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Methodology of robust stability analysis using small gain theorem applied to aeronautical HVDC network

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Abstract

The air transport is undergoing a period of transition, responding to recommendations aimed at drastically reducing its polluting emissions. As a result, the electrical energy vector is taking an increasingly important part on board of aircraft, requiring significant changes to the electrical network. The trend is to develop High Voltage Direct Current (HVDC) networks 540VDC / 800VDC to replace the traditional 115VAC / 230VAC electrical system (ATA24) and support the synergy between non-propulsive and propulsive functions. These new aeronautical HVDC architectures may be subject to network quality issues due to the interconnection of many devices driven by power electronics. In particular, instabilities can occur under certain operating conditions, which needs to be anticipated and avoided. This is the purpose of the network specification step, carried out by the aircraft manufacturer. This paper explores the use of Small Gain Theorem to tackle stability issue and to specify the different stakeholders in such a way as to achieve an optimum sizing of the system's components. The proposed methodology tries to provide a relevant response to this subject by linking potential network uncertainties (variable parameters, etc...) to the formulation of the requirement.

Introduction

The new applications of the aeronautics domain, such as electrification of non-propulsive functions (More Electric Aircraft = MEA) [1] [2], turbofan hybridization (More Electric Engine = MEE) [3] and electrification of aircraft propulsion (Conventional Take-Off and Landing = CTOL, Vertical Take-Off and Landing = VTOL) [4] lead to major challenges regarding Electrical Power Systems (EPS). In particular, these new architectures allow regulated sources (e.g. battery-chopper) and regulated loads (e.g. inverter-machine) to be connected together on a same bus. As a result, special attention must be given to the control of system's electrical voltage and current variables, especially by means of a stability study [5]. The positioning about this topic is to ask whether, once it has moved from its equilibrium position, the system tends to return to it [6]. Considering the vicinity of the equilibrium point under consideration, it is possible to approximate the dynamics of the complex system by linearizing it around an operating point. We deal in this case with small signal stability and it is therefore necessary to keep in mind that the assumptions of validity cover the spectrum up to switching frequencies (beyond this limit, it will be necessary to consider an harmonic study and its appropriate models) [7].

The role of aircraft manufacturer is thus to ensure a proper functioning of the system by building network requirements and ensuring that the suppliers meet them. The small signal stability is for the first time explicitly described in the AS7499 standard [8] by the Society of Automotive Engineers (SAE), still in draft. We find there the notion of impedance ratio analysis, which is a traditional approach in the state of art [9]. If this way to proceed is well adapted to specification, some limits can be identified in terms of robustness. In fact, Gain Margin (GM) and/or Phase Margin (PM) are

usually introduced to be robustly stable but without correlation to real world sources of errors. To be more precise on this point, advanced automatic approaches like the Small Gain Theorem (SGT) [10] allows to include the relevant uncertainties of the entire network (various wire lengths, unknown bandwidths, etc...) in the stability analysis.

In the first section, the modelling of the architecture under consideration will be presented as well as the main objectives of this small signal stability analysis. In the second section, the impedance ratio approach is applied, before the SGT approach in the third section. Finally, the potential of the proposed methodology will be discussed in the last section.

1. Modelling of the system

We intend to study the generic architecture where a synchronous generator is connected to the high pressure shaft of the engine, as shown on Fig 1.

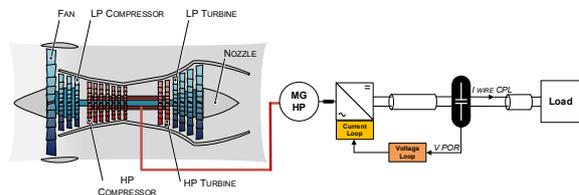


Fig 1 : Overview of the power distribution chain

Source and load sub-systems can be defined and connected together from the point of view of the Point Of Regulation (POR) respecting the current/voltage causality as shown on the Fig 2.

In the next sections, the notion of uncertainties will be introduced. By definition, they represent an error with the nominal configuration of the sub-systems; therefore the subscript "nom" has been added below and it will be used in this way until the end of the paper.

1.3. Context of small signal stability analysis

The positioning will be to follow the principles presented on Fig 7 and Fig 8.

In general, it is quite natural to consider that the detailed topology of each part of the system is not exactly known at the beginning of the design. In fact, the detailed internal models of suppliers are not easily accessible; therefore, they are seen as “black boxes” from the point of view of the network. In our case, the initial condition will be to recover a CPL “black box” model instead of the “white box” WIPS load model.

We want then to investigate the means at the disposal of the aircraft manufacturer to ensure the network robustness facing uncertainties in the load side (this choice is arbitrary). In the first part of the study, an uncertainty of $\pm 20\%$ will be considered around the nominal value of the wire length from the POR to the CPL (i.e. $\pm 6m$ around the nominal 30m).

$$\begin{cases} R_{wire-CPL} = 1m\Omega/m (30m) \pm 20\% \\ L_{wire-CPL} = 0.2\mu H/m (30m) \pm 20\% \end{cases} \quad (1)$$

The second step is to consider these uncertainties as well as possible in the specification building process. This is where the main difference between the impedance ratio approach (Fig 7) and the SGT approach (Fig 8) comes in. It will be seen that for the first cited, the robustness is arbitrarily addressed thanks to margins whereas for the second the robustness is directly linked to the uncertainties variation range.

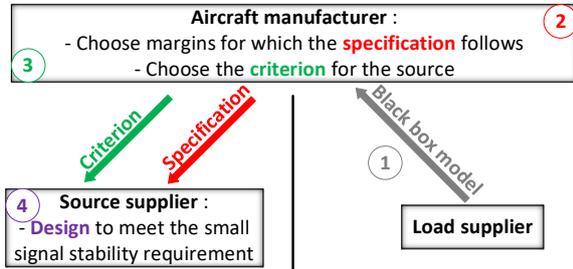


Fig 7 : Source specification according to the impedance ratio principle

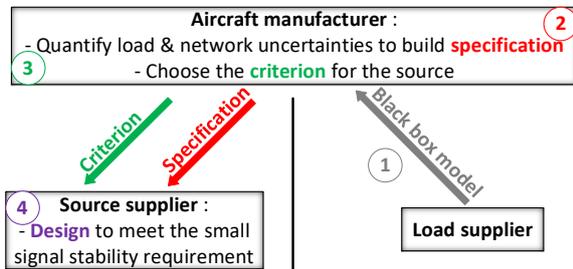


Fig 8 : Source specification according to the SGT principle

The third step comes from the specification definition: this is the characterization of the associated stability criterion.

Finally, all this information is given to the source supplier who is in charge of the design with the

objective to minimize the mass impact through the POR capacitor sizing in this example. Obviously and only for this last phase, the model is certain or “white box” (critical data are known such as machine parameters) but this does not change anything for the aircraft manufacturer, which plays no role at this step.

2. Impedance ratio approach

The basic principle of impedance ratio approach is to define an interconnection point (in our case the POR interface) at which the output impedance (in our case $G_{source-nom}$) of the source sub-system and the input admittance (in our case $G_{load-nom}$) of the load sub-system are defined.

Assuming that both source and load sub-system are individually stable, the small signal stability condition of Middlebrook involves restricting the impedance ratio in terms of gain only :

$$|G_{source-nom}(p) * G_{load-nom}(p)| < 1 \quad (2)$$

However, it is necessary to introduce the notion of margins in order to ensure that the equation (2) is still respected in case of uncertainties around the nominal configuration, in particular those considered in the equation (1).

2.1. Specification

We then introduce a GM of 6dB and a PM of 45° to move towards robust stability. These values are usual and supposed to be sufficient facing uncertainties. However, we will see that depending on the considered uncertainties, the final design made thanks to the GMPM specification can become “over-stable” (too conservative). In fact, the stability margins are not directly linked to the load sub-system uncertainties.

2.2. Criterion

The small stability criterion based on the impedance ratio approach becomes:

$$\begin{cases} |G_{source-nom}(p) * G_{load-nom}(p)| < \frac{1}{GM} \text{ or} \\ Arg(G_{source-nom}(p) * G_{load-nom}(p)) > 180^\circ - PM \end{cases} \quad (3)$$

The transfer function to design is thus:

$$G_{source-nom}(p) * G_{load-nom}(p) \quad (4)$$

If the gain of this latter exceeds the imposed GM, the specification ensures that its phase will respect the PM and vice-versa.

2.3. Design

The idea is to look for the limit value of C_{POR} that meets the previous criteria (3). Because the POR capacitor value is part of the source sub-system, $G_{source-nom}$ will change depending on its value, just as the transfer function to design (4). This is what can be seen on the

Fig 9.

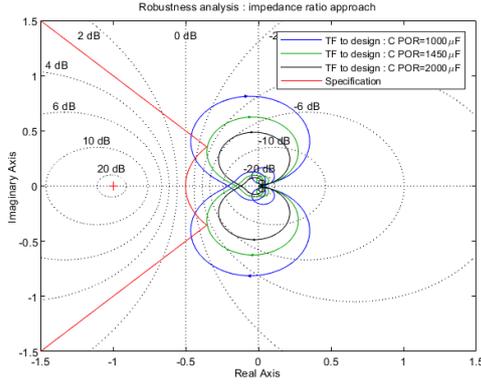


Fig 9 : Design through impedance ratio approach

The impedance ratio approach gives an optimal design of $C_{POR} = 1450 \mu F$ considering $GM = 6dB$ and $PM = 45^\circ$ through the GMPM specification. This is a final value that will not change whatever the uncertainties are. In our case of $\pm 20\%$ around the nominal 30m of feeder between the POR and the CPL, there is no way to know at this time if the design is too conservative or not.

3. SGT approach

The SGT applies to the generic situation described in Fig 10.

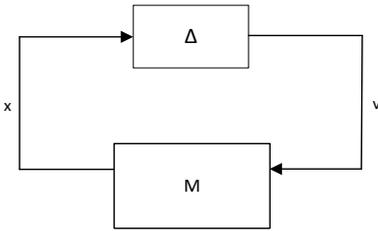


Fig 10 : SGT stability statement

M represents the known part of the system whereas the unknown part is outsourced from the nominal configuration in Δ .

The stability condition provided by the SGT is :

$$\|\Delta(p)\|_\infty < \gamma \Leftrightarrow \|M(p)\|_\infty < \frac{1}{\gamma} \quad (5)$$

The $\|H\|_\infty$ norm extends the notion of gain to Multiple Input Multiple Output (MIMO) systems. In our case, we address a Single Input Single Output (SISO) system, therefore the equation (5) becomes :

$$\max_\omega |\Delta(j\omega)| < \gamma \Leftrightarrow \max_\omega |M(j\omega)| < \frac{1}{\gamma} \quad (6)$$

We will see how to obtain the right conditions to apply the SGT in our case.

3.1. Specification

Because the load sub-system is uncertain, some errors are done in comparison with the nominal configuration.

The Fig 11 shows this phenomenon, especially in the frequency area where the $(L_{wire-CPL}, C_{CPL})$ pair acts. The $\pm 20\%$ variation of $L_{wire-CPL}$ explains the horizontal movement of the samples on each side of the nominal transfer function, whereas the $\pm 20\%$ variation of $R_{wire-CPL}$ damps more or less these curves.

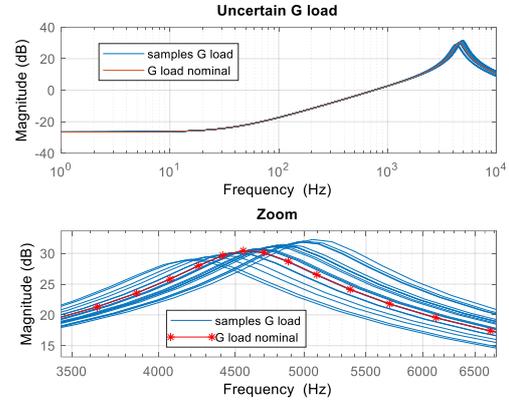


Fig 11 : Uncertain load sub-system

The errors between the nominal load sub-system and the samples of the uncertain load sub-system can be calculated in several ways, such as the Multiplicative Direct form (MD) [14]. This leads to the Fig 12.

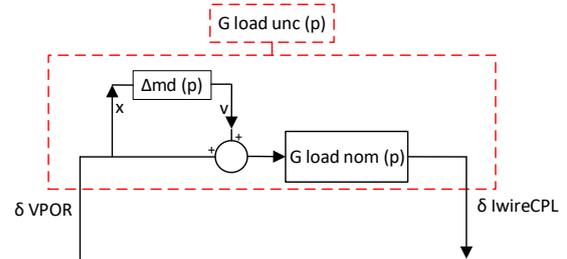


Fig 12 : Uncertain load sub-system under MD form

By construction, we have:

$$\frac{\delta I_{wire-CPL}}{\delta V_{POR}}(p) = G_{load-unc}(p)$$

It is also possible to write:

$$\delta I_{wire-CPL}(p) = G_{load-nom}(p) * \delta V_{POR}(p) * (\Delta_{md}(p) + 1)$$

So,

$$G_{load-unc}(p) = G_{load-nom}(p) * (\Delta_{md}(p) + 1)$$

Finally, it comes:

$$\Delta_{md}(p) = \frac{G_{load-unc}(p) - G_{load-nom}(p)}{G_{load-nom}(p)}$$

Thus, Δ_{md} represent a set of relative errors evolving with the frequency, as the Fig 13 shows it.

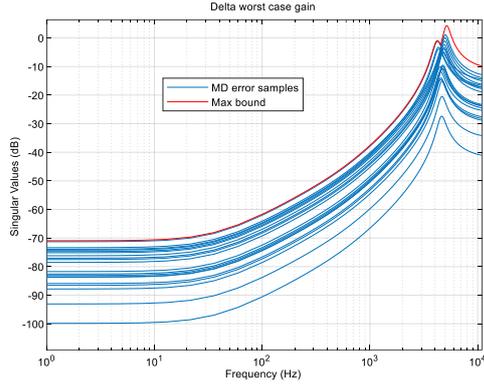


Fig 13 : Upper envelope of the errors

The errors are then majorate by a shape β_{md} (in red above). In this way, the worst gain is ensured at each frequency point.

This error maximization curve has been realized helped by the robust control toolbox of *Matlab* and more precisely the "wcgain" function.

β_{md}^{-1} will then be used as the specification of robustness as part of the SGT stability criterion whose explanation follows.

3.2. Criterion

From the Fig 12 follows immediately the Fig 14.

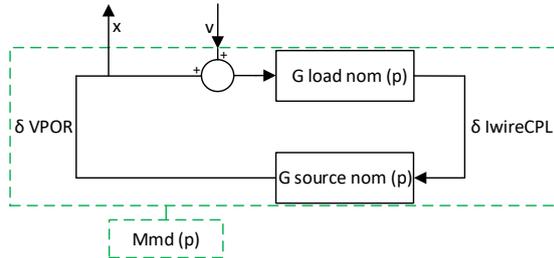


Fig 14 : Nominal closed loop associated to the MD form

Then, it is possible to calculate the transfer function to design $M_{md} = \frac{x}{v}$, resulting from the presence of Δ_{md} :

$$\begin{aligned} x &= G_{source-nom}(p) * G_{load-nom}(p) * (v + x) \\ \Leftrightarrow M_{md}(p) &= \frac{G_{source-nom}(p) * G_{load-nom}(p)}{1 - G_{source-nom}(p) * G_{load-nom}(p)} \quad (7) \end{aligned}$$

The small stability criterion based on the SGT is the application of the equation (6) to this situation :

$$\begin{aligned} \max_{\omega} |\Delta_{md}(j\omega)| &< \max_{\omega} |\beta_{md}(j\omega)| \\ \Leftrightarrow \max_{\omega} |M_{md}(j\omega)| &< \min_{\omega} \left| \frac{1}{\beta_{md}(j\omega)} \right| \quad (8) \end{aligned}$$

3.3. Design

The idea is to search for the limit value of C_{POR} that meets the previous criterion (8). Because the POR capacitor value is part of the source sub-system, $G_{source-nom}$ will change depending on its value, just as the transfer function to design (7). This is what can be

seen on the Fig 15.

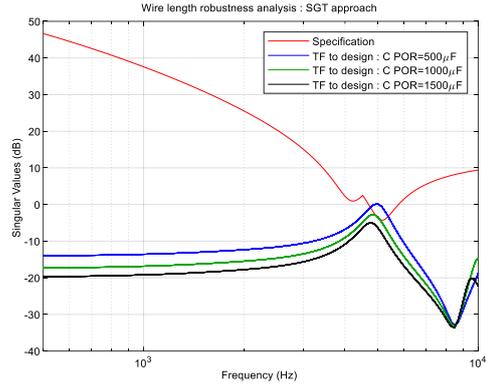


Fig 15 : Design through SGT function of uncertainties described in (1)

The SGT approach gives an optimal design of $C_{POR} = 1000\mu F$ to ensure the small signal stability of the system facing $\pm 20\%$ of wire length variation.

The impedance ratio approach was indeed too conservative for this case of uncertainties, because the obtained value was $C_{POR} = 1450\mu F$. The margins were too high, resulting in a over-sizing of the POR capacitor.

It is now essential to go further in the robustness to other uncertainties and to check the limits of the approach.

4. Discussion about the methodology

One takes place at iso-design, i.e. $C_{POR} = 1000\mu F$. It will allow to compare each domain of parameter variation on the load side in the same conditions.

Let's consider a $\pm 50\%$ variation around the nominal value of the differential filter.

$$\{C_{CPL} = 200\mu F \pm 50\% \quad (9)$$

We then check on the Fig 16 that this amount of uncertainty meets the POR capacitor design wanted at $1000\mu F$.

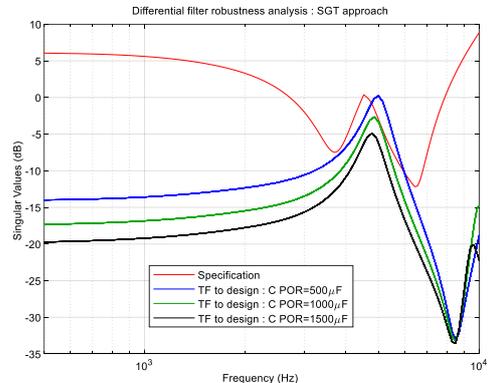


Fig 16 : Design through SGT function of uncertainties described in (9)

Based on these results, we can deduce that the small signal stability of the system is less sensitive to a

differential filter variation than a change in the wire length between the POR and the CPL. We could therefore prioritize the criticality of these two parameters. However, only a single parametric variation was addressed ; it will be seen in the last example that a coupled variation of several parameters makes it more difficult to draw such conclusions.

We now look the load regulation parameters that are the bandwidth f_{0-I} and the damping ξ_I . Despite a strong variation around their nominal values,

$$\begin{cases} f_{0-I} = 1kHz \pm 80\% \\ \xi_I = 0,707 \pm 80\% \end{cases} \quad (10)$$

They do not impact the stability of the system. In fact, the POR capacitor value is moving towards a few μF while staying away from the specification on the Fig 17.

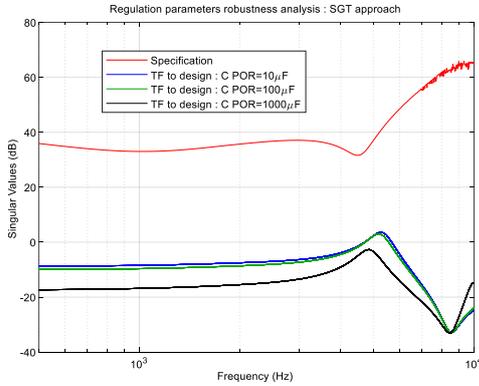


Fig 17 : Design through SGT function of uncertainties described in (10)

It is important at this stage to keep in mind the fact that the small signal stability analysis is only one constraint among others. These conclusions must be necessarily compared with other subjects.

As mentioned in the introduction, the harmonic study can potentially go against the trend of minimizing the C_{POR} value : the filtering action of the network harmonic pollution would be then be impacted.

It was also previously considered not to deal with the external disturbances. However, we can easily imagine that reducing the value of the POR capacitor will impact the source voltage regulation performance.

This is why it can only be said that the load control parameters are not impactful regarding the small signal stability study.

Finally, we seek to understand what happens when all uncertain parameters vary together. In order to reach the design of $1000\mu F$ as we can see on the Fig 18, the new considerations are:

$$\begin{cases} R_{wire-CPL} = 1m\Omega/m (30m) \pm 12\% \\ L_{wire-CPL} = 0.2\mu H/m (30m) \pm 12\% \\ C_{CPL} = 200\mu F \pm 12\% \\ f_{0-I} = 1kHz \pm 12\% \\ \xi_I = 0,707 \pm 12\% \end{cases} \quad (11)$$

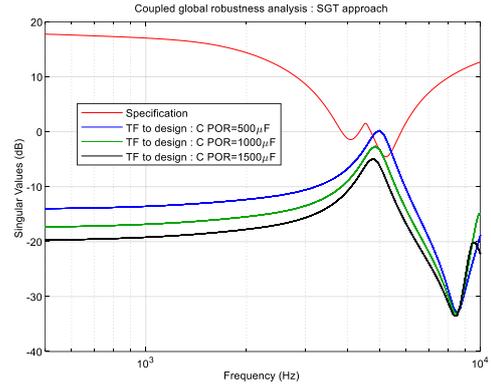


Fig 18 : Design through SGT function of uncertainties described in (11)

The range of variation around parameters has been reduced compared to the previous single parametric considerations but we can still draw interesting conclusions. As a reminder, the impedance ratio approach gives us a design of $1450\mu F$ regardless of the amount of uncertainty. With the SGT approach, a smaller value of C_{POR} ($1000\mu F$) ensures a coupled variation of $\pm 12\%$ around all critical parameters at the load side without reaching unstable conditions.

We could increase even more the variation around the nominal conditions and thereby exceed the constant C_{POR} value designed by the impedance ratio approach, but it is not necessarily relevant. Indeed, these approaches are based on small signal modeling, therefore too much gap with the nominal operating point does not make sense.

Conclusion

The robust stability study is a critical point to consider in the scope of new aeronautical systems pre-design. In this paper, we have proposed a methodology based on the SGT whose the main feature is the uncertainties-based specification building process. It helps to understand what the right trends of design are, which uncertainties acts in which frequency domain and to what extent. In that way, the sense of this approach is to search for an optimized network design depending on key parameters of the architecture. In comparison with the classical approaches, represented by the impedance ratio analysis, the counterpart is the fact that the network designer must be able to have a minimal knowledge of the system to build a suitable requirement. Thus, this way of doing will not lead to an universal standard but remains applicable within the framework of an aircraft program. Moreover, for significant disturbances around the equilibrium point under consideration, we deal with large signal issues. In this case, the analytical criteria are quite complex because of non-linearity phenomena (the reader may refer to [15] for more details). Therefore, we usually address this topic thanks to the simulation tool. We rely on transient norms on the network voltage, such as for example the MIL-STD-704 for 270VDC [16] or the prEN2282_CD [17] being

updated for the 540VDC. It should be relevant to investigate if this large signal subject lead to more restrictive constraints than the results from small signal methodologies.

Finally, the next step would be to take a more realistic network and analyse the specification process of one single load facing other loads uncertainties as well as the source uncertainties. We can also think about another prospect that would be to study the case of turbofan hybridization by adding a Low Pressure source in the architecture and observe the consequences on the small stability analysis.

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