlinearities. The authors in (Sun, Xu, & Li, 2017b) dealt with the T-S fuzzy model-based finite-time sampled-data control for a flexible spacecraft system. For finite-time attitude stabilization purposes of the flexible spacecraft, a fault-tolerant fuzzy switching controller was developed under the presence of stochastic actuator failures and sampled-data control inputs. The authors in (Aslam, Tiwari, Pandey, & Band, 2023) investigated an  $H_\infty$  attitude control method for sustainable hypersonic vehicles with Markovian jump parameters, whose dynamics were approximated using Taylor's expansion and T-S fuzzy linearization approaches. A coupled Lyapunov function was used to derive the conditions for stochastic T-S fuzzy controller design while guaranteeing a predefined  $H_\infty$  performance level. A fuzzy adaptive sliding mode control method for attitude tracking control of a flexible spacecraft in (Li, Liu, & Shi, 2020), where the authors explicitly considered configuration misalignment and input dead-zone in the control

design while achieving the global asymptotic stability of attitude tracking errors. Based on the online estimation of actuator faults, an active fault-tolerant tracking control scheme was developed in (Jiang, Gao, Shi, & Xu, 2010) for a near-space vehicle, represented by a T-S fuzzy model. Note that the proposed fault-tolerant tracking control method does not depend on any fault detection and isolation mechanism, making it more suitable for real-time implementation in aerospace engineering applications. The distributed attitude control for spacecraft formation flying systems was investigated in (Zhang, Zhang, & Zhang, 2017), where T-S fuzzy modelling was used to represent the nonlinear spacecraft attitude dynamics. Using Lyapunov stability, a modified distributed fuzzy-based  $H_\infty$  controller was proposed to stabilize the attitude of the multispacecraft. In (Li, Dai, Song, Wang, & Du, 2019), the authors presented a T-S fuzzy fault-tolerant attitude tracking control method for Mars entry vehicles subject to disturbances and actuator failures, where a fuzzy line-integral Lyapunov function was used to guarantee the closed-loop stability with satisfactory disturbance attenuation level under actuator faulty situations. Based on an event-triggered mechanism, a networked T-S fuzzy controller was proposed in (Ma, Nie, Yu, Hu, & Peng, 2020) for underactuated unmanned surface vehicles, where LMI-based control design conditions were established using a Lyapunov-Krasovskii functional. Considering the network-induced characteristics, a network-based T-S fuzzy dynamic positioning controller was developed for an unmanned marine vehicle in network environments in (Wang, Han, Fei, & Peng, 2018a), where the asynchronous difference of the normalized membership functions between the marine vehicle and its controller was explicitly considered in the control design. In (Zhang, Ye, Feng, & Li, 2021), the authors developed an event-based T-S fuzzy dynamic positioning control method for unmanned marine vehicles under network communication constraints and Deny-of-Service (DoS) attacks. The event-

triggering threshold of the event-triggering mechanism was optimized using a Q-learning algorithm to reduce the communication loads, and the observer and controller design conditions were derived via a piecewise Lyapunov function to guarantee the closed-loop global exponential stability. Based on a quantized feedback sliding mode control technique, a fault-tolerant control method for dynamic positioning of unmanned marine vehicles, represented by a T-S fuzzy model with unknown membership functions, was proposed in (Hao, Zhang, Li, Lin, & Chen, 2021). The thruster faults were dealt with using a switching mechanism, while the time-varying delay effects were considered via an improved dynamic quantization parameter adjustment strategy. T-S fuzzy event-triggered sliding mode control was also proposed in (Zhang, Yao, Xing, & Feng, 2022) for unmanned underwater vehicles under multiple practical constraints, where LMI-based design conditions were established to ensure the existence of a practical sliding mode and the stability of the sliding motion while avoiding the Zeno's effect in the proposed event-triggering mechanism. The authors in (Cheng, Zhang, Xie, Zhang, & He, 2023) designed an adaptive T-S fuzzy dynamic positioning control of unmanned surface vehicles against DoS attacks, where an adaptive multievent-triggered mechanism was proposed to dynamically adjust the event-triggered thresholds for data transmission improvements. T-S fuzzy modelling was considered in (Shi, Sun, & Hou, 2023) to represent the nonlinear dynamics of unmanned surface vehicles subject to external disturbances and actuator faults. Then, a disturbance observer and an extended state observer were developed to estimate external disturbances, unmeasured states and actuator faults for sampled-data dynamic positioning output feedback controller design. Another sampled-data fuzzy controller was designed in (Kim, Lee, & Joo, 2021) for a quadrotor unmanned aerial vehicle, whose dynamics can be represented as a decentralized T-S fuzzy model. The LMI-based sampled-data tracking control design

conditions were derived using a time-dependent Lyapunov-Krasovskii functional and a reference model. A fault-tolerant T-S fuzzy controller was developed in (Li, Xu, & Yu, 2022) for underwater vehicles, where the event-triggering mechanism was designed using multiple past sampled data to decide the next release instant. Based on the canonical Bessel-Legendre inequality, the proof for the asymptotic stability of the T-S fuzzy delayed underwater vehicle system was established.

Due to the high complexities of aerospace and marine systems, most of the existing T-S fuzzy control approaches were validated in simulations. Further experimental or hardware-in-the-loop tests must be carried out to further evaluate the practical performance and the real-world applicability of these control results.

**Automotive and transportation applications.** The authors in (Tang, Du, Sun, Ning, Xing, & Li, 2016) developed a state-observer-based T-S fuzzy control method for a semi-active quarter-car suspension installed with a magnetorheological damper. Experimental tests were performed with a quarter-car test rig under different road excitations to demonstrate the effectiveness of the proposed fuzzy control algorithm. To consider simultaneously the varying sprung and unsprung masses, the unknown actuator nonlinearity, and the suspension performances in the control design, an adaptive sliding-mode controller was developed in (Li, Yu, Hilton, & Liu, 2012), whose design conditions are expressed in the form of an optimization problem. Other  $H_\infty$  T-S fuzzy controllers were also proposed for active suspension systems with random actuator delay in (Han, Zhong, Chen, & Tang, 2019), and in the presence of sprung mass variation while reducing the motor wear in (Shao, Naghdy, & Du, 2017). A disturbance-observer-based T-S fuzzy controller was proposed in (Ning, Sun, Zhang, Du, Li, & Zhang, 2017) for an active seat suspension. The disturbance observer was used to compensate the disturbances caused by frictions, and modelling errors. Moreover, the

T-S fuzzy technique was leveraged to improve the control performance by considering different drivers' weights in the control design. Extensive simulations and experiments were carried out to validate the effectiveness of the proposed T-S fuzzy controller. A switched T-S fuzzy method was proposed in (Qing, Hongliang, Songlin, Weiwei, & Yongfeng, 2023) for continuous damping control of semi-active suspensions, where the asymmetry saturation of the control current was explicitly considered in the PDC-based state feedback controller design. Extensive simulation and experimental results were carried out under various transient excitation conditions to demonstrate the improvements of the proposed fuzzy control method in terms of ride comfort and road handling. The authors in (Jeong, & Choi, 2021) proposed a magnetorheological damper fault diagnosis algorithm using only two accelerometers, which are commonly available for commercial vehicles. To this end, a T-S fuzzy unknown input observer was developed for the estimation of the vehicle suspension system subject to damper hysteresis and unknown road elevation. Moreover, a data-driven machine learning algorithm was used to generate a fault flag, which not only minimizes design efforts but also ensures optimal performance as illustrated by experimental results obtained with a quarter-car test rig. A fuzzy reduced-order observer was developed in (Zhu, & Li, 2019) to estimate the unknown shaft torque of an integrated motor-transmission system, where the T-S fuzzy modelling was used to deal with the system nonlinearity caused by the air drag torque. As shown by the simulation results, the transient estimation performance can be enhanced with the pole placement technique, while the robustness with respect to the road condition variations was guaranteed with a robust  $H_\infty$  filtering approach. A T-S fuzzy unknown input observer was proposed in (Li, Liu, & Shi, 2020) to estimate the transmission input-output shaft torque and the drive wheel speed for a hybrid powertrain system. The effectiveness of the proposed reduced-order Luenberger



observer was experimentally validated under different operating modes. Based on another unknown input T-S fuzzy observer scheme, the authors in (Losero, Lauber, & Guerra, 2018) developed a virtual strain gauge, where the observer design was performed in the angular domain. This fuzzy observer scheme was applied to estimate the engine torque and the clutch torque via the angular deflection of a dual-mass flywheel, which is crucial for powertrain management in the automotive industry. T-S fuzzy control techniques have been successfully applied to various robust path tracking or lateral control problems of intelligent vehicles with experimental validations (Nguyen, Sentouh, & Popieul, 2018; Guo, Wang, Luo, & Li, 2020; Nguyen, Sentouh, Zhang, & Popieul, 2019b; Liang, Feng, Lu, Yin, Zhuang, & Mao, 2023). In particular, to reduce communication resources at the channel level, a decentralized event-triggered scheme was recently investigated in (Zhang, Hu, Zhang, Bian, Nguyen, & Ding, 2023) for path-tracking control of autonomous vehicles. The vehicle lateral dynamics were described using T-S fuzzy framework and the uncertainties of the cornering stiffness coefficients were considered in the LMI-based control design via a norm-bounded approach. The proposed event-triggered control method was validated with a high-fidelity CarSim vehicle model to highlight its effectiveness in terms of path-tracking performance and communication resources reduction. Moreover, to avoid using expensive on-board vehicle sensors for control purposes, T-S fuzzy output feedback control with or without using observer structures has been investigated while considering the actuator saturation constraints (Hu, Chen, & Wang, 2020; Nguyen, Rath, Guerra, Palhares, & Zhang, 2020) or the effects of friction forces (Alcalá, Puig, & Quevedo, 2019). T-S fuzzy model-based observers have been proposed for the estimation of vehicle states and various related unknown variables. For instance, based on a N-TS fuzzy representation of the vehicle nonlinear dynamics, fuzzy unknown input

observers were developed and real-time validated in (Nguyen, Campos, Guerra, Pan, & Xie, 2021; Nguyen, Dinh, Guerra & Pan, 2021) to cope with nonlinear descriptor systems and simultaneously estimate the lateral speed, steering input, and effective engine torque, which are key components influencing vehicle handling, stability control, and fault diagnosis in autonomous ground vehicles (Pan, Nguyen, Guerra, Sentouh, Wang, & Popieul, 2022). To further improve the estimation performance in the case where the vehicle system is subject to modelling uncertainties and unknown inputs, a feedforward neural network, designed with a data-driven uncertainty identification approach, was incorporated in a T-S fuzzy reduced-order observer scheme in (Nguyen, Nguyen, & Delprat, 2023). The effectiveness of the proposed neural-network-based T-S fuzzy observer was experimentally validated with an autonomous vehicle on a real test track. To deal with the time-varying driver characteristics while considering the human-machine interaction involved in the driving process, T-S fuzzy state feedback shared controllers were developed in (Nguyen, Sentouh, & Popieul, 2016; Benloucif, Nguyen, Sentouh, & Popieul, 2019; Fang, Wang, Wang, Liang, Liu & Yin, 2023) for driver-automation cooperative path tracking control. To avoid the use of costly vehicle sensors, fuzzy output feedback shared controllers have been also investigated for co-driving control with and without requiring an observer, see for instance (Li, Xie, Zhao, Gao, Hu, & Wong, 2021) and (Nguyen, Sentouh, & Popieul, 2017; Ding, Shan, Han, Jiang, Peng, & Liu, 2022), respectively. The driver-in-the-loop test results, obtained with these fuzzy human-machine cooperative control methods, showed that the path tracking performance, the vehicle stability and the driving comfort are significantly improved while the potential driver-automation conflicts can be mitigated when the driver-automation interaction can be explicitly considered in the T-S fuzzy shared controller designs. Apart from automotive and intelligent vehicles applications, T-S

fuzzy control techniques were also successfully explored in railways engineering, e.g., for high-speed train control with experimental tests (Tasiu, Liu, Yan, Chen, Hu, & Wu, 2020; Tasiu, Wang, Liu, Zhang, Zhang, Meng, & Zhao, 2023).

**Networked nonlinear systems.** Nowadays, the fuzzy control problem for networked nonlinear systems is a hot topic. As stated in (Precup, Preitl, Petriu, Bojan-Dragos, Szedlak-Stinean, Roman, & Hedrea, 2020), the presence of time delay in both the control signal and the sensor measurement transmission in networked control systems creates challenging control problems. One such problem is the transcendental characteristic equation of the control system, which is not simple even if linear controlled processes and controllers are assumed, and numerical problems have to be solved. Networked nonlinear systems can be viewed in the framework of mechatronics, exemplified with telesurgical applications (Precup, Haidegger, Preitl, Benyó, Paul, & Kovács, 2012), event-triggered path tracking control considering roll stability under network-induced delays for autonomous vehicles (Viadero-Monasterio, Nguyen, Lauber, Boada, & Boada, 2023), fuzzy model-based nonlinear networked control systems (Qiu, Gao, & Ding, 2016), and event-triggered control with unreliable communication links (Li, Chen, Wu, & Lam, 2017). The up-to-date literature on this topic includes the representative results dealing with the distributed PI state estimation problem for nonlinear systems over sensor networks (Wang, Wang, Zou, Chen, & Yue, 2023), T-S fuzzy systems under FlexRay communication protocol (Wang, Wang, Zou, Ma, & Dong, 2023), PID-fuzzy control of nonlinear systems subjected to controller parameter disturbances over mixed fading channels (Wang, Wang, Zou, & Dong, 2022), and finite-horizon energy-to-peak state estimation of networked linear time-varying systems (Zou, Wang, Shen, Dong, & Lu, 2023).

**Robotics, biomechanics and other mechatronics applications.** Based on the PDC concept, a T-S fuzzy controller was proposed in (Guechi, Lauber, Dambrine, Klančar, & Blažič, 2010) for non-holonomic vehicles subjected to delayed measurements. A nonlinear predictor observer was designed to estimate the robot state variables while accounting for the constant time-delayed outputs. Both simulation and experimental tests were performed to illustrate the effectiveness of the proposed T-S fuzzy tracking-error model-based approach. Sun et al. developed a switched T-S fuzzy control method for wheeled mobile robots in (Sun, Chen, Wang, & Huang, 2017a), where the visual odometry was leveraged for robot localization purposes. Moreover, the robot stabilization and practical constraints on visual odometry can be practically ensured using Lyapunov stability theory as illustrated with suitable experimental tests. Using a T-S fuzzy descriptor control approach, PDC-based controllers were investigated in (Vermeiren, Dequidt, Afroun, & Guerra, 2012; Nguyen, Nguyen, Dequidt, Vermeiren, & Dambrine, 2019) for the tracking control of planar parallel robots and serial robots, respectively, where parametric uncertainties were considered in the fuzzy control design using norm-bounded techniques. Extensive comparative studies using computer simulations were performed together with different conventional control strategies in robotics to evaluate the tracking control performance. Nikdel et al. investigated the fuzzy model-based control problem for flexible joint robots with experimental validations in (Nikdel, Hosseinpour, Badamchizadeh, & Akbari, 2014). To improve the control performance of the proposed PDC-based state feedback controller, the Hybrid-Taguchi genetic algorithm was employed to select the control parameters. A T-S fuzzy model-based controller was developed in (Wen, Hu, Lv, Wang, & Peng, 2019) for the trajectory tracking of a humanoid robot NAO manipulator, where a Q-learning reinforcement learning algorithm was used for robot trajectory planning with

the possibility of obstacle avoidance. Both simulation and experimental results were presented to illustrate the tracking control effectiveness of the proposed T-S fuzzy controller and Q-learning-based trajectory planning algorithm. For biomechanics and rehabilitation applications, in (Guelton, Delprat, & Guerra, 2008), a descriptor fuzzy proportional-integral (PI) observer was proposed to real-time estimate the joint torques and angular velocities in human stance from angular positions. By experimental results, the authors showed that the proposed T-S fuzzy observer can outperform the well-known inverse dynamics joint torques estimation method in terms of robustness with respect to disturbances/uncertainties. PI descriptor fuzzy observers and descriptor fuzzy controllers were also investigated in (Blandeau, Estrada-Manzo, Guerra, Pudlo & Gabrielli, 2018; Guerra, Blandeau, Nguyen, Srihi, & Dequidt, 2020) to study the sitting control strategies of persons living with spinal cord injury (SCI). Despite some preliminary test results, further experimental investigations should be performed to thoroughly validate the proposed model-based approaches for these challenging SCI biomechanics applications with a highly complex system modelling level. In order to control the knee joint angle movement of paraplegic patients through electrical stimulations, a T-S fuzzy regulator was developed in (Gaino, Covacic, Cardim, Sanches, De Carvalho, Biazeto, & Teixeira, 2020), where a simple method was used for system discretization and then T-S fuzzy modelling. Experiments with a paraplegic volunteer and a healthy person showed that predefined design specifications (stability, decay rate, and input constraints) can be achieved under small sampling periods. T-S fuzzy-model-based approaches using Lyapunov stability theory and LMI-based design techniques have been successfully applied to many other mechatronics applications, including two-wheel inverted pendulum systems (Huang, Wang & Chiu, 2010), overhead crane systems (Aguiar, Leite, Pereira, Andonovski, & Škrjanc, 2021), variable

speed wind turbines (Bououden, Chadli, Filali, & El Hajjaji, 2012; Schulte, & Gauterin, 2015), and hydraulic turbines (Ma, & Wang, 2021; Tian, Wang, Chen, & Yang, 2021).

Robustness with respect to parametric uncertainties is a crucial closed-loop property for control engineering applications. An alternative solution to overcome the design conservatism due to the classical norm-bounded approach (Tanaka, & Wang, 2004) complex nonlinear systems, is based on the interval type-2 fuzzy modelling (Lam, & Seneviratne, 2008). Different from the classical T-S fuzzy system, whose grades of membership are fixed, those of the interval type-2 fuzzy system may vary within a certain range to capture the system uncertainties. Considerable research efforts have been devoted to developing a systematic control framework for interval type-2 fuzzy systems (Lam, 2018). However, despite their theoretical interests, the application of interval type-2 fuzzy-model-based approaches to real-world engineering systems is still limited. Some notable mechatronic applications based on LMI-based design techniques with experimental validations can be mentioned such as the robust control of a bolt-tightening tool mounted on a robot arm (Lam, Li, Deters, Secco, Wurdemann, & Althoefer, 2013), a planar snake robot (Bhandari, Raj, Pathak, & Yang, 2022), and a vehicle sensor fault estimation application (Liu, Chen, Na, Luo, & Zhang, 2020).

### **3. Data-driven fuzzy control**

As stated in (Precup, Preitl, Petriu, Roman, Bojan-Dragos, Hedrea, & Szedlak-Stinean, 2020), in contrast to model-based control, data-driven control or data-based control avoids the system (namely the process) identification by constructing controllers directly from data without identifying a system (or process) model. For this reason, data-driven control is also referred to as model-free control, i.e., model-free in controller tuning. In addition, Li, Yuan, Li, & Zhu (2022) consider that model-free control achieves model-free operation such that the dynamics and the stability

performance of a control system are robust to process model variations. This paper uses the term “data-driven” instead of the term “model-free” because there are data-driven or data-based, or model-free control techniques that use non-parametric process models or closed-loop control system models expressed in the time domain or the frequency domain. Nevertheless, the book (Precup, Roman, & Safaei, 2021) considers both “data-driven” and “model-free” terms in the discussion of several controllers. In addition to this book, a useful discussion on model-based versus data-driven control, which has inspired and continues to inspire future research directions, is provided in (Hou, & Wang, 2013). A thorough analysis of data-driven control techniques, focusing on a representative one, is presented in (Chi, Hui, & Hou, 2022).

As stated in (Precup, Preitl, Petriu, Roman, Bojan-Dragos, Hedrea & Szedlak-Stinean, 2020), the model-free tuning of fuzzy controllers is an alternative approach to their model-based design discussed in the previous section, in order to benefit from the advantages of data-driven control and fuzzy control and, if possible, to mitigate their shortcomings. Indirect data-driven fuzzy control in the sense of model-free tuning of fuzzy controllers was first proposed and applied in (Preitl, Precup, Fodor, & Bede, 2006; Preitl, Precup, Preitl, Vaivoda, Kilyeni, & Tar, 2007; Precup, Preitl, Rudas, Tomescu, & Tar, 2008) and continued in (Roman, Precup & David, 2018; Roman, Precup, Bojan-Dragos, & Szedlak-Stinean, 2019; Precup, Roman, & Safaei, 2021; Roman, Precup & Petriu, 2021; Precup, Preitl, Bojan-Dragos, Hedrea, Roman, & Petriu, 2022), mainly dealing with hybrid data-driven and fuzzy controllers through structures that combine data-driven control and fuzzy control to incorporate model-free features in fuzzy control system structures. The direct data-driven fuzzy control ensures the data-driven tuning of the parameters of the fuzzy controllers.

This section will focus on the classification of data-driven controllers conducted in (Precup, Roman, & Safaei, 2021) in terms of analysing mechatronics applications of these controllers in fuzzy control. Both indirect and direct data-driven fuzzy control techniques are discussed. A classical indirect data-driven fuzzy control technique will be briefly presented in the last part of this section.

A convenient way to guarantee the performance of data-driven fuzzy systems is to use optimal control, i.e., to express the performance specifications in terms of optimization problems where the variables are represented by the parameters of the fuzzy controllers. These optimization problems are then solved in various one-shot or iterative formulations, which are briefly discussed in the context of the data-driven control techniques discussed below. The robustness and stability guarantees of data-driven fuzzy systems are challenging. These will be discussed in Section 5, and the information provided in (Precup, Roman, & Safaei, 2021) can be used for the stability analysis and guarantee.

In the literature, different perspectives are used to classify the data-driven control techniques. For example, the structure of the control system is considered in (Hou, & Zhou, 2013), which leads to two categories. In the first category, the controller structure with one or more unknown parameters is assumed to contain the optimal controller obtained from experimental knowledge of the process or the process structure; the controller design is then transformed into a direct identification problem to compute the controller parameters. The second category concerns controllers designed based on certain functional approximations or equivalent descriptions of the process, such as neural networks, fuzzy models, or Taylor series approximation; the controller parameters are then tuned by minimizing a specified objective function (which plays the role of performance criterion) using the input-output data, including



both offline and online data. This section will discuss, as carried out in (Precup, Roman, & Safaei, 2021), the techniques associated with these controllers, divided into two categories that aim their implementation, namely iterative ones and non-iterative or one-shot ones. The most successful data-driven control techniques considered in (Precup, Roman, & Safaei, 2021) will be briefly discussed below, supporting them with classical and recent results in combination with fuzzy control in mechatronics applications.

Iterative Feedback Tuning (IFT) (Hjalmarsson, Gunnarsson, & Gevers, 1994; Hjalmarsson, Gevers, Gunnarsson, & Lequin, 1998; Hjalmarsson, 2002) is a well-established iterative data-driven technique that iteratively tunes controller parameters along the gradient direction of an objective function. IFT is applicable when an initial appropriately parameterized controller that ensures a finite value of the objective function is assumed to be known (Preitl, Precup, Preitl, Vaivoda, Kilyeni, & Tar, 2007; Jung, Jeon, Kang, & Oh, 2021). IFT is applied in (Precup, Preitl, Rudas, Tomescu, & Tar, 2008; Precup, Radac, Tomescu, Petriu, & Preitl, 2013) in the indirect data-driven fuzzy control approach for tuning PI fuzzy controllers for servo systems with experimental validation. IFT is also applied in (Precup, Tomescu, Radac, Petriu, Preitl, & Dragos, 2012) in the same indirect approach to the tuning of state feedback fuzzy controllers for three tank systems with experimental validation.

Model-Free Adaptive Control (MFAC) is attractive because it uses online input-output data collected from the process (Hou, & Huang, 1997; Yu, Wang, Bu, & Hou, 2020). The MFAC structures are based on local dynamic linearized models of the process, and the control algorithms are formulated in a similar manner to model-based predictive control. Three types of dynamic linearization data models are included in MFAC structures: Compacted Form Dynamic Linearization (CFDL) (Hou, & Jin, 2011a; Hou & Jin, 2011b), Partial Form Dynamic Linearization (PFDL) (Hou, & Jin,

2011b) and Full Form Dynamic Linearization (FFDL) (Hou, 1999). A fuzzy logic-based adjustment strategy is utilized in (Wang, Yang, Liang, Liu, Wang, & He, 2018b) to select either MFAC or data-driven Iterative Learning Control (ILC) and appropriate control algorithm parameters at different control stages, and applied to control of a spray fluidized bed granulation process with simulated validation. MFAC is modelled as a fuzzy relational model in (Kadri, & Hussain, 2010) and applied to cooling coil control with simulated validation. The performance of MFAC with CFDL is improved in (Roman, Precup, Bojan-Dragos, & Szedlak-Stinean, 2019; Roman, Precup, Petriu, Hedrea, Bojan-Dragos, & Radac, 2019) by replacing the linear PD component of the control law with a PD-fuzzy component with experimental validation on tower crane systems. Hybrid model-free adaptive fuzzy controllers with tower crane systems experimental validation are reported in (Precup, Roman, & Safaei, 2021).

According to (Preitl, Precup, Preitl, Vaivoda, Kilyeni, & Tar, 2007), Iterative Learning Control (ILC) ensures that the performance of control systems that repeatedly perform the same tasks can be improved using the experience gained from previous experiments in control system operation. This performance improvement is important for two reasons. First, ILC can be formulated as the iterative solution of a parametric optimization problem that ensures the minimization of an objective function to meet the performance indices imposed on the control system; this can be viewed in the general context of learning. Second, since ILC generally does not act on the controller parameter but on the control system structure outside the controller, it can be applied to reference input tuning in two-degree-of-freedom control system structures, with beneficial effects on the control system behaviour with respect to both reference and disturbance inputs.

ILC has been treated recently in the framework of data-driven control, and a thorough analysis of both theory and applications is conducted in (Chi, Hui, & Hou, 2022). Other relevant results in this regard are optimal data-driven ILC (Chi, Hou, Huang, & Jin, 2015), constrained data-driven optimal ILC (Radac, Precup, & Petriu, 2015; Chi, Liu, Zhang, Hou, & Huang, 2018), MIMO ILC (Bolder, Kleinendorst, & Oomen, 2018), data-driven terminal ILC (Bu, Zhu, Hou, & Liang, 2020), and the analysis of ILC in the condition of incomplete information (She, 2018). An adaptive fuzzy ILC algorithm with learning capabilities offered by the fuzzy logic part is designed in (Yu, & Hou, 2021) and applied to the control of high-speed trains with simulated validation. A fuzzy model is included in (Liu, Illian, Leonhardt, & Misgeld, 2023) in the ILC algorithm for rehabilitation exoskeletons with compliant joints with experimental validation. A fuzzy logic-based gain scheduling of PD type ILC scheme is applied in (Yan, Guo, Zhang, Yan, & Liu, 2020) to air-conditioning control systems with experimental validation. A fuzzy dynamic model of a human-exoskeleton is embedded in (He, Li, Li, Liu, & Wu, 2022) in an adaptive robust ILC scheme that also includes a certain neural network architecture, and the control system structure is validated by digital simulation results. Three ILC schemes are applied in (Precup, Preitl, Tar, Tomescu, Takács, Korondi, & Baranyi, 2008) in the indirect data-driven fuzzy control approach to the tuning of PI-fuzzy controllers for servo systems with experimental validation.

According to (Precup, Roman, & Safaei, 2021), Reinforcement Learning (RL) is a data-driven and machine learning (ML) technique whose specific feature is the use of information gathered from interactions with the environment. The RL problem is formulated in the Markov decision process framework using dynamic programming to solve the optimization problem that ensures optimal reference tracking. An RL agent

performs actions in the environment and adjusts its knowledge about itself and the environment based on the reward it receives. This process is applied iteratively and incrementally, so that the RL agent gets better and better at choosing actions that maximize or minimize rewards (Sutton, & Barto, 2017). As pointed out in (Sutton, Barto, & Williams, 1992), RL is a viable technique that solves optimal reference tracking problems and bridges the gap between ML and control. In this context, the RL agent is the controller that automatically learns how to modify its parameters and how to control a process based on the feedback (i.e., reward) it receives from it (Busoniu, de Bruin, Tolić, Kober, & Palunko, 2018). A neural fuzzy controller is designed in (Guo, Lama, Jiang, & Zhou, 2014) to combine the strengths of fast online learning and self-adaptation of neural networks and fuzzy control for automated server parameter tuning with experimental validation. A fuzzy integral RL-based fault-tolerant control algorithm is proposed in (Zhang, Zhang, Cai, & Han, 2019) to combine an RL technique and a fuzzy-enhanced model, and applied to a single-link robotic arm system and pitch-rate control of F-16 fighter aircraft with simulated validation. A deep Q-network is combined with fuzzy logic in (Chen, Hu, Tang, & Cheng, 2022) to address autonomous driving with experimental validation. The inputs of a Q-learning scheme are fuzzified in (Yin, & Li, 2022) to ensure generation control of power systems with simulated validation. A fuzzy integral RL-based tracking control algorithm is proposed in (Zhang, Zhang, Mu, & Sun, 2019) and applied to a single-link robotic arm system with simulated validation. An adaptive neuro-fuzzy controller is constructed in (Tooranjipour, & Vatankhah, 2018) to reduce the number of rules using the quaternion backpropagation concept and using RL to evaluate the output value produced by the critical neural network; this controller is applied to chaotic spinning disk control with simulated validation.

Model-Free Control (MFC) (Fliess, & Join, 2009; 2013) combines the popular and widely used PI and PID controllers, as well as Proportional (P) controllers, with an intelligent term that compensates for the effects of nonlinear dynamics, disturbances, and uncertain parameters. These controllers are referred to as intelligent P, intelligent PI, and intelligent PID controllers. An integral local model of the process is used and its identification is not required. The experimental validation of hybrid model-free fuzzy controllers with tower crane systems is reported in (Precup, Roman, & Safaei, 2021). Fuzzy logic is exploited in a second-order intelligent PI controller with twin rotor aerodynamic systems experimental validation in (Roman, Precup, & David, 2018). Merah, Hartani, Yazid, & Chikouche (2022) combined a hybrid fuzzy system to control the suspension damper with an intelligent PID-fuzzy controller to manage the in-wheel dynamic vibration absorber. They developed an integrated full-vehicle suspension system aimed at jointly improving ride comfort and road holding. To enhance passenger comfort and vehicle stability regardless of road conditions, fuzzy logic was applied in (Unnithan, & Subramaniam, 2022) to control the semi-active stability augmentation system of a large van. A type-1 fuzzy attitude controller was developed to mitigate loop interactions and to address limitations in optimizing control gains between heave and pitch with roll motions. In the inner loop, a Mamdani interval type-2 fuzzy logic controller was used for ride control to accommodate system uncertainties and nonlinearities.

Active Disturbance Rejection Control (ADRC) (Gao, 2006) uses an extension of the system model with an additional and fictitious state variable that models the unmodeled dynamics of the controlled process. This virtual state is estimated online by an extended state observer and then used in the control signal to decouple the system from the actual disturbance acting on the controlled process. Fuzzy control replaces PID

control in (Feng, Zhang, Gao, & Li, 2021) in ADRC applied to hypersonic aircraft with simulated validation. Fuzzy control adjusts online the parameters of the state error nonlinear control law in (Shen, Xu, Chen, & Xia, 2023) in ADRC applied to an unmanned helicopter with simulated validation. Fuzzy logic is used in (Li, Sung, Guo, & Liu, 2022) to smoothly correlate ADRC and MFAC in the control of two-degrees-of-freedom inflatable robotic arms with experimental validation. The performance of ADRC is improved in (Roman, Precup, & Petriu, 2021) by replacing the linear PD component of the control law with a PD-fuzzy component with experimental validation on tower crane systems.

A direct data-driven fuzzy controller based on fuzzy arithmetic operations and the identified fuzzy model of the controlled process is proposed in (Dombi, & Hussain, 2019) and applied to vehicle lateral dynamics with simulated validation. A direct data-driven fuzzy controller based on interval type-2 distending function and the identified fuzzy model of the controlled process is proposed in (Dombi, & Hussain, 2023) and applied to a quadcopter with simulated validation. An optimal fuzzy controller was proposed in (Yuan, Nguyen, & Zhou, 2021) for semi-active air suspensions of heavy trucks, where the fuzzy control rules are optimized by the genetic algorithm. Based on the optimal fuzzy control scheme and the data map of the random road surfaces, an adaptive network-based fuzzy inference system was developed to enhance the ride comfort and road friendliness of heavy trucks.

As specified in (Precup, Roman, & Safaei, 2021), IFT is a representative data-driven technique, which aims to minimize objective functions  $J$  that are specific to Linear Quadratic Gaussian (LQG) controllers expressed as

$$J(\boldsymbol{\chi}) = \frac{1}{2N} E\left\{ \sum_{k=1}^N \{ [L_y(q^{-1})\delta y(k, \boldsymbol{\chi})]^2 + \lambda [L_u(q^{-1})u(k, \boldsymbol{\chi})]^2 \} \right\}, \quad (24)$$

where  $\boldsymbol{\chi}$  is the parameter vector of the controller,  $L_y(q^{-1})$  and  $L_u(q^{-1})$  are weighting filters that penalize the output error (or the tracking error)  $\delta y$  and the control input (or the control signal)  $u$  to give importance to certain frequency regions,  $q^{-1}$  is the unit delay operator, referred to also as the backward shift operator,  $\lambda$  is the control signal weighting parameter,  $k$  is the discrete time index, and  $N$  is the number of samples or the length of the experiment (the trial). The mathematical expectation  $E\{\Xi\}$  is taken with respect to the stochastic probability distribution of the disturbance inputs applied to the process and thus affects the control system behaviour. The disturbance inputs are assumed to be zero mean discrete-time stochastic processes, and it is also assumed that sequences in different experiments are mutually independent in order to obtain unbiased estimates of the gradient.

The expression of the vector variable of the objective function in (24), which is also the parameter vector  $\boldsymbol{\chi}$  of the controller, is

$$\boldsymbol{\chi} = [\chi_1 \quad \chi_2 \quad \dots \quad \chi_n]^T \in \mathfrak{R}^n, \quad (25)$$

where  $\chi_l, l=1\dots n$ , are the controller tuning parameters. The expression  $J(\boldsymbol{\chi})$  of the objective function in (2.1) should be specified, including the weighting filters  $L_y(q^{-1})$ ,  $L_u(q^{-1})$  and the weighting parameter  $\lambda$ , such that the minimization of the objective function will ensure the fulfilment of the performance specifications imposed on the control system. The expression of the output error (or the tracking error)  $\delta y(k, \boldsymbol{\chi})$  is

$$\delta y(k, \boldsymbol{\chi}) = y(k, \boldsymbol{\chi}) - y_d(k), \quad (26)$$

where  $y(k, \boldsymbol{\chi})$  is the controlled output and  $y_d(k)$  is the output of the reference model, i.e., the desired output of the control system to be tracked by the controlled output.

The objective of IFT is to compute the optimal parameter vector  $\boldsymbol{\chi}^*$  as the solution to the optimization problem

$$\boldsymbol{\chi}^* = \arg \min_{\boldsymbol{\chi}} J(\boldsymbol{\chi}), \quad (27)$$

which minimizes the objective function  $J$  expressed in (2). The expression of the optimal parameter vector  $\boldsymbol{\chi}^*$  is

$$\boldsymbol{\chi}^* = [\chi_1^* \quad \chi_2^* \quad \dots \quad \chi_n^*]^T \in \mathfrak{R}^n, \quad (28)$$

where  $\chi_l^*, l = 1 \dots n$ , are the optimal tuning parameters of the controller.

As pointed out in (Precup, Roman, & Safaei, 2021), the major hint in solving the optimization problem in (27), which represents an optimal control problem, is the computation of the gradient of the objective function with respect to the controller parameters. The specific feature of IFT is that an estimate of the gradient of the objective function with respect to the controller parameters can be obtained by conducting special “gradient” experiments on the control system at each iteration of IFT, thus obtaining the data-driven estimation of the gradient.

In the IFT algorithm, the solution is approached iteratively using different gradient-based search algorithms as, for example, the Gauss-Newton scheme expressed as the following IFT algorithm, which represents the parameter update law

$$\boldsymbol{\chi}^{[i+1]} = \boldsymbol{\chi}^{[i]} - \gamma^{[i]} (\mathbf{R}^{[i]})^{-1} \text{est} \left[ \frac{\partial J}{\partial \boldsymbol{\chi}} (\boldsymbol{\chi}^{[i]}) \right], \quad (29)$$

where  $i, i \in \mathbf{N}$ , is the iteration number, the superscript  $[i]$  indicates the value of a certain scalar or vector or matrix at iteration  $i$ , the vector  $\text{est} \left[ \frac{\partial J}{\partial \boldsymbol{\chi}} (\boldsymbol{\chi}^{[i]}) \right]$  is the estimated



gradient, the parameter  $\gamma^{[i]}$ ,  $\gamma^{[i]} > 0$ , is the step size of the current iteration, and  $\mathbf{R}^{[i]}$  is a positive definite regular matrix, which typically chosen to be equal to an estimate of the Hessian matrix of  $J$  or to the identity matrix.

The Single Input-Single Output (SISO) control system structure with the IFT algorithm is presented in Figure 2, where: IFTA – the Iterative Feedback Tuning algorithm, PS – the performance specifications applied to IFT in order to define the objective function in (2), RM – the reference model,  $r$  – the reference input or the set-point,  $e = r - y$  – the control error

$$e = r - y, \quad (30)$$

$u$  – the control signal or the control input, and  $d$  – the disturbance input. As illustrated in Figure 2, the controller should be parameterized by the parameter vector  $\chi$ . In addition, all signals in the control system are assumed to be differentiable with respect to  $\chi$ .

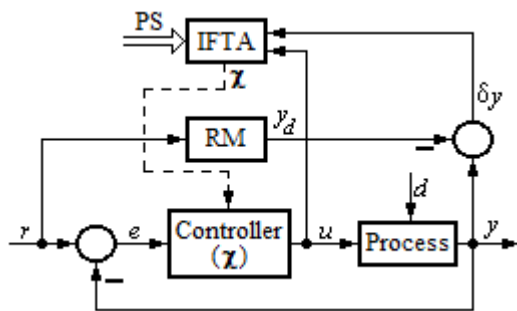


Figure 2. SISO control system structure with IFT algorithm (Precup, Roman, & Safaei, 2021).

Considering a linear controller in Figure 2, the general IFT algorithm dedicated to one-degree-of-freedom SISO controllers is organized in terms of the following steps (Precup, Roman, & Safaei, 2021):

*Step IFT1.* The reference model is set and the expression of the objective function in (24) is formulated and its parameters are set such that the minimization of the objective function by solving the optimization problem defined in (27) ensures the fulfilment of the performance specifications imposed to the control system. The parameters specific to the parameter update law given in (29) are set. An initial appropriately parameterized controller, which ensures a finite value of the objective function in (2), is designed and tuned. The number of iterations is set.

*Step IFT2.* A first experiment, referred as to the normal experiment, is conducted using the reference input  $r_{\{1\}} = r$  applied to the control system, where the notation  $\{j\}$ ,  $j \in \{1,2\}$ , is used for the subscript that is inserted to certain variables in order to specify the number of the experiment conducted with the control system. The normal experiment is conducted in terms of the control system structure given in Figure 2. The control signal  $u_{\{1\}}$ , the controlled output  $y_{\{1\}}$  and the control error  $e_{\{1\}}$  are measured.

*Step IFT3.* A second experiment, referred to as the gradient experiment, is conducted using the reference input  $r_{\{2\}} = e_{\{1\}}$  applied to the control system in terms of the control system structure given in Figure 2. The control signal  $u_{\{2\}}$  and the controlled output  $y_{\{2\}}$  are measured.

*Step IFT4.* The estimated expressions of  $\frac{\partial \delta y}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi})$  and  $\frac{\partial u}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi})$ , namely

$est \left[ \frac{\partial \delta y}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi}) \right]$  and  $est \left[ \frac{\partial u}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi}) \right]$ , respectively, are computed using (Precup, Roman,

& Safaei, 2021)

$$est \left[ \frac{\partial \delta y}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi}) \right] = \frac{1}{C(q^{-1}, \boldsymbol{\chi})} \cdot \frac{\partial C}{\partial \boldsymbol{\chi}}(q^{-1}, \boldsymbol{\chi}) \cdot y_{\{2\}}(k, \boldsymbol{\chi}), \quad (31)$$

$$\text{est} \left[ \frac{\partial u}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi}) \right] = \frac{1}{C(q^{-1}, \boldsymbol{\chi})} \cdot \frac{\partial C}{\partial \boldsymbol{\chi}}(q^{-1}, \boldsymbol{\chi}) \cdot u_{(2)}(k, \boldsymbol{\chi}), \quad (32)$$

where the controller is characterized by the discrete transfer function  $C(z^{-1}, \boldsymbol{\chi})$ , which illustrates that it should be suitably parameterized by the parameter vector  $\boldsymbol{\chi}$  expressed in (25). The gradient of the controller transfer function,  $\frac{\partial C}{\partial \boldsymbol{\chi}}(q^{-1}, \boldsymbol{\chi})$ , is a column matrix

expressed as

$$\frac{\partial C}{\partial \boldsymbol{\chi}}(q^{-1}, \boldsymbol{\chi}) = \left[ \frac{\partial C}{\partial \chi_1}(q^{-1}, \boldsymbol{\chi}) \quad \frac{\partial C}{\partial \chi_2}(q^{-1}, \boldsymbol{\chi}) \quad \dots \quad \frac{\partial C}{\partial \chi_n}(q^{-1}, \boldsymbol{\chi}) \right]^T, \quad (33)$$

where all derivatives are assumed to exist. The two estimates in (31) and (32) are next substituted in the following relationship to calculate the estimated gradient of the objective function, i.e.,  $\text{est} \left[ \frac{\partial J}{\partial \boldsymbol{\chi}}(\boldsymbol{\chi}) \right]$  as in (Precup, Roman, & Safaei, 2021):

$$\begin{aligned} \text{est} \left[ \frac{\partial J}{\partial \boldsymbol{\chi}}(\boldsymbol{\chi}) \right] &= \frac{1}{N} \sum_{k=1}^N \{ L_y(q^{-1}) \delta y(k, \boldsymbol{\chi}) \text{est} \left[ \frac{\partial \delta y}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi}) \right] \\ &+ \lambda L_y(q^{-1}) u(k, \boldsymbol{\chi}) \text{est} \left[ \frac{\partial u}{\partial \boldsymbol{\chi}}(k, \boldsymbol{\chi}) \right] \}. \end{aligned} \quad (34)$$

The values of  $\delta y(k, \boldsymbol{\chi})$  and  $u(k, \boldsymbol{\chi})$  are taken from the normal experiment and next used in (34).

*Step IFT5.* The next parameter vector  $\boldsymbol{\chi}^{[i+1]}$  is computed in terms of applying the parameter update law expressed in (29).

Step IFT1 is carried out once. Steps IFT2 to IFT5 are repeated for a predefined number of iterations set in step IFT1. The above steps clearly show that the controller parameter tuning is carried out without using the process model in terms of executing

two experiments per iteration and additional computation, thus ensuring the iterative experiment-based performance improvement of the control system via the numerical solving of the optimization problem defined in (27).

The steps IFT1 to IFT5 are applied to linear controllers. However, in the indirect IFT approach applied to fuzzy controllers, the linear controllers are replaced by fuzzy controllers. As proceeded in (Precup, & David, 2016), cost-effective PI-fuzzy controllers are next briefly discussed because they can be extended relatively easily to PID-fuzzy controllers and transformed in particular forms of PD-fuzzy controllers. Two versions of PI-fuzzy controllers are next considered (Precup, & Preitl, 1999a): the PI-fuzzy controller with integration of controller output (PI-FC-OI), and the Proportional-Integral-fuzzy controller with integration of controller input (PI-FC-II), with the structures given in Figure 3.

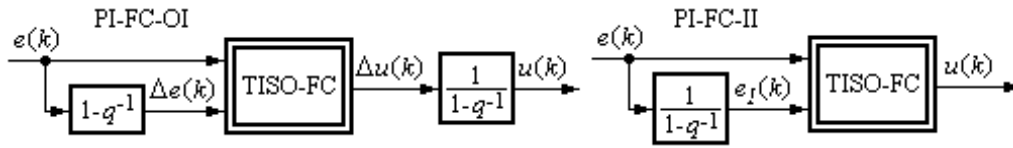


Figure 3. Structures of Proportional-Integral-fuzzy controllers (Precup, & David, 2016; Precup, & David, 2022).

As shown in Figure 3, the two PI-fuzzy controllers are built around the Two Inputs-Single Output Fuzzy Controller (TISO-FC), which is a nonlinear subsystem without dynamics. In addition, the input variables are also scheduling variables. The dynamics are inserted in PI-FC-OI by the numerical differentiation of the control error  $e(k)$ , leading to the increment of control error  $\Delta e(k) = e(k) - e(k-1)$ , and the numerical integration of the increment of control signal  $\Delta u(k) = u(k) - u(k-1)$ , which gives the control signal  $u(k)$ . The dynamics is inserted in PI-FC-II by the numerical integration

of  $e(k)$  producing the integral of control error  $e_I(k) = e_I(k-1) + e(k)$ . It is assumed that the nonlinear scaling factors of the input and output variables specific to TISO-FC are inserted in the controlled process.

As specified in (Precup, & David, 2016; Precup, & David, 2022), the fuzzification in TISO-FC that belongs to PI-FC-OI is done in terms of the input (and also scheduling) and output membership functions illustrated in Figure 4 for Mamdani PI-fuzzy controllers. The fuzzification in TISO-FC that belongs to PI-FC-II is done in terms of the same membership functions as those specific to PI-FC-OI, but the input variable  $e_I(k)$  is used instead of  $\Delta e(k)$ , and the output variable  $u(k)$  is used instead of  $\Delta u(k)$ . The relatively simple shape of the membership functions depicted in Figure 4, which is reflected in a few parameters, contributes to ensuring the cost-effective implementation of the fuzzy controllers. Other distributions of membership functions can lead to the desired modification of FC nonlinearities. The Takagi-Sugeno-Kang PI-fuzzy controllers make use of only the input membership functions illustrated in Figure 4.

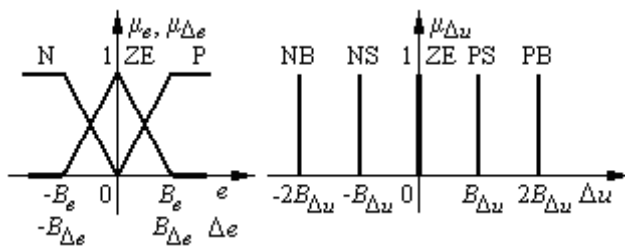


Figure 4. Input and output membership functions of Mamdani PI-fuzzy controllers with integration on controller output (Precup, & David, 2016; Precup, & David, 2022).

Both versions of Mamdani PI-fuzzy controllers, namely PI-FC-OI and PI-FC-II, employ Mamdani's MAX-MIN compositional rule of inference assisted by the

following rule base, exemplified for PI-FC-OI (Precup & David, 2016; Precup & David, 2022):

- Rule 1: IF  $e(k)$  IS N AND  $\Delta e(k)$  IS P THEN  $\Delta u(k)$  IS ZE,
- Rule 2: IF  $e(k)$  IS ZE AND  $\Delta e(k)$  IS P THEN  $\Delta u(k)$  IS PS,
- Rule 3: IF  $e(k)$  IS P AND  $\Delta e(k)$  IS P THEN  $\Delta u(k)$  IS PB,
- Rule 4: IF  $e(k)$  IS N AND  $\Delta e(k)$  IS ZE THEN  $\Delta u(k)$  IS NS,
- Rule 5: IF  $e(k)$  IS ZE AND  $\Delta e(k)$  IS ZE THEN  $\Delta u(k)$  IS ZE, (35)
- Rule 6: IF  $e(k)$  IS P AND  $\Delta e(k)$  IS ZE THEN  $\Delta u(k)$  IS PS,
- Rule 7: IF  $e(k)$  IS N AND  $\Delta e(k)$  IS N THEN  $\Delta u(k)$  IS NB,
- Rule 8: IF  $e(k)$  IS ZE AND  $\Delta e(k)$  IS N THEN  $\Delta u(k)$  IS NS,
- Rule 9: IF  $e(k)$  IS P AND  $\Delta e(k)$  IS N THEN  $\Delta u(k)$  IS ZE,

and the centre of gravity method for singletons is used in the defuzzification module of the fuzzy controllers. The inference engines of both versions of Takagi-Sugeno-Kang PI-fuzzy controllers are based on the SUM and PROD operators assisted by the following rule base, exemplified for PI-FC-OI (Precup, & David, 2016; Precup, & David, 2022):

- Rule 1: IF  $e(k)$  IS N AND  $\Delta e(k)$  IS P THEN  $\Delta u(k) = K_p^1[\Delta e(k) + \mu^1 e(k)]$ ,
- Rule 2: IF  $e(k)$  IS ZE AND  $\Delta e(k)$  IS P THEN  $\Delta u(k) = K_p^2[\Delta e(k) + \mu^2 e(k)]$ ,
- Rule 3: IF  $e(k)$  IS P AND  $\Delta e(k)$  IS P THEN  $\Delta u(k) = K_p^3[\Delta e(k) + \mu^3 e(k)]$ ,
- Rule 4: IF  $e(k)$  IS N AND  $\Delta e(k)$  IS ZE THEN  $\Delta u(k) = K_p^4[\Delta e(k) + \mu^4 e(k)]$ ,
- Rule 5: IF  $e(k)$  IS ZE AND  $\Delta e(k)$  IS ZE THEN  $\Delta u(k) = K_p^5[\Delta e(k) + \mu^5 e(k)]$ , (36)
- Rule 6: IF  $e(k)$  IS P AND  $\Delta e(k)$  IS ZE THEN  $\Delta u(k) = K_p^6[\Delta e(k) + \mu^6 e(k)]$ ,
- Rule 7: IF  $e(k)$  IS N AND  $\Delta e(k)$  IS N THEN  $\Delta u(k) = K_p^7[\Delta e(k) + \mu^7 e(k)]$ ,
- Rule 8: IF  $e(k)$  IS ZE AND  $\Delta e(k)$  IS N THEN  $\Delta u(k) = K_p^8[\Delta e(k) + \mu^8 e(k)]$ ,
- Rule 9: IF  $e(k)$  IS P AND  $\Delta e(k)$  IS N THEN  $\Delta u(k) = K_p^9[\Delta e(k) + \mu^9 e(k)]$ ,

and the weighted average method is applied in the defuzzification module.

Using the PI-fuzzy controller structures described above, the rule bases given in (35) and (36) make these controllers behave as bumpless interpolators between separately designed PI controllers. The maximum number of such controllers is nine,

and the following conditions ensure the interpolation between only two separately designed PI controllers in the case of Takagi-Sugeno-Kang PI-fuzzy controllers (Precup, & David, 2016; Precup, & David, 2022):

$$\begin{aligned} K_p^1 = K_p^2 = K_p^4 = K_p^5 = K_p^6 = K_p^8 = K_p^9 = K_p, \quad K_p^3 = K_p^7 = \eta K_p, \\ \mu^1 = \mu^2 = \mu^3 = \mu^4 = \mu^5 = \mu^6 = \mu^7 = \mu^8 = \mu^9 = \mu. \end{aligned} \quad (37)$$

The unified design methodology of these fuzzy controllers consists of the following design steps for Mamdani and Takagi-Sugeno-Kang PI-fuzzy controllers considered in their PI-FC-OI versions (Precup, & David, 2016):

*Step FC1.* The continuous-time design and tuning of the linear PI controller with the transfer function  $C(s)$

$$C(s) = \frac{k_c(1 + T_i s)}{s} = k_c \left(1 + \frac{1}{T_i s}\right), \quad k_c = k_c T_i \quad (38)$$

is carried out, and it leads to the controller gain  $k_c$  (or  $k_C$  depending on the expression of the PI controller transfer function) and integral time constant  $T_i$ .

*Step FC2.* The sampling period  $T_s$  is set according to the requirements of quasi-continuous digital control. Tustin's method is next applied to discretize the continuous-time linear PI controller, and the recurrent equation of the incremental digital PI controller is

$$\Delta u(k) = K_p [\Delta e(k) + \mu e(k)], \quad (39)$$

where the expressions of the parameters  $K_p$  and  $\mu$  that appear in (37) and (39) are (Precup, & David, 2016)

$$K_p = k_c (T_i - 0.5T_s), \mu = \frac{T_s}{T_i - 0.5T_s}. \quad (40)$$

The parameter  $\eta$ , with typical values  $0 < \eta < 1$ , is introduced in (37) to alleviate the overshoot of the fuzzy control system when both inputs have the same sign. These PI-fuzzy controllers can also be applied to the control of non-minimum phase systems with right half-plane zeros, where such rule bases produce the alleviation of the downshoot as well.

*Step FC3.* The modal equivalence principle (Galichet & Foulloy, 1995) is applied to map the linear controller parameters onto the PI-fuzzy controller ones. The application of this principle to the Takagi-Sugeno-Kang PI-FC-OI leads to the tuning condition (Precup, & David, 2022)

$$B_{\Delta e} = \mu B_e, \quad (41)$$

and the application to the Mamdani PI-FC-OI leads to the tuning conditions (Precup, & David, 2016)

$$B_{\Delta e} = \mu B_e, \quad B_{\Delta u} = K_p \mu B_e. \quad (42)$$

The tuning conditions for the Mamdani PI-FC-II are (Precup, & Preitl, 1999b)

$$B_{\Delta e} = \frac{1}{\mu} B_e, \quad B_{\Delta u} = K_p B_e, \quad (43)$$

and the tuning condition for the Takagi-Sugeno PI-FC-II is (Precup, & David, 2016)

$$B_{\Delta e} = \frac{1}{\mu} B_e. \quad (44)$$



As specified in (Precup, & David, 2016; Precup, & David, 2022), the value of the parameter  $B_e$  in (41) – (44) must be set by the designer. This can be carried out according to the designer's experience, and the stability analysis of the fuzzy control system can be conducted in this regard in terms of the results presented in the previous section, but the data-driven feature of the fuzzy controller could be lost. The systematic tuning of the parameter  $B_e$  can be done in terms of defining and solving optimization problems, where this parameter is one of the elements of the vector variables of the objective functions.

Concluding, the indirect IFT approach applied to fuzzy controllers consists of the steps FCIFT1 and FCIFT2:

*Step FCIFT1.* The steps IFT1 to IFT5 of the IFT technique are applied to the linear PI controllers.

*Step FCIFT2.* The parameters of the linear controllers are mapped on to the parameters of the PI-fuzzy controllers in terms of the steps FC1 to FC3.

This approach has been applied successfully in (Precup, Preitl, Rudas, Tomescu, & Tar, 2008; Precup, Radac, Preitl, Petriu, & Dragos, 2009) to the IFT-based performance improvement of fuzzy controllers for servo systems. Additional details on the application of the step FCIFT1 are given in (Roman, Precup, Hedrea, Preitl, Zamfirache, Bojan-Dragos, & Petriu, 2022). Although this indirect approach affects the optimality, which actually represents a shortcoming, the experimental results reported in this context clearly show the performance improvement of the control system as a direct result of applying IFT.

#### **4. Evolving fuzzy control**

Fuzzy control systems have proven countless times that they can effectively control

complex nonlinear systems, as shown in Sections 2 and 3. The field has matured over time and has established links with several related research areas. The introduction of the Takagi-Sugeno fuzzy model, which has similarities with linear systems, has facilitated the introduction of the established approaches to the control of linear systems. The adaptive approach was one of the first to be adopted. Adaptive fuzzy control offers several advantages because it is known for its adaptability, allowing it to adjust parameters in real time as system conditions change. This adaptability leads to improved performance, especially for nonlinear and time-varying systems, and operation in the presence of uncertainties. Additionally, it reduces the need for manual tuning and can lead to better control results.

However, Figure 1 (b) shows that there are several points in the control loop where adaptation can be introduced:

- The simplest approach is to introduce adaptation into the defuzzification module or, equivalently, into the parameters of the fuzzy rule consequents. This is particularly straightforward when the consequents take the form of affine functions. This approach led to adaptive fuzzy control, for which there exist many direct and indirect schemes in the literature (Wang, 1993; Yin, & Lee, 1995; Blažič, Škrjanc, & Matko, 2003). It soon became clear that the full potential of adaptive fuzzy control could not be achieved without extending adaptation to other components of the fuzzy control system.
- The next step towards evolving systems is to introduce the ability to adjust membership functions so that they effectively capture the input-output relationships of the system. In the case of nonlinear systems, these relationships can vary across different regions of the input space. Consequently, a more accurate fuzzy approximation can be achieved with fewer rules compared to

static membership functions. However, there are potential risks associated with implementing these on-line tunings, particularly as their success depends on the quality of the data. The process of adjusting membership functions to better represent input data is similar to clustering techniques, which group data points based on similarity, revealing inherent clusters, and the latter has often been used in evolving systems.

- When adaptation is introduced to all components of the fuzzy control system, we refer to such a system as an evolving intelligent control (EIC) system. Like any evolving intelligent system (EIS), an EIC uses machine learning algorithms to learn from data streams, but the specific task of an EIC is to adapt and evolve the control strategy accordingly. The main difference with adaptive control is that not only the parameters but also the structure of the control systems are adapted and improved over time in response to changing environment or new information. In particular, this refers to the case where the number of rules in the inference module adapts over time. New rules may be added, obsolete rules may be removed, and existing rules may be combined or split. Not all EICs have all of these features, but at least one of these operations must be implemented to classify a control system as an EIC.

#### **4.1. Learning in EIS**

An EIS is essentially a nonlinear mapping that transforms its inputs into corresponding outputs. One of the key steps in the implementation of the evolving fuzzy system is the appropriate parameterization of the three modules: fuzzification, defuzzification, and the inference system. As a result, the approximation of the EIS output is expressed as a function of the inputs and the initially unknown parameters. The learning capabilities of the EIS are introduced through adaptive laws that govern

gradual parameter updates and a mechanism that allows changes in the structure by adjusting the number of fuzzy rules. If properly designed, the evolving nature of the EIS allows it to continuously refine its input-output mapping to the desired one with an arbitrarily small error.

Learning the parameters in the rule consequents of Takagi-Sugeno models is generally straightforward, as these parameters are often linear with respect to the output. Established adaptive laws from adaptive fuzzy systems derived by gradient descent of the cost function or using Lyapunov stability theory can be applied. This typically results in a form of recursive least squares estimation.

The fuzzification module offers considerable flexibility in the development of evolving algorithms, with various options available for calculating the degree of fulfilment of fuzzy rules. In the field of evolving fuzzy systems, these approaches can be broadly categorized into two main groups:

- Degrees of membership are calculated using traditional membership functions, e.g. Gaussian, triangular, and trapezoidal.
- Membership degrees are calculated based on the relative position of the current data to the cluster centres (closer data points have higher membership in the respective cluster). Some of the most commonly used approaches are fuzzy c-means fuzzification, which uses Euclidean distance, and Gustafson-Kessel fuzzification, which uses Mahalanobis distance. A slightly different approach is kernel-based fuzzification, which uses a kernel function centred on each cluster to determine the degree of membership (Graves, & Pedrycz, 2010).

Adaptation of fuzzification parameters typically uses adaptive laws or optimization techniques, with gradient descent being a common approach. For

membership functions, parameter updates involve adjusting their shape, width and position. In the case of clustering techniques, the parameters that define the position and shape of the clusters are updated accordingly.

Evolving fuzzy systems include mechanisms for adding new membership functions or new clusters based on estimated improvements in the cost function. However, these algorithms tend to be conservative in deciding when to expand the fuzzy model structure, requiring a significant improvement in the cost function before introducing changes. Fuzzy rules can also be deleted. This is typically done when a rule contradicts other rules in its vicinity. In addition, rules may be deleted if they have had a consistently low degree of fulfilment in the past, and/or if they have been created and updated with “old” data. Some evolving fuzzy systems also provide mechanisms for merging and splitting rules. However, it is important to note that similar results can often be achieved by using adding and pruning mechanisms alone. Because there is so much freedom in designing an EIS, almost every paper proposes some variation of an existing evolving approach.

#### **4.2. Stability considerations**

The stability of control systems is a fundamental aspect that determines their behaviour and performance over time. Feedback, a key concept in control theory that makes control so powerful, can induce system instability if not appropriately designed. The introduction of adaptation into control systems inherently introduces non-linearity, even within linear control systems. This can lead to significant stability challenges when seemingly small deviations from the initial assumptions occur. The initial excitement about the practical applications of adaptive control waned after the tragic accident of the X-15-3 aircraft in 1967. Early indications of the vulnerabilities of adaptive control led to a shift in research focus to stability. Addressing stability is complex due to the

multiple feedback loops present in adaptive control systems. Most research relies on Lyapunov functions for stability analysis. Robust adaptive control aims to develop control strategies and techniques that can effectively deal with uncertainties, unmodeled dynamics, variations and disturbances in a system while ensuring stability and performance. Many ideas borrowed from the theory of robust adaptive control have been introduced into adaptive fuzzy control to ensure system stability for a certain class of nonlinear systems.

As adaptation is integrated into more control subsystems, the complexity of the stability analysis increases. In the context of EICs, stability analysis becomes more complex due to potential changes in control structure and strategies. Another important aspect of EICs is the quality of available training data, which is essential to ensure system stability and robustness. As a result, conservative updates to control strategies that have undergone at least some validation may be preferable to introducing poor models obtained during transients, where the quality of the data may be questionable.

### **4.3. Control laws in EIC**

Evolving intelligent systems were originally developed to model systems in supervised contexts where input/output data are accessible during learning (Angelov, & Buswell, 2001; Kasabov, & Song, 2002) with rare early papers dealing with control (Angelov, 2004). The evolving approach has been successfully used in several applications where stability is not the primary concern, such as prediction and forecasting, fault detection and diagnosis, data mining, pattern recognition, and classification, and can be extended to other types of fuzzy models, such as type-2 fuzzy models (Juang, & Tsao, 2008).

In control, however, the fundamental presence of a control loop requires that the control design remains based on stability considerations. Looking at this problem from a machine learning perspective, the control of dynamic systems, unlike modelling,

represents an unsupervised learning scenario since the target control law is not available during learning. However, the control must be applied to the actuators at each sampling time, and a poor choice of control at successive sampling times will result in degraded performance. Therefore, the use of EIS for control purposes is inherently more challenging than its use in various other contexts.

Various approaches are used to address this issue, but most often the control is built around a relatively simple and robust control structure that does most of the work to achieve stabilization, while EISs are used to reduce modelling errors. In the following, the approaches in the literature are classified according to this fixed-structure primary means of stabilization.

#### ***4.3.1. Early (conservative) approaches***

Some early papers dealing with EIC did not really solve the fundamental problem of simultaneous control and controller evolving, but somehow simplified it. For instance, in (Juang, & Lin, 1998), the structure of the evolving neural fuzzy inference network was initially pre-trained using data collected during open-loop operation. Afterwards, complete adaptation, with or without evolving mechanisms, was initiated to guide the system toward the control objective. A similarly conservative approach was taken in (Cara, Pomares, Rojas, Lendek, & Babuška, 2010), where the data are first kept in the buffer before a special supervisory mechanism decides whether to form a new membership function and on which variable. The modification of the structure of the evolving controller is therefore based on the analysis of the error surface and the determination of the input variables that are most responsible for the error. The proposed algorithm was implemented to control a servo system consisting of a DC motor with an extra weight.

Evolutionary algorithms and evolving systems have sometimes been confused. Evolutionary algorithms, commonly used in optimization tasks, typically operate in an offline fashion, iteratively refining solutions based on pre-defined objectives using historical data. However, some systems that incorporate evolutionary algorithms as part of their learning or adaptation mechanisms could be considered evolving systems. An example can be found in (Lee, & Hallam, 1999) where the controller structure evolves in real time based on a growing tree of logical operators and the fitness function assigned to individual controllers. The approach has been successfully tested on a real Khepera miniature robot performing various tasks such as obstacle avoidance, box pushing and exploration.

A major problem with many evolving controllers is that the antecedent part of the rule, especially when the membership functions are added, is completely uninterpretable. This often leads to overlapping of the rules. In (Cara, Herrera, Pomares, & Rojas, 2013), this problem is addressed by using a scatter partitioning of the input space. Together with a singleton in the consequent of the fuzzy controller rules, this leads to a nice interpretable surface of the controller mapping. The structure of the controller is thus much more controlled and sudden loss of stability is unlikely, although stability is not formally proven. The proposed controller is applied to an industrial mechanical suspension system.

#### ***4.3.2. Sliding mode control***

Perhaps the most common approach to solving the EIC design problem is to integrate an evolving fuzzy system to emulate the “ideal controller” and a sliding mode controller (SMC) to ensure system stability. Being stability-oriented, this method is based on the construction of a Lyapunov function consisting of a term that is quadratic with respect to the control error states  $\mathbf{e}$  and a term that is quadratic with respect to the parameter



estimation errors (deviations from the "ideal" parameterization of the evolving fuzzy control system). The task of control design is to find adaptive laws for parameter estimates and a control law that together ensure that the derivative of the Lyapunov function is negative (at least everywhere except in the region where the errors are relatively small). The problem is that the effect of modelling errors (fuzzy modelling errors, disturbances, parasitic dynamics) needs to be compensated. In the case of SMC, this compensation is achieved by introducing an additive robustifying control input that contains a switching or soft-switching function (usually the sign function). As a result, the derivative of the Lyapunov function becomes less than a negative quadratic form of  $\mathbf{e}$ , ensuring that the error states  $\mathbf{e}$  converge to  $\mathbf{0}$  along the so-called sliding surface. The drawback of this approach is that the control signal exhibits undesirable chattering. All the methods discussed in this section guarantee closed-loop stability because the control design is based on the concept of Lyapunov stability theory.

In (Chang, 2010) the combination of adaptive fuzzy control and sliding mode control was proposed where the number of fuzzy rules is kept constant. The approach was implemented to control a 2-DOF rehabilitation robot actuated by pneumatic muscle actuators. In an early implementation of EIC (Park, Park, Kim, & Moon, 2005), the approach follows similar lines to other papers dealing with adaptive fuzzy control, i.e., the algorithms are designed within the Lyapunov stability framework. However, unlike the other approaches at the time, this paper introduced the mechanism of creating new triangular membership functions, and thus new fuzzy rules, as needed. The stability is achieved by adding the control term  $\psi \operatorname{sgn}(\mathbf{k}^T \mathbf{e})$ , where  $\mathbf{k}$  is a constant vector and  $\psi$  is determined by the adaptive law. In the case of (Lu, Chang, & Tsai, 2011), the following sliding surface is used:

$$s(t) = e^{(n-1)}(t) + k_n e^{(n-2)}(t) + \dots + k_2 e(t) + k_1 \int_0^t e(\tau) d\tau, \quad (45)$$

where  $e(t)$  is the control error while the constants  $k_i$ ,  $i = 1 \dots n$ , are chosen so that all roots of the polynomial  $s^n + k_n s^{n-1} + \dots + k_1$  are located in the open left half-plane. Similarly as before, the additive term takes the form of  $\psi \operatorname{sgn}(s(t))$  where  $s(t)$  is defined by (45). This is a very common solution that is used in many works discussed in this section. Gaussian membership functions are utilized in (Lu, Chang, & Tsai, 2011), and fuzzy rules can be either added or deleted from the rule base. The approach was successfully validated through hardware implementation on an inverted pendulum system.

In (Lin & Li, 2013), the evolving fuzzy system is used to replicate an ideal control that results in perfect tracking of the reference model. Gaussian membership functions with adaptive parameters are used, and a method for generating and eliminating fuzzy rules is introduced. In the SMC term, soft switching is used (implemented with the tanh function). The approach has been evaluated on a simulated anti-lock braking system (ABS), demonstrating effective tracking and robust performance under different road conditions and transitions between them. In (Hsu & Wong, 2016), the evolving fuzzy system is again used to tune the primary controller, while the additive SMC term includes the sign function. The number of fuzzy rules can be increased or decreased based on the tracking performance. The algorithm is implemented in a microcontroller to control a voice coil motor. The evolving fuzzy control algorithm proposed in (Lin, Ramarao, & Gopalai, 2019) again gradually constructs the main controller by adding and pruning the rules and adapting their parameters, while an additive term contains another three-rule fuzzy system that serves as a compensator to ensure the robust stability. The approach was successfully tested on

two complex simulated mechatronic systems, namely a double inverted pendulum system and a biped robot.

The approach of (Han, Zhou, Qiao, & Feng, 2015) uses a linear controller to stabilize the linearized dynamics, the EIC with changing structure is used to account for the nonlinearity, while the third term again includes the sign function of the linear combination of control error states.

The method proposed in (Ferdaus, Pratama, Anavatti, & Garratt, 2019; Ferdaus, Pratama, Anavatti, Garratt, & Pan, 2020) introduces an EIS with the addition and removal of fuzzy rules for primary control, supported by a sliding-mode controller acting as an auxiliary robustifying control. The parameters of the sliding surface also adapt in this approach. The algorithm is implemented using the C programming language to facilitate easy transition to dedicated hardware. While testing has been limited to simulated autonomous vehicles like bio-inspired flapping wing micro air vehicles, quadcopters, and hexacopters, future plans involve experiments with real flying devices.

The combination of an EIS and an SMC is also proposed in (Al-Mahasneh, Anavatti, & Garratt, 2020), where they also introduce a dedicated database to store the record of deleted nodes for future retrieval. The algorithm is used to control the angular position of the NXT DC motor. A similar approach is used in (Huynh, Lin, Le, Le, Vu, & Chao, 2022), where a self-organising structure can automatically add or prune the layers to achieve an efficient network structure, while adapting the parameters of the fuzzification and defuzzification module. The approach was successfully tested on a simulated nine-link biped robot and a real magnetic levitation device. Another version of the EIC supported by an SMC is proposed in (Ngo, Hoang, Tran, Nguyen, Nguyen, & Le, 2022), where they also show the results of experimental validation on the

Quanser 2-DOF robot. In (Wang, & Fei, 2022), they also propose the combination of the EIC and an SMC but the latter is a fractional-order SMC. As a result, the size of the chattering is smaller than that of an ordinary integer-order SMC. The approach was tested on a detailed simulated model of a MEMS gyroscope.

The approach in (Le, Lin, & Huynh, 2018) uses type-2 fuzzy functions. Similar to the above approaches, an evolving fuzzy system with an adaptive number of fuzzy rules plays the main role in the control. The additive term with the sign function again helps to achieve stability. It is worth noting that the learning rates for the adaptive laws are optimized using the PSO algorithm. A type-2 evolving fuzzy controller has also been proposed in (Al-Mahturi, Santoso, Garratt, & Anavatti, 2023). It also uses an evolving system to approximate the ideal controller, while robustness is ensured by the term using the saturation function applied to  $s(t)$  defined by (45). The algorithm was tested on simulated tracking of a mobile robot.

#### ***4.3.3. Model reference adaptive control***

A natural approach to the design of an EIC is to extend an adaptive fuzzy controller by introducing the mechanisms to adjust the number of rules. In the case of a direct adaptive fuzzy controller with fuzzy blending of individual control rules designed in the Lyapunov stability framework, the extension with more fuzzy rules does not introduce changes in the control law. Consequently, the stability of the whole system is not compromised if the new rules are based on a sufficient amount of data. This statement is an oversimplification of a very complex system and can only serve as a basic idea. A thorough stability proof is required to provide guarantees.

In an early implementation of this approach (Phan, & Gale, 2008), the direct adaptive fuzzy model reference control algorithm using triangular membership

functions was extended with the ability to add new membership functions. Their number is limited by the user. Adaptive laws use parameter projection to prevent parameter drift. The stability of the control system was proven and tests on an inverted pendulum model showed good performance.

The EIC proposed in (Blažič, Škrjanc, & Matko, 2014) can be seen as an extension of the direct fuzzy model reference adaptive control proposed in (Blažič, Škrjanc, & Matko, 2003). It uses the same control law and the same adaptive law for the control gains. The evolving part introduced in (Blažič, Dovžan, & Škrjanc, 2014) uses the eFuMo method based on the Gustafson-Kessel clustering algorithm. This EIS is the representative of the so-called incremental methods, which only implement the mechanisms for adding new rules. The stability of the proposed EIC has been demonstrated. In (Dovžan, Blažič, & Škrjanc, 2014), new evolving mechanisms were introduced, allowing not only the addition of new rules but also their deletion, splitting into two, or merging of two rules. The method was applied to the control of a detailed simulated model of the helio-crane laboratory pilot device (Blažič, & Zdešar, 2017).

#### ***4.3.4. Auxiliary robust control***

Research papers dealing with EIC often aim to simplify the core control stability problem by proposing a dual-controller solution. The first controller, typically a robust linear controller (referred to as the auxiliary controller), focuses on stabilizing the feedback loop. Meanwhile, the second controller, which evolves over time, deals with system nonlinearity and unmodeled dynamics.

The above approach was used in (Gao, & Er, 2003). This work applies to a certain class of MIMO systems. The evolving part is based on a fuzzy network with Gaussian membership functions and allows the addition and deletion of rules. The

stability of the system is proven in the paper. The approach has been used to control simulated models of an inverted pendulum and a two-link robotic manipulator.

If it is looked carefully at the control law in (Gao, & Er, 2003), it becomes clear that the inputs to both the linear (constant gain) auxiliary controller and the evolving controller consist of error signals and their derivatives. Consequently, both controllers can be considered as PD-like controllers. It will be shown that nonlinear PID control is prevalent in the papers referred to in this section.

In (Chen, & Lin, 2011), a linear PD controller was used in the feedback, while the evolving interval type-2 neural fuzzy network was used in the feedforward part. Thus, the stability problem of the EIC was not an issue because the PD controller was chosen by the designer. The proposed approach was used for high-precision motion control of permanent magnet linear synchronous motor drives. A PD controller used as a stabilizing controller is also proposed in (Ngo, & Phuong, 2015). However, in their approach, an evolving wavelet fuzzy cerebellar model articulation controller was applied in the feedback. The stability of the proposed solution is shown, and the control is implemented on a three-link de-icing robot manipulator.

In (Hsu, 2013), an evolving functional-linked wavelet neural network is proposed for control. The wavelet base functions are used at the input of the network. The evolving algorithm can generate additional hidden neurons. Stability is proven in the paper. The proposed control is applied to a DC motor. The control part of the algorithm again has one input – the so-called sliding surface

$$s(t) = \dot{e}(t) + k_2 e(t) + k_1 \int_0^t e(\tau) d\tau. \quad (46)$$

The sliding surface  $s(t)$  can also be interpreted as the output of the PID controller. The signal  $s(t)$  is fed into the supervisory compensator, which is simply an adaptive gain. The signal  $s(t)$  also serves as the only input to the evolving part, which shows that the total control input is the linear combination of the three components of  $s(t)$ . In other words, the controller is a nonlinear PID one.

A very similar approach from the control law perspective which can also be interpreted as a nonlinear PID, is proposed in (El-Sousy, 2014), while the evolving part features a different algorithm based on a recurrent radial-basis function network-based evolving system. All control algorithms are implemented in a TMS320C31 DSP-based control computer and experimentally tested on the torque control of the rotor position for a permanent-magnet synchronous motor servo drive. The control algorithm proposed in (El-Sousy, 2016) is very complex as it consists of three parts: a mixed  $H_2/H_\infty$  controller, an evolving recurrent fuzzy wavelet neural network controller, and a robust controller (again a PD). It is important to note, however, that the vector entering all three parts is the same, namely a filtered tracking error vector. The paper includes stability proof. The proposed approach has been implemented on a two-axis motion control system (X-Y table) driven by two permanent-magnet linear synchronous motor servo drives.

The following approaches use fuzzy rules with PID control actions in the rule consequents. When these rules are combined, a nonlinear PID is again obtained. The main question is how to define the control objective. This influences the design of the parameter adaptive laws and evolving mechanisms. A very useful technique is to introduce the reference model, i.e., the output of the plant should follow the reference signal filtered through a reference model (typically a first-order linear system). In other words, the evolving PID attempts to linearize the control system. However, in most

cases, the linearization is only approximate due to the limited degrees of freedom of the controller. In (Škrjanc, Blažič, & Angelov, 2014), a robust evolving cloud-based controller (RECCO) is proposed. This approach can be seen as an extension of direct adaptive fuzzy control, where each fuzzy rule consists of a PID controller with adaptive control gains in the consequent. The structure of the controller evolves over time based on the distances in the input space where so-called data clouds are defined. The approach also introduces several techniques from robust adaptive control (dead zone in the adaptation law, parameter projection, leakage in the adaptation law, interruption of adaptation) to improve the robustness of the system. After introducing normalizations in the data and changes in the adaptation laws of the consequent parameters (Andonovski, Blažič, Angelov, & Škrjanc, 2015; Andonovski, Angelov, Blažič, & Škrjanc, 2016), the approach was also implemented on the plate heat exchanger device. A similar approach where structure evolving is based on data clouds is used in (Andonovski, Mušič, Blažič, & Škrjanc, 2016), where a new mechanism for removing the "less active" and "less informative" clouds is introduced to prevent the addition of new clouds based on outliers, or at least to help remove existing clouds with little information. Another approach where evolving is based on data clouds is presented in (Angelov, Škrjanc, & Blažič, 2013). The approach also features PID controllers in the rule consequents, but new rules are added not only regarding the local density, but also considering the global density, which indicates how representative a measurement is with respect to the entire data distribution. The approach has been successfully applied to the laboratory pilot plant of two tanks (Costa, Škrjanc, Blažič, & Angelov, 2013). In (Chen, & Liu, 2017), an evolving probabilistic fuzzy neural network controller is proposed. The distinctive feature is that the membership functions are asymmetric (different "standard deviations" are used on either side of a Gaussian function). The algorithm includes the addition and



pruning of fuzzy rules. From a control point of view, the controller is a PD with adaptive and nonlinear gains. The approach was implemented for position servo control of a permanent magnet linear synchronous motor servo drive system with two position commands tracking under different operating conditions.

#### ***4.3.5. Model predictive control***

Another option in the feedback loop, when combined with an EIC, is model predictive control (MPC). It has been applied in (Zdešar, Dovžan, & Škrjanc, 2014), where it complements the feedforward part consisting of the inversion of the Takagi-Sugeno fuzzy model. This approach does not require explicit optimization of the cost function, as the optimization problem is solved analytically. Evolving of a fuzzy model is based on the eFuMo model, which has the mechanisms for adding, deleting, merging and splitting rules. The proposed approach was implemented on a helio-crane laboratory pilot system.

In (Han, Zhang, Hou, & Qiao, 2016), a nonlinear MPC is developed based on an evolving recurrent radial basis function neural network. The optimization problem is formulated as minimizing the cost function over the horizon, which is achieved by gradient optimization. A stability proof of the proposed scheme is given.

## **5. Conclusions and research challenges**

A brief overview of mechatronics applications of fuzzy control has been presented in this paper. The following classification of control systems was proposed and used for this purpose: (i) model-based fuzzy control, (ii) data-driven fuzzy control, and (iii) evolving fuzzy control. The applications highlighted in this paper cover several areas beginning in 2011. However, papers published before 2011 have also been considered if the authors believe they are representative of the topics discussed. There are several

challenges that deserve more study when considering fuzzy control from the point of view of its applicability. Some of these challenges are discussed as follows.

Concerning fuzzy model-based approaches, a significant research effort has been directed towards mitigating the conservatism in stability analysis and control design. As previously discussed, this has been achieved by exploring more versatile Lyapunov function candidates and employing effective convexification techniques to reformulate the stability analysis and control design problems within an LMI framework. An alternative approach to achieving more relaxed stability analysis results involves improving the extraction and exploration of information about membership functions during the design procedure. In spite of recent advances, these two research directions still have the potential to yield highly impactful results in the field of fuzzy model-based stability analysis (Lam, 2018).

Despite the maturity of fuzzy model-based approaches from a theoretical perspective, with numerous existing results, their real-world applications remain significantly limited in practice. Indeed, most of the existing engineering applications of T-S fuzzy control and estimation are primarily limited to academic demonstrations, with no notable industrial applications. This is primarily due to the fact that the numerical complexity of the related stability analysis, observation and control design conditions exponentially grows with respect to the number of premise variables (Tanaka, & Wang, 2004). Hence, T-S fuzzy model-based results are mostly suitable for systems with only a few nonlinearities, which is generally not the case in complex industrial plants. As previously discussed, several approaches have been developed to overcome this major practical limitation (Dehak, Nguyen, Dequidt, Vermeiren, & Dambrine, 2022). However, it is necessary to further investigate the trade-off between numerical complexity, design conservatism, and the modelling uncertainties introduced by these

model-reduction approaches. Recently, data-based approaches have been proposed to reduce the dimensionality of the premise variables, for instance, principal component analysis (Kwiatkowski, & Werner, 2008) and deep neural networks (Koelewijn, & Tóth, 2020). However, these modelling approaches also suffer from significant drawbacks related to approximation errors and their performance dependence on collected data. We strongly believe that only a T-S fuzzy control/estimation framework with a low level of numerical complexity, reasonable design conservatism, and simple tuning methods can facilitate its practical application. The quest for such a T-S fuzzy framework remains an open-ended question for the fuzzy control community, and it will undoubtedly inspire a significant amount of research efforts in the future.

As suggested by Sugeno (1999), fuzzy models with singleton consequents, also referred to as Mamdani-like fuzzy models, can be employed in nonlinear control and analysis to mitigate the design conservatism associated with T-S fuzzy model-based approaches. This is possible because the information from the triangular membership functions of Mamdani-like fuzzy models can be explicitly incorporated into stability analysis (Sugeno, & Taniguchi, 2004). Moreover, Mamdani-like fuzzy models are particularly useful within industrial contexts to study complex plants with no acceptable analytical descriptions as they can be conveniently implemented using look-up tables. However, despite some recent advances to achieve convex LMI-based stability analysis conditions for continuous-time case (Nguyen, Sugeno, Campos, & Dambrine, 2016) as well as discrete-time counterpart (Nguyen, Dehak, Guerra, & Sugeno, 2023), Mamdani-like fuzzy model-based approaches still remain computationally heavy, especially when the number of piecewise regions becomes large. Moreover, the existing control design of Mamdani-like fuzzy models is reformulated in a non-convex optimization problem (Nguyen, Taniguchi, Eciolaza, Campos, Palhares, & Sugeno, 2019), which poses

additional challenges for real-time applications. Therefore, identifying pertinent research directions aimed at reducing the computational load and establishing convex control design conditions for Mamdani-like fuzzy models would constitute significant contributions to fuzzy model-based approaches from both theoretical and practical standpoints.

As stated in (Precup, Roman, & Safaei, 2021), the main shortcoming of data-driven control is the difficulty of systematic stability and robustness analyses. In other words, tuning to ensure tracking of the reference trajectory does not guarantee robust stability and robust performance. This is normal, since these analyses require detailed mathematical models of the controlled process. Nevertheless, the term “robustness”, which in model-based control refers to the property of a controller to have low sensitivity to modelling errors, is usually avoided in data-driven control. Since the tuning process here is model-free, the use of the term “robustness” sounds ambiguous and should be clarified. The stability analysis of a data-driven fuzzy control system is challenging and the information given in (Precup, Roman, & Safaei, 2021) can be used in this regard.

There is still a lot of room to develop various direct data-driven fuzzy controllers in terms of proper modelling of the fuzzy logic subsystems of the controller so as to successfully replace the linear subsystems in various data-driven controllers. The following three data-driven control techniques that have the potential to be merged with fuzzy control in direct and indirect approaches are presented next, the rest of them are described in Chapter 2 in (Precup, Roman, & Safaei, 2021):

- Simultaneous Perturbation Stochastic Approximation (SPSA) (Spal, & Cristion, 1998; Wang, & Spall, 1998) is supported by the fact that, unlike deterministic steepest descent algorithms, the gradient-based stochastic approximation

algorithms employed in IFT and SPSA use estimated gradients of the objective function. If the gradients cannot be computed from real-time experimental data, they must be estimated from the noisy measurements of the objective function using finite difference approximations around the current operating point. SPSA is advantageous because it reduces the implementation cost by requiring only two evaluations of the objective function per iteration.

- Iterative Correlation-based Tuning (CbT) (Karimi, Mišković, & Bonvin, 2004; Mišković, Karimi, Bonvin, & Gevers, 2007) operates in the model reference control framework. The relationship between the reference input and the tracking error is highlighted in the correlation function of the two signals when the quasi-stationary framework is realistically assumed (Ljung, 1999). A decorrelation procedure is applied to make the tracking error converge to zero. The objective function depends on the correlation function of the two signals (i.e., the reference input and the tracking error) and is minimized in an iterative fashion. The initial controller in iterative CbT is usually linear and appropriately parameterized, and it should ensure a finite value of the objective function.
- Frequency-domain Tuning (FdT) (de Bruyne, & Kammer, 1999; Kammer, de Bruyne, & Bitmead, 1999; Kammer, Bitmead, & Bartlett, 2000) makes use of a Linear-Quadratic (LQ) objective function that penalizes the tracking error in the model reference control system, and it is minimized in the form of a variant of the classical gradient-based stochastic approximation algorithm that is based on estimating the gradient of the objective function. The objective function is expressed in the frequency domain using Parseval's theorem and next spectral analysis techniques, which allow the calculation of different autocorrelation and cross-correlation sequences of the control system signals. Spectral estimates are

obtained for the transfer functions involved in the FdT algorithm. The advantage of FdT is the use of frequency response functions that are non-parametric models. Thus, the derivatives of the objective function with respect to the controller parameters are obtained in the frequency domain (Khadraoui, Nounou, Nounou, Datta, & Bhattacharyya, 2014; da Silva Moreira, Acioli Júnior, & Rezende Barros, 2018). Stability between two consecutive iterations is ensured by calculating the Vinnicombe metric and the generalized stability margin using nonparametric models obtained by spectral analysis.

The structures shown in Figure 1 can also be adapted to two-degree-of-freedom (2-DOF) fuzzy controllers, which have been proposed in (Precup & Preitl, 1999b; Precup, & Preitl, 2003) as fuzzy controllers with non-homogeneous dynamics with respect to the input channels, and then applied to servo systems and electric drives (Precup, Preitl, Petriu, Tar, Tomescu, & Pozna, 2009; Preitl, Stinean, Precup, Preitl, Petriu, Dragos, & Radac, 2012). These structures can provide very good performance in mechatronics applications with respect to both reference and disturbance inputs.

As discussed earlier, ensuring the stability of the control system is a crucial requirement in the design of nonlinear control systems. Stability analysis of fixed-structure fuzzy control systems is a challenging task, which becomes even more difficult when the evolving nature of the controller is considered. Considering that the controller evolving depends on the streaming data taken from different sources in the complex control system, we realise that this process should not be left to run without proper supervisory mechanisms installed.

Some challenges for future research on evolving control systems include:

- Introducing more robust adaptive approaches to the learning process. This means that the adaptation of parameters and structure should be based on conservative decisions to avoid the known mechanisms (such as parameter drift) that lead to instability of adaptive control (Ioannou, & Kokotovic, 1984). The proposed solutions to improve the robustness of the system, such as the use of a dead zone, parameter projection, leakage, etc., should also be tailored to the adaptation of the structure.
- Improving the interpretability of evolving control systems. Evolving control systems have moved away from the initial concept of early fuzzy control systems, where linguistic rules were translated into fuzzy logic rules. Interpretability may not be a major concern for most automation users. However, a significant problem related to poor interpretability arises from rules that are obviously flawed and should not be there, but which produce unfavourable effects in specific operating regions. These problematic rules are typically created from short sequences of bad data or data taken during transients. They are activated in limited operating ranges, making it difficult to detect poor control. Therefore, continuous monitoring of the rule base is necessary to improve system robustness.
- Long-term autonomy and learning. Evolving control systems should be able to continuously learn and adapt over long periods of time. This includes mechanisms to retain knowledge, adapt to new circumstances and avoid forgetting previously learned information. The aim is to ensure that evolving control systems can provide efficient, accurate and contextually appropriate control decisions throughout their operational lifetime, minimising the detrimental effects of memory degradation or outdated knowledge. Achieving

long-term autonomy is fundamental to the sustainable and effective use of these systems in various domains.

These challenges will attract both researchers and practitioners. The authors are convinced that the number of successful mechatronics applications of fuzzy control will continue to grow.

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#### **Data availability statement**

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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