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Statistical Modeling of soft magnetic composites' permeability

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Abstract

Soft magnetic composites (SMC) also known as powder cores are widely used in many applications because of their linearity, controllable permeability and isotropy. Because of the distributed air gap, the permeability is governed by the inner demagnetizing fields. To model the permeability dependence on the filling fraction, essentially two approaches are used both based on spatial periodicity hypothesis: the non magnetic grain boundary model and the effective medium theory. Actually, the first one works only for dense materials and the second works at low concentration. Both need fitting of two parameters, the inner demagnetizing factor and the particle susceptibility, just because the periodicity hypothesis is never verified for magnetic filling factors larger than 20%. A breakaway model is proposed based on the computation of mathematical esperance of the statistical distribution of magnetic chains and the subsequent determination of demagnetizing coefficient of an ellipsoid of equivalent aspect ratio. The model is collated with permeability data from the literature for spherical or non-spherical particles based SMCs and shows a excellent agreement with only one or even without fitting parameter in the whole concentration range.

Keywords: soft magnetic composite, powder core,

1. Introduction

Magnetic powder cores have been developed in the late 30s' for the need of telecommunication industry. Load coils used in telephone lines needed low loss linear materials with permeability from several tens to several hundreds. Metal powders were mixed with few % of diluent and fine ceramic powder and pressed with various pressures to manufacture the cores with the desired permeability. In order to obtain low loss material, zero magneto-cristalline anisotropy and zero magnetostriction was necessary, so molybdenum permalloy was mostly used. Today, molybdenum permalloy powder cores (MPP) are still widely used in telecommunications and power electronics, and different powders are also used for temperatures above 50°C such as Isoperm (Fe₅₀Ni₅₀), Sendust (Fe with 10% Si and 5.5% Al in mass), 6.5% silicon steel or iron carbonyl for high frequency applications. During the 90's two companies proposed new soft magnetic composites (SMC) based on iron powder compacted under high pressure and high temperature that exhibits permeability above 100 together with high induction and high Curie point. SMC are not only compatible with electrical machines magnetic parts, they also give one more degree of freedom in the design of machines by allowing the flux to circulate in 3D [1] and make recycling easier. Since the early 2000's many efforts have been made to improve and create new SMCs based on different fillers and binders [2, 3, 4, 5, 6, 7] with a recent focus on nanostructured SMCs [8].

However, the problem of modeling the magnetic per-

meability of heterogeneous media, despite considerable work done since the XIXth century, still needs a model able to address composites with concentrations from 0 to 1 with good accuracy compared to experiment, eventually taking the particle shape into account in a straightforward way. Indeed, from the beginning of electromagnetism science the concept of demagnetizing field remained one of the most difficult. Already touched experimentally by Michael Faraday about 1845, Maxwell got deeper inside by deriving the expression of the demagnetizing factor of revolution ellipsoid. However, they were effectively computed in a large range of aspect ratio only in 1945 by Stoner [9] and the demagnetizing factors of general ellipsoids were calculated by Osborn [10] the same year. For other shapes (cylinders, prisms) some approximated expressions were known or deduced from experiment by various authors (Du Bois, Ewing, Tanakadaté, in the 1885-1895 decade, see *e.g.* [11],) but because the demagnetizing field is not homogeneous in a non ellipsoidal body, the demagnetizing factor depends on the susceptibility of the material. Demagnetizing coefficients of cylinders were exactly calculated by D.X. Chen only in 1991 and improved in 2006 including negative susceptibilities [12, 13], that of rectangular prisms by Aharoni for $\chi = 0$ in 1998 [14] and by D.X. Chen for $\chi \neq 0$ [15] in 2006. Non exhaustively, different recent contribution may be quoted to show how the problem is current: tensor approach, for a collection of ellipsoids with a distribution of aspect ratio [16]; computation by finite element method and subsequent mathematical fitting [17]; microscopic approach, by Monte Carlo simulations of model

spin Hamiltonians [18]; derivation from Newtonian potential [19]; 2D reluctance modeling [20]; superposition principle applied to magnetic/non-magnetic disks stacks [21]; mixing of two powders with different shapes [22] and one should add that the discussion on the demagnetization in hard magnetic solid bodies is still controversial [23, 24].

The different models can be classified in two radically different approaches: the reluctance model and the effective medium theory (EMT).

The reluctance model is often called non magnetic grain boundary model (NMGB) [25] though the result is similar to that of a magnetic ring with an air gap. Indeed, applying Hopkinson’s law, a similar result was already obtained by J.A. Ewing to determine the gap of cut cores [26].

The EMT model is based on the classical concept originally developed by Rudolf Clausius [27] which consist in considering a dielectric media as an electric dipole located in a spherical cavity inside an homogenized medium. The concept was developed by Garnett ¹ to model the optical properties of glasses with metal inclusions by in 1904 [28] and Bruggeman for the effective permittivity or conductivity of mixed media in 1935 [29]. Despite the age of the theory, there is still very active researches in this fields concerning homogenization of layered media with periodicity smaller than wave length [30], permittivity of non linear media [31], quantum dot arrays in graphene [32], electro-magnetic behavior of cement paste [33] or magneto-electrics [34].

EMT has been applied to SMCs by Mattei et al. [35] introducing the effect of particle shape [36]. They have shown the existence of a magnetic percolation threshold at 15% [37] and proposed different validations and improvements [38, 39, 40, 41]. More recently a tensor approach was proposed to take cubic magneto-crystalline anisotropy of magnetic particles into account [42] and a solution was given by Skomski for 2 types of particles [22]. However, all these studies were validated with relatively low particles concentrations (< 50%). Good agreement with the EMT approach has been shown studying up to 80% [36] keeping the concentration dependent demagnetizing coefficient as a fitting parameter [43, 44, 45].

The main problem of these models is that they consider an isolated particle in a medium so they are applicable only to diluted media (filling factor is below the percolation threshold, typically 20% for spheres). For interacting particles media the effect of the demagnetizing factor has to be fitted.

In this paper the two existing models will be first reviewed. A revision of the reluctance models will be proposed and it will be compared to the homogenization model and experimental data from low to high concen-

tration, showing the limits of these models. In the next section, a new approach is proposed in which the SMC is not considered periodic but as a statistical distribution of spherical particles forming magnetic percolation paths which mean length depends explicitly on the filling factor. Then the demagnetizing factor is computed from which the effective permeability is deduced and the results are compared with experimental data in the whole range of concentration. Finally, the model is extended to non spherical particles.

2. Current models

2.1. Reluctance based models

The reluctance model, is from far the simplest. The principle is to consider the powder as a magnetic circuit with a gap. One should consider a simple cubic pack of cubes having a edge D and separated by a non magnetic layer of thickness δ . The total reluctance of the circuit is the sum of the particle and the gap reluctance:

$$\mathcal{R} = \frac{D}{\mu_0\mu_r D^2} + \frac{\delta}{\mu_0 D^2} \quad (1)$$

Writing the magnetomotive force according to Hopkinson’s law $nI = \mathcal{R}\Phi$ with $\Phi = BD^2$ and $nI = H(D + \delta)$, one easily gets the effective permeability ²:

$$\mu_{eff} = \frac{D}{D + \delta} \cdot \frac{\mu_r}{1 + \frac{\delta}{D}\mu_r} \quad (2)$$

which is equivalent to the result given in [25] apart from the unimportant geometrical factor under the condition $D \gg \delta$. The ratio δ/D is actually a demagnetizing coefficient associated to this geometry, it is related to the filling fraction x by:

$$N = \frac{\delta}{D} = (x^{-1/3} - 1) \quad (3)$$

Experience shows that NMGB model agrees well with MnZn ferrites which are composed of MnZn grains with CaO ultra-thin insulating layer though, usually, the magnetic barrier is estimated from the model. Typically this barrier is about few nm as shown by TEM in high permeability Mn-Zn ferrite [47]. However, in Mn-Zn ferrites, the magnetic fraction is over 95% and each grain is separated by the paramagnetic layer. In the usual form, the model has major drawbacks:

- it systematically underestimate the effective permeability if the fraction is lower than 90%

¹The model is usually referred as ‘Maxwell Garnett’, Maxwell is the usual surname of Pr Garnett.

²The effective permeability of powder cores is usually defined as the initial one μ_i , so the intrinsic bulk susceptibility/permeability χ/μ_r is the initial one too. Alternatively, the models can be used for the maximum (reversible) permeability and the bulk value should be taken as the max (reversible) one. In most cases, due to the demagnetizing effect, μ_i and μ_{max} differs by less than 5%

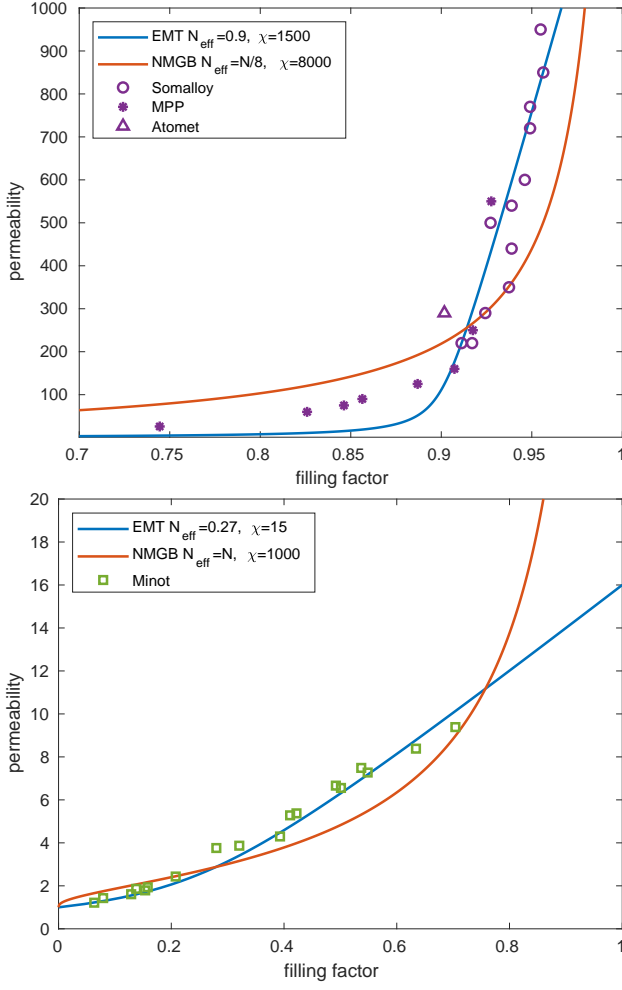


Figure 1: NMGB and EMT models compared to values of some high permeability commercial SMCs (see references in [Appendix A](#)) and to an experimental composite composed of $3\mu\text{m}$ iron spheres [46].

- it has no physical meaning for small fractions because when $\delta > D$, $N > 1$, *i.e.* $x < \frac{1}{8}$.
- $\mu_{eff} \rightarrow 0$ when $x \rightarrow 0$
- $\mu_{eff} < 0$ if $\frac{\delta}{D} > 1 - \frac{1}{\mu_r}$
- SMCs have imperfect insulation, which implies the presence of magnetic contact between particles.

A more general approach would deduce the applied field from Ampere's theorem $H(D + \delta) = H_i D + H_g \delta$ where the subscript i is for internal and g for gap. By continuity of the normal component of B , one can write that $\mu_0 H_g = B_i = \mu_0 H_i + M$. Combining these two equations leads to:

$$H = H_i + \frac{\delta}{D + \delta} M \quad (4)$$

Substituting $M = \chi H_i$ and putting $B = \mu_0 \mu_{eff} H = \mu_0 (1 + \chi) H_i$ yields a slightly different expression of the effective permeability

$$\mu_{eff} = \frac{1 + \chi}{1 + \frac{\delta}{D + \delta} \chi} = \frac{1 + \chi}{1 + (1 - x^{1/3}) \chi} \quad (5)$$

Within the limit of small gap and large susceptibility this expression is equivalent to Eq. (2), but now it can be seen that the demagnetizing coefficient has a different expression and is never greater than 1. Also, the effective permeability $\mu_{eff} \rightarrow 1$ if $\delta \gg D$. This correction of the NMGB model improves the physical meaning and the range of applicability of the model. It is well known that the problem with NMGB model is due to the fact that the main assumptions (*i*) the uniformity of the field in the gap is clearly not justified if the filling fraction is not very close to one, (*ii*) the regular cubic packing, *i.e.* 1D approximation, is unrealistic.

A 2D extension of the model has already been proposed by the author to apply to bcc packing of square flakes [20]. Despite good agreement with nanocrystalline flakes composites, it lacks by generality since it works only for oriented square flakes with in-plane field.

2.2. Homogenization models

The concept of homogenization was first applied to dielectrics. In the original model, each individual electric dipole is supposed to be located in a spherical cavity in a uniformly polarized homogeneous medium corresponding to the mean field produced by the contributions of all dipoles. The effective permittivity deduced from this concept is given by Clausius-Mossotti relation (see e.g. [48].) There are different improvements of the model such as Garnett's, to take into account the permittivity of the host medium, or Bruggeman's, which considers two (or more) particles in an effective medium. Among the different variants of the model [49], Bruggeman's is mostly used for magnetic composites [36] and usually referred as effective medium theory (EMT). In its scalar form, the effective permeability μ_{eff} is defined from the relative permeability of each media μ_i , their concentration c_i and their demagnetizing coefficients N_i by the expression:

$$\sum_i \frac{c_i N_i (\mu_i - \mu_{eff})}{\mu_{eff} + N_i (\mu_i - \mu_{eff})} = 0 \quad (6)$$

In the present case, the material has 2 components only one of which is ferromagnetic, $\mu_1 = 1 + \chi$, $\mu_2 = 1$, $N_1 = N_2 = N$, $c_1 = x$ and $c_2 = 1 - x$. Written in terms of susceptibility, the above equation is simplified as a second order equation:

$$(1 - N) \chi_{eff}^2 + [1 + (N - x) \chi] \chi_{eff} - x \chi = 0 \quad (7)$$

which has only one physical solution

$$\chi_{eff} = \frac{-[1 + (N - x) \chi] + \sqrt{[1 + (N - x) \chi]^2 + 4(1 - N) x \chi}}{2(1 - N)} \quad (8)$$

For larger concentrations, χ_{eff} has the same order of magnitude as χ_i so the last term of Eq. (7) is negligible and a trivial solution is

$$\lim_{x \rightarrow 1} \mu_{eff} = 1 + \frac{(x-N)\chi-1}{1-N} \quad (9)$$

This linear dependence for high x versus the hyperbolic one with NMGB is revealing the lack of coherence of existing models.

On the other edge, for low concentrations, first order expansion of Eq. (8) yields

$$\lim_{x \rightarrow 0} \mu_{eff} = 1 + \frac{x\chi}{1+(N-x)\chi} \quad (10)$$

furthermore if $N \gg x - i.e.$ any particle shape except needles – and $\chi_i \gg 1$:

$$\mu_{eff} \approx 1 + \frac{x}{N} \quad (11)$$

As mentioned in the introduction, the problem is that N_{eff} has to be fitted from experiment. Mattei has proposed an expression of N_{eff} but it depends on the magnetic contrast $\alpha = (\mu_i - \mu_{eff})/\mu_{eff}$ leading to non explicit solution of the problem [38].

Furthermore, the EMT can't be applied for large particle fractions because the volume fraction itself is limited by the shape of the particles. In the case of spheres, the highest density is 74% for hcp or fcc packing, 63.6% for random close packing. and even 53.6% for random loose packing with a connectivity of 4 [50]. Indeed, it is impossible to keep the particles spherical above the limit of 60% so for any larger x , N is changed.

2.3. Comparison of the models with experiment

Concerning high permeability SMCs, it is observed that none of these models can fit the data without a coefficient adjustment corresponding to unreasonable particle geometries. In Fig. 1 (left box) reasonable agreement is found in a very limited range by fitting the demagnetizing factor and the susceptibility. For NMGB model, it has to be reduced — compared to a sphere — by a factor 8 which may be justified qualitatively by the contact between particles. However the experimental agreement remains poor. For EMT, a good fitting is found for $N = 0.9$ which seems quite unreasonable since N is expected to be close to 1/3 and this value would corresponds to a prolate ellipsoid of $r = 5$ with field parallel to short axis. Furthermore, the permeability is below 10 for $x < 0.8$. Besides the models disagree on the value of the susceptibility up to a factor of 5 or more. For low concentrations (Fig. 1 right box), EMT is in very good agreement with experience but needs two fitting parameters, here $N = 0.27$ and $\chi = 15$. In contrast, NMGB needs no fitting parameter, here $\chi = 1000$, but 10 times less or more wouldn't make appreciable difference. Again the experimental agreement is poor for NMGB.

This clearly shows that the two existing models hardly agree on the value of N and χ . Furthermore, there is a gap for fractions between 60 and 90% where none of these models apply whereas powder cores have mostly a magnetic particle concentration in this gap.

3. Model based on the Statistical Mean Chain Approximation (SMCA)

3.1. Equivalent chain length

In order to take into account the random nature of real SMCs, we first consider an infinite chain of spheres composed of a fraction x of magnetic particles, $1-x$ of non magnetic particles or voids, both of diameter D . If the voids are randomly distributed, the chain is actually composed of series of magnetic and non-magnetic monomers, dimers, trimers, etc, separated by voids.

For example, if $x = 0.5$, the random chain can be decomposed in ordered chains alternating single particles/voids units, particle/voids dimers, particle/voids trimers, etc. with associated probability as illustrated in Fig. 2. In contrast an ordered chain would be composed of alternating single particles and voids.

In order to obtain the average chain length as a function x , the number of chains composed of n particles has to be calculated as a function of x .

The event A_n corresponding to a chain of n particles surrounded by 2 voids has the following probability to be realized:

$$P_n(A_n) = x^n(1-x)^2 \quad (12)$$

where x^n corresponds with the particles and $(1-x)^2$ with the 2 voids.

The sum over all chain lengths is calculated from Eq. (12)

$$\begin{aligned} (1-x)^2 \sum_{n=1}^{+\infty} x^n &= (1-x)^2 \left(\sum_{n=0}^{+\infty} x^n - 1 \right) \\ &= (1-x)^2 \left(\frac{1}{1-x} - 1 \right) \\ &= (1-x)x \end{aligned} \quad (13)$$

so Eq. (12) can be normalized with respect to Eq. (13)

$$P_n(A_n) = (1-x)x^{n-1} \quad (14)$$

The average chain length is the expected value of Eq. (14)

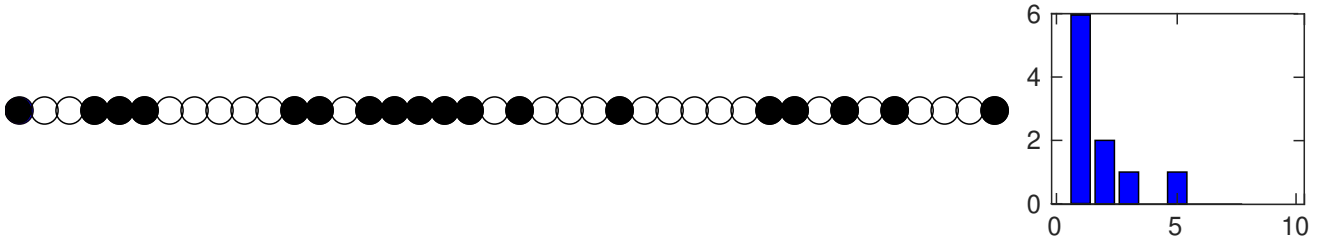


Figure 2: Example of a random chain generated by a uniform probability with a threshold of 0.5 – black and white balls are equiprobable – and the corresponding histogram.

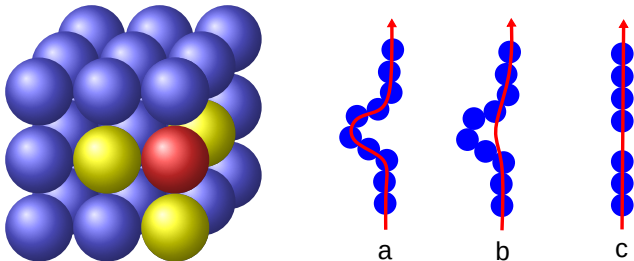


Figure 3: 3 possible position of a magnetic particle (red ball) nearest void (yellow balls). Scheme of a percolation path (a), the mean magnetic path for same geometry (b), the 1D equivalent magnetic path (c).

$$\begin{aligned}
 \mathbb{E}[x] &= \sum_{n=1}^{+\infty} n P_n(A_n) \\
 &= (1-x) \sum_{n=1}^{+\infty} n x^{n-1} \\
 &= (1-x) \frac{1}{(1-x)^2} \\
 &= \frac{1}{1-x}
 \end{aligned} \tag{15}$$

So finally, for any value of the filling fraction x , the aspect ratio of the mean chain takes a remarkably simple form:

$$r = \frac{\langle \ell \rangle}{D} = \frac{1}{1-x} \tag{16}$$

were D is the diameter of the spheres. Going back to the example when $x = 0.5$, $r = 2$ is found instead of 1 for an ordered chain.

If $x \rightarrow 0$, the chains are single particles, and if $x \rightarrow 1$, the chain is infinite.

In 3 dimensions, voids are no more distributed along the chain but in the volume. In a simple cubic packing, voids around a magnetic particle can be localized in the 3 directions (see left sketch in Fig. 3). However, if the magnetic field is homogeneous and directed parallel to one of the cube edges, we may consider only the voids along a 1D chain, this is to say that percolation path perpendicular to the field is not taken into account.

This situation is totally different from that of electrical percolation. Indeed, in the scheme Fig. 3 (a) shows the electrical percolation path and (b) the magnetic path for the same geometry providing the susceptibility is relatively small which is often the case for small particles. So path (b) would be equivalent to (c). If the voids are distributed with the same probability in the 3 directions, the equivalent chain will have an aspect ratio:

$$r = \frac{\langle \ell \rangle}{D} = \frac{1}{1-x^{1/3}} \tag{17}$$

3.2. Effective demagnetizing factor

Now the problem is to determine the demagnetizing factor associated with the equivalent chain. Here a classical assumption is made saying that a chain composed of r particles (see Fig. 4a) exhibits a demagnetizing factor equivalent to a prolate ellipsoid of aspect ratio r (see Fig. 4c) which is known from Eq. (17). Eventually this model avoids the problem of compaction limits since particles may be distorted inside a chain with limited effect on N_{eff} (see Fig. 4b.)

Using one of the well known expression of the demagnetizing factor of a prolate ellipsoid (here as formulated by Stoner [9]),

$$N(r) = \frac{1}{r^2 - 1} \left[\frac{r}{\sqrt{r^2 - 1}} \ln(r + \sqrt{r^2 - 1}) - 1 \right] \tag{18}$$

one can easily compute the equivalent demagnetizing factor as a function of the filling fraction thru Eq. (17). This computation is compared with effective demagnetizing coefficients determined experimentally by Forrer with iron balls [51] in Fig. 5. In this experiment Forrer considered that the demagnetizing factor can be determined as the inverse of the initial slope of the anhysteretic curve, χ_{an} witch is equivalent to the slope of the major hysteresis loop at $H = H_c$. For low concentrations as N is low, the loop may be not saturated so the χ_{an} may be underestimated and by consequence N may be overestimated, which may explain the discrepancy between the present model and the measurements. On the other side, the demagnetizing coefficient of a n -spheres chain is obviously larger than the ellipsoid having an aspect ratio of n and the difference is more visible for small n . This

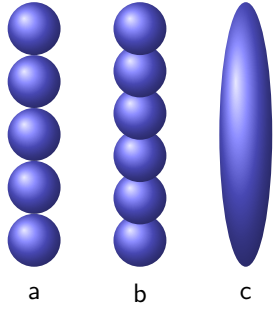


Figure 4: Schemes of (a) a hard sphere chain, (b) a soft sphere compressed chain, (c) the equivalent ellipsoid.

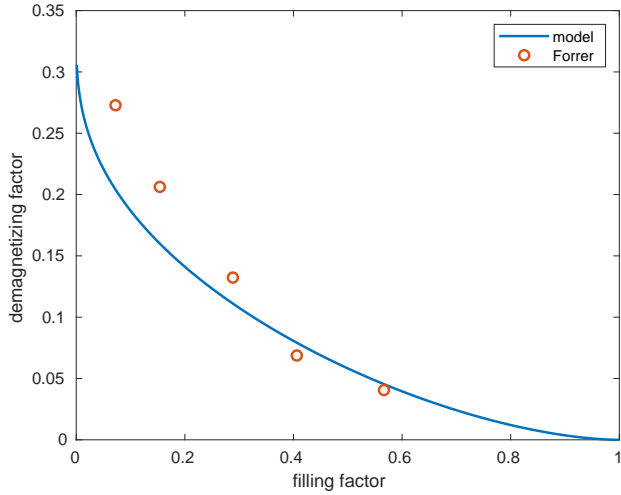


Figure 5: Average demagnetizing factor of iron spheres composite as a function of filling fraction (data are from Ferrer [51])

may explain why the gap between the model and the experiment is larger for small x , but evidently there is a good agreement between the two without any fitting parameter.

3.3. Permeability of magnetic spheres composites

Let us consider a SMC composed of spherical magnetic particles dispersed in a zero susceptibility matrix such as a para or diamagnetic binder. The field H_a required to apply to the material to achieve the internal field H_i is increased by the demagnetizing field according to:

$$H_a = H_i + N_{eff}M = (1 + N_{eff}\chi)H_i \quad (19)$$

where N_{eff} is the effective demagnetizing factor of the equivalent average chain determined statistically. The mean magnetization of the SMC can be written

$$M = \chi_{eff}H_a = x\chi H_i \quad (20)$$

Substitution of Eq. (19) in Eq. (20) leads to

$$\mu_{eff} = 1 + \frac{x\chi}{1 + N_{eff}\chi} \quad (21)$$

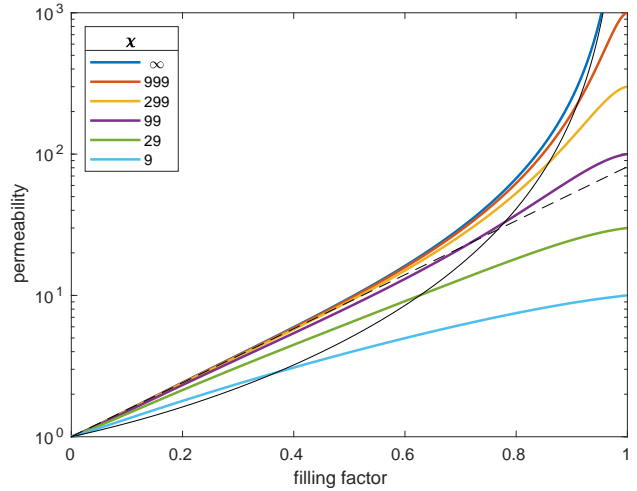


Figure 6: Effective permeability dependence on the filling factor parametrized by the susceptibility according to the present model. (---) exponential approximation Eq. (22), (—) long cylinder approximation Eq. (23).

This expression makes physical sense since for $x \rightarrow 0$ and $\chi \rightarrow \infty$ it leads exactly the same as EMT Eq. (11) – where EMT works well –, and since for $x \rightarrow 1$ it leads exactly the same expression as modified NMBG Eq. (5) – where NMBG works well – except for the new definition of the demagnetizing factor.

A computation of the effective permeability as a function of the filling factor is shown for different values of the susceptibility in Fig. 6. It is remarkable that for $x = 0.5$ and susceptibilities above 99, the present model gives an effective permeability about 10 which is significantly larger than that given by NMBG (only 2) and contrarily to EMT the linear behavior expected for $x > 1/3$ is not observed but an exponential one up to $x \approx 0.5$. A useful empirical approximation may be written:

$$\mu_{eff} = 81^x \quad \forall x \leq 0.5 \quad \& \quad \chi \geq 99 \quad (22)$$

Compared with experiment as shown in Fig. 7, a very good fit of Weidenfeller's [45] and Minot's [46] low permeability composites is obtained by only adjusting the susceptibility, contrarily to EMT that needs also N to be fitted. Note that with the present model $\chi = 21$ and 150 whereas $\chi = 16$ and 52 with EMT.

For large concentrations, comparing with commercial data (iron and permalloy based SMCs) shown in Fig. 8 highlights the efficiency of the model. Though significant spread is observed – partly due to the uncertainty on x values – data are best fitted with $\chi = 999$ for low concentrations and $\chi \rightarrow \infty$ for high concentrations. This may be partly justified by the fact high density SMCs are usually based on larger particles, thus having higher susceptibility³.

³By opposition to academic studies based on nearly spherical

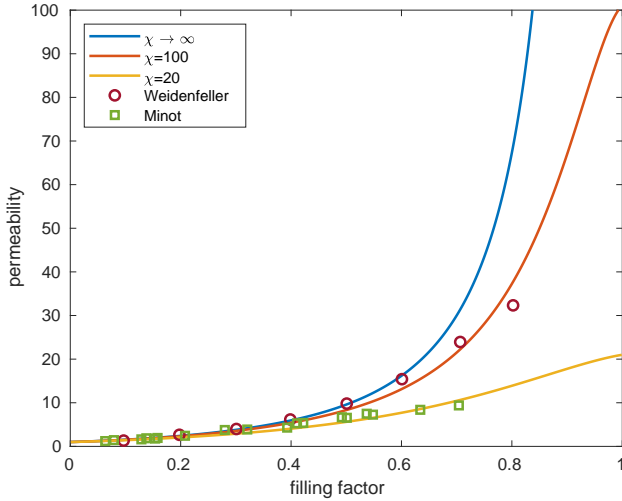


Figure 7: Comparison of the model with experience for low concentration SMCs based on spherical particles (top box). Data from Weidenfeller are for Fe-Co-V spheres [45] and Minot’s for Fe sphere [46].

For dense media, because particle chains are very long, a 1D model should be sufficient. If $x \rightarrow 1$ the SMC should be equivalent to a cylinder of aspect ratio $r = 1/(1-x)$. Its demagnetizing coefficient can be approximated using the fluxmetric demagnetizing coefficient of a cylinder with $r > 20$, $N_{eff} \approx \frac{1}{2r^2}$, so Eq. (21) becomes

$$\mu_{eff} = 1 + \frac{x\chi}{1 + \frac{1}{2}(1-x)^2\chi} \quad \forall x > 0.95. \quad (23)$$

Surprisingly, the approximated model fits better the data, but one has to be cautious because of the relatively large scattering of data and also because particles were assumed to be spherical and mono-dispersed in SMCA which is clearly not the case in dense SMCs. Indeed, the particles are not basically not spherical and hot pressing strains them appreciably (see *e.g.*[52].)

3.4. Permeability of non-spherical magnetic particles composites

If the particles are not spherical Eq. (18) should be replaced by that of the corresponding shape (*e.g.* that of a cylinder as computed by Chen [12].) However, expressions using complete elliptic integrals are rather difficult to implement. A simple approximation would be to consider that the aspect ratio dependence for any shape is the same as that of a prolate ellipsoid. In other words, if N_p is the demagnetizing coefficient of the particle, that of the chain of aspect ratio r would be $3N_p N_{eff}(r)$ verifying that in diluted media $3N_p N_{eff}(1) = N_p$. This leads to the general expression:

and mono-disperse particles, in the same series of commercial data the mean diameter of particles may depend on the filling factor

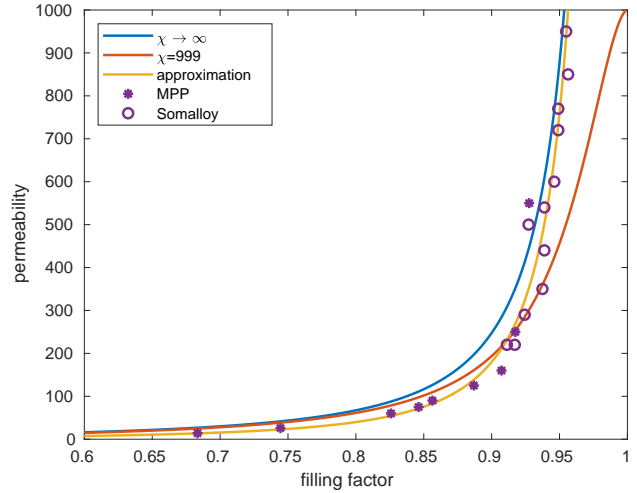


Figure 8: Comparison of the model with experience for high concentration SMCs based on nearly random shaped particles (bottom box). Data are from technical documentations, see Appendix A.

$$\mu_{eff} = 1 + \frac{x\chi}{1 + 3N_p N_{eff}\chi} \quad (24)$$

A parametric study of this equation is given in Fig. 9. Qualitatively these curves look like in [36] except the curvature which is more pronounced at same N_p .

To collate with experimental data, a fitting of N_p with $\chi \rightarrow \infty$ is displayed Fig. 10. The best fit of experimental curves is obtained for $N_p = 0.5$ corresponding to $r \approx 1.8$, a realistic result considering the micrography of SMCs [52] showing usually non equiaxis particles. In order to demonstrate the validity of Eq. (24) in a more straightforward way, the case of SMC constituted of Finemet flake is considered. Data correspond to particles sieved with 160 and 80 μm meshes, pressed in a resin and measured with a field perpendicular to pressure. Considering each particle as 20 μm thick square prisms with 80 and 160 μm edges, N_p was calculated according to [14] and Chen [15] with $\chi = 999$ with in plane field. As the distribution is not known, we took the mesh size as the flake size. Eq. (24) was applied directly considering $\chi \rightarrow \infty$ without any kind of fitting. The result shown in Fig. 11 is surprisingly good in spite of the strong geometrical approximation. Taking into account the susceptibility of the particle in the computation of N_p yields a slightly higher effective permeability but doesn’t change the order of magnitude.

To compare with EMT, fitting the curve for the samples with 80 μm particles leads to a slightly higher effective demagnetizing coefficient (≈ 0.22) – which is self-contradictory since in EMT $N_{eff} \leq N_p$ – and $\chi \approx 200$ which is 2 orders of magnitude too small for Finemet, one of the softest materials ever made ($\chi > 30\,000$). The only limitation of the model at this stage is that non-spherical particles must be oriented. This condition is however

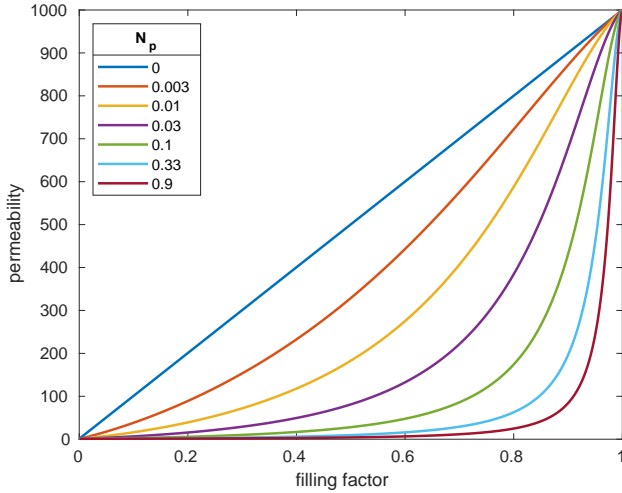


Figure 9: Effective permeability dependence on the demagnetizing coefficient of the particles according to the present model computed for $\chi = 999$. Demagnetizing coefficients correspond to aspect ratio of ∞ , 32, 15, 7.5, 3, 1, 1/30, respectively

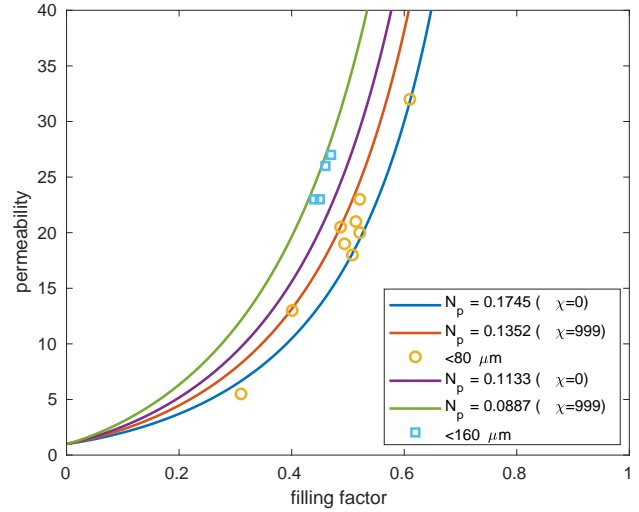


Figure 11: Permeability of Finemet SMCs as a function of filling fraction. Particles are $20 \times 80 \times 80 \mu\text{m}^3$ Finemet flakes. Markers are data from V. Léger [20], demagnetizing coefficient of rectangular prism is computed from [14] and [15].

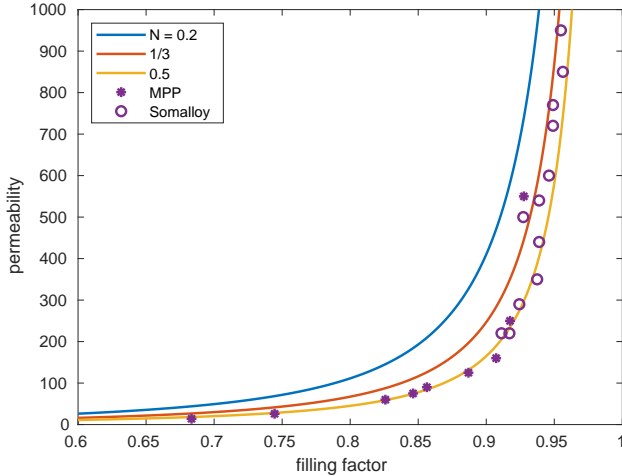


Figure 10: Effect of the shape of the particles on the model for $\chi \rightarrow \infty$ compared to high density SMC.

easily satisfied with flakes based composites since compaction under field or with a rotating piston naturally turns flakes perpendicular to the pressing direction.

4. Conclusion

A new model for the permeability of SMCs has been successfully developed based on simple statistical considerations and classical concepts in electromagnetism. The main strength of the model is the predetermination of the effective demagnetizing coefficient directly from the filling fraction and the approximate shape of the particles. For SMCs with a filling fraction less than 60% even the exact knowledge of the intrinsic susceptibility is not necessary providing its value is greater than 100. Experimental collation is very good with composites made from spherical particles, but also for that based

on flake-shaped particles like milled nanocrystalline ribbons. Because the main parameter of the model N_p is determined independently and the second one χ doesn't require accurate determination – only the order of magnitude is needed – the Statistical Mean Chain Approximation model is able to predict the permeability of SMCs made from any particle shape and any magnetic material. A small adjustment of N_p may be necessary only in the case of powders having indefinite particle shape.

To compare the models, the parameters of the models are indicated in Table 1. In some cases, parameters are obtained directly from the model (noted C in the table) and in some cases they must be fitted. An assessment of the quality of the fit is given in column Q together with an assessment of the physical meaning (Φ) of the value of the parameters. The NMGB model has clear physical meaning but needs adjustment of N_{eff} for dense SMCs and the fitting is poor. On the over hand, EMT usually yields good fitting but the two parameters must but adjusted, especially N_{eff} that may be lower than N_p for low concentrations and much higher for high concentrations. In contrast, the present model gives very good fit whatever is the concentration or the shape of the particles and requires at maximum one parameter to be fitted, χ for low permeability SMCs or N_p when the particle shape is undefined. If the particle shape is known and the intrinsic susceptibility large, even no fitting is needed. It is thought that the present model based on a statistical approach is more versatile, robust and physically coherent, the reason is clearly because usual models are based on the spacial periodicity hypothesis which is never verified in practice.

material	model	parameters			scores		
		N_{eff}	χ	N_p	Q	Φ	F
3 μ m spheres	NMGB	C	∞	\times	P	G	0
	EMT	0.27	15	\times	VG	VG	2
	SMCA	C	20	\times	VG	VG	1
High density SMCs	NMGB	C/8	8000	\times	P	P	2
	EMT	0.9	1000	\times	VG	P	2
	SMCA	C	∞	0.5	VG	VG	1
80 μ m	NMGB	C	10 ⁴	\times	G	VG	0
Nanocrystalline composite	EMT	0.22	100	C	P	P	2
	SMCA	C	∞	C	VG	VG	0

Table 1: Comparison of the models. Numbers are fitted values, except the susceptibility of nanocrystalline flakes which was measured before crushing of the ribbon; Q is quality of the fit; Φ the physical coherence; P, G, VG are for poor, good, very good respectively; F is the number of fitting parameters; C means predetermined by calculation.

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Appendix A. Commercial references

MPP	μ_i	Somalloy	μ_{max}
14 μ	14	110i 1P	220
26 μ	26	110i 5P	220
60 μ	60	130i 1P	290
75 μ	75	130i 5P	350
90 μ	90	700HR 1P	440
125 μ	125	700 1P	540
160 μ	160	700HR 5P	600
250 μ	250	1000 5P	720
550 μ	550	700 3P	850
		1000 3P	950

MPP (Molly-Permalloy Powder) references used are from Magnetics www.mag-inc.com.

Somalloy is a trade mark of Höganäs www.hoganas.com/somalloy.

Atomet is a trade mark of Rio Tinto, product reference EM1, $\mu_{max} = 290$ qmp-powders.com/product-applications/components/magnetic-applications/.