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Zitterbewegung CP Violation in a Schwarzschild Spacetime

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Neutral kaons oscillations in a Schwarzschild spacetime are analyzed. The interplay between two oscillations: (i) mixing associated with second order weak coupling and (ii) strange quark's zitterbewegung, introduces a coupling responsible for the observed CP violation. This curvature induced violation is a CPT violation with T conservation rather than a T violation with CPT conservation. The non-dissipative Hermitian evolution of the kaons system leads to the identification of this CPT violation. Then, the finite lifetime of the short-lived kaons induces a dissipative rotation of the imaginary violation parameter such that it becomes real and appears as a T violation. The consequences of this elucidation of the origin of neutral kaons CP violation are then discussed and some open perspectives are identified.

I. INTRODUCTION

Historically strange particles and the $\theta - \tau$ puzzle were the source of an in-depth analysis of elementary particles symmetries. The identification of the *strangeness* emerged from the study of the kaons behavior by Gell-Mann, Nishijima and Pais [1–5]. Lee and Yang suggestion of parity (P) non conservation in weak interactions [6] (discovered by C. S. Wu et al. [7]) finds its roots in the attempts to understand kaons decays. Beside space-inversion P , time-reversal (T) and charge-conjugation (C) studies have found also an ideal laboratory with strange mesons experiments. A laboratory that proved and remains fruitful.

Sixty years ago Christenson, Cronin, Fitch and Turlay reported the first measurement of CP violation through the observation and analysis of long-lived neutral kaons decays into two charged pions [8–10], a decay forbidden by CP conservation.

Since this discovery, the Lee, Oehme and Yang (LOY) model, based on two coupled Schrödinger equations [11], (completed by T. T. Wu and Yang [12]) provides the canonical model to understand the symmetries and describe the dynamics of neutral kaons.

Beside this phenomenological description, there was also a need to understand the origin of this violation.

The superweak hypothesis, introduced by Wolfenstein [13, 14], appeared as an explanatory framework during the early years of CP violation studies. Following decades of precision measurements this small CP asymmetry is now satisfactorily incorporated in the Standard Model with the Cabibbo-Kobayashi-Maskawa (CKM) matrix [15, 16].

Understanding the origin of CP violation is of prime importance as it might explain how our matter-dominated universe emerged during its early evolution, as suggested by Sakharov [17].

Various hypothesis have been put forward to explain this origin. Following Good's preliminary analysis [18], the impact of a gravitational field, or a new type of external field, has been considered within the framework of several models. Immediately after the experimental discovery of CP violation, Bernstein, Cabibbo and Lee [19] initiated this program of research [20–24] that was also pursued with the speculative antigravity hypothesis [25–28]. In the light of the recent results of the Gbar experiments [29], this last type of hypothesis appears now highly speculative. Among the many attempts to understand the origin of CP violation, the validity of the Weisskopf-Wigner approximation [30] was also revisited. Rather than the non-Hermitian LOY two states model, the decay and mixing can be described with three Hermitian coupling between two discrete states and a continuum set of final states [31–33].

As a result of the failure of these various attempts to predict the experimental value of the most documented observed effects: the $\langle K_S | K_L \rangle$ overlap between the CP eigenvectors the long-lived (K_L) and the short-lived (K_S) kaons, the ultimate origin of CP violation remains an open issue. However, the inclusion of CP violation in the Standard Model looks satisfactory.

In this article we revisit the impact of the earth gravitational field on neutral kaons dynamics.

Gravity is not described as a long range interacting field, but is considered as a curved spacetime geometry in accordance with general relativity. A Schwarzschild geometry is used and both (i) the requirements of covariance and (ii) the Feynman view on antiparticles as particles propagating backward in time are taken into account to adapt the LOY model to the spacetime geometry around earth.

In order to avoid any non physical interplay between fundamental first principles and the Weisskopf-Wigner approximation, we consider that K_L and K_S are stable particles and find that $\langle K_S | K_L \rangle$ is an imaginary number in a Schwarzschild spacetime. If $\langle K_S | K_L \rangle$ is a real number CPT holds and if it is an imaginary number T holds [11, 12]. Then, taking into account the finite lifetime of

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K_S , $\langle K_S | K_L \rangle$ is rotated and becomes a real number with a value fully consistent with the experimental data. The measurement of $\langle K_S | K_L \rangle$ provides this real value and thus favored, in the past, the inclusion of this effect as CP and T violations in the LOY model, rather than CPT violation and T conservation. We demonstrate here that, at the fundamental level, a massive object like earth induces a CPT violation with a T conservation, consistent with the experimental results but usually interpreted as a CP violation with T violation and CPT conservation. Within the framework of the Weisskopf-Wigner approximation spontaneous decays are described as irreversible processes, thus it is not surprising that a T violation comes into play in this class of dissipative models.

The CPT theorem was demonstrated on the basis of three main assumptions: (i) Lorentz group invariance, (ii) spin-statistics relations and (iii) local field theory. In a Schwarzschild spacetime the assumption (i) is not satisfied so CPT theorem no longer holds. The CPT theorem framework is a flat Minkowski geometry associated with a zero temperature vacuum state. A Schwarzschild geometry is not flat and the vacuum state is no longer cold but displays a small temperature, thus we must not be surprised to observe a CPT violation.

The impact of the earth gravitational field on elementary particles quantum dynamics has been predicted and measured with neutron interferometry fifty years ago [34, 35]. These experiments were compatible with Newtonian gravity. For the kaon system the observed CP violation requires Einsteinian gravity. A model of neutral kaons CP violation within a Newtonian framework leads to a factor 2 mismatch with the experimental results.

The experimental results reported in [35] are related to the influence of Newtonian gravity on the external degrees of freedom of an elementary particle. It is commonly accepted that the energy scale at which gravity becomes significant to influence fundamental particles interactions (the internal degrees of freedom) is given by the Planck mass

$$M_P = \sqrt{\hbar c / G_N} = 2.17 \times 10^{-8} \text{ kg}, \quad (1)$$

where G_N is the constant of gravitation. However, an estimate of gravity effects based solely on M_P does not take into account the possibility that an interplay between mixing and oscillations might reveal a specific coupling. For example, if we consider a particle with mass $m = 497.6 \text{ MeV}/c^2$ like a neutral kaon, the parameter $m/M_P \sim 0.40 \times 10^{-19}$. However, if the mass matrix displays a coupling δm to the antiparticle, this might increase this parameter as a result of a different scaling, for example for neutral kaons: $(m/\delta m)(m/M_P) \sim 0.11 \times 10^{-4}$. Beside the historical tests of general relativity such as: (i) the anomalous perihelion advance, (ii) the bending of light rays near a massive object and (iii) the time delay in radar sounding, we can consider the experiments dedicated to CP violation in neutral kaons as new positive tests of general relativity in the vicinity of a massive object.

General relativity remains the right theory, at the sub-fundamental length scale, to understand strange particles quantum dynamics near a massive object like earth.

The historical method used by Dirac to set up its relativistic first order equation (an iteration of the time derivation to check the matching with the relativistic energy-momentum dispersion relation) is used here to construct a relativistic LOY model in a Schwarzschild spacetime. Following a critical analysis of the LOY model (Ref. [36] provides a clear review of this model), we conclude that, in order to reveal the dominant gravitational coupling, a set of two coupled relativistic Klein-Gordon (K-G) equations is a more appropriate model than two coupled Schrödinger equations. However, coupled first order Schrödinger equations are more adapted to study discrete symmetries conservation, or violation, than K-G equations. We accommodate both descriptions in a logical and coherent way and discover that spacetime curvature is the source of a small CPT violation whose amplitude is consistent with the measured direct and indirect CP violations.

Within the framework of a K-G description quark internal dynamics comes into play in a simple way as a source of a small additional term responsible for CPT violation and T conservation. This additional coupling is due to a velocity matrix element intimately linked to the *zitterbewegung* behavior of spin 1/2 fermion/antifermion pairs, here the strange quark. Thus we call the new effect identify and described here: *zitterbewegung CP violation in a Schwarzschild spacetime*. It is to be noted that the elimination of some *zitterbewegung* effects with a canonical transforms of the Foldy-Wouthuysen (FW) type can not be put at work in a curved spacetime because FW transforms are non-local in spacetime.

This paper is organized as follow. In the next section we present the principles and methods used to identify the dominant gravitational coupling to be considered in a first order LOY model. In section III the impact of a Schwarzschild metric on the evolution of the quantum amplitudes of a kaon system is considered. The importance of the quarks internal motions is revealed in this section. The quark velocity matrix elements identified in section III are calculated in section IV. As a result of this calculation the physics of CPT violation versus T violations is identified and analyzed in section V. Section VI explore the problem of direct CP violation. Section VII gives our conclusions and present some open perspectives to be explored. The consequences for cosmological baryogenesis are briefly considered in section VII to provide a global view on the vast field of new studies opened by the identification and demonstration of the origin CP violation in neutral kaons systems.

The main results of this study are : (i) the origin of CP violation in neutral kaons systems is identified, (ii) the measured values of both the direct and the indirect CP violation parameters are predicted and (iii) the right status of the symmetry breaking induced by Schwarzschild curvature and quarks *zitterbewegung* is restored: CPT

violation with T conservation rather than T violation with CPT conservation.

This article is the companion paper of a short letter [37], thus for the clarity of the demonstrations some elementary theoretical results are recalled. Contrary to common practice we keep \hbar , c and G_N rather than setting them to one. Most of the cited references are not recent because the origin of CP violation is a long-standing problem and the subject of gravity as its origin did not attract much attention during the past three decades, despite the fact that it provides a simple and accurate mechanism. Moreover, as this is a sixty years old problem, an historical perspective was clearly needed.

II. SCHRÖDINGER AND KLEIN-GORDON DESCRIPTIONS

The logic of the demonstration given in the next sections can be summarized as follows: we start from the CPT and CP symmetric LOY model with two coupled Schrödinger equations in a Minkowski spacetime and generalize it to a Schwarzschild spacetime.

The principles for this generalization are invariance and covariance requirements with respect to the energy-momentum relation and the Feynman view of an antiparticle as a particle propagating backward in time.

The tools for this generalization are general relativity to describe spacetime and Dirac's equation to describe spin 1/2 quark dynamics inside kaons.

The first principles used in this demonstration are well established, without speculative assumptions on new fields or new interactions, so that the pertinence of the results of our model is granted by its simplicity.

Our starting point is the CPT and CP symmetric LOY model [36]: the rest frame evolution of a neutral kaons system (K^0/\bar{K}^0) is described by two amplitudes, a and b , defining the state $|\Psi\rangle$ as a function of the particle proper time τ ,

$$|\Psi\rangle = a(\tau) |K^0\rangle + b(\tau) |\bar{K}^0\rangle. \quad (2)$$

The kaons states $|K^0\rangle$ and $|\bar{K}^0\rangle$ are normalized to unity and are orthogonal. They are eigenstates of the strangeness operator with eigenvalues ± 1 . The relative phase of K^0 and \bar{K}^0 states is not observable and is fixed by convention. In this study it is convenient to take $CP|K^0\rangle = |\bar{K}^0\rangle$. The two CP and energy eigenstates, with mass eigenvalues m_1 and m_2 and CP eigenvalues ± 1 , are given by

$$\begin{aligned} |K_1\rangle &= \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}, \\ |K_2\rangle &= \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}. \end{aligned} \quad (3)$$

Without CP violation $K_L = K_2$, $K_S = K_1$ and $\langle K_S | K_L \rangle = 0$.

These particles are unstable and $\partial\langle\Psi|\Psi\rangle/\partial\tau < 0$, thus the Hamiltonian restricted to the ($|K^0\rangle, |\bar{K}^0\rangle$) Hilbert space is not Hermitian. The Weisskopf-Wigner approximation [30] is used to describe decay with a non-Hermitian Hamiltonian H_K . The (proper) time evolution of the state $|\Psi\rangle$ is given by two coupled Schrödinger equations

$$j\hbar\frac{\partial}{\partial\tau}|\Psi\rangle = H_K|\Psi\rangle, \quad (4)$$

mixing the two amplitudes a and b

$$j\frac{\hbar}{c^2}\frac{\partial}{\partial\tau}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} M & -\delta M \\ -\delta M & M \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}. \quad (5)$$

It is important to note that the Hamiltonian

$$\begin{aligned} H_K &= Mc^2 \left(|K^0\rangle\langle K^0| + |\bar{K}^0\rangle\langle\bar{K}^0| \right) \\ &\quad - \delta Mc^2 \left(|\bar{K}^0\rangle\langle K^0| + |K^0\rangle\langle\bar{K}^0| \right), \end{aligned} \quad (6)$$

which looks like relativistic because of the occurrence of c^2 , provides in fact a description which is not fully relativistic. The matrix elements M and δM ,

$$\begin{aligned} M &= m - j\frac{\hbar}{2c^2}\Gamma, \\ \delta M &= \delta m - j\frac{\hbar}{2c^2}\delta\Gamma, \end{aligned} \quad (7)$$

are the sum of an Hermitian part:

$$\begin{aligned} m &= \frac{m_2 + m_1}{2}, \\ \delta m &= \frac{m_2 - m_1}{2} > 0, \end{aligned} \quad (8)$$

plus a dissipative non-Hermitian part:

$$\begin{aligned} \Gamma &= \frac{\Gamma_1 + \Gamma_2}{2}, \\ \delta\Gamma &= \frac{\Gamma_2 - \Gamma_1}{2} < 0, \end{aligned} \quad (9)$$

where $m_{1/2}$ are the inertial masses and $\Gamma_{1/2}$ the inverse of the mean lifetimes of K_1 and K_2 . The ordering of this model is : (i) $\delta m/m \sim 3.5 \times 10^{-15}$, (ii) $\Gamma_1/\Gamma_2 \sim 600$ and (iii) $2\delta mc^2/\hbar\delta\Gamma \sim 2\delta mc^2/\hbar\Gamma \sim 1$.

In the following we restrict the Hamiltonian H_K to its Hermitian part ($\Gamma_2 = \Gamma_1 = 0$) in order to avoid unphysical *interferences* between (i) phenomenological Weisskopf-Wigner damping ($\Gamma, \delta\Gamma$) and (ii) first principles such as Dirac's equation for quarks dynamics and

Schwarzschild geometry for spacetime. We will demonstrate that the interplay between Weisskopf-Wigner decay and these first principles is in fact the source of a confusion between CPT and T violations.

The Hermitian/unitary evolution ($\Gamma = \delta\Gamma = 0$) of a stable kaon system in a flat spacetime is given by

$$j\frac{\hbar}{c^2}\frac{\partial}{\partial\tau}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} m & -\delta m \\ -\delta m & m \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}. \quad (10)$$

rather than (5). The finite lifetimes of the particles is included in section V to complete the model and will elucidate the confusion between T and CPT violations.

Several critical comments can be made about (10). The invariant relativistic energy (E)-momentum (p) dispersion relation for a particle with mass m is $E^2 - p^2c^2 = m^2c^4$. To set up a quantum description through the correspondence principle ($E \leftrightarrow j\hbar\partial/\partial\tau$) we have to consider the particle rest frame ($p = 0$) dispersion relation $\langle K^0 | E | K^0 \rangle = \pm mc^2$. The Schrödinger model (10) is based on a first order time derivation where energy is restricted to $+mc^2$, so it does not fulfill the full relativistic dispersion.

The relations $\langle K^0 | E | K^0 \rangle = \langle \bar{K}^0 | E | \bar{K}^0 \rangle = +mc^2$ used in (10) are just the rest frame restriction ($p = 0$) of the operator $\sqrt{p^2c^2 + m^2c^4}$ which is non-local in a Minkowski spacetime and so can not operate in a curved spacetime. A local evolution associated with the local invariant relation $E^2 = p^2 + m^2$ is more appropriate for a generalization to a curved spacetime and will ensure a fully covariant and local description.

An other critics of the model (10) is that nothing distinguish the \bar{K}^0 as the antiparticle of the K^0 . Nothing indicates that b is the amplitude of an antiparticle whose particle is described by the amplitude a . Both a and b describe particles and they have the same mass. This drawback will be cured with the Feynman prescription: if some additional velocity matrix element is involved in the description of the kaons rest frame evolution, we must allocate positive and negative sign to such coupling in order to describe one particle propagating forward in time and the other (the antiparticle) propagating backward time. By doing so we really describe a pair particle-antiparticle.

To set up a fully relativistic description of K^0/\bar{K}^0 oscillations there are two possibilities: either (i) to consider two weakly coupled K-G equations [38] for a and b , or (ii) to keep the first order time derivative and introduce a weak coupling between four Feshbach-Villars amplitudes $a \pm j\hbar\partial_\tau a/mc^2$ and $b \pm j\hbar\partial_\tau b/mc^2$ fulfilling a set of four first order equations within the framework of the Feshbach-Villars representation (F-V) [39]. The first order F-V equations and the second order K-G equations are two equivalent representations of the very same relativistic quantum dynamics. The K-G representation will be used in the following.

The use of a K-G representation follows the way Dirac resolved the apparent contradiction between the requirements of relativistic dynamics and quantum mechanics:

(i) energy-momentum dispersion relations are quadratic forms as the mass is a relativistic invariant, however (ii) wave equation are linear forms with respect to time derivation/energy in order to define a positive probability density associated with a unitary conservation relation. These two requirements are not incompatible if we consider a first order quantum evolution such that the iteration of the time derivation provides the relativistic dispersion relation through the correspondence principle. In doing so Dirac identified the the γ matrices algebra. Here, we use the iteration of a first order wave equation in a curved spacetime to identify the dominant gravitational coupling to be considered in the first order wave equation.

The gravitational energy of a kaon on earth is very small compared to its rest mass and kinetic energy, their ratio is of the order of 10^{-9} . This strong ordering allows the use of a K-G description in a Schwarzschild spacetime within the framework of a perturbative expansion.

Anticipating the results obtained in the next sections, the logical steps to extend the LOY model to curved spacetime can be summarized as follows.

The earth influence is described by a Schwarzschild geometry responsible for a small Hamiltonian h_\oplus . Then we perform a linear change of variables $|\Phi\rangle = |\Psi\rangle \exp(jmc^2\tau/\hbar)$, so that the inertial masses of the kaons are eliminated and we have to study coupled massless diquark states. With this representation the Hermitian part of the LOY Hamiltonian H_K (10) becomes H'_K and h_\oplus is unaltered. The evolution of $|\Phi\rangle$ is given by

$$j\hbar\frac{\partial}{\partial\tau}|\Phi\rangle = (H'_K + h_\oplus)|\Phi\rangle, \quad (11)$$

where $h_\oplus(\tau) \ll H'_K$ (the notation $h_\oplus \ll H'_K$ means the ordering between the corresponding matrix elements of both operators).

It is very important to recognize that, although $h_\oplus(\tau)$ ($\sim 10^{-15}H'_K$) describes a very small amplitude quark internal oscillations coupled to the earth gravitational field, these oscillations display a very high frequency so that $\hbar H'_K{}^{-1}\partial h_\oplus/\partial\tau$ ($\sim 10^{-3}H'_K$) is not so small in front of H'_K .

As explained previously a K-G description is more appropriate with respect to relativistic dispersion and locality. A K-G description allows to disclose the dominant gravitational coupling. This disclosure is then used to set up first order equations of the LOY type relevant to describe quantum evolution in a Schwarzschild geometry. Following Dirac's historical method, for the dominant coupling to emerge from (11), the proper time derivative is iterated

$$-\hbar^2\frac{\partial^2}{\partial\tau^2}|\Phi\rangle = \left(H'_K{}^2 + j\hbar\frac{\partial h_\oplus}{\partial\tau}\right)|\Phi\rangle + h_\oplus^2 + H'_K h_\oplus + h_\oplus H'_K |\Phi\rangle. \quad (12)$$

The strong ordering : $h_\oplus^2 \ll H'_K h_\oplus \ll \hbar\partial h_\oplus/\partial\tau$ and $\hbar H'_K{}^{-1}\partial h_\oplus/\partial\tau \ll H'_K$ is then used to neglect all the terms

very small in front of $\hbar\partial h_{\oplus}/\partial\tau$ and H'_K ($\partial h_{\oplus}/\partial\tau$ is independent of time and commute with H'_K)

$$-\hbar^2 \frac{\partial^2}{\partial\tau^2} |\Phi\rangle = \left(H'_K + \frac{j\hbar}{2} H'^{-1}_K \frac{\partial h_{\oplus}}{\partial\tau} \right)^2 |\Phi\rangle. \quad (13)$$

The solutions of the second-order equation (13) includes, of course, redundant solutions which do not satisfy the original first order equation (11). A 2×2 matrix has several square roots and the choice of the pertinent one is guided by the Minkowski limit $h_{\oplus} = 0$. The right framework to discuss quantum evolution and discrete symmetries is provided by a first order time evolution and there are two simple first order equations associated with (13)

$$j\hbar \frac{\partial}{\partial\tau} |\Phi\rangle = \pm \left(H'_K + \frac{j\hbar}{2} H'^{-1}_K \frac{\partial h_{\oplus}}{\partial\tau} \right) |\Phi\rangle. \quad (14)$$

We keep the positive sign to recover (11) in the limit $h_{\oplus} = 0$ and we adapt the resulting equations to the description of a couple particle/antiparticle according to the Feynman prescription as velocity matrix elements are involved in $\partial h_{\oplus}/\partial\tau$: a \bar{K}^0 is a K^0 propagating backward in time. Moreover this prescription will ensure real energy eigenvalues.

The Dirac relativistic position operator, involved in h_{\oplus} , displays unfamiliar and nonintuitive properties (zitterbewegung): the associated velocity operator, involved in $\partial h_{\oplus}/\partial\tau$, has two eigenvalues $\pm c$ associated with four eigenvectors describing pairs of fermion/antifermions. These properties will be presented in section IV.

The final step, in section V, is to take into account the short lifetime of the K_S which elucidates the CPT versus T confusion.

The first principles derivation, presented along these logical steps in the next sections, relates the observed CP violation to the interplay between two oscillations in a Schwarzschild spacetime : (i) quarks's zitterbewegung inherent to relativistic spin 1/2 dynamics and (ii) kaons mixing associated with second order ($\Delta S = 2$) weak interaction coupling. This interplay is neither an interference nor a resonance as the frequencies of these oscillations are rather different, this is a secular effect like ponderomotive effects which are usually described in term of an *effective* or a *dressed* mass in the field of oscillators theory [40, 41]. A discussion of the effective mass concept is presented in the last section.

Rather than an emphasis on relativistic covariance/invariance, the derivation of (14) from (11) through the intermediate step (13) can be simply viewed as the pragmatic use of a strong ordering to evaluate the impact of a small high frequency oscillation perturbation (quarks zitterbewegung) on a lower frequency $\Delta S = 2$ oscillator ($K^0 \rightleftharpoons \bar{K}^0$).

III. KAONS OSCILLATIONS IN A SCHWARZSCHILD SPACETIME

Having briefly reviewed the logic driving the back and forth transition between a first order quantum dynamics and a second order one, we address in this section the issue of the impact of spacetime curvature on the LOY model.

Near the earth surface, at a radius r from the earth center, the Schwarzschild metric is given by the line element

$$ds^2 = - \left(c^2 - \frac{2G_N M_{\oplus}}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2G_N M_{\oplus}}{rc^2}} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2), \quad (15)$$

where the notations (t, r, θ, φ) are standard and $M_{\oplus} = 5.9724 \times 10^{24}$ kg is the earth mass. This metric is fully characterized by the Schwarzschild radius

$$R_S = 2 \frac{G_N M_{\oplus}}{c^2} = 8.870 \times 10^{-3} \text{ m}. \quad (16)$$

The earth radius $R_{\oplus} = 6.3781 \times 10^6$ m is far larger than the Schwarzschild one, $R_S/R_{\oplus} \sim 10^{-9}$. In a Minkowski spacetime with time t , the energy E of a particle with mass m is:

$$E = mc^2 \frac{dt}{d\tau}, \quad (17)$$

where τ is the proper time of the particle. In a Schwarzschild spacetime described by the metric (15), the energy E of a particle with mass m is:

$$E = mc^2 (1 - R_S/r) \frac{dt}{d\tau}, \quad (18)$$

To extend (17) from Minkowski to Schwarzschild geometry (18), the covariant definition $E = -c^2 g_{0\beta} p_{\beta}$ is simply used. The kaon inertial rest frame ($t = \tau$) energy $E = mc^2$ becomes the Kaon free fall frame proper energy $E = mc^2 (1 - R_S/r)$.

In a kaon experiment r is ultimately the instantaneous *barycentric* position of the energy ($/c^2$) associated with the d/\bar{d} and s/\bar{s} quarks dynamics inside the kaons. These quarks perform unknown, small scale, high frequency, fast motions with respect to an average center of mass/energy. This average center of mass/energy of the kaon follows a geodesic which is experimentally a very slow vertical free fall combined with a fast horizontal (nearly) inertial motion as the curvature correction $mc^2 R_S/R_{\oplus} \sim 10^{-9} mc^2$ is small.

Moving to the horizontal inertial frame of the kaon, the radial position r , to be considered in (18), is decomposed as

$$r = R_{\oplus} + X + x, \quad (19)$$

the sum of: (i) the earth radius R_{\oplus} , plus (ii) the average vertical displacement $X(\tau)$, with respect to R_{\oplus} , associated with the average classical slow free fall motion of the kaon (τ is the proper time of the kaon), plus (iii) the fast, unknown, internal vertical motion x , with respect to $R_{\oplus} + X$, of the instantaneous *barycentric* position of the energy ($/c^2$) associated with the d/\bar{d} and s/\bar{s} quarks high frequency dynamics (zitterbewegung).

The instantaneous energy/mass position is described by $X + x$ and the average energy/mass position is described by X the average over the quarks dynamics, i.e., the kaon position.

The ordering $X + x \ll R_{\oplus}$ allows to expand the energy (18) in the kaon frame where $t = \tau$:

$$E = mc^2 \left[1 - \frac{R_S}{R_{\oplus}} + \frac{R_S}{R_{\oplus}^2} (X + x) \right]. \quad (20)$$

$$\hat{x} = \langle d\bar{s} | \hat{x} | d\bar{s} \rangle |K^0\rangle \langle K^0| + \langle \bar{d}s | \hat{x} | \bar{d}s \rangle | \bar{K}^0 \rangle \langle \bar{K}^0| + \langle \bar{d}s | \hat{x} | d\bar{s} \rangle | \bar{K}^0 \rangle \langle K^0| + \langle d\bar{s} | \hat{x} | \bar{d}s \rangle |K^0\rangle \langle \bar{K}^0|. \quad (21)$$

The dimension of the spinor tensorial product $|qq'\rangle$ is 16. We can neglect the dynamics of the d/\bar{d} quark component as the mass of the s/\bar{s} strange component account for

$$\hat{x} = \langle \bar{s} | \hat{x} | \bar{s} \rangle |K^0\rangle \langle K^0| + \langle s | \hat{x} | s \rangle | \bar{K}^0 \rangle \langle \bar{K}^0| + \langle s | \hat{x} | \bar{s} \rangle | \bar{K}^0 \rangle \langle K^0| + \langle \bar{s} | \hat{x} | s \rangle |K^0\rangle \langle \bar{K}^0|. \quad (22)$$

This strong ordering provides a reduction of the spinor state dimension from 16 down to 4 and allows the use of Dirac's equations to evaluate the \hat{x} matrix elements involved in (22).

The s and \bar{s} carry internal kinetic energy and potential energy in addition to mass energy and the kaon internal energy (mc^2) is attached to the strange quark internal

$$j \frac{\hbar}{mc^2} \frac{\partial |\Psi\rangle}{\partial \tau} = |\Psi\rangle - \frac{\delta m}{m} \left[| \bar{K}^0 \rangle \langle K^0| + |K^0\rangle \langle \bar{K}^0| \right] |\Psi\rangle - \frac{R_S}{R_{\oplus}^2} [R_{\oplus} - X(\tau)] |\Psi\rangle + \frac{R_S}{R_{\oplus}^2} \hat{x} \cdot |\Psi\rangle. \quad (23)$$

To simplify the analysis we eliminate the common phases of a and b associated with the inertial mass m , and the average potential energy resulting from the kaon average position $R_{\oplus} + X$, to obtain a simple representation: the

The proper time τ is the kaon rest frame time along a geodesic and $X(\tau)$ describes the vertical part of this geodesic, so it can not operate on the kaon internal state: X is not observable as an internal kaon dynamical variable in the kaon rest frame $X(\tau)$, it must be considered as an additional time dependant (negligible) energy.

The case is different for the quarks vertical motions x around $R_{\oplus} + X$ because τ is not the proper time of the quarks. The internal position operator \hat{x} operates in the tensorial product space of the d and s Dirac's spinors spaces. Kaons are spin 0 diquark states $|qq'\rangle$: $|K^0\rangle = |d\bar{s}\rangle$ and $| \bar{K}^0 \rangle = | \bar{d}s \rangle$. Quark states $|q\rangle$ and diquark states $|qq'\rangle$ are normalized to unity, diquark states are singlet combinations with respect to the spin but we keep the simple notations $|qq'\rangle$.

The representation of the vertical position operator \hat{x} in the kaon Hilbert space ($|K^0\rangle, | \bar{K}^0 \rangle$) is

96% of the sum of the d/\bar{d} and s/\bar{s} components. The instantaneous motion of the energy-barycenter inside a kaon is dominated by the strange quark dynamics,

motion. The strong mass ordering between s and d justify the restriction of the internal position x to the heaviest quark dynamics, the lighter quarks d and \bar{d} are just followers of the kaons slow free fall dynamics and passive witness of the s/\bar{s} zitterbewegung fast motion.

The coupled Schrödinger equations on earth ($R_S \neq 0$, $\Gamma = \delta\Gamma = 0$) are given by

state $|\Phi\rangle$ defined as

$$|\Phi\rangle = |\Psi\rangle \exp j \frac{mc^2}{\hbar} \tau \times \exp -j \frac{mc^2}{\hbar} \frac{R_S}{R_{\oplus}} \left[\int_0^\tau du - \int_0^\tau \frac{X(u)}{R_{\oplus}} du \right]. \quad (24)$$

The first phase on the second line of (24) is not observable in experiments because: (i) it is very small

($R_S/R_\oplus \sim 10^{-9}$) in front of the main phase on the first line $mc^2\tau/\hbar$, and (ii) as opposed to the $\delta m/m \sim 10^{-15}$ mixing correction to m , which increases the mass of K_2 and decreases the mass of K_1 , this R_S/R_\oplus correction add up in the same way to the K_2 and K_1 energies so that interferometry is impossible.

The second phase on the second line of (24) scales as $R_S X/R_\oplus^2 \sim X [\text{m}] \times 10^{-16}$ and is smaller than the first one. Kaons beamlines are essentially horizontal and X is almost constant. A small dispersion of the measurements, due to the dispersion of the small vertical path X in the kaon beam might be considered, nevertheless, both phase terms on the second line of (24) are negligible and neglected in front of $mc^2\tau/\hbar$ and unobservable with interferometric experiments.

Given the precision of the experiments, we can consider:

$$|\Phi\rangle = |\Psi\rangle \exp j \left(\frac{mc^2}{\hbar} \tau \right), \quad (25)$$

and the amplitudes u and v of $|\Phi\rangle$

$$|\Phi\rangle = u(\tau) |K^0\rangle + v(\tau) |\bar{K}^0\rangle, \quad (26)$$

describe the evolution of coupled quark-antiquark and are solutions of

$$j \frac{\hbar}{c^2} \frac{\partial}{\partial \tau} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 & -\delta m \\ -\delta m & 0 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} + m \frac{R_S}{R_\oplus^2} \times \begin{bmatrix} \langle \bar{s} | \hat{x} | \bar{s} \rangle & \langle \bar{s} | \hat{x} | s \rangle \\ \langle s | \hat{x} | \bar{s} \rangle & \langle s | \hat{x} | s \rangle \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}, \quad (27)$$

where we have used the relations (23,24,22) under the assumption of the strange quark dynamical dominance.

The Hamiltonian H'_K and h_\oplus , introduced in (11) to present the logical steps of the model, can be identified as the first and second 2×2 matrix on the right hand side of (27). An in-depth discussion of the four matrix elements $\langle \hat{x} \rangle$ is not needed as the final result will not involve them. It is sufficient to evaluate an upper bound of their orders of magnitude. The Compton wavelength ($\lambda_C = \hbar/mc = 3.96 \times 10^{-16}$ m) of the kaon provides such an upper bound as quarks are bound states inside the volume of the kaon. The very small numerical value 8.6×10^{-32} of the ratio $R_S \lambda_C / R_\oplus^2$ leads to the occurrence of a very strong ordering fulfilled by these four matrix elements

$$m \frac{R_S}{R_\oplus^2} |\langle \hat{x} \rangle| < 10^{-15} \delta m. \quad (28)$$

On the basis of this ordering, the influence of gravity might be neglected, however before dropping this interaction we have to examine it within a fully relativistic framework.

Such a framework is provided by a K-G description and moreover, even if $mc^2 (R_S/R_\oplus^2) |\langle \hat{x} \rangle|$ describes a very small amplitude quark internal oscillations (*zitterbewegung*) coupled to the earth gravitational

field, these are very high frequency oscillations so that $\hbar (m/\delta m) (R_S/R_\oplus^2) \partial |\langle \hat{x} \rangle| / \partial \tau$ are not so small in front of δmc^2 .

Following Dirac's historical method, the relativistic K-G dynamics associated with (27) is obtained through an iteration of the proper time derivation. The orders of magnitude of two of the resulting terms, $m^2 R_S^2 \lambda_C^2 / R_\oplus^4$ $\delta m^2 \sim 10^{-34}$ and $m R_S \lambda_C / \delta m R_\oplus^2 \sim 10^{-17}$, ensure that all the ordering listed below the relation (12) are fulfilled. Among the various gravitational coupling we keep only the dominant one and the iteration of ∂_τ on (27) gives

$$-\frac{\hbar^2}{c^4} \frac{\partial^2}{\partial \tau^2} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \delta m^2 & 0 \\ 0 & \delta m^2 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} + j \frac{m\hbar}{c^2} \frac{R_S}{R_\oplus^2} \times \begin{bmatrix} \frac{\partial \langle \bar{s} | \hat{x} | \bar{s} \rangle}{\partial \tau} & \frac{\partial \langle \bar{s} | \hat{x} | s \rangle}{\partial \tau} \\ \frac{\partial \langle s | \hat{x} | \bar{s} \rangle}{\partial \tau} & \frac{\partial \langle s | \hat{x} | s \rangle}{\partial \tau} \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}. \quad (29)$$

The evolution described by (29) requires the knowledge of spinor velocity matrix elements of the type $\partial \langle \hat{x} \rangle / \partial \tau$. The next section is dedicated to this calculation.

IV. BOUND QUARKS VELOCITY MATRIX ELEMENTS

We define the set of 4×4 matrices: $\alpha = (\alpha_x, \alpha_y, \alpha_z)$ and β [38], expressed in terms of the 2×2 Pauli matrices $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ and the 2×2 identity matrix I as

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \quad (30)$$

These matrices are involved in the Dirac equations describing $|\bar{s}\rangle$ and $|s\rangle$ spinors evolutions inside kaons. Without loss of generality we take the x direction of the Dirac representation (30) as the vertical direction. We will demonstrate in this section that the matrix

$$\begin{bmatrix} \frac{\partial \langle \bar{s} | \hat{x} | \bar{s} \rangle}{\partial \tau} & \frac{\partial \langle \bar{s} | \hat{x} | s \rangle}{\partial \tau} \\ \frac{\partial \langle s | \hat{x} | \bar{s} \rangle}{\partial \tau} & \frac{\partial \langle s | \hat{x} | s \rangle}{\partial \tau} \end{bmatrix} = c \begin{bmatrix} \langle \bar{s} | \alpha_x | \bar{s} \rangle & \langle \bar{s} | \alpha_x | s \rangle \\ \langle s | \alpha_x | \bar{s} \rangle & \langle s | \alpha_x | s \rangle \end{bmatrix} = \begin{bmatrix} 0 & c \\ c & 0 \end{bmatrix}. \quad (31)$$

The time derivative of the diagonal elements $\langle s | \hat{x} | s \rangle$ and $\langle \bar{s} | \hat{x} | \bar{s} \rangle$ are 0 and the time derivative of the non diagonal terms $\langle s | \hat{x} | \bar{s} \rangle$ and $\langle \bar{s} | \hat{x} | s \rangle$ are c . This result, independent of the charge, mass and coupling of the quarks inside the kaons, is part of the various nonintuitive behaviors of relativistic spin 1/2 particles named *zitterbewegung* [38]. The term *zitterbewegung* as used in this study is given wide meaning. It means all the properties associated with the velocity of the spin 1/2 particles, either free or bound, and cover a larger set of results than those restricted to its original meaning: *trembling motion* of relativistic free fermions.

A simple argument to sustain the validity of Eq. (31) is the fact that, for a localized bound state, the average

velocity $\partial \langle s | \hat{x} | s \rangle / \partial \tau$ and $\partial \langle \bar{s} | \hat{x} | \bar{s} \rangle / \partial \tau$ is zero and the eigenvectors of the α_x operator are of the type

$$c \begin{bmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{bmatrix} \cdot \begin{bmatrix} \lambda \\ \mu \\ \pm \mu \\ \pm \lambda \end{bmatrix} = \pm c \begin{bmatrix} \lambda \\ \mu \\ \pm \mu \\ \pm \lambda \end{bmatrix}. \quad (32)$$

These two (λ, μ) spinors are associated with fermion-antifermion pairs with the same amplitude for the particle and the antiparticle. Besides arguments, several proof of the identity Eq. (31) can be constructed. For example, a lengthy demonstration can be constructed with the help of (i) a Dirac spinor basis and (ii) a charge conjugation operator interpretation of s and \bar{s} relations. If we note \mathbf{p} the linear momentum and \pm the spin states, a complete basis $|s, \mathbf{p}\pm\rangle$ and $|\bar{s}, \mathbf{p}\pm\rangle$ can be used to expand the stationary bound state wave-function $\langle \mathbf{x} | s_K \rangle$ inside the kaon:

$$\langle \mathbf{x} | s_K \rangle = \sum_{+-} \int d\mathbf{p} \langle \mathbf{x} | \mathbf{p}\pm \rangle \langle s, \mathbf{p}\pm | s_K \rangle, \quad (33)$$

where $\langle \mathbf{x} | \mathbf{p}\pm \rangle \sim \exp i\mathbf{p} \cdot \mathbf{x} / \hbar$. The evaluations presented on pages 37-38 of reference [38] provides guidelines with Gordon's relation [38, 42], but in Ref. [38] these are space time interfering waves associated with a free particle, although here we consider a basis and the bound state assumption imply a set of relations of the type $|\langle s, \mathbf{p}\pm | s_K \rangle| = |\langle s, -\mathbf{p}\pm | s_K \rangle|$ between the Fourier coefficient of $\langle \mathbf{x} | s_K \rangle$. Gordon's relation simplify the analysis and the convection current and spin current cancel together for $\langle s | \alpha_x | s \rangle$ and not for $\langle s | \alpha_x | \bar{s} \rangle$. Rather than this lengthy demonstration in Fourier space, we address the issue of the proof of (31) starting from Dirac's equation in real space.

The relativistic dynamics of the s and \bar{s} quantum amplitudes is described by two 4 components Dirac spinors $|\phi\rangle$ and $|\varphi\rangle$. The Dirac Hamiltonians H_S and $H_{\bar{S}}$ are constructed on the basis of the interpretation of Feynman's propagator theory where antiparticles are viewed as particles propagating backward in time

$$j\hbar \frac{\partial}{\partial t} |\phi\rangle = H_S |\phi\rangle, \quad (34)$$

$$-j\hbar \frac{\partial}{\partial t} |\varphi\rangle = H_{\bar{S}} |\varphi\rangle. \quad (35)$$

The Hamiltonian of the particle and the antiparticle are:

$$H_S = c\boldsymbol{\alpha} \cdot \mathbf{p} + U(\mathbf{x})\beta + m_s c^2 \beta, \quad (36)$$

$$H_{\bar{S}} = -c\boldsymbol{\alpha} \cdot \mathbf{p} + U(\mathbf{x})\beta + m_{\bar{s}} c^2 \beta. \quad (37)$$

The position operator \mathbf{x} and momentum operator \mathbf{p} fulfill the canonical commutations rules. The unknown *scalar potential* U describes the mean field strong interactions ultimately ensuring quarks confinement in the kaons bag. The d and \bar{d} follow the \bar{s} and s so we assume that charge neutralization takes place and we neglect the impact of

a mean field electromagnetic interaction which is usually screened on average in such a globally neutral and very hot plasma.

One of the puzzling property of Dirac's equations is the expression of the velocity matrix element $\partial \langle \phi_1 | \mathbf{x} | \phi_2 \rangle / \partial t$ between two spinors solutions of (34),

$$\frac{\partial}{\partial t} \langle \phi_1 | \mathbf{x} | \phi_2 \rangle = \frac{j}{\hbar} \langle \phi_1 | H_S \mathbf{x} - \mathbf{x} H_S | \phi_2 \rangle. \quad (38)$$

The commutator $[H_S, \mathbf{x}]$ is easily evaluated as the canonical commutator $[x_i, p_j]$ is equal to $j\hbar \delta_j^i$. The position \mathbf{x} commutes with α and β and any well behaved function of \mathbf{x} . The final result is that $[H_S, \mathbf{x}]$ is simply equal to $-j\hbar c\boldsymbol{\alpha}$ thus

$$\frac{\partial}{\partial t} \langle \phi_1 | \mathbf{x} | \phi_2 \rangle = c \langle \phi_1 | \boldsymbol{\alpha} | \phi_2 \rangle. \quad (39)$$

It is very important to note that this result is independent of the unknown potential U and of the mass of the quark. Historically, the name *zitterbewegung* was introduced by Schrödinger to characterize the fast oscillations of free particles when Eq. (39), restricted to the particular case of free fermions ($U = 0$), is integrated. Here *zitterbewegung* is given wide meaning: the very general relation (39) for bound quarks.

A probability current and a probability density $|\langle \mathbf{x} | \phi \rangle|^2$ are associated with a state $|\phi\rangle$ solution of (34) [38]. The conservation equation between current and density is given by the usual divergence relation

$$\frac{\partial}{\partial \mathbf{x}} \cdot c [\langle \phi | \mathbf{x} \rangle \cdot \boldsymbol{\alpha} \cdot \langle \mathbf{x} | \phi \rangle] + \frac{\partial}{\partial t} |\langle \mathbf{x} | \phi \rangle|^2 = 0. \quad (40)$$

The time evolution of the density, $\partial |\langle \mathbf{x} | \phi \rangle|^2 / \partial t$, cancels for an energy eigenstate $\langle \mathbf{x} | \phi \rangle$ localized in space. Thus for stationary bound states the probability current is the rotational of a localized vector field \mathbf{F} :

$$\langle \phi | \mathbf{x} \rangle \cdot \boldsymbol{\alpha} \cdot \langle \mathbf{x} | \phi \rangle = \nabla \times \mathbf{F}. \quad (41)$$

Considering a very large volume \mathcal{V} , bounded by a surface \mathcal{S} enclosing this localized bound state $\langle \mathbf{x} | \phi \rangle$:

$$\int_{\mathcal{V}} \nabla \times \mathbf{F} dV = \int_{\mathcal{S}} \mathbf{n} \times \mathbf{F} dS = \mathbf{0}, \quad (42)$$

(\mathbf{n} is the outward normal on \mathcal{S}) if $\langle \mathbf{x} | \phi \rangle$ is assumed to decay sufficiently rapidly at large distance $|\mathbf{x}|$. This spatial decay is exponential for quarks bound states. The relations (41,42) can be rewritten

$$\int_{\mathcal{V}} \langle \phi | \mathbf{x} \rangle \cdot \boldsymbol{\alpha} \cdot \langle \mathbf{x} | \phi \rangle d\mathbf{x} = \langle \phi | \boldsymbol{\alpha} | \phi \rangle = \mathbf{0}. \quad (43)$$

The case of $H_{\bar{S}}$ is similar and with the relations (39,43) we have demonstrated the nullity of the two diagonal elements in (31).

The evaluation of non diagonal elements requires some additional specifications. Consider now the velocity matrix element $\partial \langle \varphi | \mathbf{x} | \phi \rangle / \partial t$ between a spinor ϕ solution of (34), and a spinor φ solution of (35)

$$\frac{\partial}{\partial t} \langle \varphi | \mathbf{x} | \phi \rangle = \frac{j}{\hbar} \langle \varphi | H_{\bar{S}} \mathbf{x} - \mathbf{x} H_S | \phi \rangle. \quad (44)$$

Despite the fact that the two spinors φ and ϕ are the solutions of two different equations, each being the time reversed of the other, a simple expression similar to Eq.(39) is obtained,

$$\frac{\partial}{\partial t} \langle \varphi | \mathbf{x} | \phi \rangle = c \langle \varphi | \boldsymbol{\alpha} | \phi \rangle. \quad (45)$$

To construct the appropriate spinor states $|s\rangle$ and $|\bar{s}\rangle$ to be used in (29) we have to consider the bound states of strange quark s and \bar{s} describing stationary solution of (34,35). Without loss of generality we have taken the x direction of the Dirac representation (30) as the vertical direction. The spacetime geometry has revealed a coupling associated with the dynamics along this x vertical direction independently of the y and z directions, thus we can forget these directions and take $p_y = p_z = 0$ in the horizontal rest frame of the quark (the case $p_{y,z} \neq 0$ is obtained with a simple (y, z) plane wave Fourier expansion and gives the same final result).

We consider two energy eigen-spinors of the type $\langle x | s \rangle \exp -jEt/\hbar$ and $\langle x | \bar{s} \rangle \exp -jEt/\hbar$. The 8 first order coupled differential equations (34) and (35) are in fact 4 times the repetition of a reduced set of 2 equations describing the coupling between two wave-functions $\chi(x)$ and $\zeta(x)$

$$j\hbar c \frac{\partial \chi}{\partial x} + [E + U(x) + m_s c^2] \zeta(x) = 0, \quad (46)$$

$$j\hbar c \frac{\partial \zeta}{\partial x} + [E - U(x) - m_s c^2] \chi(x) = 0. \quad (47)$$

Considering localized solutions such that $\zeta(\pm\infty) = \chi(\pm\infty) = 0$, the general steady state solutions of the 8

coupled differential equations (34) and (35) can be written

$$\langle x | s \rangle = N \begin{bmatrix} \lambda \chi(x) \\ \mu \chi(x) \\ \mu \zeta(x) \\ \lambda \zeta(x) \end{bmatrix}, \quad \langle x | \bar{s} \rangle = N \begin{bmatrix} \lambda \zeta(x) \\ \mu \zeta(x) \\ \mu \chi(x) \\ \lambda \chi(x) \end{bmatrix}. \quad (48)$$

The factor N is adjusted to ensure normalization: $\langle s | s \rangle = 1$ and $\langle \bar{s} | \bar{s} \rangle = 1$. The two complex numbers (λ, μ) specify the spin state. The expected property assumed in (31) is demonstrated as

$$\langle \bar{s} | \alpha_x | s \rangle = \langle s | \alpha_x | \bar{s} \rangle = \langle s | s \rangle = 1. \quad (49)$$

The results of this section on strange quark and antiquark confined in a kaons is summarized by Eq.(31) which is independent of the quark mass, charge and confining potential.

V. NEUTRAL KAONS CPT VERSUS T INDIRECT VIOLATIONS

On the basis of the previous sections, we introduce the small parameter $\kappa \ll 1$ defined by

$$\kappa = \frac{m\hbar}{\delta m^2 c} \frac{R_S}{4R_{\oplus}^2} = \left(\frac{m}{2\delta m} \right)^2 \frac{R_S \lambda_C}{R_{\oplus}^2} = 1.7 \times 10^{-3}, \quad (50)$$

so that the relations (29) and (31) can be written

$$-\frac{\hbar^2}{c^4} \frac{\partial^2}{\partial \tau^2} \begin{bmatrix} u \\ v \end{bmatrix} = \delta m^2 \begin{bmatrix} 1 & 4j\kappa \\ 4j\kappa & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}. \quad (51)$$

The right hand side matrix in (51) can be reduced to a simple square with an accuracy $O[\kappa^3] - O[\kappa^4] \sim 10^{-9} - 10^{-12}$

$$-\frac{\hbar^2}{c^4} \frac{\partial^2}{\partial \tau^2} \begin{bmatrix} u \\ v \end{bmatrix} = \delta m^2 \left(\pm \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \pm \begin{bmatrix} 2j\kappa & 2\kappa^2 \\ 2\kappa^2 & 2j\kappa \end{bmatrix} \right)^2 \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \delta m^2 \times O \begin{bmatrix} \kappa^4 & \kappa^3 \\ \kappa^3 & \kappa^4 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}. \quad (52)$$

Despite the fact that such a K-G evolution is associated with the complete relativistic energy-momentum dispersion, the right quantum framework to discuss discrete symmetries is a set of first order equations of the LOY type. The solutions of the second-order equation Eq.(52) includes redundant solutions which do not satisfy the original first order equation. Among the first order equations leading to Eq.(52) we have to consider the one whose limit is (10) when $\kappa = 0$. From (52) we go back to

(u, v) first order evolutions

$$j \frac{\hbar}{c^2} \frac{\partial}{\partial \tau} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 & -\delta m \\ -\delta m & 0 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \delta m \begin{bmatrix} 2j\kappa & -2\kappa^2 \\ -2\kappa^2 & -2j\kappa \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}. \quad (53)$$

We have implemented the Feynman prescription, in going from Eq. (52) to Eq.(53), to distinguish the \bar{s}/K^0 from its antiparticle the s/\bar{K}^0 as $\kappa \sim \partial \langle \bar{s} | \hat{x} | s \rangle / \partial \tau$ is proportional to a velocity matrix element. By doing so we

describe the dynamics of a pair particle-antiparticle, but the iteration of the time derivation in (53) no longer gives (52). This is not surprising because (53) is a reduced description as (52) should be first recast as four coupled F-V equations in order to set up a first order quantum evolution [39]. Then two of the four F-V relations should be selected and interpreted as a Schrödinger LOY model. The direct reduction to two coupled Schrödinger equations is a shortcut validated by the limit $\kappa = 0$ and the Feynman rule.

Then, going back to the initial kaons amplitudes (a, b) defined by (25,26,2), the LOY model taking into account the lowest order spacetime curvature effect, revealed by (52), is given by

$$j\hbar \frac{\partial}{\partial \tau} \begin{bmatrix} a \\ b \end{bmatrix} = c^2 \begin{bmatrix} m & -\delta m \\ -\delta m & m \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} + \delta m c^2 \begin{bmatrix} 2j\kappa & -2\kappa^2 \\ -2\kappa^2 & -2j\kappa \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}. \quad (54)$$

The flat Minkowski spacetime limit $\kappa = 0$ (10) is recovered. The energy eigenvalues of equation (54) are real and equal to m_1 and m_2 up to a negligible $\delta m \times 10^{-12}$ correction $m_{1/2} = m \mp \delta m (1 + O[\kappa^4])$.

The eigenvectors of (54): K_L^\oplus and K_S^\oplus , are respectively associated with the time evolutions $\exp -j(m + \delta m) c^2 \tau / \hbar$ and $\exp -j(m - \delta m) c^2 \tau / \hbar$:

$$|K_L^\oplus\rangle = \frac{1 + j\kappa}{\sqrt{2}} |K^0\rangle - \frac{1 - j\kappa}{\sqrt{2}} |\bar{K}^0\rangle, \quad (55)$$

$$|K_S^\oplus\rangle = \frac{1 - j\kappa}{\sqrt{2}} |K^0\rangle + \frac{1 + j\kappa}{\sqrt{2}} |\bar{K}^0\rangle. \quad (56)$$

We have neglected $O[\kappa^2]$ terms, a better accuracy, where $O[\kappa^3]$ are neglected, can be achieved with an higher order κ expansion of the eigenvectors of (54) followed by a normalization of these eigenvectors. This $O[\kappa^3]$ result is obtained by simply adding $\kappa^2/2$ to $j\kappa$ in (55,56) and then normalize. It does not change the results below.

These two superpositions (55) and (56) are energy eigenstates but no longer CP eigenstates.

The very first consequence of these relations is the identification of the origin of CP violation in the mixing and the prediction of the amplitude of this violation

$$\left. \frac{\langle K_S^\oplus | K_L^\oplus \rangle}{2} \right|_{\Gamma=0} = j\kappa. \quad (57)$$

This relation demonstrates that, in a Schwarzschild spacetime, T conservation holds but CPT no longer holds [36].

The two stables kaons $|\tilde{K}_L\rangle$ and $|\tilde{K}_S\rangle$ solutions of Eq.(54) evolve over time according to

$$|\tilde{K}_L\rangle = \exp -j(m + \delta m) \frac{c^2}{\hbar} \tau |K_L^\oplus\rangle, \quad (58)$$

$$|\tilde{K}_S\rangle = \exp -j(m - \delta m) \frac{c^2}{\hbar} \tau |K_S^\oplus\rangle. \quad (59)$$

This result is restricted to stable kaons ($\Gamma = \delta\Gamma = 0$) in a Schwarzschild spacetime. On earth the experiments take place in a curved spacetime and, moreover, they are highly constrained by the short lifetime of the unstable K_S .

To predict the measured values of the different parameters we have to take into account this K_S finite lifetime. Decays are described as irreversible processes by the Weisskopf-Wigner approximation, so we expect that T will be violated.

We assume that the K_L is stable and that the depletion of its amplitude, associated with K_S gravitational regeneration, is negligible. In the following a star index $(*)$ indicates the *dressing* resulting from the opening of the various K_S decay channels (the star exponent $(*)$ means complex conjugation). The *dressed* kaons energy eigenstates, $(|\tilde{K}_{L*}\rangle, |\tilde{K}_{S*}\rangle)$ and $(|K_{L*}^\oplus\rangle, |K_{S*}^\oplus\rangle)$, are related through relations similar to the relations (58,59)

$$|\tilde{K}_{L*}\rangle = \exp -j(m + \delta m) \frac{c^2}{\hbar} \tau |K_{L*}^\oplus\rangle, \quad (60)$$

$$|\tilde{K}_{S*}\rangle = \exp -j(m - \delta m) \frac{c^2}{\hbar} \tau |K_{S*}^\oplus\rangle. \quad (61)$$

Within the framework of the Weisskopf-Wigner approximation we consider that the evolution of K_{S*} is described by a balance between: (i) a decay with a rate $(\Gamma - \delta\Gamma)/2 \approx -\delta\Gamma \approx \Gamma$ (7,9) and (ii) a gravitational regeneration at a rate $|\tilde{K}_L\rangle \partial \langle \tilde{K}_L | \tilde{K}_S \rangle / \partial \tau$ with

$$\frac{\partial}{\partial \tau} \langle \tilde{K}_L | \tilde{K}_S \rangle = 2j\delta m \frac{c^2}{\hbar} \langle K_L^\oplus | K_S^\oplus \rangle \exp 2j\delta m \frac{c^2}{\hbar} \tau, \quad (62)$$

where we have used (58,59).

The Schrödinger equations for the dressed $|\tilde{K}_{S*}\rangle$ defined in (61) is

$$\frac{\partial |\tilde{K}_{S*}\rangle}{\partial \tau} = -j \frac{c^2}{\hbar} (m - \delta m) |\tilde{K}_{S*}\rangle - \Gamma |\tilde{K}_{S*}\rangle + |\tilde{K}_{L*}\rangle \left[\frac{\partial}{\partial \tau} \langle \tilde{K}_L | \tilde{K}_S \rangle \right] \quad (63)$$

The overlap of the *dressed* energy eigenstates $|K_{S*}^\oplus\rangle$ and $|K_{L*}^\oplus\rangle$, i.e. the observed (indirect) CP violation parameter, is obtained from the solution of the steady state balance between decay and gravitational regeneration on the second line of (63),

$$-\Gamma \langle K_{L*}^\oplus | K_{S*}^\oplus \rangle + 2j\delta m \frac{c^2}{\hbar} \langle K_L^\oplus | K_S^\oplus \rangle = 0, \quad (64)$$

where we have used $\langle K_{L*}^\oplus | K_{L*}^\oplus \rangle = \langle K_L^\oplus | K_L^\oplus \rangle = 1$ and the relations (62,60,61). We conclude that the measured *dressed* CP violation parameter $\langle K_{L*}^\oplus | K_{S*}^\oplus \rangle$ is a real number despite the fact that the *bare* $\langle K_L^\oplus | K_S^\oplus \rangle$ is an

imaginary number

$$\left. \frac{\langle K_{L^*}^\oplus | K_{S^*}^\oplus \rangle}{2} \right|_{\Gamma \neq 0} = \left(\frac{\delta m c^2}{\hbar \Gamma / 2} \right) \kappa = 1.6 \times 10^{-3}, \quad (65)$$

in good agreement with the experimental data [43]. As expected T no longer holds.

Note that, if the restriction of the dynamics to the strange quark is taken into account through a lowering of the mass m down to $(m_s/m_d + m_s) \times m$, this provides the value

$$\frac{m_s}{m_d + m_s} \left(\frac{\delta m c^2}{\hbar \Gamma / 2} \right) \kappa = 1.60 \times 10^{-3}. \quad (66)$$

The numerical relation $m g \hbar / (2 \delta m)^2 c^3 \approx \text{Re } \varepsilon / 2$ ($g = c^2 R_S / 2 R_\oplus^2 = 9.8 \text{ m/s}^2$ and ε is the experimental CP violation parameter) was identified four decades ago by E. Fischbach [44]. At that time Fischbach was exploring the impact of a gravitational coupling of the order of $\hbar g / 2 \delta m c^3$ and he noted the puzzling relation: a simple *ad hoc* increase by a factor $m / 2 \delta m$ gives $\text{Re } \varepsilon / 2$ (at that time the experimental value of $\text{Re } \varepsilon$ was slightly larger).

The confusion between gravitational CPT violation and T conservation on the one hand and the usual view of CPT conservation and T violation on the other hand, comes from the fact that the usual LOY model in a flat spacetime overlooked the necessity to separate: (i) first the overlap of bare-stable states due to fundamental interactions (weak and gravity) and (ii) then the overlap of dressed-unstable states described within the framework of the Weisskopf-Wigner approximation. This is why, on the basis of the experimental results, the observed real CP violation parameter (65) was added as a non diagonal CPT conserving term rather than a diagonal T conserving term in the LOY Hamiltonian (6).

That the inclusion of decays induces a T violation is not surprising as the Weisskopf-Wigner approximation describes an irreversible process.

At the fundamental level the CP violation observed on earth in the K^0/\bar{K}^0 system is a CPT violation and the K_S finite lifetime makes it appear as a T violation.

VI. INTERPRETATION OF NEUTRAL KAONS DIRECT CP VIOLATION

According to the previous section, indirect CP violation find a simple explanation when (i) spacetime curvature is considered and then (ii) the K_S finite lifetime is taken into account.

We consider now both K_L finite lifetime and K_S finite lifetime as direct CP violation is precisely the issue of K_L CP violating decays.

A way to extract the indirect (ε) and direct (ε') CP violations parameters is to analyze the 2π decays of K_L and K_S through the amplitude ratios η_{00} and η_{+-} which are phase convention dependant quantities [36]. Unlike

$K_L \rightarrow 2\pi$ decays, semileptonic decays, $K_L \rightarrow \pi^+ l^- \bar{\nu}_l$ and $K_L \rightarrow \pi^- l^+ \nu_l$, are allowed even without CP violation and are sensible to indirect CP violation only. In the following analysis we do not consider semileptonic decays and we restrict the study to 2π decays: $K_L \rightarrow \pi^0 \pi^0$ and $K_L \rightarrow \pi^+ \pi^-$.

When CPT is not conserved, their amplitude ratios η_{00} and η_{+-} are defined as

$$\eta_{00} = \frac{A(K_{L^*} \rightarrow \pi^0 \pi^0) \langle K^0 | K_{S^*}^\oplus \rangle}{A(K_{S^*} \rightarrow \pi^0 \pi^0) \langle K^0 | K_{L^*}^\oplus \rangle}, \quad (67)$$

$$\eta_{+-} = \frac{A(K_{L^*} \rightarrow \pi^+ \pi^-) \langle K^0 | K_{S^*}^\oplus \rangle}{A(K_{S^*} \rightarrow \pi^+ \pi^-) \langle K^0 | K_{L^*}^\oplus \rangle}. \quad (68)$$

The *rephasing factors* $\langle K^0 | K_{S^*}^\oplus \rangle / \langle K^0 | K_{L^*}^\oplus \rangle$ in (67) and (68) are mandatory to define phase convention independent amplitude ratios η_{00} and η_{+-} and obtain phase convention invariant quantities, i.e. meaningful physical quantities.

This factor $\langle K^0 | K_{S^*}^\oplus \rangle / \langle K^0 | K_{L^*}^\oplus \rangle$ is one under the assumption of CPT conservation.

To evaluate the rephasing factor $\langle K^0 | K_{S^*}^\oplus \rangle / \langle K^0 | K_{L^*}^\oplus \rangle$ we use Eqs.(55,56) before dissipation and calculate $\langle K^0 | K_S^\oplus \rangle / \langle K^0 | K_L^\oplus \rangle$, the result is $1 - \langle K_S^\oplus | K_L^\oplus \rangle$. Then this rephasing factor is dressed by the short-lived kaons decays (65) and becomes

$$\frac{\langle K^0 | K_{S^*}^\oplus \rangle}{\langle K^0 | K_{L^*}^\oplus \rangle} = 1 - 2\kappa \left(\frac{\delta m c^2}{\hbar \Gamma / 2} \right). \quad (69)$$

We use now the Bell-Steinberger (B-S) unitarity relations [45] to study the relations between the amplitude ratios, η_{00} and η_{+-} , and the parameters of the LOY model. The depletion of the kaon state $|\Psi\rangle$ (2) is described by the set of amplitudes $\langle f | \mathcal{T} | \Psi \rangle$ according to

$$\left. \frac{\partial \langle \Psi | \Psi \rangle}{\partial \tau} \right|_{\tau=0} = - \sum_f |\langle f | \mathcal{T} | \Psi(\tau=0) \rangle|^2, \quad (70)$$

where f runs over the set of all the allowed final states. The kaon state $|\Psi\rangle$ is a linear superposition of the *dressed* unstable states

$$|\Psi\rangle = A \exp -j(M + \delta M) \frac{c^2}{\hbar} \tau |K_{L^*}^\oplus\rangle + B \exp -j(M - \delta M) \frac{c^2}{\hbar} \tau |K_{S^*}^\oplus\rangle, \quad (71)$$

where M and δM are given by the definitions (7,8,9). The relation (70) is evaluated with (71) and the coefficients of AA^* , BB^* and AB^* give the three relations:

$$\Gamma_S = \sum_f |\langle f | \mathcal{T} | K_{S^*}^\oplus \rangle|^2, \quad (72)$$

$$\Gamma_L = \sum_f |\langle f | \mathcal{T} | K_{L^*}^\oplus \rangle|^2, \quad (73)$$

$$2j \frac{c^2}{\hbar} \delta m + \frac{\Gamma_S}{2} = \sum_f \frac{\langle f | \mathcal{T} | K_{L^*}^\oplus \rangle \langle f | \mathcal{T} | K_{S^*}^\oplus \rangle^*}{\langle K_{S^*}^\oplus | K_{L^*}^\oplus \rangle}, \quad (74)$$

where we note the inverse of the mean lifetimes Γ_S and Γ_L rather than Γ_1 and Γ_2 ($\Gamma_S/2 = \Gamma$). For K_S the sum over f is dominated by $K_S \rightarrow 2\pi$ decays, and more precisely by the K_S decay to the isospin-zero combination of $|\pi^+\pi^- \rangle$ and $|\pi^0\pi^0 \rangle$ noted $|I_0 \rangle$. Neglecting the others decays, and restricting the right hand side of (74) to this single 2π isospin $I = 0$ amplitude, we obtain

$$\sum_f \langle f | \mathcal{T} | K_{L^*}^\oplus \rangle \langle f | \mathcal{T} | K_{S^*}^\oplus \rangle^* = \langle I_0 | \mathcal{T} | K_{L^*}^\oplus \rangle \langle I_0 | \mathcal{T} | K_{S^*}^\oplus \rangle^* \quad (75)$$

which is accurate to within 5 %. The right hand side of (75) can be expressed as a function of the amplitude ratios (67) and (68) with the help of the isospin decomposition of $|\pi^+\pi^- \rangle$ and $|\pi^0\pi^0 \rangle$ states:

$$\left(\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00} \right) \Gamma_S = \langle I_0 | \mathcal{T} | K_{L^*}^\oplus \rangle \langle I_0 | \mathcal{T} | K_{S^*}^\oplus \rangle^*. \quad (76)$$

The resulting approximate B-S unitarity relation [45] in a Schwarzschild spacetime is

$$\left(2j \frac{\delta mc^2}{\hbar} + \frac{\Gamma_S}{2} \right) 2\kappa \left(\frac{\delta mc^2}{\hbar\Gamma/2} \right) = \left(\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00} \right) \Gamma_S, \quad (77)$$

where we have used (65) to express $\langle K_{S^*}^\oplus | K_{L^*}^\oplus \rangle$ in (74).

A number of interesting conclusions can be drawn from the approximate B-S relation (77).

The parameter ε is defined as

$$\varepsilon = \frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}. \quad (78)$$

The parameter κ is a real number thus (77) leads to the usual phase

$$\arg(\varepsilon) = \arctan \left(\frac{2\delta mc^2}{\hbar\Gamma} \right) = 43.4^\circ, \quad (79)$$

which complete the relationship (65) obtained in the previous section,

$$\text{Re}(\varepsilon) = \left(\frac{2\delta mc^2}{\hbar\Gamma} \right) \kappa = 1.6 \times 10^{-3}. \quad (80)$$

The parameter ε' is defined as

$$\varepsilon' = \frac{1}{3}\eta_{+-} - \frac{1}{3}\eta_{00}. \quad (81)$$

To predict ε' some additional properties of η_{+-} or η_{00} are needed. We assume that the $2\pi_0$ amplitudes fulfill the simple relation

$$A(K_0 \rightarrow \pi^0\pi^0) = A(\bar{K}^0 \rightarrow \pi^0\pi^0), \quad (82)$$

because the interaction between a $(\pi^0\pi^0)$ state and a neutral kaon state, K_0 or \bar{K}^0 , can not differentiate the K^0 from the \bar{K}^0 and a final state phase can be absorbed by a proper phase convention between the two kaons states.

With: (i) this additional relation (82), (ii) the definition (67), (iii) the value of the rephasing factor (69) and (iv) (55,56) we obtain

$$\eta_{00} = \kappa \left(\frac{\delta mc^2}{\hbar\Gamma/2} \right) \left[1 - 2\kappa \left(\frac{\delta mc^2}{\hbar\Gamma/2} \right) \right] \quad (83)$$

These results (80,83) and the definitions (78,81) imply

$$\frac{\text{Re} \varepsilon'}{\text{Re} \varepsilon} = \kappa \left(\frac{\delta mc^2}{\hbar\Gamma/2} \right) = 1.6 \times 10^{-3}, \quad (84)$$

which is different from the definition of the experimental parameter

$$\text{Re} \frac{\varepsilon'}{\varepsilon} = 1.6 \times 10^{-3}, \quad (85)$$

but finally provides a result in agreement with the experimental data as $\arg(\varepsilon) \approx \arg(\varepsilon') \approx \pi/4$ so that $\text{Re} \varepsilon' / \text{Re} \varepsilon = \text{Re} \varepsilon' / \varepsilon$ [43].

It is to be noted that the usual *a priori* statements on phase convention and *CPT* conservation are to be revisited to construct the complete interpretation of direct *CP* violation measurements in a Schwarzschild spacetime.

The study of the 2π decays presented here to extract $\text{Re} \varepsilon' / \text{Re} \varepsilon$ is only an illustrative example of the vast program of re-interpretation of the commonly assumed *CPT* conserving neutral kaons data, within the framework of a *CPT* violation induced by spacetime curvature.

VII. CONCLUSION AND PERSPECTIVES

The spacetime environment of earth is described by a Schwarzschild metric. Taking this fact into account, the agreements between the predicted values and the experimental data allow to conclude that the set of relations (54,55,56) and (57,64,65) is an operational framework to explain indirect *CP* violation and interpret direct *CP* violation experimental results on earth.

These explanations and interpretations require that: (i) the dissipative dressing of the various amplitude, and (ii) their dependance/independence on phase conventions, must be carefully taken into account and, finally, (iii) the identification of the implicit *a priori CPT* conservation assumptions are to be clearly identified.

One of the main drawback of the usual view on *CP* violation is the puzzling small *T* violation at the microscopic level. With the *zitterbewegung CP violation* mechanism the preservation of *T* at the fundamental level is restored.

The violation of the time reversal symmetry is macroscopic and not microscopic, it comes from the very large phase space offered to the final states of a decay, compared with the initial state of a particle at rest. The decay can be described as a spontaneous irreversible process with the Weisskopf-Wigner approximation because, at the classical level of the final particles orbits, the

Poincare recurrence time needed for the particles to come back and concentrate nearby the initial decay point is probably larger than the age of the universe [46]; moreover they are also unstable.

A simple toy model, avoiding any entropy production due to decays, and describing $K \rightarrow 2\pi$ with T conservation, can be constructed as a gedanken experiment as follows. The kaons Hilbert space is completed with the 2π isospin zero state from $(|K^0\rangle, |\bar{K}^0\rangle)$ to $(|K^0\rangle, |\bar{K}^0\rangle, |I_0\rangle)$. These three quantum states are confined into a spherical cavity with a boundary able to reflect π^0 and π^\pm de Broglie waves without losses or additional dispersive phase changes. Then we consider a particular initial condition: at $\tau = 0$ we prepare a K_0 at rest at the center of the spherical cavity. The quantum state (2) is replaced by

$$|\psi\rangle = a(\tau) |K^0\rangle + b(\tau) |\bar{K}^0\rangle + p(\tau) |I_0\rangle. \quad (86)$$

to describe also 2π isospin $I = 0$ states. The evolution of the amplitudes a , b and p are given by

$$j \frac{\hbar}{c^2} \frac{\partial}{\partial \tau} \begin{bmatrix} a \\ b \\ p \end{bmatrix} = \begin{bmatrix} m & -\delta m & \delta \mathcal{M} \\ -\delta m & m & \delta \mathcal{M} \\ \delta \mathcal{M}^* & \delta \mathcal{M}^* & \mathcal{M} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ p \end{bmatrix}, \quad (87)$$

where \mathcal{M} is an appropriate energy eigenvalue of a Schrödinger equation in a spherical cavity. The coupling $\delta \mathcal{M}$ is of the order of δm as Γ is of the order of δm . The extension of this model (87) to a Schwarzschild spacetime leads to a CPT violation without a T violation. But this toy model remains a gedanken experiment as (i) the preparation of the initial quantum state and (ii) the design of perfect de Broglie wave reflecting walls are outside the scope of experimental state-of-the-art.

Time reversal symmetry is restored, at the microscopic level, by the results presented in section V, but at the cost of a reformulation of the equivalence principle between the effective inertial mass of a particle and the effective inertial mass of its antiparticle in a curved spacetime.

We consider a stable particle N and its antiparticle \bar{N} , the internal interaction Hamiltonian H defines the inertial mass $M_{N/\bar{N}}$ in terms of its rest frame energy eigenstates $|N\rangle$ and $|\bar{N}\rangle$ ($\langle N|H|\bar{N}\rangle = 0$) as

$$M_N c^2 = M_{\bar{N}} c^2 = \langle N|H|N\rangle = \langle \bar{N}|H|\bar{N}\rangle. \quad (88)$$

This inertial mass is equal to the gravitational mass [29].

This definition (88) holds: (i) if the particles and antiparticles can be observed separately without any ambiguities and (ii) if the spacetime is a flat Minkowski geometry with a cold vacuum state.

If the particle and its antiparticle are coupled through a set of virtual states accessible *via* the Hamiltonian H such that $\langle N|H|\bar{N}\rangle = \langle \bar{N}|H|N\rangle^* \neq 0$, and if the spacetime curvature does not cancel, then the definition of the inertial mass can be extended with the introduction of an

effective inertial mass. At a distance $R > R_S$ from the center of a spherical massive object whose environment is described by a Schwarzschild metric

$$M_N = \frac{\langle N|H|N\rangle}{c^2} + j \frac{\hbar}{2c} \frac{\langle N|H|N\rangle}{\langle \bar{N}|H|N\rangle} \frac{R_S}{R^2}, \quad (89)$$

$$M_{\bar{N}} = \frac{\langle \bar{N}|H|\bar{N}\rangle}{c^2} - j \frac{\hbar}{2c} \frac{\langle \bar{N}|H|\bar{N}\rangle}{\langle N|H|\bar{N}\rangle} \frac{R_S}{R^2}. \quad (90)$$

The mass is an invariant and can not be a function of a relative position R , thus the imaginary factor of these expression is to be considered as an effective mass similar to the effective mass associated with the ponderomotive potential [40, 41].

An equivalence principle between the effective inertial mass of a particle and the effective inertial mass of its antiparticles can be stated as

$$M_N = M_{\bar{N}}^*. \quad (91)$$

An equivalence principle between the gravitational mass and these effective inertial masses can be stated as

$$M_{GN} = M_{G\bar{N}} = \frac{M_N + M_{\bar{N}}}{2}. \quad (92)$$

An interesting question is the generalization of (89) and (90) to others metrics, either of the Kerr type to study the impact of rotation (which will probably gives simply a relation where the Schwarzschild *surface gravity* is to be replaced by the Kerr *surface gravity* if additional coupling between particle spin and rotation are neglected), or of the Robertson-Walker type to set up cosmological models aimed at identifying the origin of asymmetric baryogenesis.

If the selection rules allow the intermediate virtual states, responsible for $\langle N|H|\bar{N}\rangle \neq 0$, to become observable free states, then the CPT violating dynamics must be dressed with dissipative decays such that $\partial \langle N|N\rangle / \partial t < 0$ and $\partial \langle \bar{N}|\bar{N}\rangle / \partial t < 0$.

It is interesting to note that for a Schwarzschild black hole we can introduce the Hawking temperature $k_B T_H = \hbar c / 4\pi R_S$ conjugated to the Bekenstein-Hawking entropy so that the effective mass (89) on the event horizon (EH) can be expressed as

$$M c^2|_{EH} = \langle N|H|N\rangle \left[1 \pm j 2\pi \frac{k_B T_H}{\langle \bar{N}|H|N\rangle} \right]. \quad (93)$$

To complete this study on kaons systems, the issues of heavy (charmed and bottom) neutral mesons systems should be addressed. The dissipative dressing of the various amplitudes and their dependence/independence on

phases conventions must be very carefully taken into account because the B^0/\bar{B}^0 and D^0/\bar{D}^0 do not display a strong ordering between the width Γ_H/Γ_L (H heavy and L light) similar to the strong ordering of the K^0/\bar{K}^0 system. Such a careful analysis of the interplays between the fundamental zitterbewegung CPT violation and the dissipative evolution associated with fast decays will be presented in a forthcoming study.

The mechanism of zitterbewegung CPT violation might change the way we understand how our matter-dominated universe emerged during its early evolution. The observed present level of CP violation on earth is at least ten orders of magnitudes smaller than the one needed to explain the observed matter-antimatter asymmetry. The amplitude of CP violation (65) is the product of three factors at a distance $R > R_S$ from the center of a spherical massive object or a large energy fluctuation,

$$\left(\frac{m}{2\delta m}\right)^2 \left(\frac{\delta mc^2}{\hbar\Gamma/2}\right) \left(\frac{R_S\lambda_C}{R^2}\right). \quad (94)$$

The first two factors are associated with fundamental interactions: electromagnetic, weak and strong, the third one is associated with spacetime geometry.

The amplitude of violation is maximum for $R \sim R_S$ and can be larger than on earth, for example in the vicinity of Schwarzschild black holes. Note that the ordering $\kappa < 1$ must be fulfilled for the previous various expansions to be relevant. For a small quasi-static universe with radius R_u we can speculate that the scaling for the third factor in Eq. (94) will be λ_C/R_u . An evolving universe provides a more appropriate model than a static one to study the cosmological implications of this new mechanism. For example, rather than the Schwarzschild metric (15), a two-dimensional Robertson-Walker universe with line element $ds^2 = c^2 dt^2 - a^2(t) dx^2$ and conformal time η such that $ds^2 = a^2(\eta) [d\eta^2 - dx^2]$ can be considered. The adaptation of the LOY model to this metric and its extension to two K-G equations is similar to its adaptation to the Schwarzschild metric and reveals new terms violating CPT invariance, moreover the occurrence of a $\partial a/\partial \eta$ term is to be considered under an appropriate ordering. All these fundamental issues will be explored in forthcoming studies.

We hope that the new mechanism presented and advocated in sections III and V will help to clarify our un-

derstanding of the strong asymmetry between the abundance of matter and the abundance anti-matter in our present universe. Among the three factors in (94), the third one is the one which will be determined by the geometry of the early universe and it offers opportunities to identify a scenario to fill up the ten orders of magnitude gap between earth CP violation and the cosmological observational data.

Neutrinos oscillations should also be revisited in a Schwarzschild spacetime to explore the impact of the interplay between curvature and mixing, even if neutrinos are assumed massless as massless particles, such as photons, are deflected by spacetime curvature.

The inclusion of the present results into the standard model can be performed as follows: (i) the CKM matrix in its present form provides the best framework to understand the hierarchy, dynamics and symmetries of elementary particles in the Schwarzschild spacetime describing the vicinity of earth's surface, we can call this matrix CKM^\oplus , (ii) in a flat Minkowski spacetime, far from any massive object, we have to consider a more simple CKM matrix without CP and T violations, we can call this more symmetric matrix CKM° . Both CKM^\oplus and CKM° are operational tools, but in different environments.

To summarize our findings: (i) spacetime curvature induces CP violation in kaons systems, (ii) the measured values of CP violations parameters are predicted within the framework of a simple Schwarzschild geometry, (iii) the right status of the symmetry breaking induced by a Schwarzschild curvature and quarks zitterbewegung is restored: CPT violation with T conservation rather than T violation with CPT conservation.

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