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► **To cite this version:**

Pierre Moussou, Adrien Guilloux, Eric Boccaccio, Guillaume Ricciardi. Fluid damping in fuel assemblies. ASME 2017 - Pressure Vessels and Piping Conference, Jul 2017, Waikoloa, United States. pp.PVP2017-65664, V004T04A048, <10.1115/PVP2017-65664>. <hal-04497186>

**HAL Id: hal-04497186**

**<https://hal.science/hal-04497186v1>**

Submitted on 9 Mar 2024

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## PREPRINT PVP2017-65664

### FLUID DAMPING IN FUEL ASSEMBLIES

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#### ABSTRACT

Damping is known to be a major parameter in the seismic design of nuclear facilities. Of special interest is the case of fuel assemblies in PWR plants, which, unlike other components, are submitted to axial flows: it has been known since the late 80s that their frequency response to lateral excitations was largely dependent on the flow velocity, and the issue raised by this observation is to determine a consistent fluid force model which could be used in seismic design.

In the scientific literature, the standard model of fluid forces exerted upon an oscillating slender body was originally derived by Lighthill, and it involves a lift coefficient which, up to a reference frame shift, describes the force generated by a small angle of inclination of the body axis against the flow direction. Recent works by Divaret et al. have provided a value of this lift coefficient equal to 0.11 for a single cylinder, and to 0.18 for a square array of 5 by 8 cylinders, the Reynolds numbers being in the range of  $10^4$ .

Sticking to the idea that the damping stems from the local angle of inclination of the structure against the flow direction, the present study revisits recent tests performed in the Hermes test rig of CEA Cadarache, where a fuel assembly was submitted to incipient flow velocities varying from 1.5 to  $5\text{m}\cdot\text{s}^{-1}$ , and to a lateral force exerted upon the middle grid, generating displacements in the ranges of a few mm and of a few Hz. Under the assumption that the fuel assembly behaves in an approximately linear manner and that it undergoes harmonic deformations close to its first natural mode shape, the dissipative fluid force can be expressed by an adequate combination of the hydraulic cylinder force and of the structure displacements. A lift coefficient equal to 0.3-0.4 is obtained with this procedure, which stands for the overall fuel bundle, rods and grids included.

#### NOMENCLATURE

$C$  lift coefficient of the fuel assembly (-)

$\mathbf{D}$	structure damping matrix (in N.s/m)
$\mathbf{D}_{add}$	added mass matrix (in N.s/m)
$E$	Young modulus of the structure (in MPa)
$f$	average frequency (in Hz)
$F_c$	force exerted by the hydraulic cylinder (in N)
$F_n^{damping}$	equivalent damping fluid force exerted upon the n-th grid (in N)
$\mathbf{F}_{ext}$	vector of the external forces (in N)
$I$	inertia of a guide tube (in m <sup>4</sup> )
$\mathbf{Id}$	identity matrix (-)
$k$	modal stiffness of the first mode (in N/m)
$\mathbf{K}$	structure stiffness matrix (in N/m)
$\mathbf{K}_{add}$	fluid stiffness matrix (in N/m)
$L_{equ}$	average span from one grid to the next (m)
$m$	modal mass (in kg)
$m_{add}$	fluid added mass (in kg)
$\mathbf{M}$	structure mass matrix (in kg)
$\mathbf{M}_{add}$	added mass matrix (in kg)
$ND$	sum of the rod diameters (in m)
$t$	time (in s)
$U$	fluid velocity (in m/s)
$\mathbf{X}$	grid displacement vector (in m)
$\Phi_1$	first natural mode of the structure (-)
$\varphi$	sweeping phase of the hydraulic cylinder displacement
$\rho$	fluid density (kg/m <sup>3</sup> )
$\omega$	circular frequency (in Hz)

#### INTRODUCTION

Tests in the late 80s have shown that an axial flow generates a significant damping in nuclear fuel rods submitted to lateral oscillations [1, 2, 3, 4]. Two procedures have been used by different teams, the first consists in submitting a rod to a forced displacement, and the second in letting it oscillate freely. In both cases, large damping values have been observed,

which increase with the flow velocity [5, 6, 7, 8]. The drawback of these structural mechanics approach is that the overall damping coefficient results from a combination of the fluid forces and of the structural response of the fuel assembly. One way to overcome this difficulty consists in elaborating a structure model, and, by inverse calculation, in estimating the fluid force distribution. This operation is in theory achievable [9], but it requires a carefully tuned mechanical model of the structure to provide accurate results.

The purpose of the present study is to derive a direct estimation of the lift force exerted upon a fuel assembly in a series of forced displacement tests, taking benefit from the invariance of the deformation shape in the range of fluid velocities of interest. In these conditions, a theoretical expression of the lift force can be provided which depends only on measured structural data, and does not require the elaboration of a mechanical model of the fuel assembly. A lift force coefficient can on this basis be deduced from a series of measurements in a test rig.

### DISSIPATIVE FLUID FORCES EXERTED UPON A FUEL ASSEMBLY IN AXIAL FLOW

Since the pioneering works of Taylor [10] and Lighthill [11], it is established that a slender structure oscillating in axial flow is submitted to a lateral fluid force, which, at least in the limit of the low frequencies, is equivalent to a lift force when the structure is slightly inclined. An illustration of this equivalence is provided for a straight cylinder in Fig. 1, where a lateral oscillation at a velocity  $\dot{X}$  appears, throughout a frame shift, comparable to an inclination by an angle equal to  $\dot{X}/U$ .

A major feature of such a lift force is that it cannot be deduced from an ideal potential flow, so that following Lighthill, the most achieved theories of axial flow instability incorporate a dissipative force term based on this very angle [12]. It is proposed to adapt the same procedure to a fuel assembly, as illustrated in Fig. 2.

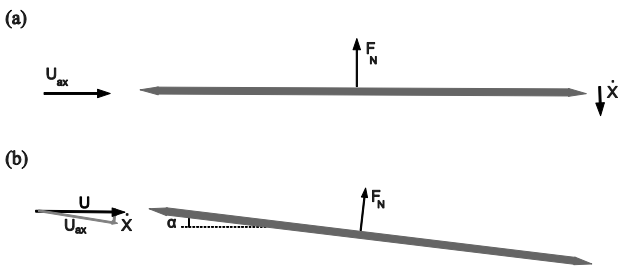


Fig. 1: Equivalence of an oscillating structure in axial flow and of a steady inclined structure in the limit of low frequencies, after Divaret et al. [13]

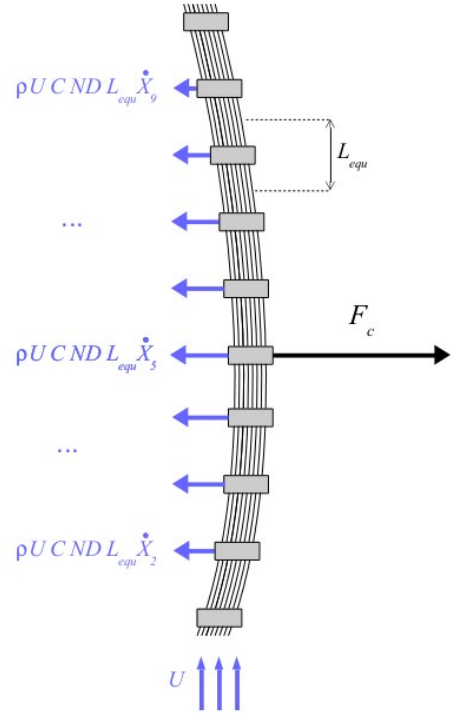


Fig. 2: Fuel assembly submitted to a forced displacement of its fifth grid in axial flow, and lift forces induced by its oscillation

In the representation of Fig. 2, the fluid generates point forces localized at each grid. Though the actual pressure forces are distributed along each fuel rod and each grid, it is convenient to replace them by these averaged forces. Indeed, as long as the displacement of each grid is reasonably close to its neighbor, the fuel assembly can be split in a thought experiment into eight series of grids with two half spans of rods, all elements being identical, and moving as a series of rigid bodies. Within these approximations, all damping fluid forces can be described by a lumped model, as shown in Fig. 2:

$$F_n^{damping} = i\omega 0.5 \rho U C N D L_{equ} \dot{X}_n \quad (1)$$

To avoid any confusion in the following, it should be highlighted that the fluid generates also forces related to the inclination angles of the rods, because during one oscillation cycle, the fuel assembly alternatively undergoes a deformation without lateral velocity and a lateral velocity with the same profile, while in straight position. The forces related to the deformation are in phase with the structure displacement, so that they do not contribute to the overall damping, and they can hence be discarded in the framework of the present study without consequences.

### EXPERIMENTAL PROCEDURE

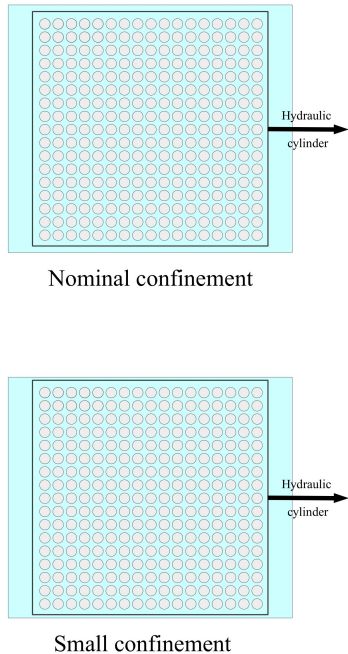
Experiments were performed in HERMES T, a single phase hydraulic loop in CEA Cadarache designed for testing full scale PWR fuel assemblies. During the tests, a 14 feet fuel assembly was arranged inside the rig, with its upper and lower ends clamped. This fuel assembly is made up of 25 guide tubes and 264 fuel rods in a square array, for a total height of 4.5m. At regular spacing intervals, the guide tubes are welded to ten grids, which firmly hold the fuel rods. Each fuel rod has a diameter equal to 9.5mm and a pitch equal to 12.5mm.

The assembly cross section is square, with a width equal to about 210mm. The test section is 20mm wider than the fuel assembly in the excitation direction and either 5mm or 10mm wider in the orthogonal direction in order to test the influence of a reduced confinement, as shown in Fig. 3.

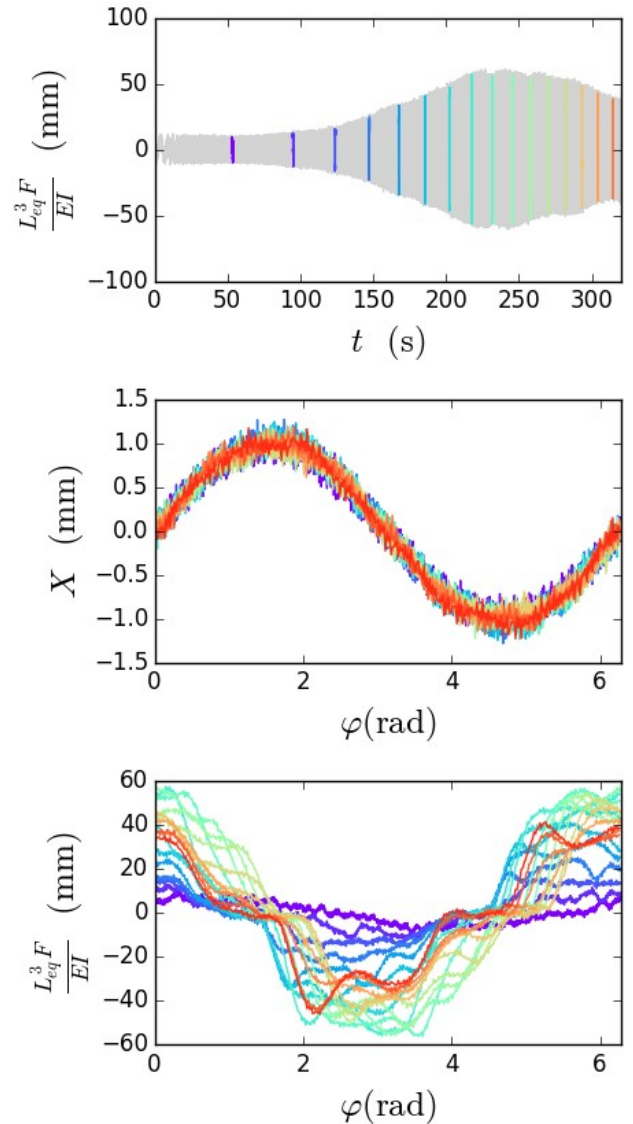
A swept sine displacement ranging from 0.2 to 3 Hertz is imposed to the fifth grid, which is approximately in the middle of the fuel assembly. This displacement is controlled by a hydraulic actuator, also called hydraulic cylinder in the following. The tests are carried out with displacement amplitudes ranging from 1 to 10mm. An illustration is provided in Fig.4, where the force is divided by a reference stiffness term for the sake of readability.

A load cell arranged between the fuel assembly and the hydraulic actuator measures the force applied to the grid. The displacements of the 2nd to the 9th grid are simultaneously recorded by Linear Variable Differential Transformer sensors.

Tests are performed at 50°C in still water, and under axial flow for average fluid velocities equal to 1.5m/s, 3m/s and 5m/s. Further details about the experiments can be found in the references [14, 15].



**Fig. 3: Horizontal confinements of the fuel assembly during the different tests**



**Fig.4: Example of a sweep sine measured force envelope during a test (upper curve), and samples of cycles at different frequencies: hydraulic cylinder displacement (middle plot) and reduced force (lower plot) against the phase during several cycles**

The sweeping sine technique performed during the experiments requires some data processing in order to elaborate a consistent harmonic analysis. Furthermore, it is assumed that the non-linear behavior of the fuel rod, illustrated by the lower curve in Fig.4 does not have an influence on the estimation of the lift coefficient. One argument in favor of this assumption is that the overall dissipation during one cycle is estimated by the average value of the product of the force and of the hydraulic cylinder velocity. As this latter term is an almost perfect sine

(see Fig.4), the non-linear components of the force do not interfere with an estimation of the lift coefficient. It may be worth mentioning here that the non-linear behavior is in large proportions due to the mechanical friction inside the grids, a phenomenon easy to observe by performing a slow cycle of deformation, which generates a hysteretic displacement against force curve [5]: an accurate description of this effect ranges far beyond the scope of the current study, so that it is not detailed any further.

Let then the reference displacement be assumed to follow a sine-like law:

$$x_{ref}(t) = X_{ref} \cos \varphi(t)$$

where the phase  $\varphi$  increases approximately as the second power of time, so that the instantaneous frequency can be conveniently defined by:

$$f(t) = \frac{1}{2\pi} \frac{d\varphi}{dt}$$

For practical purposes, the phase  $\varphi$  can be approximated by a spline function, increasing by  $\pi$  each time a zero of  $x_{ref}$  is found. The key idea consists in using  $\varphi$  as a time-like variable instead of the usual Fourier term  $\omega t$ , every measurement being expanded in a series of harmonic terms:

$$x_n(t) = \text{Re} \left( \sum_p X_n^p e^{ip\varphi(t)} \right)$$

the coefficients being determined by a straightforward integration upon the variable  $\varphi$ , turned into an integration upon time by replacing the increment  $d\varphi$  by  $\varphi'(t)dt$ :

$$X_n^p = \frac{1}{\pi} \int (\cos p\varphi - i \sin p\varphi) x_n(t) \frac{d\varphi}{dt} dt \quad (2)$$

This procedure provides the best estimation of the harmonic amplitude of the  $n$ -th grid displacement, the associated frequency being the average value of  $f$  along the cycle.

## LUMPED MODEL ESTIMATION OF THE LIFT COEFFICIENT

Keeping in mind that the objective consists in deriving a model of fluid damping, the mechanical behavior of the fuel assembly is assumed to be linear, for the sake of simplicity.

Moreover, as only the grid displacements and the hydraulic cylinder force are recorded, a lumped model is proposed in order to match the degrees of freedom with the available data. Let then the positions of the grids be assembled into a state vector  $\mathbf{X} = (X_1, \dots, X_{10})$ , the dynamic equation of the fuel assembly in the presence of flow and under the effect of the hydraulic cylinder force  $F_c$  can be *a priori* written in harmonic regime by a Galerkin approach:

$$-(\mathbf{M} + \mathbf{M}_{add})\omega^2 + i\omega(\mathbf{D} + \mathbf{D}_{add}) + (\mathbf{K} + \mathbf{K}_{add}) \mathbf{X} = \mathbf{F}_{ext} \quad (3)$$

where  $\mathbf{M}$  and  $\mathbf{M}_{add}$ ,  $\mathbf{D}$  and  $\mathbf{D}_{add}$  and  $\mathbf{K}$  and  $\mathbf{K}_{add}$  are the mass and the added mass matrices, the structure and fluid damping matrices and the stiffness and the added stiffness matrices, respectively, and where the external force vector  $\mathbf{F}_{ext}$  has only one non-vanishing component, associated with the hydraulic cylinder. Such a representation overlooks the non-linear behavior of the grids which is the root cause of the hysteretic damping in the absence of flow. Some authors have tried to grasp this effect by introducing a complex stiffness, yet this approach raises the issue of causality [16], and special care must be taken to match the real and the imaginary parts of the transfer function in Eq. (3). Fortunately, it is here enough to assume that the structural matrices  $\mathbf{K}$  and  $\mathbf{D}$  have such values that the hysteretic damping is adequately described, without entering into further details. For the sake of completeness, it may be worth mentioning that the local force equations proposed by Païdoussis [12] for axial flow instabilities are more general than Eq. (3), this latter can be considered as a low order expansion in powers of  $\omega$  of the dynamics of fluid-coupled beams.

Sticking to the low frequency lumped model approach, the mass matrices can fairly be assumed to be proportional to the identity matrix, because the spans between the grids are practically identical. Hence, each degree of freedom carries the same mass of structure and of water, which is denoted  $m + m_{add}$  in the following. As regards the fluid damping, the lift force described in the third section scales with the lateral velocity of the structure, which implies that the fluid damping matrix is also proportional to the identity matrix:

$$\mathbf{D}_{add} = 0.5 \rho U C N D L_{equ} \mathbf{Id} \quad (4)$$

where  $L_{equ}$  is the average span from one grid to the next, where  $ND$  stands for the summation of the rod diameters,  $C$  being the lift coefficient defined in such a way that it can easily be compared to the single cylinder one.

The identification approach relies on the assumption that the deformation shape generated by the hydraulic cylinder displacement is very close to the first natural mode of the fuel assembly, and that this deformation remains unchanged in the frequency range close to the first natural and for various fluid

flows. Such an assumption is sound in the sense that the first natural mode of a fixed-fixed beam is known to be close to the static deformation due to a point force applied in its middle, and it can be validated by comparing all the deformations in the range of parameters of interest. Let in the framework of this assumption  $\Phi_1$  be the first natural mode of the structure, normalized by equating the displacement of the hydraulic cylinder to unity. By definition,  $\Phi_1$  is an eigenmode of both the mass and the stiffness matrices [17]:

$$(\mathbf{M} + \mathbf{M}_{add})\Phi_1 = (m + m_{add}) \Phi_1 \quad \text{and} \quad (5)$$

$$\mathbf{K} \Phi_1 = k_1 \Phi_1$$

Furthermore, a similar relation holds for the fluid damping matrix:

$$\mathbf{D}_{add} \Phi_1 = 0.5 \rho U C N D L_{equ} \Phi_1 \quad (6)$$

Unfortunately, no theoretical reason supports a similar relation for the added mass matrix nor for the structure damping matrix. One further approximation is now needed to provide an estimation of the damping coefficient: let the added stiffness generate a real term  $\mathbf{K}_{add} \Phi_1$ . Hence, provided that the deformation  $\mathbf{X}$  remains proportional to the natural mode  $\Phi_1$ , the matrix projection of Eq. (3) along the displacement  $\mathbf{X}$  has an imaginary part equal to:

$$\text{Im}(i\omega {}^t\mathbf{X}\mathbf{D}\mathbf{X}) + 0.5 \omega \rho U C N D L_{equ} {}^t\mathbf{X}\mathbf{X} = \text{Im}({}^t\mathbf{X}\mathbf{F}_{ext}) \quad (7)$$

Assuming that the structure dissipation term  ${}^t\mathbf{X}\mathbf{D}\mathbf{X}$  does not depend on the flow velocity, and observing that the scalar product  ${}^t\mathbf{X}\mathbf{F}_{ext}$  is simply equal to the product of the grid displacement with the hydraulic cylinder force, the lift coefficient can be estimated by a straightforward difference between a transfer function in the presence of flow and in quiescent fluid:

$$0.5 \omega \rho U C N D L_{equ} = \text{Im} \left( \frac{X_c F_c(U)}{{}^t\mathbf{X}\mathbf{X}} - \frac{X_c F_c(0)}{{}^t\mathbf{X}\mathbf{X}} \right) \quad (8)$$

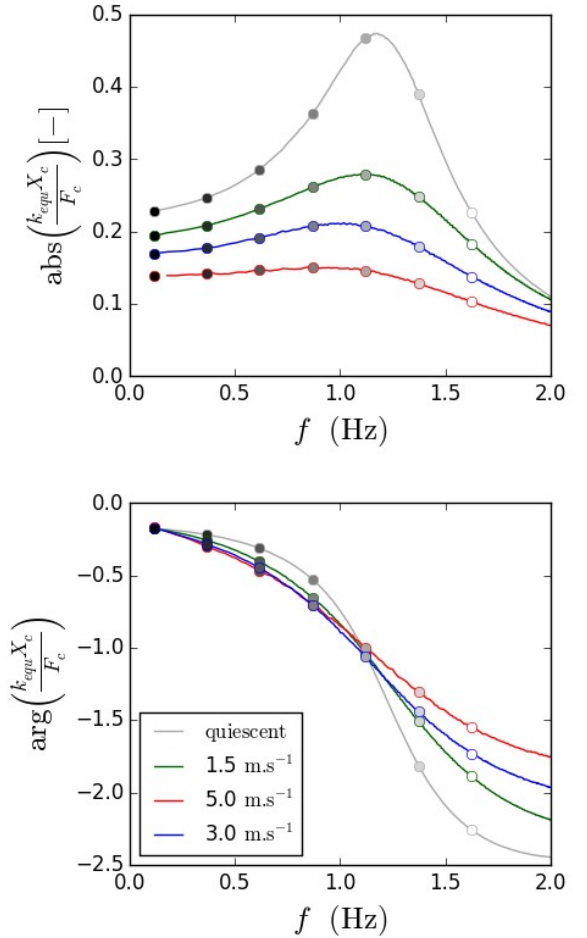
Eq. (8) provides a direct estimation of the lift coefficient of a grid to grid span inside a fuel assembly. Its main interest is that it is based on measured data and does not require any explicit modeling of the structure.

## RESULTS AND DISCUSSION

For assessing its performance, the previous procedure is first applied step by step to an example of test, performed with the nominal confinement at a prescribed amplitude of 10mm.

The transfer function shown in Fig.5 illustrates the considerable influence of the flow velocity upon the overall damping coefficient, a result coherent with the scientific literature given in introduction.

A very satisfying result is the independence of the deformation with the flow and the frequency, illustrated by Fig.6, which supports the fundamental assumption of the procedure. The slight phase variation observed in the lower curve is less than a few degrees, and is mostly due to the frequency variation during one cycle. Hence, it can reasonably be assessed that the deformation during the tests is invariant, and it is assumed to be very close to the first natural mode shape.



**Fig. 5: Transfer function of the fuel assembly in quiescent flow and for three fluid velocities. Upper plot: modulus, lower plot: argument. Continuous line: successive cycles of the sweeping process, circles: selection of frequencies for further illustrations**

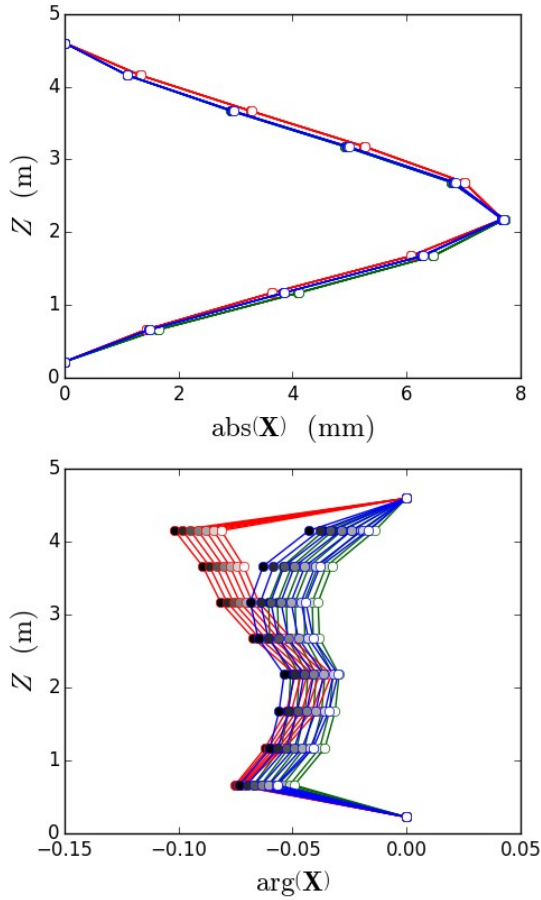


Fig. 6: Deformations of the fuel assembly, with the same color code as in Fig. 5

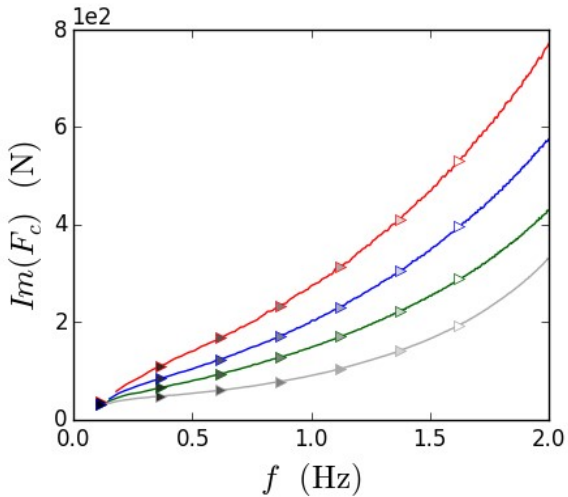


Fig. 7: Imaginary part of the hydraulic cylinder force, with the same color code as in Fig. 5

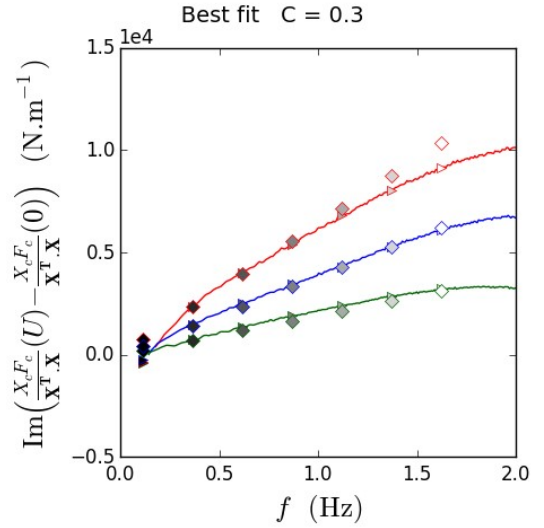
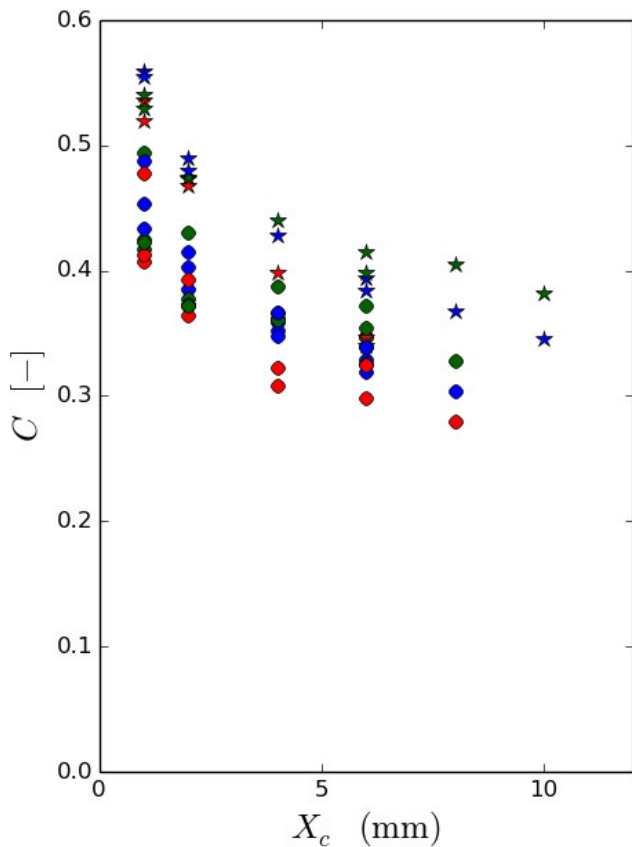


Fig. 8: Linear fit of the reduced force vs. frequency graph, with the same color code as in Fig. 5

The damping effect of the flow is easily highlighted by plotting the imaginary part of the force: as is well known in vibrations, the conservative forces associated with stiffness and inertia are in phase with the displacement, and the dissipative forces which are the root cause of damping exhibit a phase shift of  $\pi/2$ . As can be seen in Fig. 7, for a prescribed displacement of the hydraulic cylinder, the dissipative force increases with the frequency, and also with the flow rate.

The contribution of the mechanical friction to the dissipative force can be removed by subtracting the imaginary force in quiescent fluid to the imaginary forces measured in flow regime. Following this procedure, the r.h.s. term of Eq. (8) can be plotted as shown in Fig. 8. As can be seen, the three curves fairly follow a linear trend, with a slope depending on the flow velocity. A least square estimation of the lift coefficient can easily be obtained, and an overall value of this coefficient equal to 0.3 is obtained. Such a value is significantly higher than the one obtained by Divaret et al. with an array of cylinders [13], a result which makes sense because the grid generates an extra contribution to the fluid force.

Similar figures can be obtained with other tests, which are not reproduced here. Be it enough to mention that in most cases, the deformations have the same constant shape, exceptions being due to gripping effects caused by mechanical friction and not related to the fluid forces. All the results of these other tests are compiled in Fig. 9, which represents the least-square lift coefficients as a function of the forced displacement amplitudes for various flow rates.



**Fig. 9: Lift coefficients against the hydraulic cylinder amplitude for different flow velocities (same color code as in Fig. 5). Circles: two series of tests with a nominal confinement, stars: one series of test with a small confinement**

As can be seen, the lift coefficient associated to the fuel assembly is in the range 0.3-0.5 for all test conditions. It decreases with the oscillations amplitude, and to a lesser extent, with the flow rate. Furthermore, a reduction of the lateral confinement significantly increases the lift coefficient, an effect which can reasonably be explained by an increase of the oscillation-driven fluid velocity around the grids, and, in smaller proportions, around the rod bundle.

## PERSPECTIVES

An experimental procedure has provided a series of values of the equivalent lift coefficient for a PWR 14 ft fuel assembly. These values range from 0.3 to 0.4 in the case of nominal confinement, and it is proposed that they should be considered for seismic design purposes.

For future studies, a more elaborate modelling of the fuel assembly could be undertaken, based on the Païdoussis representation [12]. The measurements could be further processed, beyond the first natural frequency, when other modes contribute to the deformation. A challenging task will be

to revisit the combination of friction related non linearities and of unsteady flow effects.

From a more academic perspective, further investigations should be undertaken to examine the flow pattern along slender structures, in order to confirm the validity of the lift coefficient approach: it was found by Divaret et al. [13] that the pressure distribution was not perfectly uniform along a single cylinder in axial flow.

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