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Neuromorphic machine learning photonic computing based on wavelength-division multiplexing technologies

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Abstract

Optical neural networks (ONNs), or optical neuromorphic hardware accelerators, have the potential to dramatically enhance the computing power and energy efficiency of mainstream electronic processors, due to their ultra-large bandwidths of up to 10's of terahertz together with their analog architecture that avoids the need for reading and writing data back-and-forth. Different multiplexing techniques have been demonstrated to demonstrate ONNs, amongst which wavelength-division multiplexing (WDM) techniques make sufficient use of the unique advantages of optics in terms of broad bandwidths. Here, we review recent advances in WDM-based ONNs, focusing on methods that use integrated microcombs to implement ONNs. We present results for human image processing using an optical convolution accelerator operating at 11 Tera operations per second. The open challenges and limitations of ONNs that need to be addressed for future applications are also discussed.

I. Introduction

Artificial Neural Networks (ANNs), inspired by the human biological brain, have achieved unprecedented success in a wide range of applications ranging from image recognition to sophisticated board games [1-10], due to their capability for learning to be able to process unknown data intelligently. ANNs are mathematical network models formed by densely interconnected neurons, with the capability of addressing complicated tasks, limited by the scale of the network (i.e., number of neurons and synapses). As such, the number of operations and parameters of ANNs, which determine the hardware's computing power, scale exponentially with performance, including key attributes such as accuracy [11].

However, while more advanced applications of ANNs bring about ever-higher demands of the hardware's computing capabilities, the performance density and energy efficiency of leading electronic hardware platforms face severe limitations, as reflected by Moore's law [11-14]. The performance density (i.e., number of operations performed within a given chip scale) can no longer increase beyond where Moore's law ends, at device feature sizes of around 5nm. As a result of reaching this limit, the energy efficiency of memories has shown little improvement since 2015. This is due to two fundamental limitations—the so-called electronic bandwidth bottleneck and the von-Neumann bottleneck [15, 16]: the former limits the clock rate of electronic devices to ~ 2 GHz, while the latter introduces high energy consumption during the process of reading and writing data back and forth.

Optical neural networks (ONNs), are promising next-generation neuromorphic accelerators for ANNs, since they can potentially offer ultra-large bandwidths of >30THz in order to reach dramatically accelerated computing speeds, together with low power consumption due to operating inherently in the analog regime [17-64]. To realize

ONNs, weighted synapses forming the interconnections between neurons need to be implemented with multiple physical paths established within the temporal-, spatial- and wavelength-division parallelism [17-25]. For example, time-delayed optical loops are employed to achieve reservoir computing or spiking neural networks [26-35] that can store over one thousand nodes within the cavity. Integrated photonic waveguides [36-39] and free-space optics [40-45] have been employed in order to achieve synaptic connections spatially. Further, multi-wavelength sources combined with weight bands [46-52] or wavelength-sensitive elements [53-55] can be used to achieve synaptic connections within the wavelength domain. Wavelength division multiplexing (WDM) techniques are critical to fully exhaust the wideband advantages of optics. While the loaded data generally occupies an electronic bandwidth $< 50\text{GHz}$ limited by analog-to-digital converters and generic opto-electronic interfaces, the $>30\text{THz}$ optical bandwidth needs to be fully exploited by introducing multiple parallel wavelength channels to cover the full optical bandwidth.

Optical frequency combs (OFCs), offering a large number of equally spaced wavelength channels, are powerful tools for communications systems and neuromorphic computing to significantly enhance the capacity or parallelism of the system [65-77]. Classic methods for OFC generation, such as those based on discrete laser arrays or electro-optic (EO) modulation, face limitations of one form or another, such as the bulky size and high cost, or the limited number of wavelengths. Kerr optical frequency combs [78-83], or “microcombs”, originated from the parametric oscillation within high Q nonlinear micro-ring resonators (MRRs), are promising on-chip OFC sources, as they can offer a large number of wavelengths in an integrated platform with much smaller footprint and higher scalability, performance, and reliability. Microcombs have enabled significant breakthroughs in frequency synthesis [128, 129], ultrahigh capacity communications [130-134], spectroscopy and measurement [135-144], microwave photonics [145-154], and neuromorphic optics [52, 54, 55].

In this paper, we review recent progress of the use of microcombs for neuromorphic optics. We introduce a convolution accelerator based on time-wavelength interleaving that operates at 11 Tera operations per second (TOPS), and discuss the potential and challenges for ONNs. The paper is structured as follows. Section II presents recent advances in optical frequency combs and microcombs, in terms of device platforms, nonlinearity and oscillation states, and generation methods. Section III reviews recent work of ONNs based on temporal-, spatial- and wavelength-division multiplexing (SDM, TDM and WDM). Section IV presents the theory and operation principle of a WDM-based convolution accelerator and experimental results of human image processing. The open challenges and limitations of ONNs are discussed in Section IV.

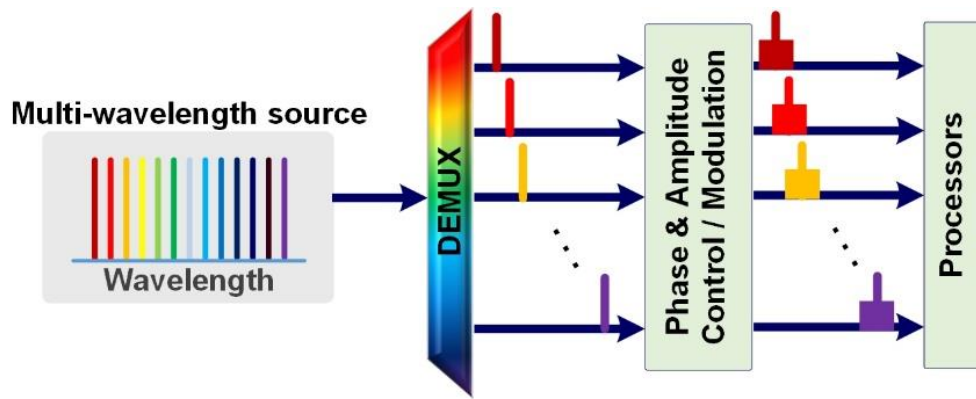


Figure 1. The overall architecture of WDM systems

II. Integrated optical frequency combs for wavelength-division multiplexing

As shown in Figure 1, typical WDM systems consist of a multi-wavelength source that establishes parallel wavelength channels carrying identical or different data for subsequent data transmission or processing, followed by wavelength multiplexers and demultiplexers that enable different wavelength channels to be modulated or processed separately.

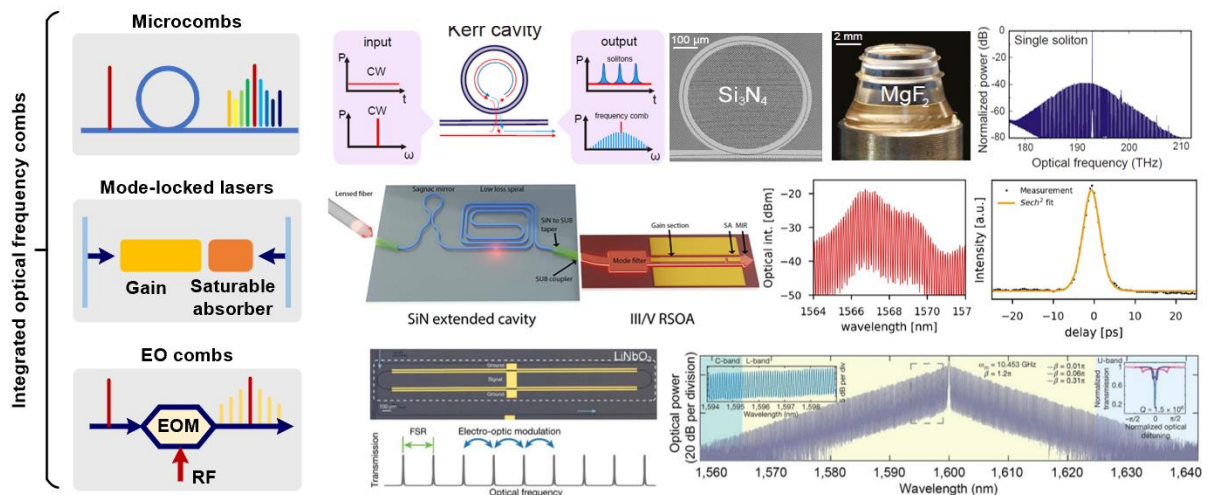


Figure 2. Integrated optical frequency combs. Figures are adapted from [84, 85, 103]

OFCs are key to implementing the multi-wavelength source in these systems, due to their relatively compact architecture, in contrast to discrete laser arrays, which inherently offers equal frequency intervals between the comb lines that enable the easy frequency domain manipulation of the wavelength channels [65-77]. During the past two decades, the advance of photonic nanofabrication techniques has led to the production of integrated OFCs in different forms, offering remarkable advantages in terms of the system's size, weight and power consumption, and cost [77]. Existing integrated OFCs can be divided into several categories based on the underlying physical origins, including: a) Kerr frequency combs, or microcombs [78-83], that originate from parametric oscillation in an integrated micro-ring resonator (MRR); b) mode-locked lasers that employ gain media, such as Erbium-doped fiber amplifiers, in order to sustain oscillations and mode-locking mechanisms, such as saturable absorbers to yield pulsed outputs [84]; and c) electro-optic modulators that employ the second-order nonlinearity to introduce sidebands centered around a certain optical carrier [85]. While the latter requires external RF sources/oscillators

to drive the modulator, microcombs and mode-locked lasers are more compact and cost-effective, although at the expense of having to tailor the underlying physics in order to obtain the desired oscillating states/performance. For mode-locked lasers, the gain needed to sustain oscillations comes from optical amplifiers, which restricts the operating bandwidth to within a certain range, such as the Erbium-doped fiber amplifier gain bandwidth that operates in the optical C band ranging from 1535 to 1570 nm. Hence, this limits the parallelism or data capacity for applications such as optical computing and communications.

Microcombs are powerful integrated OFC sources, due to their compact footprint and ultra-wide bandwidths capable of octave-spanning operation, supported by the broadband nonlinear gain [78-83]. Microcombs originate from parametric oscillation within high-Q micro-resonators, which are achieved either in integrated form, such as micro-ring resonators [83], or in 3-dimensional form, such as spheres or rods [78]. The key to microcomb generation is to obtain sufficiently high parametric gain, which is directly determined by the strength of the third-order nonlinearity of the material platform and the Q factor of the resonator (i.e., low linear and nonlinear losses) [79]. Within high Q-factor micro-resonators, the optical intra-cavity field can be significantly resonantly enhanced in order to initialize nonlinear phenomena that otherwise generally requires high optical power — such as the modulation instability gain and thus parametric oscillation. In 2010, parametric oscillation based on integrated platforms was first reported [83, 86, 87], which revealed the ultimate potential of microcombs to be mass produced together with other optical components on a single chip, employing well-established CMOS platform fabrication techniques.

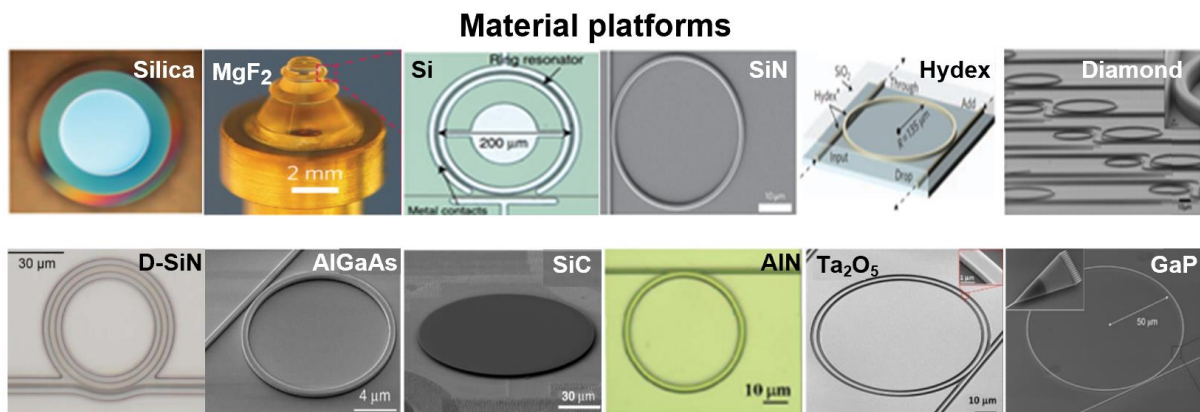


Figure 3. Material platforms of microcombs, including silica [88], MgF₂ [89], silicon [90], silicon nitride [87], Hydrex [92], diamond [93], deuterated silicon nitride [94], aluminum gallium arsenide [95], silicon carbide [97], aluminium nitride [98], tantalum pentoxide [99] and gallium phosphide [100]. Figures are adapted from [88-100]

While microcomb generation generally requires a high external pump power that brings about limitations in terms of energy efficiency and footprint (i.e., high-power amplifiers are unavoidably needed), significant effort has been made to reduce the parametric oscillation threshold. On the one hand, novel material platforms [88-100], such as SiC [97] and AlGaAs [95, 96] can exhibit significantly higher third-order optical nonlinearities, while on the other hand, advances in nanofabrication techniques, such as the Damascene reflow process and multi-mode waveguides [101-102], increase the Q factor to enhance the build-up optical fields in the micro-resonator. To date, a nonlinear coefficient of $n_2 = 2.6 \times 10^{-17} \text{ m}^2 \text{ W}^{-1}$ has been achieved with AlGaAs waveguides, resulting in an ultra-low threshold power of 0.036mW [95]. Q factors of over 10 million have been achieved with the Damascene reflow process for integrated micro-resonators [101], and these advances indicate that microcombs can be directly

generated with a generic laser source where high-power amplifiers are no longer needed, albeit specific pump detuning control mechanisms are still necessary.

Governed by the Lugiato-Lefever equation [79], multiple parameters contribute to the rich dynamics of microcombs. These include the pump power and detuning (perturbed by thermal effects) that determine the intra-cavity pump power as well as the nonlinearity that determines the oscillation threshold and comb states. Also important is the dispersion that balances the nonlinearity, which together affect the comb bandwidth. Different nonlinear effects and their impact on microcomb generation have been investigated, including the basic third-order nonlinearity (degenerate and non-degenerate four-wave mixing, or FWM) that lead to primary combs, also termed Turing patterns (with comb spacings at multiple free spectral ranges (FSRs), of the micro-resonator). Using delicate pump detuning control, single solitons [89], multiple solitons [103, 104], and breathers [105-108] can be generated. On the other hand, mode crossings, can lead to soliton crystals [109-112]. Finally, using a combination of gain and loss, normal dispersion and mode crossings can lead to dark solitons that offer higher energy output [113]. Combs based on Raman scattering, a molecular-scale process where a Stokes photon and an optical phonon are generated from a pump photon, can enable broadband Raman gain and Stokes solitons coexisting with Kerr combs [114]. Brillouin scattering, a lattice-scale process where a backward scattering Stokes photon and an acoustic phonon are generated from a pump photon, that introduces narrowband Brillouin gain at ~ 10 GHz away from the pump. As such Brillouin combs are observed in large resonators with FSRs matching with the Brillouin gain [115, 116]. Making use of the second-order nonlinearity can leads to the generation of frequency-doubled combs, with tailored dispersion to achieve phase matching at both oscillating wavelengths [117].

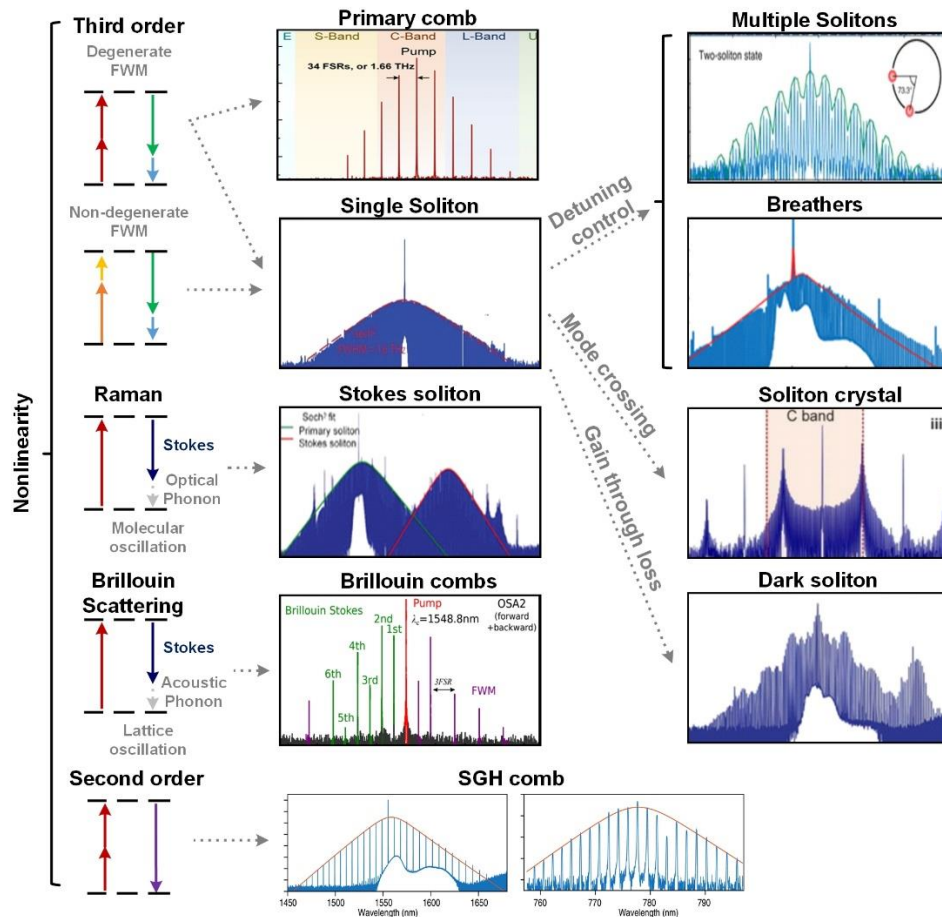


Figure 4. Diverse states of microcombs based on different types of nonlinear effects. Figures are adapted

from [89, 103, 105, 113-115, 117, 146].

In parallel with the development of microcomb material platforms, advanced pumping methods have been demonstrated aiming at, on the one hand, overcoming the thermal effects of the micro-resonator that severely perturb the wavelength detuning control for soliton generation, and on the other hand, further reducing the complexity and footprint of the overall comb setup. Typically, the pump can be illustrated in three forms: continuous-wave (CW) lasers that operate at a single wavelength [89, 103, 118, 119]; dual CW lasers with one serving as an auxiliary laser [120, 121]; and optical pulses that feature much higher peak power to initialize parametric oscillation [122, 123].

Soliton generation requires that the pump wavelength be swept from the blue to red shifted side of the microresonator resonance, finally landing in the soliton step region, during which the thermal effects of the micro-ring resonator shift the resonance with respect to the intra-cavity power [89]. Since the single soliton state features a much lower intra-cavity power compared to the originating chaotic state, deterministic soliton generation remains challenging and requires delicate external control of the pump-resonance detuning because of the inherent resonance shift that happens at the onset of soliton generation. Classic detuning control methods that have been widely employed and verified include fast wavelength/resonance tuning to reduce the accumulated heat during the detuning sweeping process [103, 119], as well as power kicking methods to manipulate the resonance shift induced by the thermal effects and pump power [108]. Recently, advances have been made, following the development of hybrid integration techniques, including more delicate control of the detuning in both forward and backward directions that lead to accurate control of the single and multiple soliton states [89]. Dual pump approaches that employ an auxiliary laser to balance the thermal effects induced by the pump laser [120, 121] have been successful. Heterogeneously integrated laser and micro-resonators enable massive production of the microcomb system [124] while injection locking approaches that lock the detuning via cross- and self-phase modulation effects to achieve turnkey microcomb generation [125-126] have been very successful. Finally, self-oscillating pump generation methods significantly reduce the pump power and enable high energy efficiency laser cavity soliton states [127, 128].

All of these significant advances have collectively led to microcombs that exhibit an ever-increasing maturity for practical applications, providing a wideband, high-energy-efficiency, compact, turnkey and mass-producible comb source for wavelength-division multiplexing systems [129-158]. The development of novel 2D nonlinear materials such as graphene oxide may offer significant new opportunities for achieving novel methods of generating microcombs. [159-165]

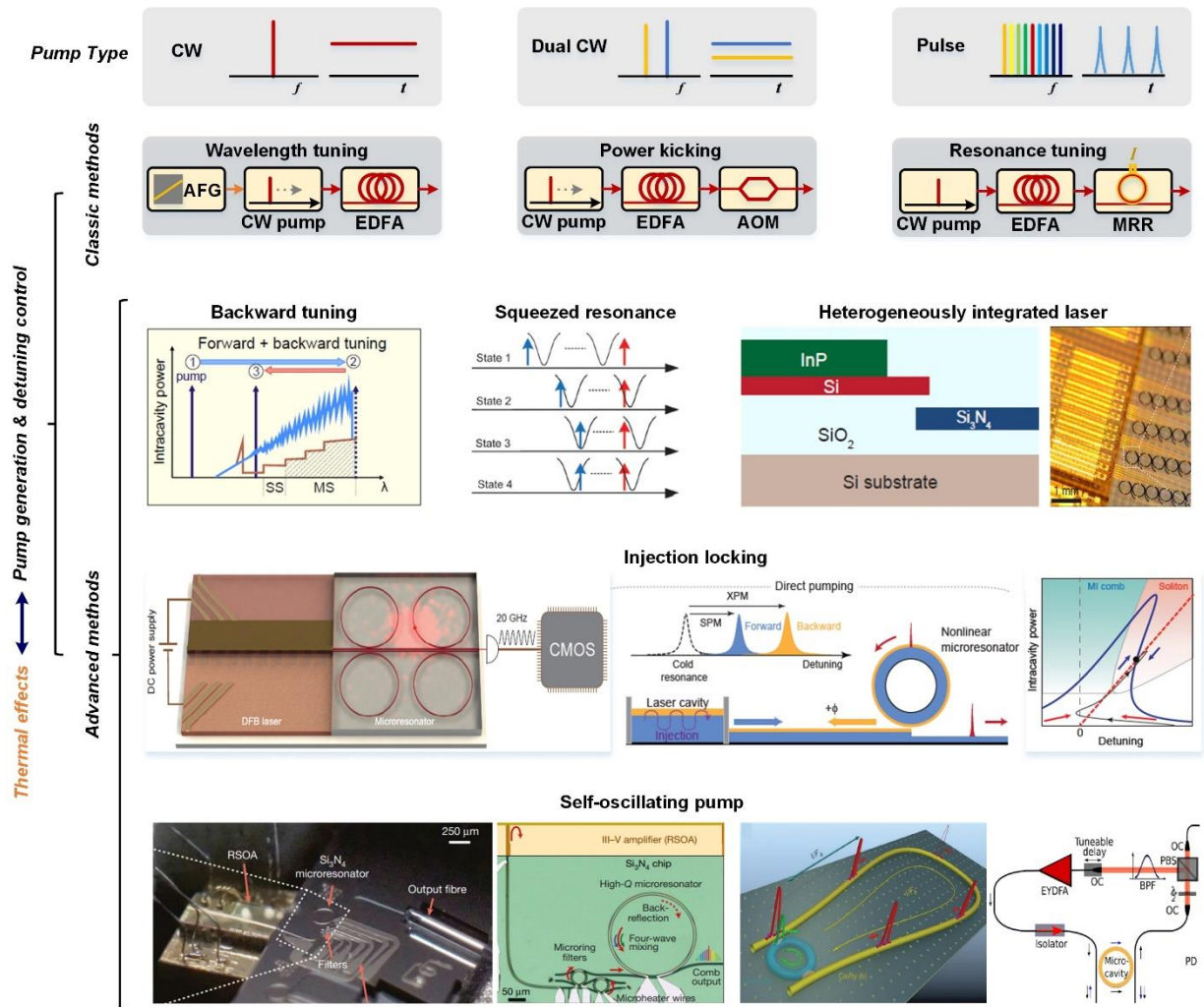


Figure 5. Pumping methods of microcombs. Figures are adapted from [103, 120, 124-127].

III. Optical neural networks based on wavelength-division multiplexing

While electronic neuromorphic hardware face increasingly large gaps between the desired and achievable performance in terms of computing density and energy efficiency, bounded by Moore’s law, optical neuromorphic accelerators have attracted significant interest over the past decade, mainly due to their ultrawide optical bandwidth and low power consumption enabled by their inherently analog architecture [17-25].

The key to accelerate computing for artificial intelligence applications is to achieve the basic mathematical operations optically in a highly controlled manner, such that the overall parallelism and data throughput can be significantly enhanced for high computing power and large-scale fan-in/-out. High data throughput can be achieved with high-speed electro-optical interfaces, including electro-optical modulators and photodetectors that can reach over 50 GHz in bandwidth. High parallelism, needed to process large-scale data such as images or speech, can be implemented with multiplexing techniques that have been widely used in optical communications [17-18].

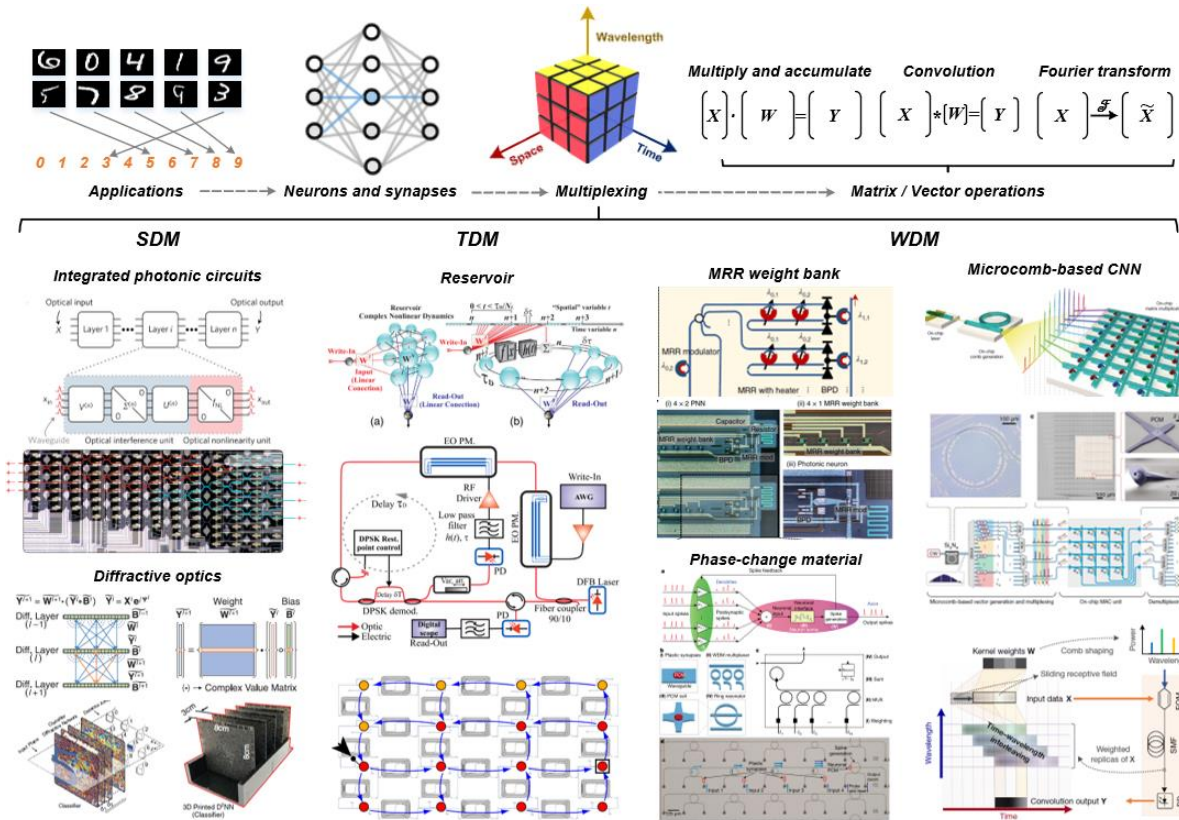


Figure 6. ONNs based on different multiplexing techniques. Figures are adapted from [28, 31, 36, 37, 40, 46, 52, 54]

The parallelism of optical neural networks mainly determines the fan-in/out of the network, which determines the networks' capability of processing large-scale data. Likewise, the number of connections between the neurons denotes the networks' capability in terms of processing complicated tasks. Typical techniques to enhance the parallelism employ space-, time-, or wavelength-division multiplexing. ONNs based on space-division multiplexing (SDM) have been achieved with integrated photonic circuits [36-39], diffractive lens and others [40-45], where parallel input nodes are realized spatially in the form of different optical waveguide ports or pixels of the lens. Matrix multiplication operations have been achieved using intensity modulation/loss management of the optical field and mutual interference paths. ONNs based on time-division multiplexing (TDM) convert the input data or/and synaptic weights into temporal waveforms for matrix/vector operations [26-35], which requires additional optical buffers to achieve the accumulation process. The advantage of TDM techniques is that they are potentially capable of updating the synaptic weight at high speed in order to significantly accelerate the training processing of the neural networks. However, they generally require other multiplexing techniques to enhance the computing power, due to their inherent sequential/serial operational methods.

Wavelength-division multiplexing (WDM) is a unique technique enabled by optics, in contrast to electronics. Supported by the ultra-wide optical bandwidths up to 10s' of THz, 100's of wavelength channels can be established for parallel data processing of neural networks, thus leading to significantly enhanced computing speed—similar to the significantly enhanced data transmission capacity for WDM-based communications systems. Current WDM-based ONNs [46-55] can be generally categorized into two types, according to whether the wavelength channels carry identical data. Individually modulated wavelength channels [46, 52] enable potentially higher

flexibility and parallelism, capable of performing generic matrix multiplication operations; however, they require large arrays of modulators with well-matched wavelength multiplexers/demultiplexers, which remain challenging to be integrated and synchronized on chip without significant increase in complexity and cost. For example, for a 50GHz-spacing microcomb source, over 80 wavelength channels can be established in the C band, which in turn requires over 80 modulators. In parallel, simultaneously modulated wavelength channels [54, 55] are potentially much more straightforward to implement and integrate, as only one modulator is needed to broadcast the input data. The drawback to this approach is that, despite the increase in parallelism, the overall computing power is limited for generic matrix operations. This is true for all broadcast-and-weight systems used for matrix multiplication [55], where the input data is implemented as serial temporal waveforms. Here, WDM does not in fact lead to an enhancement in the computing parallelism and so the computing speed is similar to the data rate [54].

IV. WDM-BASED convolution accelerators

Recently, [54] convolution accelerators have been proposed based on a time wavelength interleaving approach, which avoids the trade-offs mentioned above and has achieved high computing power within a compact footprint. The operation principle of the convolution accelerator is illustrated in Fig. 7. For vector convolutions between a $1 \times L$ data vector and a $1 \times R$ weight vector, the data vector is converted to a temporal waveform $\mathbf{X}[n]$ via digital-to-analog converters, where n denotes discrete temporal locations of the symbols; the weight vector is mapped onto the power of optical comb lines as $\mathbf{W}[R-i+1]$ ($1 \leq i \leq R$, i increases with wavelength). Via electro-optical modulation, $\mathbf{X}[n]$ can be broadcast onto the comb lines, yielding weighted replicas as $\mathbf{W}[R-i+1] \cdot \mathbf{X}[n]$. Next, the weighted replicas are progressively delayed via dispersion, with the delay step between adjacent wavelength channels equal to the symbol duration of $\mathbf{X}[n]$, thus yield delayed replicas $\mathbf{W}[R-i+1] \cdot \mathbf{X}[n-i]$. After photodetection, the replicas are summed as

$$\mathbf{Y}[n] = \sum_{i=1}^R \mathbf{W}[R-i+1] \cdot \mathbf{X}[n-i] = (\mathbf{W} * \mathbf{X})[n] \quad (1)$$

where each symbol $\mathbf{Y}[n]$ within the range of $[R+1, L+1]$ denotes the dot product between \mathbf{W} and a sliding window $[n-R, n-R+1, n-R+2, \dots, n-1]$ of \mathbf{X} , thus achieving convolution operations between the input and weight vector. The convolution accelerator can be used to form the convolutional layer of a neural network, extracting feature maps of the input data to greatly reduce the parametric complexity of the network without sacrificing the overall performance.

The output waveform \mathbf{Y} contains $R+L-1$ symbols, amongst which $R-L+1$ symbols denote the convolution results, for each computing cycle. Here, each symbol is the result of R multiplications and R accumulation operations. As such, the computing speed can be given as $(R+L-1)/(R-L+1) \times 2 \times R \times B$, where B denotes the symbol rate. For practical applications, the length of the input data vector is much larger than that of the weight vector ($L \gg R$), thus $(R+L-1)/(R-L+1) \approx 1$ and the computing speed can be given as $2 \times R \times B$.

In contrast, the same hardware architecture can be tailored to achieve matrix multiplication for the fully connected layer. Assuming that the input data vector $\mathbf{X}_{FC}[n]$ and the weight vector $\mathbf{W}_{FC}[R-i+1]$ both have a length of R ($1 \leq i \leq R$, $1 \leq n \leq R$), thus, according to Equation 1, the output waveform after photodetection is

$$\mathbf{Y}_{FC}[n] = \sum_{i=1}^R \mathbf{W}_{FC}[R-i+1] \cdot \mathbf{X}_{FC}[n-i] \quad (2)$$

By sampling at the time slot denoted by $n=R+1$, the matrix multiplication result of the two input vectors is obtained as

$$\mathbf{Y}_{FC}[R+1] = \sum_{i=1}^R \mathbf{W}_{FC}[R-i+1] \cdot \mathbf{X}_{FC}[R+1-i] = \sum_{i=1}^R \mathbf{W}_{FC}[i] \cdot \mathbf{X}_{FC}[i] \quad (3)$$

The output waveform \mathbf{Y} contains $2L-1$ symbols for each computing cycle, amongst which only one symbol denotes the vector multiplication result from L multiplications and L accumulation operations. As such, the computing speed can be given as $1/(2L+1) \times 2 \times L \times B \approx B$.

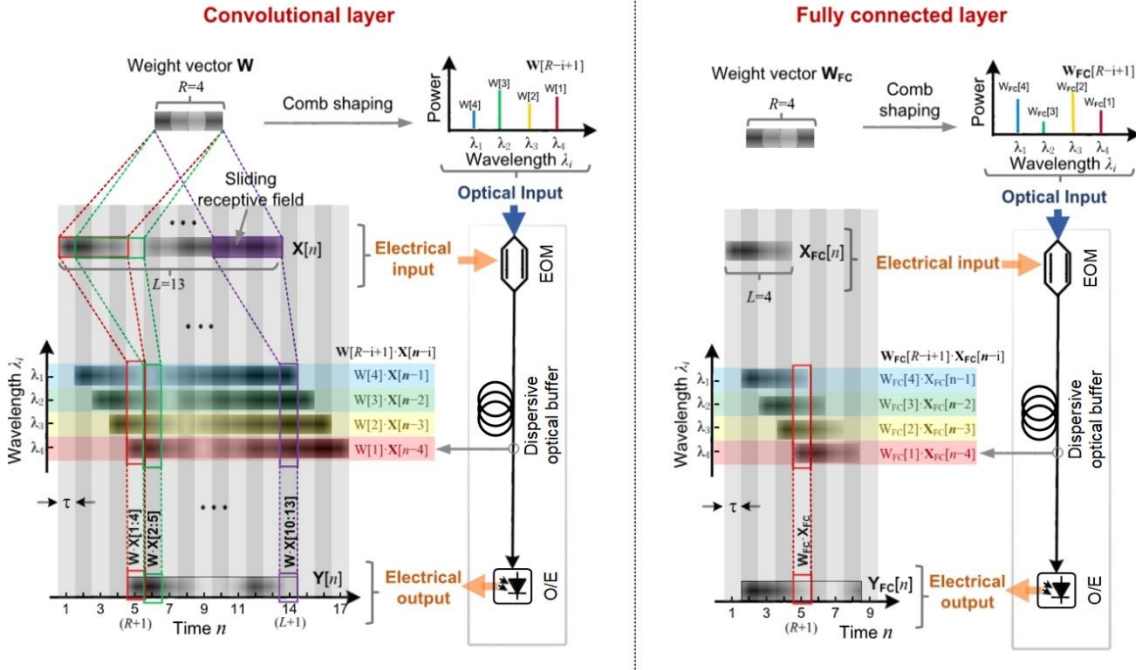


Fig. 7. The operation principle of the photonic convolution accelerator for the convolutional layer with $R = 4$ and $L = 13$, and the fully connected layer with $R = L = 4$, consisting of an electro-optical modulator (EOM), an optical buffer that has progressive wavelength-sensitive delay, and an optical-to-electrical conversion module (O/E). Figures are adapted from [54]

As illustrated above, this time wavelength interleaving method can significantly enhance the computing power with WDM and a single modulator; albeit it is applicable only for specific convolution operations, rather than general matrix operations.

The convolution accelerator can be used to form convolutional neural networks, which has shown unprecedented performance for image recognition applications. Here, we present the results of human image processing, using the convolutional accelerator as illustrated in [54]. 90 comb lines were employed to form ten 3×3 convolutional kernels, achieving diverse image processing functions, including: identical, blur, bottom/top/left/right Sobel, emboss, outline, sharpen and motion blur. A 500×500 input image was flattened into a vector and converted into an electrical input waveform via a high-speed electrical digital-to-analog converter, at a data rate of 62.9 Giga Baud. The waveform was then broadcast onto all wavelength channels and weighted via electro-optical modulation. Following this, the weighted replicas were transmitted through ~ 2.2 km of standard single mode fibre (dispersion ~ 17 ps/nm/km), which corresponds to a progressive delay of 15.9 ps that matches

with the symbol rate. The wavelength channels were then de-multiplexed into 10 sub-bands that corresponded to the 10 convolutional kernels, and finally summed upon photodetection. The output waveforms were finally sampled and rescaled to form the feature maps, denoting a diverse range of pre-defined hierarchical features of the input image. In combination with a fully connected layer, the convolutional accelerators can be used to form convolutional neural networks, where the kernels' weights are trained for specific tasks/datasets such as facial recognition.

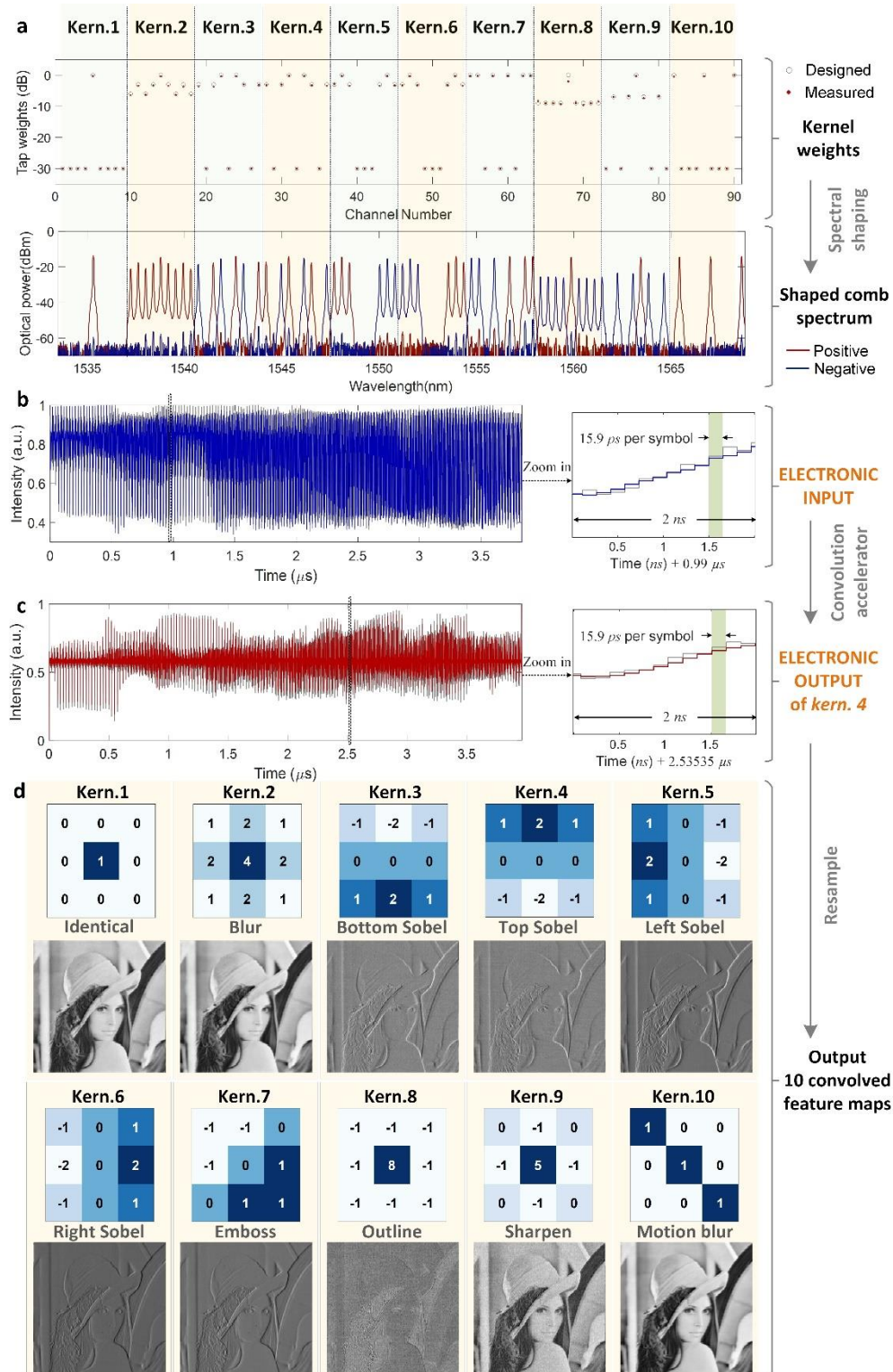


Figure 8 Experimental results of the convolution accelerator. a. The kernel weights and the shaped

microcomb's optical spectrum. b. The input electrical waveform of the image (the grey and blue lines show the ideal and experimentally generated waveforms, respectively). c. The convolved results of the fourth kernel that performs top Sobel image processing function (the grey and red lines show the ideal and experimentally generated waveforms, respectively). d. The weight matrices of the employed kernels and corresponding recovered images.

V. Discussion

While ONNs show great potential in achieving high computing power and energy efficiency, their inherent analog framework indicates that hybrid opto-electronic neuromorphic hardware is a likely optimal solution that takes advantage of both the broadband optics and the versatility of digital electronics, where optics undertakes the majority of the computing operations and electronics manages the data flow and storage. So far, although notable progress has been made on ONNs, many challenges still exist that need to be addressed for future applications. First, dense integration of the entire photonics system needs to be achieved, as this is the key to achieve competitive computing densities for ONNs in comparison to their electrical counterparts. In parallel, hybrid integration techniques, capable of integrating the comb source and subsequent multiply-and-accumulate units, are necessary to make sufficient use of optics' broad bandwidths with WDM techniques. Secondly, more categories of computing operations, such as the nonlinear functions of neurons and Fourier transforms, need to be demonstrated on chip to further enhance the universality of ONNs for diverse machine learning tasks. This will rely on advances in both novel computing architectures tailored to specific operations and the integration of high-nonlinearity components that can realize the nonlinear functions with relatively low optical power. Further, as ONNs will be achieved as assembles of massive programmable photonic units for a high spatial-division parallelism, tailored algorithms to overcome the challenges of fabrication imperfections and on-chip cross-talk are necessary for fast-converging control of on-chip elements and training of the networks.

With these challenges being fully addressed, ONNs can then be plugged into existing electronic hardware to significantly enhance the computing performance of the whole system, dramatically accelerating the training speed of computational intense neural networks, and thus in turn potentially lead to more complicated and intelligent networks for advanced machine learning tasks such as fully automated vehicles and real-time image/video processing.

VI. Conclusion

We review recent advances in WDM-based ONNs, focusing on methods that use integrated microcombs to implement ONNs. We present results for human image processing using an optical convolution accelerator operating at 11 Tera operations per second. The open challenges and limitations of ONNs that need to be addressed for future applications are also discussed.

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