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► **To cite this version:**

Rosario Patanè, Andrea Araldo, Tijani Chahed, Diego Kiedanski, Daniel Kofman. Coalitional game-theoretical approach to coinvestment with application to edge computing. 20th Consumer Communications & Networking Conference (CCNC), Jan 2023, Las Vegas, United States. pp.517-522, 10.1109/CCNC51644.2023.10060093 . hal-04492328

HAL Id: hal-04492328

<https://hal.science/hal-04492328>

Submitted on 6 Mar 2024

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Coalitional Game-Theoretical Approach to Coinvestment with Application to Edge Computing

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Abstract—We propose in this paper a coinvestment plan between several stakeholders of different types, namely a physical network owner, operating network nodes, e.g. a network operator or a tower company, and a set of service providers willing to use these resources to provide services as video streaming, augmented reality, autonomous driving assistance, etc. One such scenario is that of deployment of Edge Computing resources. Indeed, although the latter technology is ready, the high Capital Expenditure (CAPEX) cost of such resources is the barrier to its deployment. For this reason, a solid economical framework to guide the investment and the returns of the stakeholders is key to solve this issue. We formalize the coinvestment framework using coalitional game theory. We provide a solution to calculate how to divide the profits and costs among the stakeholders, taking into account their characteristics: traffic load, revenues, utility function. We prove that it is always possible to form the grand coalition composed of all the stakeholders, by showing that our game is convex. We derive the payoff of the stakeholders using the Shapley value concept, and elaborate on some properties of our game. We show our solution in simulation.

Index Terms—Coinvestment, Multi-tenancy, Edge computing, Coalitional game theory, Shapley.

I. INTRODUCTION

Coinvestment allows several stakeholders to share expenses and revenues when deploying certain projects that are otherwise non-beneficial for them. We model coinvestment via coalitional game theory. Our model can be applied in scenarios when (i) costly resources must be deployed in some nodes, (ii) only one entity, hereafter named Network Owner (NO), has access to these nodes and (iii) such resources are beneficial to third party Service Providers (SPs). One such scenario is Edge Computing (EC), which is the main application of this paper. In EC, physical nodes at the edge are possessed by an NO, which can be a network operator, like AT&T, or a tower company [1]; in such nodes, the NO deploys computational resources¹, which are used by third party SPs, e.g. video streaming services, such as YouTube, Netflix, or

¹Cloud providers, e.g. Amazon, distribute “edge” resources for rent, but their edge locations go as far as “data-centers at the edge of the 5G network”(aws.amazon.com/fr/edge). We instead consider edge nodes much closer to users, e.g. base stations or road side units, which are owned by the NO.

car manufacturers offering in the future automated driving services, such as Tesla or Renault [2], [3]. The SPs use the resources at the Edge, closer to their end-users, to distribute the load or to satisfy their low latency applications.

The deployment costs however can be very high and cannot be supported solely by the NO. This explains today’s impediment for the widespread of EC. On the other hand, SPs offering new services, can make large benefits by offering good quality services to their end-users. Hence, it is reasonable to assume that SPs may be willing to contribute to the cost of the deployment of EC resources together with the NO [4].

The goal of this paper is to understand how coinvestment can occur, i.e. how cost of deployment and benefits should be shared among the NO and SPs. Our contributions are:

- We propose a model based on coalitional game theory, and describe the discrepancy between the two categories of stakeholders in terms of revenues (brought by SPs) and infrastructure deployment (by the NO).
- We assess the existence of the core, i.e. the fact that the grand coalition composed of all the stakeholders exists and is stable, by showing the convexity of our game.
- We propose sharing of the payoff between the stakeholders based on the Shapley value solution concept, which lies in the core in this case.
- We evaluate the performance of our proposal, through numerical examples for different scenarios and configurations, considering the case of EC.

The remainder of the paper is as follows. §II introduces the related work. In §III, we formulate our model. §IV provides an analysis for the coinvestment plan between the stakeholders. In §V, we show numerical results for several scenarios and configurations. §VI concludes the paper.

II. RELATED WORK

The advantage of sharing resources between several tenants has been shown in [5], for the case of passive optical networks. However, they assume resources are already deployed and allocation to tenants is determined via auctions. We instead consider the case where no resources are deployed yet (as for EC) and thus devise a mechanism for all the players to invest

to buy and deploy resources, jointly taking into account the resource allocation among players. An example of coinvestment is given in [6]. Investors are producers/consumers of energy, and a battery exists that must be sized to meet everyone's needs. Each of them can decide whether to charge or discharge the battery to use stored energy later, when energy is more expensive (daily hours). The result of coinvestment is the price that each one has to pay to buy the battery. The problem is solved using coalitional game theory [7], as we shall do in this work. However, we cannot directly use these results, because [6] is a linear production problem, valid to model the cost of electricity, while in our case the benefit of using resources at the Edge can be non-linear, due to the diminishing return in resource deployment at the Edge [8]. Another peculiarity of our study is that we have a veto player, which corresponds to the NO. In [9] a coinvestment between Network Providers (NPs) is realized to improve energy efficiency in cellular access networks, i.e. in some parts of the day, except the peak hours, the resources of a single NP can be oversized, so, just a subset of NPs can serve all the load allowing the others to save energy. The resultant benefit of this sharing is later divided among the players. This paper is very close to the problem in this work, but does not match exactly because: (i) the players are at the same level (all NPs), while in our case players have different roles (NO and SPs), (ii) in our problem we do not care about exchanging load between players to save energy. Coalitional Game Theory has been applied to EC [10], e.g. for pricing, spectrum or content sharing in D2D communication, task offloading [11]. To the best of our knowledge we are the first to apply it for co-investment in EC, and thus there are no other works with which we can compare our proposal.

III. COINVESTMENT MODEL

Our coinvestment problem is modeled as a coalitional game with transferable payoff, i.e. players can share a common amount of utility/cost [12]. The game is defined by the tuple (\mathcal{N}, v) , where \mathcal{N} is the set of players and the *coalitional value* $v(\mathcal{S})$ is a function that associates a value to any subset $\mathcal{S} \subseteq \mathcal{N}$, called *coalition*. The players are one physical Network Owner (NO) and N Service providers (SPs). The NO is the entity that owns the network nodes. It could be a network operator, who owns the location of an antenna or a central office. It could be a separated tower company [1]. The set of players is $\mathcal{N} = \{1, \dots, N, NO\}$. The first question to be asked in coalitional games is whether the grand coalition \mathcal{N} , formed by all players, is stable, i.e. all the players have an incentive to be part of it, in terms of individual payoff. The payoff of any player $i \in \mathcal{N}$ is $x^i = r^i - p^i$, where r^i and p^i are the revenues and the payment, i.e. the capital cost, respectively.

We assume that the NO is willing to host physical resources on its nodes. To fix ideas, such nodes are edge nodes and resources are CPU in our case. They might also refer to GPU, etc. After the deployment, the NO virtualizes and allocates the resources to SPs. SPs do not own physical resources, only the NO does, in base stations for instance. Whenever players

form a coalition, they invest together in deploying an amount of computational capacity C , measured in *millicores*. Denoting by h^i the resources allocated to player i , $\sum_{i \in \mathcal{S}} h^i = C$. For d denoting the price expressed in *dollars* per resource unit, the sum of all players' payments should be such that $\sum_{i \in \mathcal{S}} p^i = d \cdot C$. As the NO is the only player that hosts the capacity, if it does not join the coalition, coinvestment cannot take place, and so, no computational capacity can be deployed and no EC can be realized. So, for any coalition \mathcal{S} , capacity is subject to:

$$C = \begin{cases} \frac{1}{d} \cdot \sum_{i \in \mathcal{S}} p^i & \text{if NO} \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

At every timeslot t , each player i has an expected load, l_t^i , i.e. the average number of requests coming from users of SP i at time t , and which is exogenous to the problem. We assume that the average load profile is the same every day. Each player i has an instantaneous utility u_t^i , in monetary units, e.g. *dollars*, which represents the revenues coming from the end users of the services. In other words, the utility is what users pay to a SP to consume its services. We assume the load and the shape of the utility function are known and truthful inputs. Similarly to [14], the utility is function of the expected load l_t^i (the more users consume the service, the more they pay) and of the allocated resources h^i (the more resources, the better the service, the more users are willing to pay):

$$u_t^i = u^i(l_t^i, h^i); \quad u^i(l_t^i, 0) = 0 \quad (2)$$

We thus assume (Eqn. (2) on the right) that the utility is null if no resources are allocated.

Observe that the case when the NO uses some Edge resources for itself can be easily modeled in our framework by introducing a fictitious SP representing the NO using resources of EC. The NO does not offer any Edge service to the end users, so, its load is null ($l_t^{\text{NO}} = 0$) and thus it does not need Edge resources for itself ($h^{\text{NO}} = 0$), which implies that $u_t^{\text{NO}} = 0$ and $\sum_{i \in \mathcal{S} \setminus \{\text{NO}\}} h^i = C$. However, the NO gets a fraction of the value of the grand coalition, if it exists.

The revenues of a coalition is the sum of the utilities of the SPs over the investment period: $\sum_{i \in \mathcal{S}} r^i = D \cdot \sum_{i \in \mathcal{S}} \sum_{t \in [T]} u_t^i$, where $D = 365 \cdot Y$, Y is the duration in years of the investment, $T = 96$ is the number of timeslots in one day, considering each timeslot has duration 15 *minutes*.

We now define the value $v(\mathcal{S})$ of any coalition $\mathcal{S} \subseteq \mathcal{N}$. When forming coalition \mathcal{S} , the players involved choose allocation vector \vec{h} and capacity C so as to maximize the coalition value $v(\mathcal{S})$:

$$v(\mathcal{S}) = \max_{\vec{h}, C} v^{\vec{h}, C, \mathcal{S}} \triangleq \max_{\vec{h}, C} D \sum_{i \in \mathcal{S}} \sum_{t=1}^T u^i(l_t^i, h^i) - d \cdot C \quad (3)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{S} \setminus \{\text{NO}\}} h^i = C; \quad h^{\text{NO}} = 0. \quad (4)$$

$$C, h^i \geq 0, \quad \forall t \in [T], \forall i \in \mathcal{S}. \quad (5)$$

Observe that NO is a **veto player**. Indeed, without NO, the investment does not take place [12, §13.2] (see (1)). Therefore,

applying (4), (2) and (3), if $\text{NO} \notin \mathcal{S}$, we get $v(\mathcal{S}) = 0$. We will see also that the SPs form altogether a veto player too.

IV. ANALYSIS

We now assess the cooperative structure of our game and its stability. We show the existence of the core, and hence the formation of the grand coalition.

A. Core and convexity of the game

Let us define a payoff vector $(x^i)_{i \in \mathcal{N}}$. The core is a set of payoff vectors, such that a payoff vector is in the core if the payoffs of each player are such that no subgroup can gain by quitting the grand coalition and forming a different coalition [12]. A well known result in coalitional game theory affirms that the core is non empty if the game is convex [13] and that a particular payoff vector having some ‘‘fairness’’ properties, i.e. the Shapley value [7], [13] lies in the core.

Theorem 1. *(The game is convex). Our game (\mathcal{N}, v) , whose value function v is described by the optimization problem (3)-(5) is convex.*

Proof. We can rewrite $v(\mathcal{S})$, via (3),(4), as

$$v(\mathcal{S}) = \sum_{i \in \mathcal{S}} \max_{h^i} v^{h^i} \quad ; \quad v^{h^i} \triangleq D \cdot \sum_{t=1}^T u^i(l_t^i, h^i) - d \cdot h^i \quad (6)$$

The maximum over h^i of Eqn. (6) gives the contribution of a single player i to the coalitional value, considering its part of the revenues due to its utility function, and the cost of the resources h^i it uses to produce this utility. Observe that such a contribution is independent from the coalition \mathcal{S} in which i participates. Eqns. (6) show that the contribution is separable, i.e. it is the summation of the value functions of the individual players.

We now prove that the game is supermodular, which implies its convexity, thanks to [15]. A game is supermodular if

$$\Delta_i(\mathcal{T}) \leq \Delta_i(\mathcal{S}), \forall \mathcal{T} \subseteq \mathcal{S} \subseteq \mathcal{N} \setminus \{i\}, \forall i \in \mathcal{N} \quad (7)$$

$$\text{where } \Delta_i(\mathcal{S}) = v(\mathcal{S} \cup \{i\}) - v(\mathcal{S}) \stackrel{(6)}{=} \max_{h^i} v^{h^i} \quad (8)$$

is the marginal contribution of player i to coalition \mathcal{S} .

Let us fix any $i \in \mathcal{N}$. Given two coalitions, \mathcal{S} and \mathcal{T} , such that $\mathcal{T} \subseteq \mathcal{S} \subseteq \mathcal{N} \setminus \{i\}$, we calculate the marginal contribution of player i to both coalitions.

We consider four cases:

- Case $\text{NO} \in \mathcal{T}$ and $i = \text{SP}^i$. For coalition \mathcal{T} we have (see (6)): $\Delta_i(\mathcal{T}) = \max_{h^i} v^{h^i}$. For coalition \mathcal{S} we have: $\Delta_i(\mathcal{S}) = \max_{h^i} v^{h^i}$ so, the marginal contributions are $\Delta_i(\mathcal{S}) = \Delta_i(\mathcal{T})$.
- Case $i = \text{NO}$. In this case the proof is trivial, in fact $\text{NO} \notin \mathcal{T} \cup \mathcal{S}$, so, for the fact that NO is a veto player

$$v(\mathcal{T} \cup \{i\}) \geq 0, v(\mathcal{T}) = 0, \forall \mathcal{T} \setminus \{\text{NO}\} \quad (9)$$

$$\text{and } v(\mathcal{S} \cup \{i\}) \geq 0, v(\mathcal{S}) = 0, \forall \mathcal{S} \setminus \{\text{NO}\} \quad (10)$$

Therefore, Eqn. (7) is verified if and only if

$$v(\mathcal{S} \cup \{i\}) - v(\mathcal{T} \cup \{i\}) \geq 0 \quad (11)$$

which is equivalent to $\sum_{j \in \mathcal{S} \setminus \mathcal{T}} \max_{h^j} v^{h^j} \geq 0$ where the last inequality is obviously true, since we have a sum of non-negative terms.

- Case $\text{NO} \notin \mathcal{T} \cup \mathcal{S}$ and $i = \text{SP}^i$. This case is easy to prove because we get $v(\mathcal{T} \cup \{i\}) - v(\mathcal{T}) = v(\mathcal{S} \cup \{i\}) - v(\mathcal{S}) = 0$, which satisfies the definition of supermodularity.
- Case $\text{NO} \notin \mathcal{T}$ and $i = \text{SP}^i$. In this case the marginal contribution of i to coalition \mathcal{T} is null and Eqn. (7) becomes $\Delta_i(\mathcal{S}) \geq 0$. To verify this, we observe that, thanks to (6):

$$\Delta_i(\mathcal{S}) = v(\mathcal{S} \cup \{i\}) - v(\mathcal{S}) = \begin{cases} \max_{h^i} v^{h^i} & \text{if } \text{NO} \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases} \geq 0$$

This completes the supermodularity proof and thus convexity. So, the grand coalition can be always formed. \square

B. Shapley value

Finding a mechanism to share the payoff among players is not trivial. One idea would be to divide the payoff equally among players. However, this would not be accepted, since some players contribute to the coalition more than others. First, the NO is a veto player, and its contribution is of primary importance. Second, the SPs do not contribute equally to the coalition: some SPs have more users than others. A second idea would be to share payoffs proportionally to the request load of each SP. However, this would be still unfair, as the benefits collected by SPs do not only depend on the quantity of requests, but also on their type (see §V-A2).

The Shapley value, a somehow fair way to share the payoff considers the marginal contribution of each player to all the possible coalitions. For convex games, which is our case thanks to h. 1, the Shapley value lies in the core [13]. It is computed as [12]: $x^i = \phi^i = \frac{1}{|\mathcal{N}|!} \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{i\}} |\mathcal{S}|! \cdot (|\mathcal{N}| - |\mathcal{S}| - 1)! \cdot \Delta_i(\mathcal{S})$.

C. Initial investment of players

Now that we derived the payoff x^i for each player, we need to calculate how much each player must pay at the beginning of the investment, i.e. p^i . This is obtained by solving the following equations:

$$r^i - p^i = x^i, \forall i \in \mathcal{N} \quad (12)$$

$$\text{s.t. } \sum_{i \in \mathcal{N}} r^i = D \cdot \sum_{i \in \mathcal{N}} \sum_{t=1}^T u^i(l_t^i, h^{*i}) \quad (13)$$

$$\text{where: } \vec{h}^*, C^* = \arg \max_{\vec{h}, C} v^{\vec{h}, C, \mathcal{N}} \text{ s.t. (4)-(5)} \quad (14)$$

D. Relevant properties of our game

If player i does not produce revenues and makes no payments, then it is a null player, i.e. $v(\mathcal{S} \cup \{i\}) = v(\mathcal{S})$ [18]. Note that there can be players that do not pay for the resources or are even paid ($p^i \leq 0$) and still positively contribute to the

coalition by collecting revenues r^i . The NO is never a null player, because it is a veto player and contributes always to any coalition.

Theorem 2. (Payoff sharing) *The Shapley outcome of the game (\mathcal{N}, v) , where v is described by the problem (3)-(5), is divided equally between the NO and the set of all SPs.*

Proof. Consider the Shapley value of the game (\mathcal{N}, v) , i.e. the payoff vector $(\phi_i)_{i \in \mathcal{N}}$. We want to prove that the Shapley value of the NO is equal to the sum of the Shapley values of all SPs. To calculate the Shapley value, we need the value of the marginal contribution of any player i to the coalition, i.e. $\Delta_i(\mathcal{S}) = \max_{h^i} v^{h^i}$. The NO is a veto player, and so, the v function is null for coalitions without it.

$$\Delta_{\text{NO}}(\mathcal{S}) = v(\mathcal{S} \cup \{\text{NO}\}) - v(\mathcal{S}) = \sum_{i \in \mathcal{S}} \max_{h^i} v^{h^i} - 0 = \sum_{i \in \mathcal{S}} \Delta_i(\mathcal{S}), \forall \mathcal{S} \subseteq \mathcal{N} \setminus \{\text{NO}\} \quad (15)$$

Now, we can show that the Shapley value of the NO is equal to the sum of the SPs Shapley values. We know that the Shapley value is in the core, which is subject to the efficiency property, $\sum_{i \in \mathcal{N}} \phi_i = v(\mathcal{N})$. Hence, $v(\mathcal{N}) = \phi_{\text{NO}} + \sum_{i \in \mathcal{N} \setminus \{\text{NO}\}} \phi_i$. The Shapley value of the NO is

$$\phi_{\text{NO}} = \frac{1}{|\mathcal{N}|!} \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{\text{NO}\}} |\mathcal{S}|! \cdot (|\mathcal{N}| - |\mathcal{S}| - 1)! \cdot \Delta_{\text{NO}}(\mathcal{S}) = \frac{1}{|\mathcal{N}|!} \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{\text{NO}\}} |\mathcal{S}|! \cdot (|\mathcal{N}| - |\mathcal{S}| - 1)! \cdot \sum_{i \in \mathcal{S}} \Delta_i(\mathcal{S}) \quad (16)$$

as $\Delta_{\text{NO}}(\mathcal{S}) = \sum_{i \in \mathcal{S}} \Delta_i(\mathcal{S}) \forall \mathcal{S} \subseteq \mathcal{N} \setminus \{\text{NO}\}$ (Eqn. (15)).

This implies the coalitional value is divided equally between the NO and the set of SPs, $\phi_{\text{NO}} = \sum_{i \in \mathcal{N} \setminus \{\text{NO}\}} \phi_i = \frac{v(\mathcal{N})}{2}$. This completes the proof. \square

The intuition behind this equal sharing of the Shapley value between the NO and the SPs is based on [16]: “each game is decomposed into a weighted sum of unanimity games in which the Shapley value assigns an equal share of a unit to each veto player”. In our case if the set of SPs is considered as one super-player, it is actually a veto player as well, because the value function is zero if no SP is in the coalition, since it would not be possible to collect revenues from users utilization.

V. APPLICATION TO EDGE COMPUTING

A. Parameters

1) *Load*: We define the load as an exogenous variable (§III). To reproduce a realistic trend, we consider the daily traffic profile of a SP serving residential users, as modeled in [17], i.e. $l_t^i = a_0 + \sum_{k=1}^K a_k \sin(2k\pi \frac{t-t_k}{T})$, where t is the timeslot and T is the number of timeslots in one day; a_k and t_k are hyperparameters determining the amplitude and the offset of each of the K sinusoidal components. We take their values from [17, Fig.2].

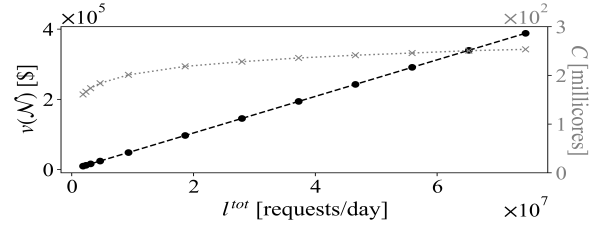


Fig. 1: Capacity and coalitional value as a function of the overall daily load

2) *Utility function and price*: As often observed in reality, we assume the utility (2) of any SPⁱ is characterized by a diminishing return effect [8]: the marginal utility increment becomes smaller by increasing the allocated resources h^i . For this reason, we model the utility with the following increasing and concave function, similar to (1) of [14]:

$$u^i(l_t^i, h^i) = \beta^i \cdot l_t^i \cdot (1 - e^{-\xi \cdot h^i}) \quad (17)$$

The term β^i is the *benefit factor* of player i which represents the benefit that a SP gets from serving one unit of load at the Edge. It is a multiplicative constant, null for the NO, $\beta^{\text{NO}} = 0$. The term ξ models the shape of the diminishing return, i.e. how fast it saturates to its upper bound $\beta^i \cdot l_t^i$. Note that this utility function follows property (2).

B. Scenario with 2 SPs of the same type

In this case there are two SPs of the same type: $\beta^{\text{SP}^1} = \beta^{\text{SP}^2} = \hat{p}$ where $\hat{p} \triangleq \frac{d}{D \cdot T}$ is the price, $d = 0.05$ dollars/millicores, amortized over each of the T time slots over the investment duration, D . SP¹ and SP² have the same temporal trends, but we take $l_t^1 = 4l_t^2 \forall t$.

In Fig. 1, we show the capacity of purchased CPU and the value of the grand coalition, as a function of the daily total load, $l^{\text{tot}} = \sum_{t=1}^T l_t^{\text{tot}}$. We observe that, the more the load the more the capacity installed to serve it. However, recall that the utility functions follow a diminishing return with respect to the resources, so, the trend of capacity C is sublinear. We observe a linear trend for coalitional value because the value function is linearly dependent on the load (see Eqn. (17)).

We observe in Fig. 2 the capacity sharing between the SPs, SP¹ receives a larger capacity: it has to serve a larger part of the requests. Note that, even if the load of SP¹ is 4 times the load of SP², the difference between the resource allocated to them is not that big: a consequence of the diminishing return.

The contribution of SPⁱ to the coalitional revenues is defined as $\hat{r}^i = D \cdot \sum_{t=1}^T u^i(l_t^i, h^i)$. We denote the grand coalitional revenues as $r^{\mathcal{N}} = \sum_{i \in \mathcal{N}} r^i$, where r^i is the result obtained in (12)-(14) given by $r^i = D \cdot \sum_{t=1}^T u^i(l_t^i, h^{*i})$. The term \hat{r}^i is the amount of revenues produced by SP i , due to the served load during the overall duration of the coinvestment.

Fig. 2 shows that most contribution comes from SP¹, since its load is four times higher than that of SP², and the utility of any SP (and thus its contribution to the grand coalitional

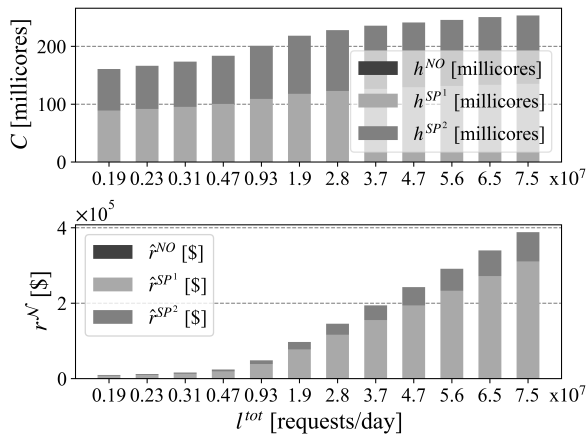


Fig. 2: Split of capacity and contributions to coalitional value

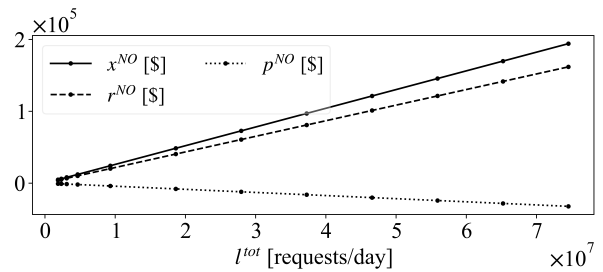
revenues) is proportional to the served load; indeed, we observe that $\hat{r}^{SP^1} = 4\hat{r}^{SP^2}$. Note that h^{NO} and r^{NO} are not in the figure, as the NO does not use resources, because its load is null; this implies that its utility is null so it does not produce revenues to the grand coalition by serving a load.

Fig. 3 shows the outcome of the game, i.e. the payment p^i , the revenue r^i and the payoff x^i of each player $i \in \mathcal{N}$ given by the Shapley value. We first observe that the payoff of each of them increases with the total load, which is obvious as the revenues of the grand coalition are the sum of the utilities of each player, which in turn increase with the number of served requests. It is interesting to notice that only SP^2 actually pays to deploy the resources at the Edge, while the NO and SP^1 have negative payments, so, they are not paying, but are being paid. The ‘‘privilege’’ of the NO and SP^1 can be explained by the fact that they are the most important for the coalition: NO is the veto player and SP^1 brings to the coalition most of the revenues collected from users (Fig. 2).

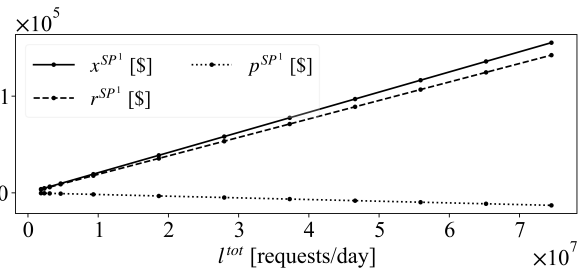
C. Scenario with 2 SPs of different types

In this case we have two SPs offering different types of services. For instance, SP^2 may offer an extremely low-latency service, e.g. augmented reality, while SP^1 may offer a less stringent service, e.g. online gaming. In this case, the utility coming from serving a request at the Edge is much higher for SP^2 than for SP^1 , since the latter could serve some of the requests (for instance requests not related with interactions with the player) from the Cloud without degrading too much the perceived user experience. Therefore, we consider now that $\beta^{SP^2} \geq \beta^{SP^1}$. As in the previous scenario, $\beta^{tot} = \beta^{SP^1} + \beta^{SP^2} = 2\hat{p}$. We assume further that $\beta^{SP^1} = (1 - \omega) \cdot \beta^{tot}$ and $\beta^{SP^2} = \omega \cdot \beta^{tot}$. We make ω vary in $[0.5, 1]$. The case $\omega = 0.5$ corresponds to the previous scenario, where the SPs were of the same type. By increasing ω , the two SPs become more heterogeneous, and in particular SP^2 has higher benefits per unit of load than SP^1 . We keep the load as before.

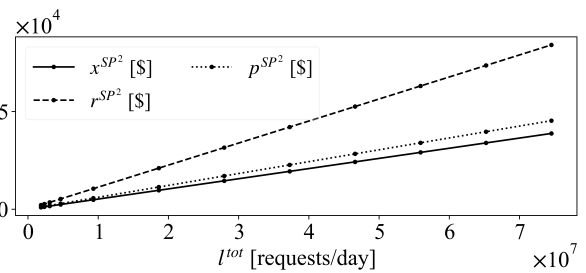
We now show how resource allocation and payoff change with ω . We observe in Fig. 4a that increasing ω , the percentage



(a) Payoff, revenues and payment by NO



(b) Payoff, revenues and payment by SP^1



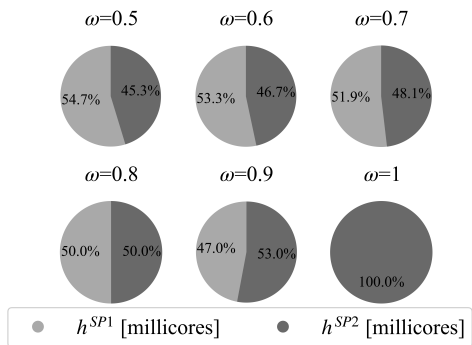
(c) Payoff, revenues and payment by SP^2

Fig. 3: Shapley value: payoffs, revenues and payments.

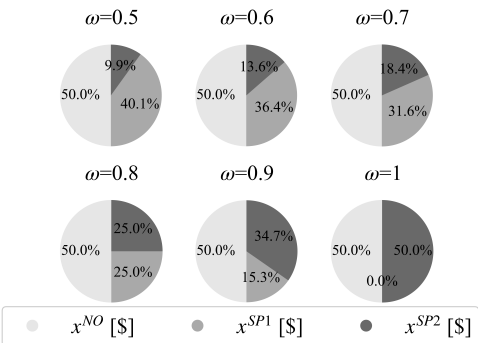
of CPU given to SP^1 decreases and is null for $\omega = 1$, at which point it is given entirely for SP^2 . This is due to the fact that, despite it attracts most of the user load, SP^1 is less useful to assign resources to, as its benefit factor becomes smaller with ω . This tells us, as expected, that resource allocation at the Edge must be taken not only based on the load, but also on the nature of the services, and in particular on time-sensitivity, which is reflected in a different benefit per unit of load satisfied at the Edge. In Fig. 4b the payoff sharing reflects what we mentioned above. The marginal contribution brought by SP^2 increases since it produces most of the coalition revenues. In all the cases, the NO has 1/2 of the coalitional value, (Th. 2).

D. Price sensitivity analysis

Fig. 5 assesses how the behavior of the coalition changes with an increasing number of SPs for different prices d . Obviously, when the price for resources increases, the purchased capacity is reduced. It is interesting to notice that this reduction is sublinear, because of (17). On the other hand, coalitional value decreases linearly. These trends remain consistent when changing the number N of SPs. Fig. 5 also confirms that adding a player in the game brings a higher benefit in terms



(a) Capacity subdivision among the players



(b) Payoff sharing

Fig. 4: Coalitional value and capacity function of ω

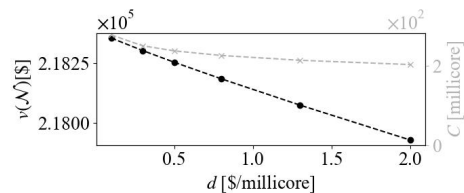
of the value of the coalition v . This is in line with the supermodularity of v which states that more value is obtained with larger number of players in the coalition, which we used in (Th. 1) to prove the convexity of our game and hence its stability.

VI. CONCLUSION

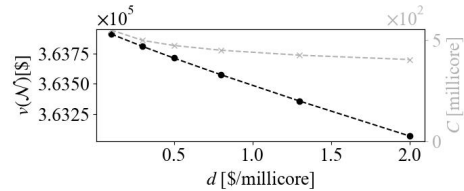
We proposed a coalitional game theory solution to enable coinvestment between heterogeneous players (NO and SPs) and applied it to the of deployment of Edge Computing. By showing the convexity of the game, we proved that the core is non-empty and that the Shapley value, which provides a fair way to divide income among players, lies in the core. So, it is always possible to form the grand coalition, made of all players. We studied numerically the solution, in terms of purchased capacity and its sharing between SPs as well as their revenues, payments and payoffs under the Shapley value, and this for different scenarios, homogeneous and heterogeneous SPs, in terms of so-called benefit factor which may be related for instance to how time-critical is the service they offer. We eventually studied the sensitivity of capacity and coalitional value to the number of SPs and to resource unitary price. For future work, we will consider adding a strategy-proof enforcement feature to ensure that players are truthful.

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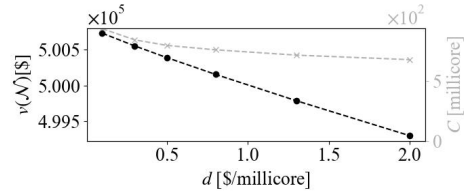
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(a) NO and $N = 2$ SPs



(b) NO and $N = 4$ SPs



(c) NO and $N = 7$ SPs

Fig. 5: Coalitional value and capacity as function of price d and number of SPs.

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