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# Unilateral Dominance and Social Discounting

Bach Dong-Xuan<sup>\*</sup> and Xiangyu Qu<sup>†</sup>

## Abstract

This paper addresses the intricate challenge of establishing social discount rates across far-reaching generations, particularly in the face of divergent social viewpoints. We propose a principle of *future-improved unilateral dominance* to characterize social discount rates. Despite its divergence from traditional principles, our principle prioritizes the welfare of distant generations, resonating with a minority concern within society. Our findings indicate that society adhering to this principle exhibit greater patience and future-oriented concern than any individual. This approach, contingent upon the preferences of current generations, offers theoretical pathways to enhance considerations for the welfare of the distant future in the context of long-term environmental projects or activities.

## 1 INTRODUCTION

The discourse surrounding social discounting remains fiercely debated, highlighted prominently by contrasting viewpoints from economists ([Weitzman \(2001\)](#); [Drupp et al. \(2018\)](#)). [Stern \(2007\)](#), for instance, advocates for a near-zero social discount rate, emphasizing the imperative for immediate action against climate change. This stance posits that the welfare of future generations should weigh heavily in present decisions. However, the paternalistic nature of this method disregards the individual opinions and impedes practical implementation in democracies ([Marglin \(1963\)](#); [Feldstein \(1964\)](#)). In contrast, [Nordhaus \(2007\)](#) proposes the adoption of market rates as a more pragmatic and balanced

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approach. The deficiency of parameters, particularly for far-reaching market, hinders the use of conventional methodologies in determining the social rate. The divergence in opinions and the absence of a convergent consensus complicate the establishment of a unanimous methodology. This discord defies easy resolution through conventional principles, as highlighted by Zuber (2011) and Jackson and Yariv (2015), who discover that discount rates founded on unanimity principle either flout time consistency or become subservient to paternalistic decision rule. This ongoing discourse illustrates the pressing need to reconcile divergent views, striving for a methodological framework that effectively harmonizes various perspectives to formulate an impactful social discount rate.

This paper aims to confront the challenge of determining social discount rates across far-reaching generations. Specifically, it delves into the quandary of establishing a novel and rational principle when social individuals hold divergent perspectives on the discount rate. The objective is to devise a non-paternalistic approach that ensures both time consistency and effective safeguarding of the welfare of far-reaching distant generations, facilitating environmental preservation and sustainable development. It is worthy to note that environmental protection often involves sacrifices or costs in the present for long-term benefits, mainly for future generations. In a democracy where policies are often shaped by short-term interests and the will of majority, advocating for long-term benefit, such as environmental protection, using classical principles like *unanimity* can be misleading.

In this paper, we present a novel principle termed *future-improved unilateral dominance*. This principle posits that when society compares two consumption streams, if one stream demonstrates an improvement in consumption towards more distant future generations compared to the other, and at least some individuals within society prefer this future-improved stream, then society should also favor the future-improved option. This principle exhibits two primary characteristics. Firstly, society's comparison of consumption streams is limited to specific pairs, isolating one stream as inherently more beneficial for the further future generations. Variances in individual preferences for consumption streams stem solely from discrepancies in individual discount rates. This focused evaluation exclusively addresses disparities in individual discount rates, shaping the social discount rate independently of individual consumption values. Secondly, social preference for the future-improved consumption stream does not require recognition from the entire or even the majority of social individuals. If some individuals in society hold this preference, it becomes imperative for society to align with it. In the specific case

we are discussing, although it may initially seem less compelling than the principle of unanimity, its primary focus is on prioritizing the well-being of far-reaching generations, which, however, is typically a shared concern of only a marginalized minority in society. Within this framework, this principle emerges as reasonable and meaningful. In fact, the concept of minority voices wielding decisive power, as seen in issues like disability protection<sup>1</sup> and biodiversity, is already widely prevalent.

This paper reaches a significant conclusion within our established framework, emphasizing the novel principle of unilateral dominance. The key finding asserts that adopting this principle results in a social discount rate that is smaller than that of any individual within society (Theorem 1). In simple terms, a society embracing this principle demonstrates greater patience and concern for the well-being of future generations than any single individual within that society. As uncertainties surrounding the far distant future make reconciling disparities in individual discount rate judgments challenging, the social discount rate, lower than that of every individual, reflects society's heightened caution towards the future—a rational and sensible stance in the face of an immensely distant future. Consequently, the paper's conclusions provide theoretical avenues for societies to more effectively consider the welfare of exceedingly distant future generations.

In fact, [Caplin and Leahy \(2004\)](#); [Farhi and Werning \(2007\)](#); [Feng and Ke \(2018\)](#) also claim that the social discount rate should lag behind individual rates. However, their conclusions hinge upon assumptions about future generation preferences. Conversely, our paper refrains from considering non-existent and unobservable future generation preferences. Instead, we establish an exceptionally patient social discount rate grounded solely in the current generation's preferences through the simple premise of unilateralism.

The methods proposed in the literature to address the impossibility result of Zuber-Jackson-Yariv generally fall into two categories. One approach aims to ensure the time consistency of social preferences by imposing certain restrictions on the scope of the unanimity principle ([Chambers, Echenique and Miller \(2023\)](#)) or relaxing the requirements for the time invariance of social preferences ([Millner and Heal \(2018\)](#)). The other approach, to maintain the unanimity principle, relaxes the demand for time consistency of social preferences ([Chambers and Echenique \(2018\)](#)). Although these two methods alleviate the impact of the impossibility theorem to some extent and can be applied in certain

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<sup>1</sup>For instance, Americans with Disabilities Act in the United States or similar legislations worldwide require certain facilities, including transportation hubs like stations, to provide accessibility features for individuals with disabilities. These laws often prioritize the accommodation of marginalized or minority groups to ensure inclusivity.

frameworks, the resulting social discounting remains a compromise between individual discounting and falls short of Ramsey (1928)'s ethical aspiration to equate the welfare of future generations with that of the present generation as much as possible. To delineate the maximum range of compromise in social discounting, we introduce a *restricted unanimity* principle that ensures the social discounting falls between the maximum and minimum individual discounting (Proposition 1). However, in the presence of uncertainty, Weitzman (1998) deems any form of compromise unacceptable when considering the welfare of far-distant future generations. A prudent society should choose the most patient individual discount rate as the social discount rate. Remarkably, the restrictive unanimity principle and the unilateral dominance we propose together delineate Weitzman (1998)'s discount rate (Corollary 1).

The theoretical framework is outlined in the following section. Our primary result will be presented and discussed in Section 3. We conclude by addressing several related issues in the final section.

## 2 THE MODEL

Let  $X$  be a set of *outcomes*, formally a convex and compact subset of a vector space. Let  $\mathcal{L} = \Delta(X)$  be a consumption space, namely a set of lotteries over outcomes  $X$ . Let  $\mathcal{T} = \{0, 1, 2, \dots\}$  be the set of discrete time periods. We will use  $a$ ,  $b$ , and  $c$  to denote generic elements of  $\mathcal{L}$ . A *consumption stream* is denoted by  $x = (x_0, x_1, \dots) \in \mathcal{L}^{\mathcal{T}}$ . For any  $a \in \mathcal{L}$ , with a slight abuse of notation, we use  $a$  to denote the *constant* consumption stream  $(a, a, \dots)$ . For every  $t \in \mathcal{T}$ ,  $b_t a$  denotes the consumption stream  $x$  that is defined by  $x_t = b$  and  $x_s = a$  for all  $s \neq t$ .

Society is a set of individuals  $\mathcal{I} = \{1, \dots, n\}$ . Individual  $i \in \mathcal{I}$  has preferences  $\succeq_i \subset \mathcal{L}^{\mathcal{T}} \times \mathcal{L}^{\mathcal{T}}$ . Similarly, society's preferences are denoted by  $\succeq_0 \subset \mathcal{L}^{\mathcal{T}} \times \mathcal{L}^{\mathcal{T}}$ . We assume that individual preferences  $\succeq_i$  are represented by a *discounted utility*.

**Definition 1.**  $U$  is a *discounted utility* (DU) if there exist a linear *instantaneous* function  $u : \mathcal{L} \rightarrow \mathbb{R}$  and a decreasing *discount function*  $d : \mathcal{T} \rightarrow (0, 1)$ , with  $d(0) = 1$  and  $\sup_{t \in \mathbb{N}} \frac{d(t+1)}{d(t)} < 1$  such that, for all  $x \in \mathcal{L}^{\mathcal{T}}$ ,

$$(1) \quad U(x) = \sum_{t=0}^{\infty} d(t)u(x_t).$$

That one-period discount factor is strictly less than 1 reflects that each individual

discounts in every time periods. We assume that individuals have identical preferences over consumptions, i.e.  $u_i = u$  for all  $i \in \mathcal{I}$ . (This assumption will be relaxed in the last section.) Therefore, each individual preference  $\succsim_i$  can be represented by a pair  $(d_i, u)$ .

We assume that social preference  $\succsim_i$  is represented by an *exponential discounted utility* (EDU) of [Koopmans \(1960\)](#), which is a discounted utility whose discount function  $d_0(t) = \delta_0^t$  is exponential for  $0 < \delta_0 < 1$ . We use  $(\delta_0, u_0)$  to represent social preferences  $\succsim_0$ .

We define *unanimous relation*  $\succsim_{\mathcal{I}}$  as usual:  $x \succsim_{\mathcal{I}} y$  if and only if  $x \succsim_i y$  for all  $i \in \mathcal{I}$ .

### 3 MAIN RESULTS

In fact, when individuals have DU preferences, a conflict arises between an EUD society and unanimity. In other words, even if the EDU society adheres to a dictatorial rule, it cannot simultaneously satisfy the unanimity principle. We emphasize this observation through a simple example. Let us consider a society consisting of identical individuals  $(u, d)$ . Here,  $u(a) = a$  and  $d(t) = \beta \delta^{t-1}$ , where  $\beta = \frac{1}{3}$  and  $\delta = 0.9$ . We examine four possible consumption streams where only the first three periods can have non-zero consumptions:  $x = (1, 0, 2, \dots)$ ,  $y = (1.5, 0, 0, \dots)$ ,  $z = (2, 0, 0, \dots)$ , and  $w = (1, 2.5, 0, \dots)$ . It is evident that all individuals prefer  $x$  to  $y$  and  $z$  to  $w$ . A unanimity-respected society must also endorse these preferences and have  $u_0(a) = a$ . However, if a society adheres to exponential discounted utility, then the preference of  $x$  over  $y$  requires a social discount factor  $\delta_0 > 0.5$ , and the preference of  $z$  over  $w$  requires  $\delta_0 < 0.4$ , which is impossible.

To mitigate such impossibility, we investigate the feasibility of restricting the unanimity principle to allow the social discount factor to lie within the range of the smallest and largest individual one-period discount factors.

**Restricted Unanimity:** For any  $a, b, c \in \mathcal{L}$ , if  $b_{t+1}a \succsim_{\mathcal{I}} c_t a$  for all  $t \in \mathcal{T}$ , then  $b_{t+1}a \succsim_0 c_t a$  for all  $t \in \mathcal{T}$ .

This principle restricts exclusively on the binary consumption streams. It actually contemplates two possible variations from a constant consumption level  $a$ . One variation involves a change in consumption at relatively further time  $t + 1$  from  $a$  to  $b$ , while the other sees a change in consumption at time  $t$  from  $a$  to  $c$ . Restricted Unanimity requires that if all the individuals prefer the former consumption change, then the society should also prefer the former change. We demonstrate next that this principle restricts the social discount factor within the range of individual one-period discount factors.

**Proposition 1.** *Restricted Unanimity is satisfied iff  $\min_i \inf_t \frac{d_i(t+1)}{d_i(t)} \leq \delta_0 \leq \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ .*

The above results indicate that the restricted unanimity principle contributes to overcoming the previous impossibility results. However, this improvement is limited. The principle still provides society with an extremely broad range of choices, whether adopting very low or very high social discount rates, as long as they fall within the range of individual one-period discount factors, without violating the restricted unanimity principle. If we anticipate a guiding principle to direct society's attention toward environmental issues, favoring [Stern \(2007\)](#)'s perspective, this principle does not ensure that society will inevitably choose a high social discount factor.

Through the above discussion, we have seen the limitations of the restricted unanimity. To characterize a society that more fully concerns the welfare of far-distant future generations, we will now propose a restricted unilateral dominance principle.

**Future-Improved Unilateral Dominance:** For any  $a, b, c \in \mathcal{L}$  with  $b \succ_{\mathcal{I}} a$ , if  $b_{t+1}a \succeq_i c_t a$  for some  $i \in \mathcal{I}$  and  $t \in \mathcal{T}$ , then  $b_{t+1}a \succeq_0 c_t a$ .

This principle contemplates two possible variations from a constant consumption level  $a$ . One variation involves an improvement in consumption at time  $t+1$  from  $a$  to  $b$ , while the other sees a change in consumption at time  $t$  from  $a$  to  $c$ . The Future-Improved Unilateral Dominance principle necessitates that if there exists an individual preferring the former variation over the latter, then society should also prioritize the former. The former option undoubtedly benefits the more distant future compared to the latter one. Although the change in the latter may significantly improve welfare at time  $t$ , as long as someone favors the welfare of the more distant future, society has to respect that preference. Initially, one might question why the preference of a single individual can supersede that of the entire society. However, the rationale behind this principle is specific to the ethical framework this paper addresses.

It recognizes that the decisions made by the current generation have long-term consequences that can significantly impact the well-being of future generations yet to come. Therefore, decisions should account for and mitigate potential adverse impacts on future generations. Given the significance of these long-term impacts, decisions should not only focus on short-term gains but also fully consider the welfare of future generations. Even if it is just one person or a minority within society voicing concerns about the future welfare, for instance, affected by environmental issues and sustainability, their perspective is crucial. Their understanding of the potential long-term consequences and their

advocacy for sustainable use implies a deeper consideration of the future's well-being. In essence, the argument centers on the ethical responsibility of the present generation to act as stewards for the benefit of both current and future generations. The principle of future-improved unilateral dominance plays a crucial role in emphasizing this ethical obligation. This principle characterizes the social discount factor should surpass any individual one-period discount factors.

**Theorem 1.** *Future-Improved Unilateral Dominance is satisfied iff  $\delta_0 \geq \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ .*

The maximum ratio of an individual's one-period discount factors represents the most substantial discounting that occurs across different periods for any individual in society. By setting the social discount factor greater than or equal to this maximum, the theorem ensures that the societal discounting aligns with the preferences of individuals who heavily discount the short term in favor of the more distant future. The theorem quantifies the conditions under which the Future-Improved Unilateral Dominance principle, with its ethical underpinnings, is satisfied. It establishes a mathematical criterion to ensure that societal discounting respects the preferences of individuals who prioritize the welfare of the far distant future.

Consequently, the two principles we introduce above characterize the [Weitzman \(1998\)](#) Discounting, where the social discount factor is equal to the maximum.

**Corollary 1.** *Future-Improved Unilateral Dominance and Restricted Unanimity are satisfied iff  $\delta_0 = \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ .*

One might question the limitations imposed on social discounting when a society is dedicated to uniformly endorsing future-improved consumption streams. Below, we present this unanimity principle and subsequently provide its characterization.

**Future-Improved Unanimity:** For any  $t \in \mathcal{T}$  and  $a, b, c \in \mathcal{L}$  with  $b \succ_{\mathcal{I}} a$ , if  $b_{t+1}a \succsim_{\mathcal{I}} c_t a$ , then  $b_{t+1}a \succsim_0 c_t a$ .

**Proposition 2.** *Future-Improved Unanimity is satisfied iff  $\delta_0 \geq \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$ .*

This result states that the principle of Future-Improved Unanimity is fulfilled if and only if the social discount factor ( $\delta_0$ ) is greater than or equal to the highest value across all periods of the minimum ratio of any individual one-period discount factors. In simpler



terms, for the society to adhere to this principle, the overall social discounting rate must be sufficiently patient, ensuring that even the most impatient individual's discounting factor for future periods is respected and met by the society.

## 4 CONCLUDING REMARKS

It is essential to note that we do not assert the universal applicability of the unilateral dominance principle we propose. We recognize that in a broader array of general cases, communal decision-making or prioritizing the interests of the majority or the collective often takes precedence over unilateral consent. Beyond the specific context highlighted in this paper, the principles articulated here may not be universally compelling. The acceptance or normative soundness of unilateral dominance should depend significantly on the specific ethical framework and the issues at hand.

In fact, our results can be extended to situations where individuals may differ in instantaneous utility functions. We simply need to assume the existence of uniform preferences over binary outcomes, i.e.,  $\exists a, b \in X$  such that  $a \succ_i b$  for all  $i \in \mathcal{I}$ . Now, we can restrict Future-Improved Unilateral Dominance to the domain of consumption streams where consumption belongs to  $\Delta(\{a, b\})$ . This restricted unilateral dominance requires the social discount factor to surpass individual one-period discount factors. Furthermore, we can assume the unanimity principle in the domain of constant consumption streams, implying that social instantaneous utility is a convex combination of individual utilities. Using this approach, we can address aggregation problems when double heterogeneity arises.

## APPENDIX

### A PROOF OF PROPOSITION 1

We first demonstrate **only-if part**. Assume Restricted Unanimity holds. Let  $a, b \in \mathcal{L}$  such that  $u(b) > u(a)$ . For any  $c \in \mathcal{L}$ ,  $i \in \mathcal{I}$ , and  $t \in \mathcal{T}$ , we have the following equivalence

relations:

$$\begin{aligned}
& b_{t+1}a \succsim_i c_t a \\
& \iff d_i(t+1)u(b) + \sum_{s \neq t+1} d_i(s)u(a) \geq d_i(t)u(c) + \sum_{s \neq t} d_i(s)u(a) \\
& \iff d_i(t+1)(u(b) - u(a)) \geq d_i(t)(u(c) - u(a)) \\
& \iff \frac{d_i(t+1)}{d_i(t)} \geq \frac{u(c) - u(a)}{u(b) - u(a)}.
\end{aligned}$$

Define  $f : \mathcal{L} \rightarrow [0, \infty)$  by  $f(c) = \frac{u(c) - u(a)}{u(b) - u(a)}$  for all  $c \in \mathcal{L}$ . Since the function  $u$  is continuous, so is  $f$ . Clearly,  $f(a) = 0$  and  $f(b) = 1$ , and we also have  $\min_i \inf_t \frac{d_i(t+1)}{d_i(t)} \in [0, 1)$ . Thus, there exists  $c \in \mathcal{L}$  such that

$$\frac{u(c) - u(a)}{u(b) - u(a)} = \min_i \inf_t \frac{d_i(t+1)}{d_i(t)}.$$

Hence,  $b_{t+1}a \succsim_{\mathcal{I}} c_t a$  for all  $t \in \mathcal{T}$ . From Restricted Unanimity, it follows that  $b_{t+1}a \succsim_0 c_t a$  for all  $t \in \mathcal{T}$ , implying

$$\delta_0 \geq \frac{u(c) - u(a)}{u(b) - u(a)}.$$

Therefore,  $\delta_0 \geq \min_i \inf_t \frac{d_i(t+1)}{d_i(t)}$ .

Fix  $a, b \in \mathcal{L}$  with  $u(b) < u(a)$ . It is evident that  $b_{t+1}a \succsim_i c_t a$  is equivalent to the following inequality:

$$\frac{d_i(t+1)}{d_i(t)} \leq \frac{u(a) - u(c)}{u(a) - u(b)}.$$

For every  $i \in \mathcal{I}$ ,  $\frac{d_i(t+1)}{d_i(t)} \in (0, 1)$  for all  $t \in \mathcal{T}$ , which implies that  $\max_i \sup_t \frac{d_i(t+1)}{d_i(t)} \in (0, 1]$ . There exists  $c \in \mathcal{L}$  such that

$$\frac{u(a) - u(c)}{u(a) - u(b)} = \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}.$$

Hence,  $b_{t+1}a \succsim_{\mathcal{I}} c_t a$  for all  $t \in \mathcal{T}$ . From Restricted Unanimity, we get  $b_{t+1}a \succsim_0 c_t a$  for all  $t \in \mathcal{T}$ , implying  $\delta_0 \leq \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ .

Now, we show **if part**. Assume  $\min_i \inf_t \frac{d_i(t+1)}{d_i(t)} \leq \delta_0 \leq \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ . Let  $a, b, c \in \mathcal{L}$  such that  $b_{t+1}a \succsim_i c_t a$  for all  $i \in \mathcal{I}$  and  $t \in \mathcal{T}$ . If  $u(b) > u(a)$ , then  $\frac{d_i(t+1)}{d_i(t)} \geq \frac{u(c) - u(a)}{u(b) - u(a)}$  for all  $i \in \mathcal{I}$  and  $t \in \mathcal{T}$ . Hence,  $\delta_0 \geq \frac{u(c) - u(a)}{u(b) - u(a)}$ , concluding  $b_{t+1}a \succsim_0 c_t a$  for all  $t \in \mathcal{T}$ . Similarly, we can show that Restricted Unanimity holds when  $u(a) > u(b)$ , completing the proof.

## B PROOF OF THEOREM 1

We only prove the **only-if part**, since the if part is routine. Assume Future-Improved Unilateral Dominance holds, we prove that  $\delta_0 \geq \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ . Let  $a, b \in \mathcal{L}$  such that  $u(b) > u(a)$ . It is clear that  $b_{t+1}a \succsim_i c_t a$  is equivalent to

$$\frac{d_i(t+1)}{d_i(t)} \geq \frac{u(c) - u(a)}{u(b) - u(a)}.$$

Let  $c^* \in \mathcal{L}$  such that  $\frac{d_i(t+1)}{d_i(t)} = \frac{u(c^*) - u(a)}{u(b) - u(a)}$ , then it follows from Future-Improved Unilateral Dominance that  $b_{t+1}a \succsim_0 c_t^* a$ , which is equivalent to  $\delta_0 \geq \frac{d_i(t+1)}{d_i(t)}$ . Since the inequality holds true for every  $i \in \mathcal{I}$  and  $t \in \mathcal{T}$ , we get that  $\delta_0 \geq \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ .

## C PROOF OF PROPOSITION 2

We first demonstrate **only-if part**. Assume Future-Improved Unanimity is satisfied. Let  $a, b \in \mathcal{L}$  such that  $u(b) > u(a)$ . Let  $t \in \mathcal{T}$ . For any  $c \in \mathcal{L}$  and  $i \in \mathcal{I}$ ,  $b_{t+1}a \succsim_i c_t a$  is equivalent to

$$\frac{d_i(t+1)}{d_i(t)} \geq \frac{u(c) - u(a)}{u(b) - u(a)}.$$

Let  $c^* \in \mathcal{L}$  such that  $\frac{u(c^*) - u(a)}{u(b) - u(a)} = \min_i \frac{d_i(t+1)}{d_i(t)}$ . Clearly,  $b_{t+1}a \succsim_i c_t a$  for all  $i \in \mathcal{I}$ . It then follows from Future-Improved Unanimity that  $b_{t+1}a \succsim_0 c_t a$ , which is equivalent to  $\delta_0 \geq \frac{u(c^*) - u(a)}{u(b) - u(a)}$ . By the definition of  $c^*$ , we obtain  $\delta_0 \geq \min_i \frac{d_i(t+1)}{d_i(t)}$ . Since this inequality is true for every  $t \in \mathcal{T}$ , we can conclude that  $\delta_0 \geq \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$ .

We now prove the **if part**. Assume that  $\delta_0 \geq \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$ . Let  $t \in \mathcal{T}$  and  $a, b, c \in \mathcal{L}$ , with  $u(b) > u(a)$ , such that  $b_{t+1}a \succsim_i c_t a$  for all  $i \in \mathcal{I}$ . This is equivalent to

$$\frac{d_i(t+1)}{d_i(t)} \geq \frac{u(c) - u(a)}{u(b) - u(a)}$$

for all  $i \in \mathcal{I}$ . Hence,  $\min_i \frac{d_i(t+1)}{d_i(t)} \geq \frac{u(c) - u(a)}{u(b) - u(a)}$ , which implies that  $\delta_0 \geq \frac{u(c) - u(a)}{u(b) - u(a)}$ . Thus, we establish that  $b_{t+1}a \succsim_0 c_t a$ , which ends the proof.

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