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Einstein's Special Theory of Relativity: A New Mass-Energy Equivalence

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Abstract: This paper presents a new mass-energy equation, which is an alternative to Einstein's mass-energy equivalence. The new mass-energy equation is proved mathematically using the relativistic mechanics. In the relativistic theory of the author, the relativistic velocity is the velocity of a moving particle approaching the speed of light. It is understood that the velocity of a moving particle with its non-zero mass is always less than the speed of light because light has no mass. So, the speed of light is the upper limit for all moving bodies in the Universe.

Keywords: classical mechanics, electromagnetic energy, light waves, relativistic motion

1. Introduction

Light waves are generally electromagnetic and do not require a medium for their propagation. Thus, the light can travel in a vacuum. Quanta of light are the smallest discrete packets of electromagnetic energy. A quantum of light that has no mass is called a photon. The relativistic motion [1-5] of a particle is concerned with the motion of the particle whose velocity approaches the speed of light.

2. Velocity-Momentum Relation

In classical mechanics, a variable-mass object is an object whose mass varies with time. A rocket, a moving object that burns completely at regular time intervals, and an object in motion that loses its mass at every unit of time are examples of variable-mass. In this section, the author introduces definitions of momentum-velocity equivalence [6-13].

Definition for constant-mass: The momentum (P) of a moving object with its constant-mass (m) is directly proportional to its velocity (v).

$$P \propto v \Rightarrow P = mv, \text{ where } m \text{ is constant.}$$

$$\text{Here, } \frac{dP}{dt} = F = \frac{d(mv)}{dt} = m \frac{dv}{dt}$$

Definition for variable-mass: Suppose a moving object loses its mass (m) at every unit of time and increases its velocity (v) proportionately with its mass (m). Then, the momentum (P) of the moving object is directly proportional to its velocity (v).

$$P \propto v \Rightarrow P = mv = x_i v_i \text{ at the unit } t_i \text{ of time for } i = 1, 2, 3, 4, \dots, \text{ where } 0 < x < m.$$

$$\text{Then, } \frac{dP}{dt} = F = v \frac{dx}{dt} + x \frac{dv}{dt} = \frac{d(mv)}{dt}$$

The above equation can be written as follows for our understanding:

$$\frac{dP}{dt} = F = v \frac{dm}{dt} + m \frac{dv}{dt} = \frac{d(mv)}{dt} = m \frac{dv}{dt} \quad (1)$$

For instance, let us consider two moving objects with variable mass, that is, P_1 be the momentum of a moving object with mass m and P_2 be the momentum of another moving object with mass $\frac{m}{k}$.

$$\text{Then, } kP_2 = \left(\frac{m}{k}\right) (kv) \text{ and } P_1 = kP_2.$$

If $k = 2$, then $P_1 = 2P_2$. This result shows that the velocity of the object with momentum P_2 is double of the velocity of the object with momentum P_1 because the mass of the object with momentum P_2 is half of the mass of the object with momentum P_1 .

3. Kinetic Energy

Kinetic energy is a fundamental concept in classical mechanics that quantifies the mechanical energy performed by an object due to its motion [10-13].

The equation of kinetic energy is given below:

$$E = \frac{1}{2}mv^2, \quad (2)$$

where m is the mass and v is the velocity.

By differentiating the equation of kinetic energy with respect to time t , we obtain

$$\frac{dE}{dt} = \frac{1}{2} \left(m2v \frac{dv}{dt} \right)$$

By simplifying the above differential equation, we receive

$$\frac{dE}{dt} = mv \frac{dv}{dt}$$

where the mass (m) of an object in motion is constant.

From the equation (1), we conclude that

$$dE = mv dv = v d(mv). \quad (3)$$

4. Derivation of the Lorentz Factor

The Lorentz factor [5] is the factor by which time, length, and mass change for an object moving at a speed close to the speed of light. By the following experiment, the author of this article derives the equation of the Lorentz factor.

Figure 1. depicts the relativistic speed of the metal tube (AD). Let's consider that the metal tube shown in the Figure 1 is at a stationary position and the light takes time t to travel from the point A to the point D. The length of the metal tube is measured as follows: $AB = BC = h = ct$, where c is the speed of light and t is the time taken to move from the point A to the point D.

Suppose the metal tube with mass m and velocity v moves from the point A to the point B horizontally and the light travels from the point A along with the moving metal tube. When the light reaches the point C at time t' , the metal tube moves to point B at time t' .

$$AB = vt' \text{ and } AC = ct'$$

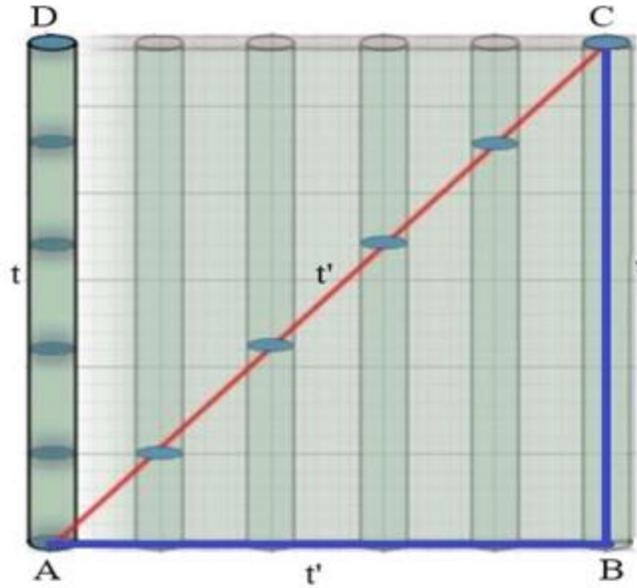


Figure 1. Relativistic speed of the metal tube.

According to the definition of the Pythagoras theorem, we can derive the Lorentz factor as follows:

$$AC^2 = AB^2 + BC^2 \quad (4)$$

By substituting $AB = vt'$ and $AC = ct'$ in the equation (4), we obtain

$$(ct')^2 = (vt')^2 + (ct)^2$$

$$t'^2 \left(1 - \frac{v^2}{c^2} \right) = t^2$$

By simplifying the above equation, we get

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{t'}{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5)$$

Hence, γ is the Lorentz factor.

5. Energy-Momentum Relation

The equation of relativistic mass is $m = m_0\gamma$, (6)

where the Lorentz factor is $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

By substituting the value of γ in the equation (6), we obtain the following equation for finding the relativistic mass.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (7)$$

Squaring both sides of the equation (7):

$$m^2 = \frac{m_0^2}{1 - \frac{v^2}{c^2}} \Rightarrow m^2(c^2 - v^2) = m_0^2 c^2$$

By simplifying the above equation, we obtain

$$m^2 c^2 - m^2 v^2 = m_0^2 c^2 \quad (8)$$

By differentiating the equation (8) with respect to time, we get

$$c^2 2m \frac{dm}{dt} - \left[2mv^2 \frac{dm}{dt} + 2m^2 v \frac{dv}{dt} \right] = 0$$

By simplifying the above equation, we receive

$$c^2 \frac{dm}{dt} = v^2 \frac{dm}{dt} + mv \frac{dv}{dt} = v \left[v \frac{dm}{dt} + m \frac{dv}{dt} \right], \text{ where } v \frac{dm}{dt} + m \frac{dv}{dt} = \frac{d(mv)}{dt}$$

$$c^2 dm = vd(mv) \quad (9)$$

According to the equation (1), the equation (9) is rearranged as follows:

$$dE = cd(cm) = vd(mv), \text{ where } v \left(v \frac{dm}{dt} + m \frac{dv}{dt} \right) = v \frac{d(mv)}{dt} \quad (10)$$

As the equation (10) is the derivation of the equation (2), we conclude that

$$E = \frac{1}{2} mc^2, \text{ where } c \text{ is the speed of light.} \quad (11)$$

$$E = \frac{1}{2} m \left(\frac{c_a}{a} \right) \left(\frac{c_b}{b} \right) = P_a \left(\frac{c_b}{2b} \right) = P_b \left(\frac{c_a}{2a} \right) = P_{2a} \left(\frac{c_b}{b} \right) = P_{2b} \left(\frac{c_a}{a} \right), \quad (12)$$

where $c^2 = \frac{c_a}{a} \times \frac{c_b}{b}$ and $P_a = m \frac{c_a}{a}$; $P_b = m \frac{c_b}{b}$; $P_{2a} = m \frac{c_a}{2a}$; & $P_{2b} = m \frac{c_b}{2b}$.

Note that P_a, P_b, P_{2a} , and P_{2b} are momentums.

6. New Mass-Energy Equation

The relativistic motion [1-13] is concerned with a moving particle whose velocity approaches the speed of light. The relativistic momentum (P) of a particle with its non-zero mass (m) cannot be equal to mc because c is the speed of light that has zero mass. That is, $P \neq mc$.

Therefore, $P = mc_v, (c_v < c)$.

If there is a value c_u such that $c_u < c$ and $c_u < c_v < c$, then $P = mc_v$.

Hence, $P = mc_v$ is the upper limit for relativistic momentum. (13)

We know that the speeds of all subatomic particles with mass are less than the speed of light. For example, neutrino speed is not faster than the speed of light because neutrino has non-zero mass. Since $P \neq mc$, the relativistic energy (E) $\neq mc^2$.

If $P = mc_v$, then $E = \frac{1}{2}mc_v^2 < \frac{1}{2}mc^2$, which is the new mass energy equation.

Here, $E = \frac{1}{2}mc^2$ is the upper limit for the energy of motion. (14)

7. Conclusion

In this article, the new mass-energy equation has been proved mathematically and also the upper limits for relativistic velocity, relativistic momentum, and relativistic energy were provided as scientific solutions. The relativistic mechanics is concerned with the motion of bodies whose velocity approaches the speed of light. This idea can enable the researchers for further involvements in the scientific research and development.

References

- [1] Annamalai, C. (2023) A Novel Mass-Energy Equation from Lorentz Factor and Energy of Motion. *SSRN Electronic Journal*. <http://dx.doi.org/10.2139/ssrn.4629897>.
- [2] Annamalai, C. (2023) Energy-Momentum Equivalence. *CoE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-g6bcz-v4>.
- [3] Annamalai, C. (2023) Momentum-Velocity Equivalence. *CoE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-g6bcz-v3>.
- [4] Annamalai, C. (2023) Energy-Work Equivalence. *CoE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-lsh2m>.
- [5] Annamalai, C. (2023) Lorentz Factor and Time Dilation on the Special Theory of Relativity. *TechRxiv, IEEE*. <https://doi.org/10.36227/techrxiv.24297325>.
- [6] Annamalai, C. (2023) Upper Limit for the Energy of Motion. *CoE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-xtxmg>.
- [7] Annamalai, C. (2023) A New Mass-Energy Equivalence from Lorentz Factor and Energy of Motion. *CoE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-cbbq0>.
- [8] Annamalai, C. (2023) Momentum-Velocity Equivalence. *CoE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-g6bcz-v2>.
- [9] Annamalai, C. (2023) Computation of Mass-Energy Equation from Lorentz Factor and Kinetic Energy. *CoE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-6nlxv-v2>.
- [10] Annamalai, C. (2023) Upper Limits for Velocity, Momentum, and Energy of Motion. *SSRN Electronic Journal*. <http://dx.doi.org/10.2139/ssrn.4660185>.
- [11] Annamalai, C. (2023) Mass-Energy Equivalence: Light Energy. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-639kj>.

- [12] Annamalai, C. (2023) Upper Limits for Relativistic Energy and Momentum. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2023-d48fx>.
- [13] Annamalai, C. (2023) Einstein's Special Theory of Relativity: A New Mass-Energy Equation. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2024-0gzht>.