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► **To cite this version:**

Stéphane Roussel, Matthieu Boffety, François Goudail. Calibration and measurement precision of division-of-focal-plane polarization sensors. 9th International Symposium on Optronics in Defence and Security, Jan 2020, Paris, France. hal-04492640

**HAL Id: hal-04492640**

**<https://hal.science/hal-04492640>**

Submitted on 6 Mar 2024

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# Calibration and measurement precision of division-of-focal-plane polarization sensors

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## ABSTRACT

The polarization imagers based on linear division-of-focal-plane (DoFP) polarization sensors are receiving much attention due to their integrated structure and their ability to capture polarization information in real time. These cameras have manufacturing defects that must be calibrated both in terms of photometry and of polarimetry. We precisely quantify the influence of these defects on the estimation precision of the Stokes vector and on such polarimetric parameters as the degree of polarization (DOP) and the angle of polarization (AOP). Moreover, commercial DoFP cameras can only measure the linear Stokes vector. To perform full Stokes measurements, one can use a retarder placed in front of them. We show that the parameters of the retarder can be optimized in order to minimize the estimation variance of the Stokes vector.

## 1 INTRODUCTION

Polarimetric imaging consists in measuring the polarization state of the light coming from each point of a scene. It can reveal contrasts that are not visible in classical images and has many applications in remote sensing, medical imaging, or industrial control. The complete polarization state can be expressed by the Stokes vector and up to now, various Stokes polarimeter architectures have been proposed. In particular, as an emerging technology, the polarization imagers based on linear division-of-focal-plane (DoFP) polarization sensors have received much attention due to their integrated structure and their ability to capture polarization information in real time. Commercial DoFP cameras are nowadays available at affordable cost. We address in this article the ways to get the most out these sensors in terms of polarization parameter estimation.

These cameras have manufacturing defects that must be calibrated both in terms of photometry and of polarimetry. We show that these defects have an influence on estimation precision of the polarimetric parameters. This influence can be precisely quantified when the camera measurements are corrupted by both additive noise and Poisson shot noise. Moreover, commercial DoFP cameras can

only measure the linear Stokes vector. In order to leverage the stable, integrated nature of available linear DoFP cameras to perform full Stokes measurements, it is possible to use a retarder placed in front of them. Such setups have been recently demonstrated and used in practice [1, 2]. We show that the parameters of the retarder can be optimized in order to minimize the estimation variance of the Stokes vector.

The paper is organized as follows. In Section 2, we describe DoFP polarization cameras and their calibration. In Section 3, we determine the estimation precision of the Stokes vectors, DOP and AOP, as a function of the calibrated parameters of the camera. In Section 4, we optimize a retarder for estimation of the full Stokes vector in 2 shots.

## 2 Polarization cameras and their calibration

In polarimetric cameras, each pixel of the sensor has a different polarimetric sensitivity linked to the element of the micropolarizer array placed just in front of it (Fig. 1). If grouped together,  $2 \times 2$  sets of neighbor pixels (called *superpixels*) can sense the whole polarization state of the incoming light. In most existing devices, the array is composed of linear polarizers, so that only the linear characteristics of the polarization states (linear Stokes vector) can be measured [3].

In order to use this type of cameras, the first step is to perform calibration. This operation is more complex than with a standard camera since the polarimetric characteristics of the micropolarizer in front of each pixel have to be calibrated. This issue has been addressed and is well documented [4, 5]. We have recently proposed a calibration approach that makes it possible to evaluate the effect of Poisson shot noise on estimation of polarimetric parameters [6]. This method yields the measurement matrix  $\mathbb{W}$  of each superpixel of the camera.

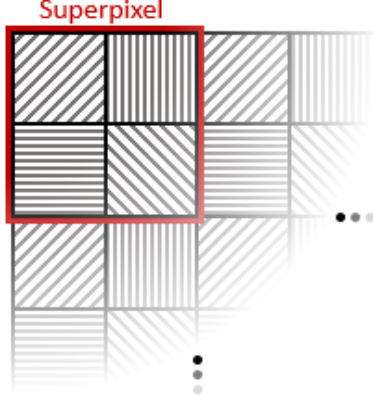


Figure 1: Schematic representation of the micropolarizers on the camera sensor. A superpixel is composed of 4 pixels with micropolarizers oriented at  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$  and  $135^\circ$ .

### 3 Estimation precision with DoFP cameras

Let us consider a superpixel with measurement matrix  $\mathbb{W}$ , and let denote the Stokes vector of the light impinging on this superpixel as:

$$\mathbf{S} = S_0[1, P \cos(2\alpha), P \sin(2\alpha)] \quad (1)$$

where  $S_0$  is its intensity,  $P$  its degree of linear polarization (DOLP) and  $\alpha$  its azimuth, or angle of polarization (AOP). The vector of the 4 intensities measured by the 4 subpixels of this superpixel is:

$$\mathbf{I} = \mathbb{W}\mathbf{S} \quad (2)$$

In practice, these intensity measurements are corrupted by two main sources of noise: additive noise and Poisson shot noise, the latter being dominant in most practical cases. To estimate  $\mathbf{S}$  from  $\mathbf{I}$ , we use the pseudo-inverse estimator:

$$\hat{\mathbf{S}} = \mathbb{W}^+\mathbf{I} \quad (3)$$

Since the measurements  $\mathbf{I}$  are perturbed by noise, the estimator  $\hat{\mathbf{S}}$  of the Stokes vector is a random vector. The precision of this estimator can thus be represented by its covariance matrix  $\Gamma^{\hat{\mathbf{S}}}$ . The variances of the three components of the linear Stokes vector are the diagonal values of this covariance matrix. One can synthetically characterize the estimation performance by the sum of these variances, i.e., the trace of the covariance matrix which is also called the *equally weighted variance* (E WV):

$$\text{E WV} = \text{trace} \left[ \Gamma^{\hat{\mathbf{S}}} \right] \quad (4)$$

Since the additive and Poisson noise sources are statistically independent, the covariance matrix can be written as

$$\Gamma^{\hat{\mathbf{S}}} = \Gamma^{\text{add}} + \Gamma^{\text{poi}} \quad (5)$$

where  $\Gamma^{\text{add}}$  is the covariance matrix in the presence of additive noise only, and  $\Gamma^{\text{poi}}$  the covariance matrix in the presence of Poisson noise only. These two matrices have the following expressions [7, 8]:

$$\Gamma_{ij}^{\text{add}} = \sigma_a^2 \delta_{ij} \quad (6)$$

$$\Gamma_{ij}^{\text{poi}} = \sum_{k=0}^2 S_k \gamma_{ij}^k \quad (7)$$

where

$$\delta_{ij} = [(\mathbb{W}^T \mathbb{W})^{-1}]_{ij}$$

$$\gamma_{ij}^k = \sum_{l=1}^4 \mathbb{W}_{il}^+ \mathbb{W}_{jl}^+ \mathbb{W}_{lk} \quad , \quad \forall (k, i, j) \in [0, 2]^3 \quad (8)$$

Moreover, it is easily shown that

$$\gamma_{ij}^0 = \frac{1}{2} \delta_{ij} \quad (9)$$

Let us define the *ideal* configuration of a superpixel as the configuration where the analysis vectors are ideal (i.e with equal transmissions and unit diattenuations) and oriented exactly at angles  $0, 45, 90, 135$  degrees. In this case, it can be shown that [6]:

$$\text{E WV}_{\text{ideal}} = 5 \left( \sigma_a^2 + \frac{S_0}{2} \right) \quad (10)$$

Consequently, in this ideal case, the EWV only depends on the input Stokes vector through its intensity  $S_0$ , and not its AOP nor its DOLP. This is no longer the case when the four analyzers are not ideal. Indeed, in the general case, that is, for an arbitrary measurement matrix  $\mathbb{W}$ , Eq. (7) leads to:

$$\text{E WV} = \left[ \sigma_a^2 + \frac{S_0}{2} \right] \sum_{i=0}^2 \delta_{ii} + \sum_{k=1}^2 S_k \beta_k \quad (11)$$

$$\text{with } \beta_k = \sum_{i=0}^2 \gamma_{ii}^k$$

Using the parametrization of  $\mathbf{S}$  defined in Eq. (1), this relation can also be written:

$$\text{E WV} = \left[ \sigma_a^2 + \frac{S_0}{2} \right] \sum_{i=0}^2 \delta_{ii} + C \cos[2(\alpha - \theta)] \quad (12)$$

with

$$\theta = \frac{1}{2} \arctan \left[ \frac{\beta_2}{\beta_1} \right] \quad (13)$$

$$C = P \frac{\sqrt{\beta_1^2 + \beta_2^2}}{\beta_0} \quad (14)$$

It is seen in Eq. (12) that the EWV consists of the sum of two terms. The first one is independent of the AOP  $\alpha$ . The second one varies sinusoidally with  $\alpha$ , the phase  $\theta$  of this sinusoid depending on the coefficient  $\beta_k$ , and its contrast  $C$  depending on the coefficients  $\beta_k$  and of the degree of polarization  $P$  of the input Stokes vector. One can define an "average" level of EWV, averaged over all the possible AOP values, in the following way:

$$\begin{aligned}\overline{\text{EWV}} &= \frac{1}{\pi} \int_0^\pi \text{EWV}(\alpha) d\alpha \\ &= \left[ \sigma_a^2 + \frac{S_0}{2} \right] \sum_{i=0}^2 \delta_{ii}\end{aligned}\quad (15)$$

Let us now consider estimation of the AOP. The expression of this parameter as a function of the Stokes vector is:

$$\alpha = \frac{1}{2} \arctan \left[ \frac{S_2}{S_1} \right] \quad (16)$$

It is a nonlinear function of the Stokes vector and it is thus difficult to obtain a closed-form expression of its variance. However, one can determine an approximate value of this variance that is valid in the case of small perturbations. In the ideal configuration, one obtains [9]:

$$\text{VAR}[\hat{\alpha}]_{\text{ideal}} = \frac{1}{2P^2} \left[ \frac{\sigma_a^2}{S_0^2} + \frac{1}{2S_0} \right] \quad (17)$$

It is seen that this variance depends on the sum of two terms:  $(S_0/\sigma_a)^2$ , which can be seen as the intensity signal to noise ratio (SNR) in the presence of additive noise, and  $1/(2S_0)$ , which is the intensity SNR in the presence of Poisson noise. Moreover, it is independent of the actual value of the AOP. This is no longer the case in a non-ideal configuration. Indeed, one has [6]:

$$\begin{aligned}\text{VAR}[\hat{\alpha}] &= \\ &\frac{1}{4P^2} \left[ \frac{\sigma_a^2}{S_0^2} + \frac{1}{2S_0} \right] \{ \delta_{11}s^2 + \delta_{22}c^2 - 2\delta_{12}cs \} \\ &+ \frac{1}{4P S_0} \{ c^2 [(\gamma_{22}^2 - 2\gamma_{12}^1)s + \gamma_{22}^1c] \\ &+ s^2 [(\gamma_{11}^1 - 2\gamma_{12}^2)c + \gamma_{11}^2s] \}\end{aligned}\quad (18)$$

where the following notation has been used:

$$c = \cos(2\alpha) \quad \text{and} \quad s = \sin(2\alpha) \quad (19)$$

It is seen in Eq. (18) that  $\text{VAR}[\hat{\alpha}]$  is a weighed sum of trigonometric functions of  $\alpha$ : the estimation variance thus depends on the actual value of  $\alpha$ . The amplitude of this variation with respect to

$\alpha$  depends on the degree of polarization  $P$  of the incident Stokes vector, on  $S_0$ ,  $\sigma_a$  and on the measurement matrix  $\mathbb{W}$  through the coefficients  $\delta_{ij}$  and  $\gamma_{ij}^k$ . One can define an "average" variance of  $\hat{\alpha}$ , averaged over the azimuth values, in the following way [6]:

$$\begin{aligned}\overline{\text{VAR}[\hat{\alpha}]} &= \frac{1}{\pi} \int_0^\pi \text{VAR}[\hat{\alpha}] d\alpha \\ &= \frac{1}{8P^2} \left[ \frac{\sigma_a^2}{S_0^2} + \frac{1}{2S_0} \right] (\delta_{11} + \delta_{22})\end{aligned}\quad (20)$$

Let us now turn to the estimation of the DOLP. In the ideal case, one obtains [9]:

$$\text{VAR}[\hat{P}]_{\text{ideal}} = \frac{\sigma_a^2}{S_0^2} [2 + P^2] + \frac{1}{2S_0} [2 - P^2] \quad (21)$$

It is seen that this variance is independent of the actual value of the AOP, and only depends on  $\sigma_a$ ,  $S_0$  and  $P^2$ . This is no longer the case in a non ideal configuration [6]:

$$\begin{aligned}\text{VAR}[\hat{P}] &= \\ &\left[ \frac{\sigma_a^2}{S_0^2} + \frac{1}{2S_0} \right] \{ P^2\delta_{00} - 2P(\delta_{01}c + \delta_{02}s) + 2\delta_{12}cs \\ &+ \delta_{11}c^2 + \delta_{22}s^2 \} \\ &+ \frac{P}{S_0} \{ P^2(\gamma_{00}^1c + \gamma_{00}^2s) \\ &- P [ +2\gamma_{01}^1c^2 + 2\gamma_{02}^2s^2 + 2(\gamma_{01}^2 + \gamma_{02}^1)cs ] + \\ &+ (\gamma_{11}^2 + 2\gamma_{12}^1)c^2s + (\gamma_{22}^1 + 2\gamma_{12}^2)cs^2 \\ &+ \gamma_{11}^1c^3 + \gamma_{22}^2s^3 \}\end{aligned}\quad (22)$$

This variance is the sum of two terms that are weighed sums of trigonometric functions of  $\alpha$ : the estimation variance thus depends on the actual value of  $\alpha$ . One can define an "average" variance of  $\hat{P}$  over the azimuth values in the following way:

$$\begin{aligned}\overline{\text{VAR}[\hat{P}]} &= \frac{1}{\pi} \int_0^\pi \text{VAR}[\hat{P}] d\alpha \\ &= \left[ \frac{\sigma_a^2}{S_0^2} + \frac{1}{2S_0} \right] \left[ P^2\delta_{00} + \frac{\delta_{11} + \delta_{22}}{2} \right] \\ &\quad - 2P^2(\gamma_{01}^1 + \gamma_{02}^2)\end{aligned}\quad (23)$$

To summarize the results obtained in this section, we have shown that it is possible to predict the polarimetric estimation performance of each superpixel of a micropolarizer grid-based camera in the presence of additive and Poisson shot noise by using the results of its calibration. It is noticed that manufacturing imperfections can lead to an increase of the estimation variance. It is also apparent that with a non-ideal superpixel, the estimation variance does depend on the AOP of the input state. This

variation can be experimentally observed, although it is in general slight compared to the average value of the variance [6].

#### 4 MEASUREMENT OF THE FULL STOKES VECTOR

Commercial DoFP cameras based on microgrids of polarizers are available, but since the array is composed of linear polarizers, only the linear Stokes vector can be measured [10, 11, 12, 6]. To measure the full Stokes vector, some imagers based on birefringent micro-grids have been demonstrated [13, 14, 15] but are still prototypes. In order to leverage the stable, integrated nature of available linear DoFP cameras to perform full Stokes measurements, it is possible to use a retarder placed in front of them (Fig. 2). Such setups have been recently demonstrated and used in practice [1, 2, 16]. Our objective is to determine the parameters that minimize estimation variance in the presence of additive white noise and Poisson shot noise.

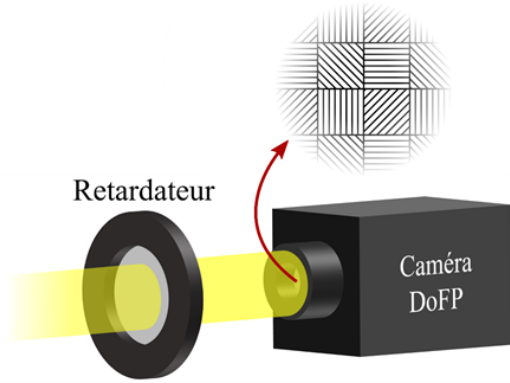


Figure 2: Configuration for measuring the full Stokes vector with a linear DoFP camera and a retarder.

Let  $\mathbf{S} = (S_0, S_1, S_2, S_3)^T$  denote the full Stokes vector to be measured. A retarder is placed in front of the DoFP camera and depends on two parameters: its retardation  $\varphi$  and its orientation  $\theta$ . For a given value of these parameters, the measurable set of Stokes vectors lies on a circle on the Poincaré sphere, since the effect of a retarder is to perform a 3D rotation of the input polarization state. Consequently, with a single measurement, it is impossible to estimate the full Stokes vector: at least two measurements are needed. One thus loses the property of instantaneous measurement. However, there exists fast polarization modulators, such as Liquid Crystal Variable Retarders (LCVRs) or rotating retarders that can switch polarization very rapidly.

These two measurements are performed with two different parameter settings of the retarder. It

can be shown that there is a continuum of optimal parameters that lead to a minimal EWV, thus to an optimal estimation precision of the Stokes vector [16]. Moreover these optimal configurations are the same in the presence of additive noise, Poisson noise or a combination of both. The two following configurations are particularly interesting:

- *Solution 1*: one measurement is performed without retarder, and one measurement with a linear retarder of retardance equal to  $\varphi = 90^\circ$  (quarter wave plate) and arbitrary orientation  $\theta$ .
- *Solution 2*, the two measurements are performed with a quarter wave plate at orientations  $\theta_1$  and  $\theta_2 = \theta_1 + 45^\circ$ ,  $\theta_1$  being arbitrary.

For these two optimal configurations, the EWV is equal to:

$$\text{EWV}_{min} = 5.5 \left( \sigma_a^2 + \frac{S_0}{2} \right) \quad (24)$$

where  $\sigma_a^2$  is the variance of additive noise. For this relation to hold, both  $\sigma$  and  $S_0$  have to be expressed in number of electrons. This equation represents the fundamental limit on estimation variance of the full Stokes vector for this type of architectures. The expressions of the estimation variances of AOP, DOP and ellipticity in this configuration can be obtained by using the formulas derived in [17, 18].

#### 5 CONCLUSION

DoFP cameras are a promising new technology that opens up new application fields to polarization imaging. Depending on the technology, they may have polarimetric defects that can be calibrated. We have derived simple closed form formulas giving the variance of estimated Stokes parameter, AOP and DOP. They take into account the actual calibrated polarization characteristics of the camera. Moreover, in applications where it is necessary to measure the full Stokes vectors, it is possible to use a DoFP camera and a retarder to get this information in 2 acquisitions instead of 4 in a classical polarization imager. These results show that commercial DoFP cameras are versatile sensors that can be easily adapted to the needs of the application at hand.

The authors acknowledge the support of the Délégation Générale de l'Armement and of the Agence Nationale de la Recherche under the POL-NOR project.

## References

- [1] J. Qi, C. He, and D. S. Elson, "Real time complete Stokes polarimetric imager based on a linear polarizer array camera for tissue polarimetric imaging," *Biomed. Opt. Express* **8**, 4933–4946 (2017).
- [2] S. Shibata, N. Hagen, and Y. Otani, "Robust full Stokes imaging polarimeter with dynamic calibration," *Optics Letters* **44**, 891–894 (2019).
- [3] J. S. Tyo, D. L. Goldstein, D. B. Chenault, and J. a. Shaw, "Review of passive imaging polarimetry for remote sensing applications." *Applied optics* **45**, 5453–5469 (2006).
- [4] S. B. Powell and V. Gruev, "Calibration methods for division-of-focal-plane polarimeters," *Optics Express* **21**, 21039 (2013).
- [5] Z. Chen, X. Wang, and R. Liang, "Calibration method of microgrid polarimeters with image interpolation," *Optical Society of America* **54**, 995–1001 (2015).
- [6] S. Roussel, M. Boffety, and F. Goudail, "Polarimetric precision of micropolarizer grid-based camera in the presence of additive and poisson shot noise," *Optics Express* **26**, 29968–29982 (2018).
- [7] D. S. Sabatke, M. R. Descour, E. L. Derejniak, W. C. Sweatt, S. A. Kemme, and G. S. Phipps, "Optimization of retardance for a complete stokes polarimeter," *Opt. Lett.* **25**, 802–804 (2000).
- [8] F. Goudail, "Noise minimization and equalization for stokes polarimeters in the presence of signal-dependent poisson shot noise," *Opt. Lett.* **34**, 647–649 (2009).
- [9] F. Goudail and B eni ere, "Estimation of the degree of linear polarization and of the angle of polarization in the presence of different sources of noise," *Appl. Opt.* **49**, 683–693 (2010).
- [10] S. B. Powell and V. Gruev, "Calibration methods for division-of-focal-plane polarimeters," *Optics Express* **21**, 21039 (2013).
- [11] Z. Chen, X. Wang, and R. Liang, "Calibration method of microgrid polarimeters with image interpolation," *Optical Society of America* **54**, 995–1001 (2015).
- [12] B. Feng, Z. Shi, H. Liu, L. Liu, Y. Zhao, and J. Zhang, "Polarized-pixel performance model for DoFP polarimeter," *Journal of Optics* **20**, 065703 (2018).
- [13] X. Zhao, A. Bermak, F. Boussaid, and V. G. Chigrinov, "Liquid-crystal micropolarimeter array for full stokes polarization imaging invisible spectrum," *Optics Express* **18**, 17776–17787 (2010).
- [14] M. Gecevicius, M. Beresna, and P. G. Kazan-sky, "Polarization sensitive camera by fem-tosecond laser nanostructuring," *Optics Letters* **38**, 4096 (2013).
- [15] W.-L. Hsu, G. Myhre, K. Balakrishnan, N. Brock, M. Ibn-Elhaj, and S. Pau, "Full-Stokes imaging polarimeter using an array of elliptical polarizer," *Optics Express* **22**, 3063 (2014).
- [16] S. Roussel, M. Boffety, and F. Goudail, "On the optimal ways to perform full Stokes measurements with a linear division-of-focal-plane polarimetric imager and a retarder," *Opt. Lett.* **44**, 2927–2930 (2019).
- [17] J. Dai and F. Goudail, "Precision analysis of arbitrary full-Stokes polarimeters in the presence of additive and Poisson noise," *J. Opt. Soc. Am. A* **36**, 1229–1240 (2019).
- [18] J. Dai and F. Goudail, "On the validity domain of approximations to estimation variance of polarization degree, azimuth and ellipticity," *J. Opt. Soc. Am. A* **36**, 1295–1305 (2019).