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Adjoint-based sensitivity and **feedback** control of noise emission

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september, 20th, 2013

- 1 Introduction and testcase
- 2 Sensitivity analysis
- 3 Direct feedback output control
- 4 Application and analysis
- 5 Summary and perspectives

Active flow control strategies

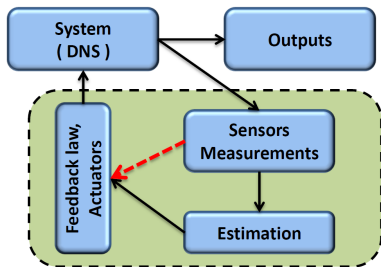
- In addition to the passive flow control and the shape optimization, it is a necessity to control flows to **enhance performances** in transportation vehicles : drag, lift, noise emission, flow instabilities, separation, ...
 - My challenge : to propose a **methodology based on theoretical and numerical** approaches for actuation law design dealing with large system ($DoF \geq 10^6$)
- 1 **Open loop control** :
 - optimization problem with full PDE (DNS, LES, ...)
 - expensive and time consuming
 - low robustness
 - boundary layers, shear layer, jet flows Airiau et al (2003), Wei & Freund (2006), Spagnoli & Airiau (2008), Sesterhenn & al (2012)
 - 2 **Feedback control** : more efficient, used in real flow and systems, robustness can be a parameter or an issue, first step towards adaptive control.

Direct feedback output control

Feedback control : to manage the huge size of fluid flow configurations

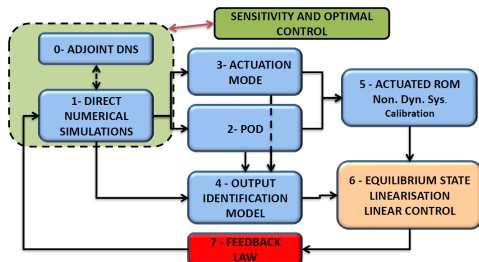
☞ Need a **Reduced Order Model**

- 1 ROM based on global stability modes : laminar flow \rightarrow turbulent flow
Bargallo, Sipp et al (2012), Raymond et al (2013) , Rowley, ... , DoF $\approx 10^5$
- 2 ROM based on **POD modes** : laminar flow \rightarrow moderate turbulent flow. Rowley (Balanced POD, 2012), Airiau & Cordier (2013), ...
- 3 POD analysis + heuristic feedback law based on physical considerations Pastoor et al (2008), ...



- Usual feedback loop
(sensors \rightarrow state estimate \rightarrow actuation)
Done with ROM
- Present work : with DNS,
direct output feedback law
sensor outputs \rightarrow actuation law

General methodology : control of a ROM



POD : Proper Orthogonal Decomposition

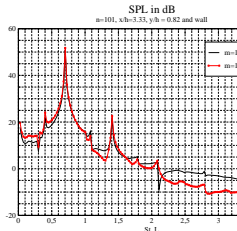
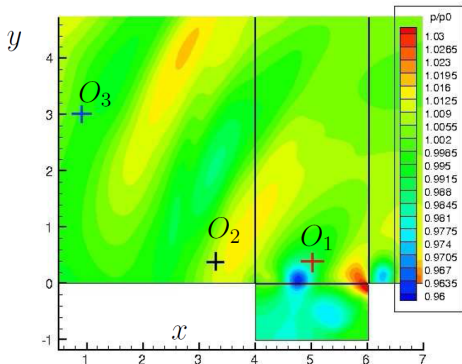
- Application to any flow control as soon as a POD is relevant
- 8 steps : large developments and programming
- DNS : large DOF ($> 10^6$)
- ROM/LINEAR CONTROL : very low DOF (< 10)
- Computational cost : 2 DNS + ROM
- Many parameters, options and choices ...
- what else ?

Issues for an efficient feedback law:

- Optimal position and type of actuators (controlability)
- Optimal position and type of sensors (observability)
- Well capturing the response of the flow (computed by DNS) to any actuation : actuation mode(s)

1a - Testcase : 2D compressible cavity flow

☞ $M_\infty = 0.6$, $R_\theta = 688$, $Re_L = 2981$, noise control is a good (severe) testcase



Testcase $L/H = 2$

(Rowley, JFM 2002)

$$St_L = \frac{n - 0.25}{M_\infty + 1.754}$$

$$St_L = 0.703$$

$$\text{Rossiter 2} : St_L = 0.74$$

- Self-sustained instability due to a feedback effects with the impingement of the shear layer on the downstream cavity corner
- Instantaneous pressure, acoustic wave directivity Rowley's case
- O_1 and O_3 : center point of the observation domain for sensitivity
- O_2 : probes for spectral analysis (SPL)

75 000 Δt

0 - Actuators : sensitivity analysis

☞ Actuator position and type provided by the sensitivity analysis

- 1 Observed quantity (functional)

$$J(\mathbf{q}, \mathbf{f}) = \int_{\Omega} \int_0^T j_{\text{observed}} d\Omega dt, \quad j_{\text{observed}} = \frac{1}{2}(p - \bar{p})^2$$

\mathbf{q} : state vector, \mathbf{f} forcing/actuation vector

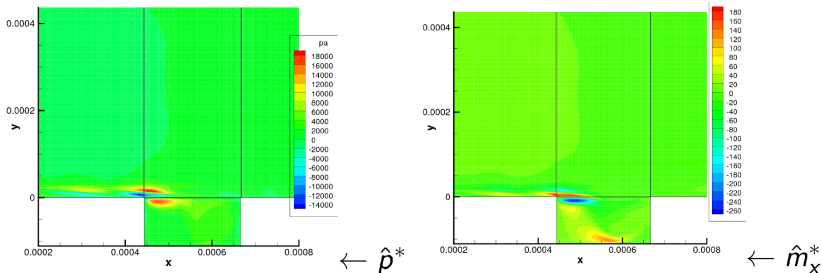
- 2 Variational problem and Fréchet derivative : sensitivity $S(\mathbf{x}, t)$

$$\delta J = \left\langle \frac{\partial j_{\text{observed}}}{\partial q_k}, \delta q_k \right\rangle_{\Omega} = \langle S_{f_i}, \delta f_i \rangle_{\Omega}$$

- 3 Adjoint Navier-Stokes equations : sensitivity is related to the adjoint state : $S_{f_i} = q_i^*$
- 4 true for wall localized forcing and global volume forcing

0 - Actuators settings

- 2D Fourier modes, adjoint state (case 10) from **adjoint DNS** (Continuous)
- stationnary mode sensitivity : steady actuation



- 2nd Rossiter mode sensitivity : unsteady actuation
- x-momentum forcing
- mass forcing

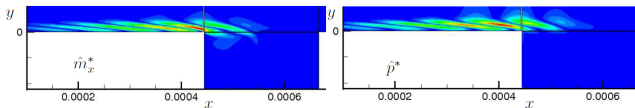


FIGURE 3 – Modes at St_1 for \hat{m}_x^*/U_∞ (blue : 0.48, red : 7.22) and \hat{p}^*/U_∞^2 (blue : 0.23, red : 3.71)

Modes localized in space associated to the controllability property

1b - DNS response to a generic actuation

☞ Need to define actuation mode in the ROM from actuated DNS

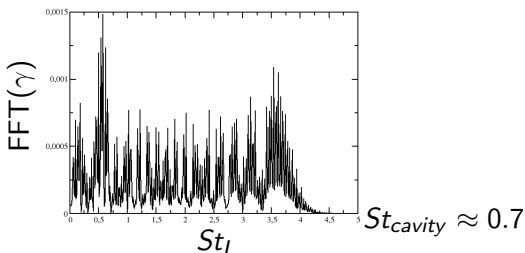
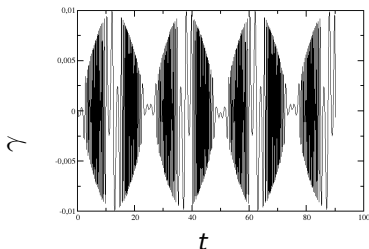
Wall normal velocity forcing

- Distributed actuation centered at $x_f = 2.72X/D$

$$f_w(\mathbf{x}, t) = \gamma(t) \exp[-r^2/\sigma^2], \quad r^2 = \|\mathbf{x} - \mathbf{x}_{for}\|^2, \sigma = 50\Delta_y$$

- Large frequency bandwidth actuation $A_1(t)$:

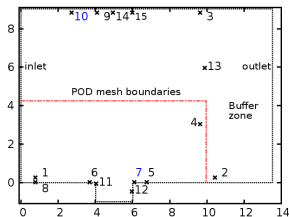
$$\gamma(t) = A_1 \sin(2\pi St_1 t) \times \sin(2\pi St_2 t - A_2 \sin(2\pi St_3 t))$$



☞ To excite and therefore later to control all possible physical unstable perturbations

3 -From DNS to POD & ROM - actuation mode

Proper Orthogonal Decomposition of unactuated flow field $\rightarrow \phi_i^u(\mathbf{x})$



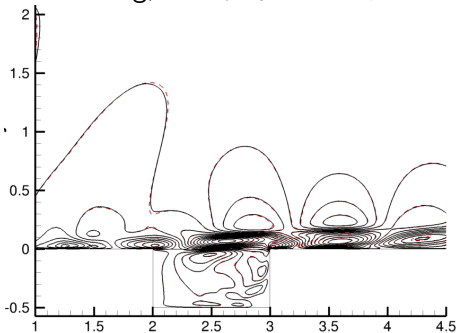
POD mesh size $<$ DNS mesh \Rightarrow
gain in CPU time and accuracy
Optimal size ?

$$\mathbf{q}^a(\mathbf{x}, t) = \bar{\mathbf{q}}^a(\mathbf{x}) + \sum_{i=1}^N a_i^a(t) \phi_i^u(\mathbf{x}) + \gamma(t) \boldsymbol{\psi}(\mathbf{x}) \quad (1)$$

- $(\phi_{i=1,N}^u, \boldsymbol{\psi})$ orthogonal basis, truncation to N modes
- Assumption : $\bar{\mathbf{q}}^a(\mathbf{x}) \approx \bar{\mathbf{q}}^u(\mathbf{x})$
- $\mathbf{q} = (\zeta = 1/\rho, u, v, p)$

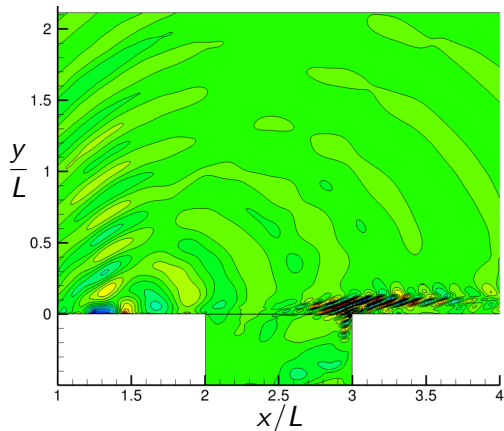
3 - Actuation mode, $A_1 = 0.001$

Sine forcing, u velocity, Vigo's and isentropic models



Distributed actuation along the wall
and in the shear layer

Chirp, u velocity, Vigo's model



Actuation close to the noise source
location

5, 6a, 6b - ROM-Galerkin projection

- **Projection** of NSE (formulation with $\zeta = 1/\rho$, *Vigo-98*, *Bourguet-09*) on $(\phi_{i=1,N}^u)$: Nonlinear forced dynamical system of low order

$$\dot{\mathbf{a}} = \mathbf{C} + \mathbf{L}\mathbf{a} + \mathbf{a}^t \mathbf{Q}\mathbf{a} + \gamma \hat{\mathbf{L}}\mathbf{a} + \gamma \hat{\mathbf{C}} + \gamma^2 \hat{\mathbf{Q}} \quad (2)$$

- Calibration of ROM (find \mathbf{C} and \mathbf{L} for $\mathbf{a}(t)_{POD} = \mathbf{a}(t)_{ROM}$)
- **Equilibrium (steady) state** (many states can exist) :

$$\text{Physical domain: } \mathbf{q}^e(\mathbf{x}) = \bar{\mathbf{q}}(\mathbf{x}) + \sum_{i=1}^N a_i^e \phi_i(\mathbf{x})$$

Equilibrium state of the NS eq.

- **Linearization** with $\tilde{\mathbf{a}} = \mathbf{a} - \mathbf{a}^e$:

$$\begin{aligned} \dot{\tilde{\mathbf{a}}} &= \mathbf{L}\tilde{\mathbf{a}} + \tilde{\mathbf{a}}^t \mathbf{Q}\mathbf{a}^e + (\mathbf{a}^e)^t \mathbf{Q}\tilde{\mathbf{a}} + (\hat{\mathbf{L}}\mathbf{a}^e + \hat{\mathbf{C}})\gamma \\ \dot{\tilde{\mathbf{a}}} &= \tilde{\mathbf{A}}\tilde{\mathbf{a}} + \tilde{\mathbf{B}}\gamma \quad \text{State equation} \end{aligned} \quad (3)$$

4 - Use of sensors : output identification model

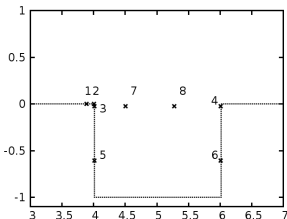
➡ Required for the feedback control law design

- Unsteady pressure sensors ($\tilde{y}_i = \tilde{p}_i$):

$$\tilde{y} = \tilde{C}\tilde{a} + \tilde{D}\gamma, \quad \tilde{y} = y - \bar{y} - \tilde{C}a^e \quad (4)$$

- sensor y_i is located on the POD mesh at \mathbf{x}_k :

$$\tilde{C}_{ij} = \phi_j^u(\mathbf{x}_k) \text{ and } \tilde{D}_i = \psi_k = \psi(\mathbf{x}_k).$$



$N_s = 6$ (1 → 6) sensors are used to build the actuation law.

Optimal positions : physical considerations, observability

6c - ROM-Linear Quadratic Regulator control

 direct output feedback control law design

- linear state space model : $\dot{\tilde{\mathbf{a}}} = \tilde{\mathbf{A}}\tilde{\mathbf{a}} + \tilde{\mathbf{B}}\gamma$
- **feedback control law** : $\gamma = -\mathbf{K}_c \tilde{\mathbf{a}}$
- minimization of $\mathcal{J} = \int_0^T (\tilde{\mathbf{a}}^t \tilde{\mathbf{a}} + \ell^2 \gamma^2) dt$.
- Ricatti equation : $\mathbf{K}_c = \frac{1}{\ell^2} \tilde{\mathbf{B}}^t \mathbf{X}$. $\left(\tilde{\mathbf{A}}^T \mathbf{X} + \mathbf{X} \tilde{\mathbf{A}} - \frac{1}{\ell^2} \mathbf{X} \tilde{\mathbf{B}} \tilde{\mathbf{B}}^T \mathbf{X} + Id = 0 \right)$.
- outputs : $\tilde{\mathbf{y}} = \tilde{\mathbf{C}}\tilde{\mathbf{a}} + \tilde{\mathbf{D}}\gamma$
- **direct feedback output control** :

$$\gamma(t) = \alpha(y(t) - \bar{y}) + \beta \quad (5)$$

- $N_{POD} = N_{Sensors}$: $\gamma(t) = -\mathbf{K}_c(\tilde{\mathbf{C}} - \tilde{\mathbf{D}}\mathbf{K}_c)^{-1}(y - \bar{y} - \tilde{\mathbf{C}}\mathbf{a}^e)$
- β imposes the mean actuation velocity, α imposes the damping of the time variation of the actuation
- implementation in DNS code

application: efficiency & robustness

- Efficient and robust feedback control law : decay of pressure fluctuation levels

Fig. 1 : control law

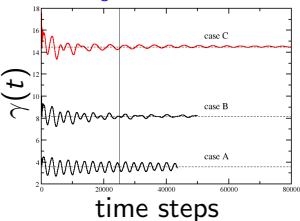


Fig. 3 : spectra

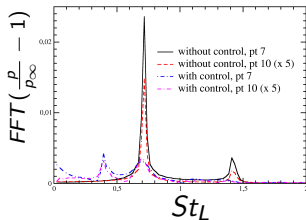


Fig. 2 : probes values

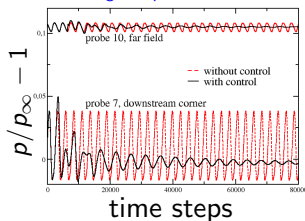
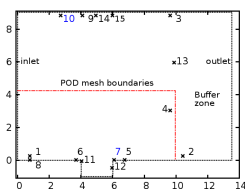


Fig. 4 : Probes location



$$\gamma(t) = \alpha(y(t) - \bar{y}) + \beta,$$

tuning β to improve the efficiency

A) $\beta \approx 5\text{m/s}$,

B) $\beta \approx 10\text{m/s}$,

C) $\beta \approx 17\text{m/s}$

Robustness, feedback law

time window : $it=25000$

spatial window : red box

near and far field noise damping

subharmonic and harmonics :

weakly nonlinear effects

Noise reduction

Fig. 1 : $SPL_{ac.} - SPL_{unac.}$

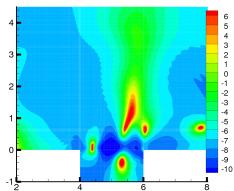


Fig. 2 : mean pressure

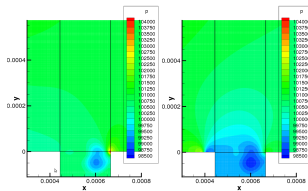
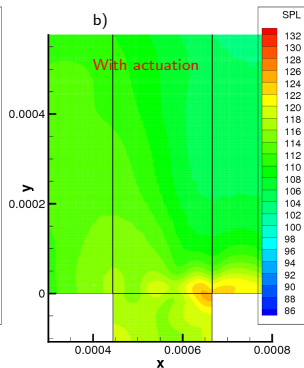
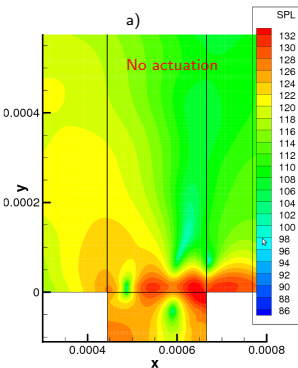


Fig. 3 : SPL levels



Case C : maximum of -10 dB

- Global noise reduction
- Few areas with increase, but SPL remains low
- wavy SPL contours: weakly nonlinear effect
- Actuation modifies the mean pressure : nonlinearity

Deeper analysis : RIC content & POD eigen values

- Actuation drastically modifies the flow dynamic: $\mathbf{q}(\mathbf{x}, t) = \bar{\mathbf{q}}(\mathbf{x}) + \sum_{i=1}^N a_i(t)\phi_i(\mathbf{x})$

Fig. 1 : POD eigenvalues

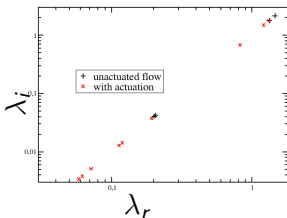
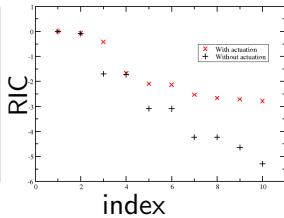
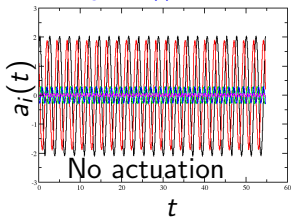
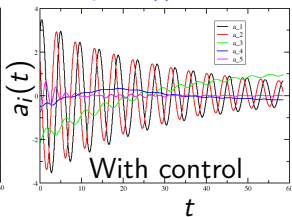


Fig. 2 : mode relevance

Fig. 3 : $a_i(t)$, no actuationFig. 4 : $a_i(t)$, with control

Relative Information Content (RIC) :

more relevant modes are required



Eigen values distribution : new spectra

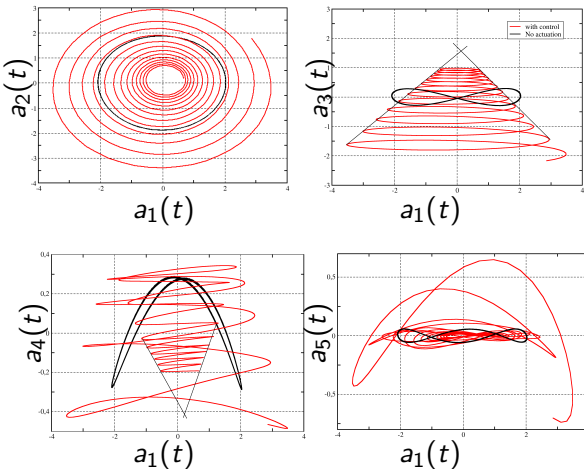


time coefficients $a_i(t) \rightarrow$ constante for

$t \rightarrow \infty$, time damping

POD time coefficients

- Phase portrait : convergence towards a steady (equilibrium) state ?



POD decompositions :

1) with unsteady actuation

$$\mathbf{q}^a(\mathbf{x}, t) = \bar{\mathbf{q}}^a(\mathbf{x}) + \sum_{i=1}^N a_i^a(t) \phi_i^u(\mathbf{x}) + \gamma(t) \psi(\mathbf{x})$$

2) with stabilization ($t \rightarrow \infty$) :

$$\mathbf{q}^\infty(\mathbf{x}) = \bar{\mathbf{q}}^{ac}(\mathbf{x}) + \sum_{i=1}^N a_i^\infty \phi_i^{ac}(\mathbf{x})$$

Next step : determine the final state $\mathbf{q}^\infty(\mathbf{x})$

Summary and perspectives

• Some concluding remarks

- 1 Feedback output control law implemented in DNS (2 DNS + ROM \rightarrow low cost)
- 2 Efficient and quite robust (when time $\rightarrow \infty$)
- 3 Several dB of noise reduction (up to -10 dB)
- 4 Possible to tune the feedback law to improve efficiency, towards nonlinearity
- 5 Independent on the DNS code : \rightarrow LES ?

• Current works

- 1 Linear Quadratic Gaussian control with ROM with state estimate
- 2 Feedback law :
$$\gamma(t) = \int_{t-t_c}^t \sum_{i=1, N_s} G_i(t-\tau) y_i(\tau) d\tau$$

• Some improvements and perspectives

- 1 Sensitivity analysis to many parameters included in the approach : N_{POD} , N_s , actuation location and type, option of the ROM or POD, ...
- 2 To test other flow decomposition to better take into account of actuation (with $\dot{\gamma}$)
- 3 To increase physical parameters (Re, Ma) and to test other flows
- 4 Nonlinear feedback analysis and robustness analysis (H_2 , H_∞)
- 5 Validation/comparisons with experiments on low Reynolds number reference flows

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