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Abstract

Swappable batteries have been deployed in several services of dockless shared e-scooters. This article provides an economic theory of swappable batteries in the production of a shared e-scooter service (S3). Explicitly modelled are the operations of battery swapping by "juicing tours", as well as the wearing law of the electric batteries depending on the depth of discharge (DoD) that triggers the next swap. In the production model, the daily number of refills and the per-refill swapping cost are key variables as they link the field implementation and the swapping logistics functions to the other production functions of battery inventory, scooter inventory, energy charging, fleet maintenance and commercial. Thus the overall "refill strategy" interplays with the respective inventory policies of batteries and of scooters. The mathematical optimization of the production cost function is addressed in four stages, by optimizing in turn (i) the swapping tours, (ii) the target DoD, (iii) the battery energy capacity (BEC), (iv) the scooter body in terms of lifespan and energy consumption rate. Characteristic equations are established for the optimal per-refill cost, DoD, BEC, scooter lifespan and energy consumption rate. Two sets of specifications, namely Constant Elasticity and Affine Linear, are specified for the battery wearing law, the battery price and the scooter price: under either set the model admits an analytical solution. In a numerical study, it is shown that the S3 cost per unit of fed-in energy is one order of magnitude greater than the out-of-the-grid electricity price.

Keywords:

shared mobility economics; electric scooters;
swappable batteries; battery wearing law;
battery energy capacity; depth of discharge;
refill tours; logistical process;
refill strategy; inventory policy; supply optimization

Highlights

- Optimal refill tour productivity as a CES function
- Refill strategy brings together juicing costs, battery and scooter prices and battery wearing law
- Battery wearing law linking lifetime number of charging cycles to Depth of Discharge
- Assuming Constant Elasticity sub-models, optimal battery energy capacity is straightforward
- Under Affine Linear sub-models, optimal battery energy capacity characterized by a quartic equation
- Scooter inventory policy involving lifespan and the energy consumption factor

1. Introduction

1.1 Background

Dockless shared e-scooter services (S3) have spread fast from their introduction in 2017 and 2018 by Bird, Spin, Lime and other operators (Button et al. 2020). Over 2018 they became the dominant mode of shared micromobility in the United States, surpassing shared bikes with docks as well as dockless ones (Zhang and Guo, 2020).

Beyond the easy deployment of dockless two-wheels in urban settings, the main reason for S3 success may well lie in its specific user experience and quality of service: ease of use, efficient speed competitive with strongly motorized modes in dense areas, comfort of the standing position little exposed to crowding, the pleasure of driving autonomously, effortlessly, smoothly and openly in the urban environment (6-t, 2019). An obvious requirement pertains to getting an available vehicle in the street: it relates to service coverage, fleet size and demand patterns (Leurent, 2021). Another requirement is the willingness-to-pay of the users, since service prices are relatively high: typically, $1 or €1 to unlock plus .15 or .2 per minute of utilization (Nawaro, 2021). On assuming average speeds of 12 to 15 km/h, i.e. 4 or 5 minutes per km, the variable fee amounts to about $1 or €1 per km, which is higher than the average cost of using one's car and much higher than typical rates of public transit (Noland, 2019).

These prices correspond more to the usage value and the users' willingness-to-pay than to the supply costs: in an economic study of an early Bird service, the BCG (2020) showed that out of average revenue of $4.1 per ride, operations costs amounted to one good half, leaving out the other half for fleet investment and margin. The perspective of making high margins in an easy business has drawn many competitors in the field: as of 2019, more than 10 operators deployed their fleets in the streets of Brussels (Moreau et al., 2020), Paris (6-t, 2019) and other cities (Button et al. 2020). To date, such competition has not led to price decreases – although it may have avoided price increases such as that observed in Los Angeles (up to $.39 per min in 2019, see BCG 2020). However the operators have endeavoured to reduce their supply costs by acting on two main levers (Kamps, 2018): first, improving scooter longevity by introducing second and beyond generations of scooters especially designed for shared usage; second, turning to more efficient battery charging processes, turning from gig-worker "juicers" collecting the scooters to swappable batteries (McKinsey, 2019). According to Gaucquelin (2020) the advantages of swappable batteries are twofold: first, increasing the uptime of scooter availability in the field and, second, reducing the weight to carry by the juicers. By fast adaptation, major service suppliers have adopted sturdier scooter bodies aimed to longer lifespans, together with swappable batteries of relatively large capacity and swapping processes either by company-operated tours (e.g. Lime, 2020) or by enticing some users to swap batteries at specific charging spots (e.g. Tier, 2020). Figure 1 exhibits shared e-scooters with swappable batteries in Paris streets. Yet, by the end of 2020, some scooter makers and S3 operators kept to batteries embedded in the e-scooters, with focus on larger battery capacity and better lifecycle management of both the batteries and the scooter bodies, each kind in its own respect (Link, 2020, Bird, 2020).

Here we shall focus on the production of an S3 service, encompassing the operations of battery charging and swapping, the inventory of e-scooter bodies and their longevity, as well as the inventory of batteries and their own longevity which is related to the depth of discharge at swapping. We will look into the interplay of these aspects and their implications on the production costs: what are the determinants of the production costs? To which extent do the batteries and their energy refills contribute to these costs? Considering that batteries as

equipment and the logistics of their refills in energy basically constitute means to feed energy in scooters, what is the unit cost of such fed-in-scooter energy? What savings can be harvested by suitable battery specification and management – in other words, what value can be created?

Fig. 1. Shared e-scooters in Paris streets: the adoption of swappable batteries

1.2 Related work

In an up-to-date review of scientific literature on electric car-sharing and micromobility, Liao and Correia (2022) addressed three folds: first on the patterns of use and travel behaviours i.e. the demand side, second on the safety impacts for modal users and also for pedestrians, third on the environmental impacts and specially the carbon footprint of such mode. As for economic issues pertaining to dockless shared e-scooters, Button et al. (2020) provided a high-level analysis addressing successively (i) the demand side of the market, (ii) the supplying industry in terms of market structure, market contestability and instability, (iii) the supplying firms and related issues of venture capital, labor side and its remuneration, as well as pricing, (iv) regulation of negative externalities, (v) stimulation of positive externalities. Concerning production cost analysis, these authors referred to “grey literature”: specifically the cost breakdown study by Trefis Team (2018) which mainly stated that the core company’s expenses pertain to scooter acquisition and charging. In turn this study is based on Kamps (2018) who analysed an operator’s margin over one scooter lifetime for three notional e-scooter models differentiated mainly by their respective longevity: (i) 300 rides, (ii) 500 rides owing to somewhat sturdier design and (iii) 1000 rides for a “super-scooter” much sturdier and of which the battery would be swappable.

Pioneering studies of early Bird implementations (Griswold, 2019, BCG, 2020) shed more light on pricing, revenues and production costs: the operational expenses (excluding scooter acquisition) of nearly $2 per trip were broken down in about 65% for charging and 20% for repair, among other expenses. Demoere (2020) built up a quantitative model of production costs and revenues for an S3 service in a city: identifying the different functions in production, he provided cost formulas for each of them with respect to a common set of parameters such as fleet size and demand flow. The model was applied to explore ranges of factors and look for conditions of service financial profitability.

To the best of our knowledge, e-scooter longevity has not yet been subjected to economic analysis. It makes a key issue in Life Cycle Assessments of the environmental impacts: after a quick study (Griswold, 2019) pointed out to very short lifetimes of 28 days owing to theft and vandalism in Louisville, Kentucky, lifespans of 2 to 7 thousand km were considered for 1st generation shared e-scooters (Hollingsworth et al. 2019, De Bortoli and Christoforou 2020, ITF 2020), whereas lifespans of 6 to 14 thousand km were put forward for 2nd generation scooters (ITF 2020, Moreau et al. 2020, De Bortoli 2021). Yet these are still provisional indications, since "as this service is very new, the lifetime is still expanding…, a final estimate of the average lifespan of those e-scooters will only be available when all e-scooters reach their end-of-life" (Moreau et al. 2020). The statement must be kept in mind on reading the claims of e-scooter makers as well as those of shared service operators, such as the 5 year lifetime target for Voi's e-scooters (Voi, 2020) or the respective lifetimes of scooter bodies and batteries (Link, 2020).

1.3 Objective and contribution

This article provides a technical and economic model of the production of a dockless shared e-scooter service with swappable batteries, considered at the city level. It encompasses swapping logistics by company-operated tours, scooter inventory, battery inventory and mutual interactions such as the influence of the target depth-of-discharge at swapping on battery wearing.

The model is a theoretical one: we postulate mathematical functions between the model variables (e.g. fleet size and scooter renewal flow) to state the technical relationships and to derive economic relationships of two kinds: first, the formation of production costs, second, the optimization of the production cost function with respect to decision variables that depend on the service operator: the productivity of swapping tours and the number of juicer duties, the target Depth-of-Discharge (DoD) and the Battery Energy Capacity, the scooter lifespan and its energy consumption rate. It is shown that the per-refill swapping cost plays a key role as it links the logistics of energy feeding, in the short run, to the medium-run setting of target DoD to trigger swapping and the long-run specification of battery capacity and scooter parameters.

Battery wearing with respect to DoD at swapping is modelled as a relationship between the lifetime number of charging cycles and the DoD. To fix ideas, two simple mathematical specifications are put forward as "specific technologies": Constant Elasticity and Affine Linear.

Under either specification the model is easy to solve. To demonstrate its application, we conduct a numerical study that explores some ranges of the parameters and gives some insights in cost-optimized conditions of service production.

1.4 About methodology

Compared to previous works by the BCG (2020) and Demoere (2020), our model is a theoretical one. Explicitly represented are the technical processes of juicing tours and battery wearing according to the target DoD. Also explicitly represented are the economic consequences of service production and its usage, i.e. its daily traffic, onto the operational expenses. Furthermore, to constitute the full service costs we also represent the inventory policies for scooter bodies, on the one hand, and removable batteries, on the other hand.

The resulting mathematical function of production costs is subjected to formal optimization, thereby modelling cost-minimizing supplier behaviour. The resulting first-order optimality conditions do constitute economic relationships between the decision variables. We will insist

on the interplay of juicing tour productivity and technical conditions, of battery conditions both short run (DoD) and long run (nominal capacity), and of the price functions of both scooter bodies and batteries.

To sum up, the scientific approach is basically theoretical. The empirical side is restricted to the setting up of the model parameters in the numerical study.

1.5 Article structure

Following this introductory section, the body of the article is organized in eight parts. Section 2 builds up the technical model: from juicing tours, to battery wearing and up to battery and scooter inventory policies. Section 3 lays out the economic model of service production: a cost function is constituted from the specific functional costs of commercial operations, field implementation, energy provision, battery logistics, battery inventory and scooter inventory. The next three sections deal with cost optimization with respect to, respectively, juicing tour productivity in Section 4, battery depth of discharge and energy capacity in Section 5, scooter lifespan and energy consumption rate in Section 6. Section 7 provides a numerical application. In Section 8, we discuss the outreach and limitations of specific sub-models, namely the unit swapping cost, the swapping policy, the juicing strategy, the battery price and the scooter body price. Lastly, Section 9 summarizes the economic theory and points out to further developments.

Main Notation

Q demand volume: number of rides per day at the service level

ℓ_R average length of e-scooter ride requested by a customer

Q_L daily commercial traffic at service level

Q_E daily energy consumption of service usage

Q_F number of refills per day

Swapping logistics

t_{SB} or $\dot{t}$, Battery swap time per scooter on refill tour

$t_{0\mathrm{C}}$ Base run time per refill tour

t_{D} Depot time per refill tour

$t_0 = t_{\mathrm{D}} + t_{0\mathrm{C}}$ Base time per refill tour

T_{J} Average time of refill tour

H_{J} Daily duration of swapper duty

n_{B} Number of battery swaps per refill tour

φ_{J} Daily frequency of refill tours

n_{J} Number of swapper duties

w_{J} Daily cost per swapper duty

H Period of service operations in a typical day

γ_{N} daily maintenance cost per scooter, from function Γ_{N} of scooter lifespan

$\tilde{p}$ per-vehicle costs on a daily basis

V_{J} Per-refill cost of swapping strategy

Battery issues

σ Depth of discharge triggering battery removal

A battery nominal energy capacity

f_{B} Daily flow of battery renewals

p_{B} Battery price, from price function $\mathrm{P_B}$

$\tilde{B}$ number of batteries

R_{B} Number of charging cycles (number of refills) over battery lifetime

$\mathrm{R_B}$ Battery wearing law associated to Lifetime energy intensity function $\mathrm{E_B}$ and auxiliary function ψ_{B}

E_1, α Parameters of Constant Elasticity $\mathrm{E_B}$ function

E_0, b Parameters of Affine Linear $\mathrm{E_B}$ function

Scooter issues

$\widetilde{N}$ fleet size (number of vehicles)

N number of in-field scooters in sufficient energy charge state

f_{S} Daily flow of scooter renewals

L_{S} Scooter lifespan

χ_E energy consumption of an e-scooter per ride length unit

p_{S} Scooter price, from price function $\mathrm{P_S}$

r_{S} Number of battery refills over scooter lifetime

2. Technical Model

Here we consider one service of shared e-scooters operated in a given urban area.

2.1 From service traffic to refill needs

Let Q denote the average daily number of rides made using the service and ℓ_R their average length. Thus, the traffic quantity produced daily amounts to

$$Q_L = Q.\ell_R \tag{2.1}$$

Indices R and L stand for Ride and Length respectively.

Denote also χ_E the average electricity consumption per unit of travelled length. Thus, per day, the energy consumption of the e-scooters is simply

$$Q_E = \chi_E.Q_L = \chi_E.Q.\ell_R \tag{2.2}$$

Let now A denote the nominal battery capacity in amount of energy. The current battery state of charge (SoC) is measured as the proportion of A that is still available. A depth of discharge (DoD) is defined as 1-SoC. Let us define $\sigma \in]0,1[$ as the maximum DoD that is allowed in field operations by the service supplier: when the battery DoD reaches σ then after the end of the on-going ride the scooter is made unavailable to customers until the low-charge battery is removed and replaced with a fresh one. For brevity we shall refer to σ as the target DoD or just "the DoD".

Then, the product $\sigma.A$ measures the amount of energy that is consumed per battery over a cycle of usage: on average it is equivalent to the amount received by the battery over a charging cycle. Per day, the number of such cycles, Q_F, induces an energy influx of $Q_F.\sigma.A$ that will balance the in-the-field energy consumption: thus,

$$Q_F = \frac{Q_E}{\sigma.A} = \chi_E.\frac{Q.\ell_R}{\sigma.A} \tag{2.3}$$

On a daily average, the number of charging cycles is also the number of battery swaps, hence of scooter refills with fresh batteries. We shall call it the daily number of refills.

2.2 Juicing as a logistical process

An elemental battery swap consists in replacing a battery with low energy level $\leq (1-\sigma).A$ by a full one under nominal energy capacity (load level about A). By assumption, battery swapping makes the specific duty of specific service agents called swappers or juicers.

Denote as t_{SB} the elementary time of scooter identification on the roadway, battery swapping and also juicer approach from the traffic flow to the scooter location and return: it pertains to the juicer and his or her vehicle.

Denote t_{0C} the base run time of the juicing tour. Assuming that there are n_B scooters to be refilled in that cycle, the juicing cycle time amounts to the following

$$t_C = t_{0C} + n_{\mathrm{B}}.t_{\mathrm{SB}} \tag{2.4}$$

Furthermore, the Refill task involves recharging the depleted batteries. Assuming that this is done off-line in a depot, the juicing tour also involves to drop and plug the depleted batteries and to pick a number of recharged batteries at that depot; some rest time may also be allowed, leading to a depot time denoted t_D per juicer tour.

Overall, a juicing tour time amounts to

$$T_{\mathrm{J}} = t_D + t_{0C} + n_{\mathrm{B}}.\, t_{\mathrm{SB}} \tag{2.5}$$

Index J stands for "Juicing". To keep notation simple, the average depot time per battery for drop-off and pick-up may be included in t_{SB}. The key thing is that the juicing tour time is a linear affine function of the number of refills, n_{B}. As n_{B} is the refill production at the tour level, it measures the productivity of the refill tour.

Denote as n_{J} the number of swapper duties, by H_{J} the time period of daily activity for each of them. Per day, the number of juicing tours amounts to

$$\varphi_{\mathrm{J}} = n_{\mathrm{J}}.\, H_{\mathrm{J}}/T_{\mathrm{J}} \tag{2.6}$$

Denoting as H the daily time length of service operations (in most cases we can expect that H =24h), φ_{J}/H is the frequency of refill tours. As n_{B} is the average number of refills per tour the total number $\varphi_{\mathrm{J}}.\, n_{\mathrm{B}}$ is supplied to match the refill demand Q_{F}: then it must hold that

$$\varphi_{\mathrm{J}}.\, n_{\mathrm{B}} = Q_{\mathrm{F}} \tag{2.7}$$

or equivalently that

$$n_{\mathrm{J}}.\, H_{\mathrm{J}}.\, n_{\mathrm{B}} = T_{\mathrm{J}}.\, Q_{\mathrm{F}}.$$

Thus n_{J} stands as a function of Q_{F} and n_{B}, among other parameters of the refill process. Combining (5) and (7), and denoting $t_0 \equiv t_D + t_{0C}$ and $\dot{t} \equiv t_{\mathrm{SB}}$ for short, we get that

$$n_{\mathrm{J}} = \frac{Q_{\mathrm{F}}}{H_{\mathrm{J}}}\left(\dot{t} + \frac{t_0}{n_{\mathrm{B}}}\right) \tag{2.8}$$

2.3 On battery wearing and renewal flow

The wearing or ageing of an electric battery consists in the progressive decrease of its effective capacity: from the nominal level at the beginning of the battery lifetime down to some pre-defined level. Electric Vehicle suppliers advise their customers to replace their batteries when the effective capacity goes below 80% of nominal (Battery University, 2019), while e-scooter suppliers warrant battery replacement of the effective capacity falls below 70% of nominal in less than 500 charging cycles (De Bortoli, 2021). This reduction is achieved after a number of charge and discharge cycles that depends on the target DoD. Let us model the number of cycles "up to battery dismissal" as a function $\mathrm{R}_{\mathrm{B}}(\sigma)$. It is the number of refills experienced by a battery. In other words, it indicates the battery lifetime in number of charging cycles.

Then, on an average basis, the daily flow of battery renewals is equal to the daily number of refills divided by the battery lifetime:

$$f_{\mathrm{B}} = \frac{Q_{\mathrm{F}}}{\mathrm{R}_{\mathrm{B}}(\sigma)} = \frac{Q_{\mathrm{E}}}{A.\, \sigma.\, \mathrm{R}_{\mathrm{B}}(\sigma)} \tag{2.9}$$

In the formula, Q_{E} depends neither on A nor on σ. The influence of σ on f_{B} is twofold as it involves first its own magnitude and second the associated battery lifetime. Both influences combine in product form.

2.4 Scooter inventory

Coming to the scooters, let us model the endurance of the scooter to wear and tear as a total length to be travelled over its technical lifetime, denoted L_{S} and called lifespan. Then, on an average basis, the daily flow of scooter renewals f_{S} is equal to the daily travelled length Q_{L} divided by the scooter lifespan L_{S}:

$$f_{\mathrm{S}} = \frac{Q_{\mathrm{L}}}{L_{\mathrm{S}}} = \frac{Q.\ell_R}{L_{\mathrm{S}}} \tag{2.10}$$

As for the total fleet size, let us denote it as $\widetilde{N}$: this number adds up the number N of scooters that can be used by customers with no restriction of low energy or bad state and the number N' of scooters waiting for refill. On average during the period of operations, there are $n_{\mathrm{J}}.H_{\mathrm{J}}/H$ juicing tours running simultaneously, each of which dealing with n_{B} scooters. Per tour, on average half of the scooters have just been refilled, so that the current number of low charge scooters is equal to

$$N' = n_{\mathrm{J}}.n_{\mathrm{B}}.\frac{H_{\mathrm{J}}}{2\mathrm{H}} = Q_{\mathrm{F}}\frac{T_{\mathrm{J}}}{2\mathrm{H}} \tag{2.11}$$

We thus obtain that

$$\widetilde{N} = N + Q_{\mathrm{F}}\frac{T_{\mathrm{J}}}{2\mathrm{H}} \tag{2.12}$$

2.5 Battery inventory

The number of batteries involved in the service, $\tilde{B}$, comes from (i) those deployed in the field, in number $\widetilde{N}$, (ii) those transported by juicers, in number n_{B} per tour, (iii) the batteries at the depot either being charged or waiting before or after charging.

The second part is also proportional to the number of juicing tours running simultaneously: we can also measure it as N'. According to Little's law, the third part may be modelled as the product of the daily flow rate of refills, Q_{F}, times an average depot time per battery, say t_{R} for Recharging.

On combining the three parts, we obtain that

$$\tilde{B} = \widetilde{N} + N' + \frac{Q_{\mathrm{F}}}{\mathrm{H}}t_{\mathrm{R}}$$

Thus,

$$\tilde{B} = N + \frac{Q_{\mathrm{F}}}{\mathrm{H}}(T_{\mathrm{J}} + t_{\mathrm{R}}) \tag{2.13}$$

3. The economic model

3.1 Service production and related costs

Service production requires (i) the holding of production means, including scooters, batteries and depot, (ii) back office processes of administration and maintenance, (iii) battery refilling, (iv) the juicing logistical process, (v) commercial functions, of which the magnitude will be roughly proportional to the demand volume.

All of these functions give rise to specific production costs. We shall model them on a daily basis, be they of capital or operational kind of expenditure.

For the e-scooters the capital expenditures have daily amount as follows:

$$C_{\mathrm{S}} = p_{\mathrm{S}}.f_{\mathrm{S}} \tag{3.1}$$

Wherein p_{S} is the scooter market price. We model p_{S} as a function $\mathrm{P_S}$ of scooter features L_{S}, χ_{E}, A (since the scooter has to accommodate the battery of such capacity, hence such size and mass), together with other features denoted as ξ:

$$p_{\mathrm{S}} = \mathrm{P_S}(L_{\mathrm{S}}, \chi_{\mathrm{E}}, A, \xi) \tag{3.2}$$

Factor ξ could represent driving controls (handles, braking controls), safety devices (lights), motor power, tyres and their comfort etc. Of course, under competitive conditions the price will reflect the manufacturing costs.

Similarly, the daily amount of capital expenditures pertaining to batteries is modelled as

$$C_{\mathrm{B}} = p_{\mathrm{B}}.f_{\mathrm{B}} \tag{3.3}$$

Wherein p_{B} is the battery market price. We shall model p_{B} as an increasing function $\mathrm{P_B}$ of battery capacity A:

$$p_{\mathrm{B}} = \mathrm{P_B}(A) \tag{3.4}$$

Again, under competitive conditions the price is expected to reflect the manufacturing costs.

Depot costs are denoted as C_{D}, administration costs as C_{A}, maintenance costs as C_{M} and we aggregate them in a function $C_{\mathrm{DAM}} \equiv C_{\mathrm{D}} + C_{\mathrm{A}} + C_{\mathrm{M}}$. Dependencies onto fleet size $\tilde{N}$, battery number $\tilde{B}$ and number n_{J} of swapper duties have sensitivity coefficients denoted γ_{N}, γ_{B} and γ_{J} respectively. The daily per scooter cost γ_{N} includes maintenance and insurance costs, telecommunication costs for scooter connectivity in interaction to the service platform, and also parking costs if applicable. As a linear approximation we will consider a base back office cost of C^0_{DAM} and an effective back office cost of

$$C_{\mathrm{DAM}} = C^0_{\mathrm{DAM}} + \gamma_{\mathrm{N}}\tilde{N} + \gamma_{\mathrm{B}}\tilde{B} + \gamma_{\mathrm{J}}n_{\mathrm{J}}$$

For model completeness we relate γ_{N} to L_{S} in a decreasing way, because longer lifespan comes with sturdier body that will decrease maintenance requirement per length unit: let then

$$\gamma_{\mathrm{N}} = \Gamma_{\mathrm{N}}(L_{\mathrm{S}}) \tag{3.5}$$

Depending on the city of implementation, specific fees (or subsidies) may apply to the service: a fixed daily fee that turns C^0_{DAM} into C^0_{F}, plus a per-scooter daily fee of p_{F}. Then, the field implementation cost of the service amounts to

$$C_{\mathrm{F}} = C^0_{\mathrm{F}} + (\gamma_{\mathrm{N}} + p_{\mathrm{F}})\tilde{N} + \gamma_{\mathrm{B}}\tilde{B} + \gamma_{\mathrm{J}}n_{\mathrm{J}} \tag{3.6}$$

It is hereafter called the "Field cost" function.

Coming to the juicing logistical process, let us denote as w_J the daily cost of one juicer duty: it includes the wages of all employees for that duty (taking into account vacations etc) as well as the cost of the duty vehicle. Per day, the juicing process costs an amount of

$$C_J = w_J . n_J \tag{3.7}$$

As for energy costs, we assume that electricity is bought at unit market price p_E and that the charging process involves some losses. Denoting θ_E the efficiency rate of this process, per unit of fed-in energy the effective price is $p'_E = p_E/\theta_E$. Then, the daily energy costs of service operations amount to:

$$C_E = p'_E . Q_E \tag{3.8}$$

Commercial costs denoted C_U are linked to the ride volume Q: assuming a unit cost of c_u,

$$C_U = c_u . Q \tag{3.9}$$

Overall, the service costs on a daily basis are measured by the following function:

$$C_\Sigma \equiv C_S + C_B + C_F + C_J + C_E + C_U \tag{3.10}$$

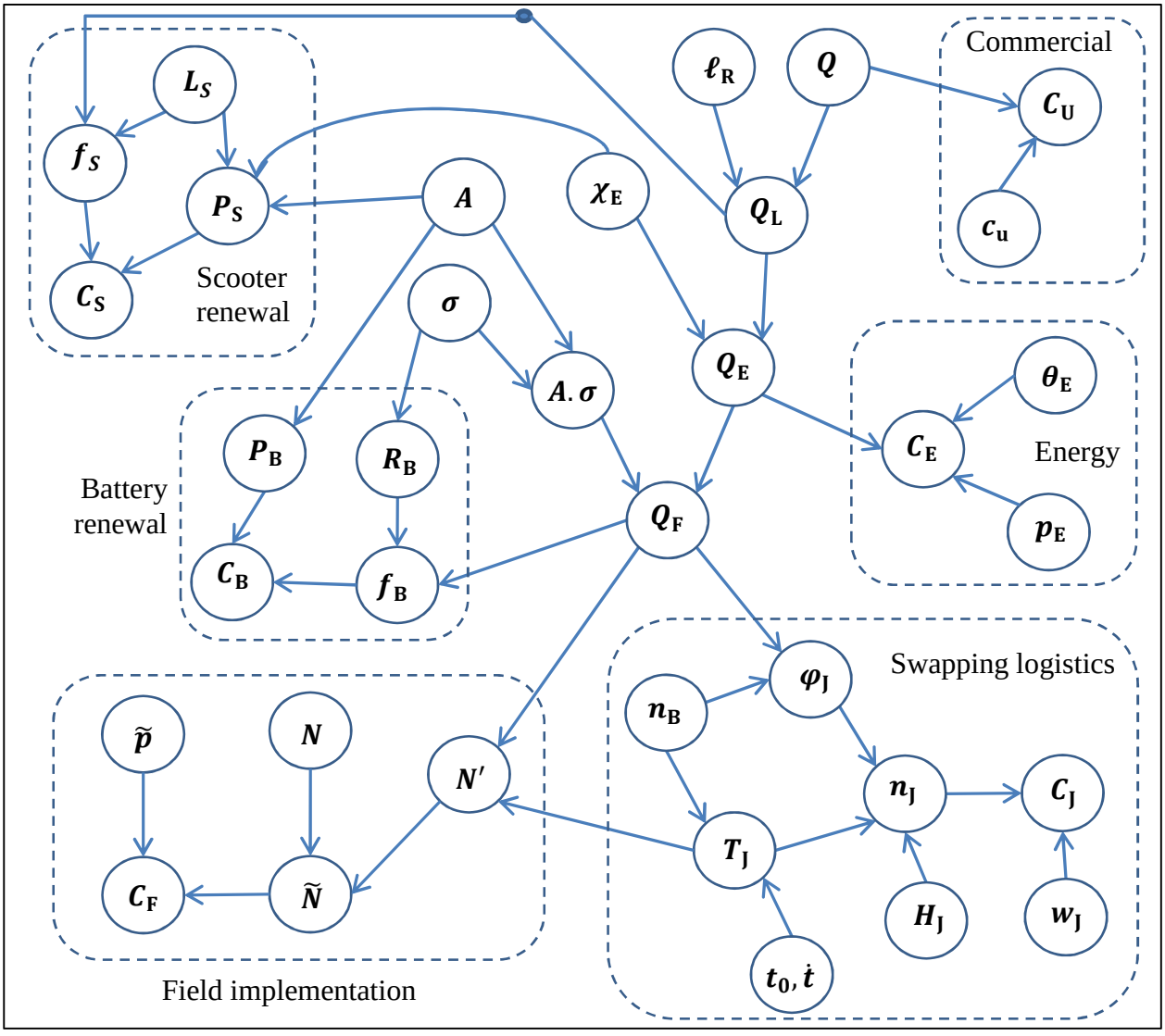


Fig. 2: Influence tree of the technical-economic model.

3.2 The per-refill swapping cost

Figure 2 traces out the influences of the variables $N, n_\mathrm{B}, \sigma, A, L_\mathrm{S}, \chi_\mathrm{E}$ which are decision variables of the service supplier, under technical conditions $t_0, \dot{t}$ of juicing tours and demand conditions Q and ℓ_R. The number of refills, Q_F, holds a remarkable position at the interface between the renewals, energy and commercial functions, on one hand, and field implementation and swapping logistics, on the other hand.

A remarkable property of the model is that Q_F fully conveys the influences of all of its upstream factors onto the field and swapping sub-models. It means that the influences of $Q, \ell_\mathrm{R}, \sigma, A, L_\mathrm{S}, \chi_\mathrm{E}$ onto C_F and C_J are all conveyed by Q_F. The sensitivity coefficient of $C_\mathrm{FJ} \equiv C_\mathrm{F} + C_\mathrm{J}$ to Q_F constitutes the per-refill swapping cost, denoted as V_J:

$$V_\mathrm{J} \equiv \frac{\partial C_\mathrm{FJ}}{\partial Q_\mathrm{F}} \tag{3.11}$$

From the respective constitution of C_F in (3.6) and of C_J in (3.7), we have that

$$C_\mathrm{FJ} = C_\mathrm{F}^0 + (p_\mathrm{F} + \gamma_\mathrm{N})\widetilde{N} + \gamma_\mathrm{B}\tilde{B} + \gamma_\mathrm{J}.n_\mathrm{J} + w_\mathrm{J}.n_\mathrm{J}$$

$$= C_\mathrm{F}^0 + (p_\mathrm{F} + \gamma_\mathrm{N} + \gamma_\mathrm{B})N + (p_\mathrm{F} + \gamma_\mathrm{N} + 2\gamma_\mathrm{B})N' + \gamma_\mathrm{B}Q_\mathrm{F}\frac{t_\mathrm{R}}{\mathrm{H}} + (\gamma_\mathrm{J} + w_\mathrm{J}).n_\mathrm{J}$$

From the constitution of N' in (2.11) and that of n_J in (2.8), and denoting $\widetilde{w}_\mathrm{J} \equiv w_\mathrm{J} + \gamma_\mathrm{J}$, $\tilde{p} \equiv p_\mathrm{F} + \gamma_\mathrm{N} + 2\gamma_\mathrm{B}$ and $V_\mathrm{J}^0 \equiv \gamma_\mathrm{B} t_\mathrm{R}/\mathrm{H}$, it comes out that

$$C_\mathrm{FJ} = C_\mathrm{F}^0 + (\tilde{p} - \gamma_\mathrm{B})N + Q_\mathrm{F}.\left(V_\mathrm{J}^0 + T_\mathrm{J}.\left(\frac{\tilde{p}}{2\mathrm{H}} + \frac{\widetilde{w}_\mathrm{J}}{H_\mathrm{J}.n_\mathrm{B}}\right)\right). \tag{3.12}$$

Given N and n_B, the coefficient of Q_F in the previous formula is precisely V_J: thus,

$$V_\mathrm{J} = V_\mathrm{J}^0 + T_\mathrm{J}.\left(\frac{\tilde{p}}{2\mathrm{H}} + \frac{\widetilde{w}_\mathrm{J}}{H_\mathrm{J}.n_\mathrm{B}}\right) \tag{3.13}$$

This formula of the per-refill swapping cost involves the swapping logistical process in terms of juicing tour time T_J and tour productivity n_B, together with the augmented juicing wages $\widetilde{w}_\mathrm{J}$ and the composite per-scooter cost $\tilde{p}$. The fixed part V_J^0 pertains to the refill time t_R and the marginal battery cost γ_B.

To sum up, independently of the amount of energy that will be fed in the battery, each refill entails a specific cost that is the unit cost of the swapping strategy.

The other influence of Q_F on the rest of service production costs, $C_\Sigma - C_\mathrm{FJ}$, is limited to that on battery renewal costs, C_B, which is conveyed by f_B. It holds that

$$\frac{\partial C_\mathrm{B}}{\partial Q_\mathrm{F}} = p_\mathrm{B}\frac{\partial f_\mathrm{B}}{\partial Q_\mathrm{F}} = \frac{p_\mathrm{B}}{R_\mathrm{B}} \tag{3.14}$$

Then, the overall sensitivity of the service production costs to the number of refills has coefficient

$$\frac{\partial C_\Sigma}{\partial Q_\mathrm{F}} = \frac{\partial C_\mathrm{FJ}}{\partial Q_\mathrm{F}} + \frac{\partial C_\mathrm{B}}{\partial Q_\mathrm{F}} = V_\mathrm{J} + \frac{p_\mathrm{B}}{R_\mathrm{B}} \tag{3.15}$$

This holistic per-swapping cost adds up the per-refill swapping cost and the per-refill share of the battery price, in a lifecycle perspective.

3.3 Production cost as a function of action levers

Most of the variables and parameters represent action levers for different actors as economic agents, according to their respective action levers. If energy provision was monopolistic then the electricity price p_{E} would be controlled by the energy provider. If the scooter market (resp. the battery market) was oligopolistic then any scooter supplier (resp. battery supplier) would have some control over p_{S} (resp. p_{B}). The in-field per-scooter daily cost, p_{F}, is expected to depend on local regulation and it may serve as an instrument of the local mobility policy.

Yet we shall essentially focus on the S3 service provider. Its action levers include N and $n_{\mathrm{B}}, \sigma, A, L_{\mathrm{S}}, \chi_{\mathrm{E}}$. Their respective outreach on the production functions and the associated costs is illustrated in figure 3. The order given to the components in the action vector reflects a temporal hierarchy of their respective outreach: n_{B} can be set up in the short run, while L_{S} and χ_{E} are long run factors. Factors σ and A may be considered as medium run factors: say short-medium run for σ and medium-long run for A.

Active fleet size N is determined by the supplier according to not only production costs but also its revenues: it influences the service density in the field, hence the quality of service and in turn the demand volume. Demand volume Q will also depend on the service tariff say τ. The joint influence of N and τ onto Q is the topic of another article. Here we will trace out the direct influences of N and Q onto service costs, alongside the influences of the cost-only levers $n_{\mathrm{B}}, \sigma, A, L_{\mathrm{S}}, \chi_{\mathrm{E}}$. Table 1 provides the sensitivity coefficients of all specific costs to all action levers available to the service supplier, save for N which only influences the field costs C_{F} with sensitivity coefficient $\tilde{p} - \gamma_{\mathrm{B}}$.

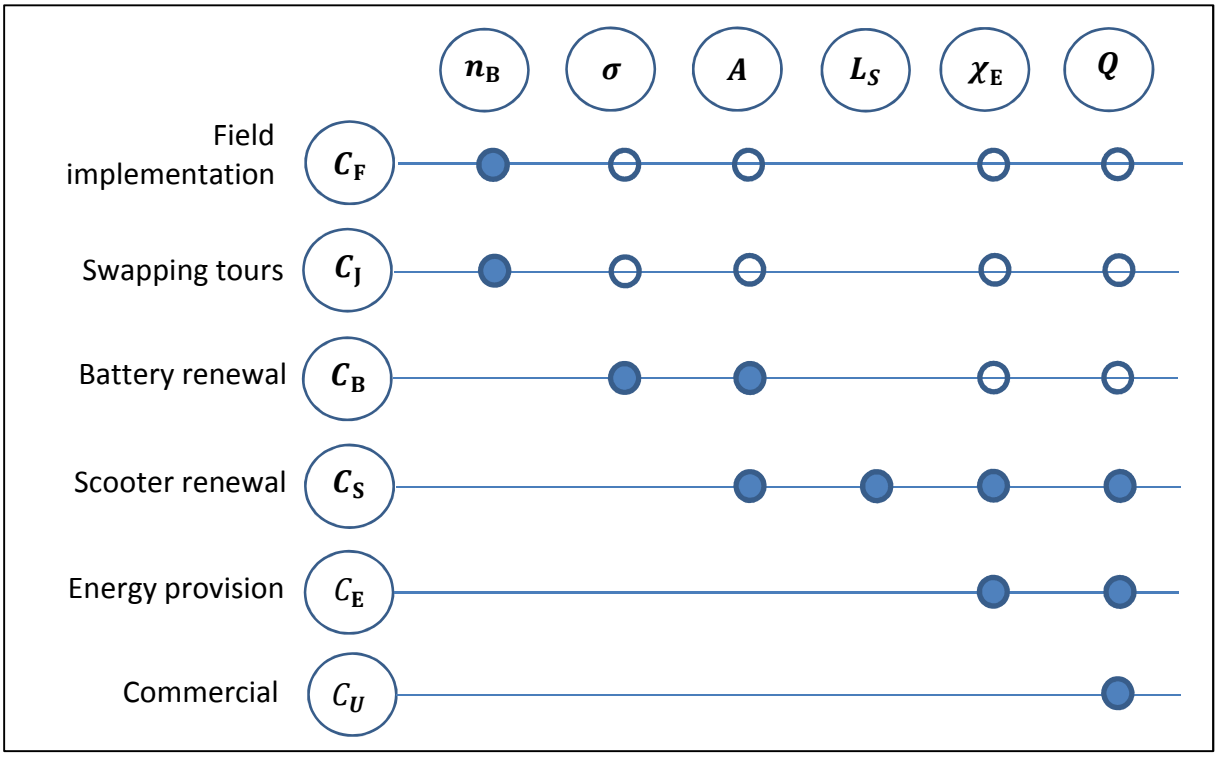


Fig. 3: The outreach of action levers of the S3 operator on its production costs (a circle marks an influence; it is empty if the influence is conveyed by Q_{F})**.**

Given N and Q, we can model the behavior of the service supplier as that of cost minimization with respect to the remaining action levers, $n_\mathrm{B}, \sigma, A, L_\mathrm{S}, \chi_\mathrm{E}$. In the next sections we shall consider first the short-run optimization of juicing tours with respect to n_B, then the medium-run optimization of batteries with respect to σ and A, next the long-run optimization of scooters with respect to L_S and χ_E.

Table 1: Sensitivity coefficients of specific Costs to Factors.
Wherein: $V_\mathrm{J}^{(1)} \equiv V_\mathrm{J}^0 + (\tilde{p}\, T_\mathrm{J})/(2\mathrm{H})$ and $V_\mathrm{J}^{(2)} \equiv V_\mathrm{J} - V_\mathrm{J}^{(1)} = \left(\widetilde{w}_\mathrm{J}.\ T_\mathrm{J}\right)/(H_\mathrm{J}.n_\mathrm{B})$.

Factor Cost	n_B	σ	A	L_S	χ_E	Q
$C_\mathrm{S} = p_\mathrm{S}.f_\mathrm{S}$	-	-	$f_\mathrm{S}.\dot{P}_{\mathrm{S}\mid A}$	$f_\mathrm{S}(\dot{P}_{\mathrm{S}\mid L} - \frac{P_\mathrm{S}}{L_\mathrm{S}})$	$f_\mathrm{S}\dot{P}_{\mathrm{S}\mid\chi}$	C_S/Q
$C_\mathrm{B} = p_\mathrm{B}.f_\mathrm{B}$	-	$-\frac{p_\mathrm{B}Q_\mathrm{F}}{\sigma}\frac{R_\mathrm{B}+\sigma\dot{R}_\mathrm{B}}{R_\mathrm{B}^2}$	$f_\mathrm{B}(\dot{P}_\mathrm{B} - \frac{P_\mathrm{B}}{A})$	-	$\frac{p_\mathrm{B}}{R_\mathrm{B}}\frac{Q_\mathrm{F}}{\chi_\mathrm{E}}$	C_B/Q
$\tilde{C}_\mathrm{F} \equiv C_\mathrm{FJ} - \tilde{C}_\mathrm{J}$	$\frac{\tilde{p}.\dot{t}}{2\mathrm{H}}Q_\mathrm{F}$	$-V_\mathrm{J}^{(1)}Q_\mathrm{F}/\sigma$	$-V_\mathrm{J}^{(1)}Q_\mathrm{F}/A$	$\widetilde{N}\,\frac{\partial\Gamma_\mathrm{N}}{\partial L_\mathrm{S}}$	$V_\mathrm{J}^{(1)}Q_\mathrm{F}/\chi_\mathrm{E}$	$V_\mathrm{J}^{(1)}Q_\mathrm{F}/Q$
$\tilde{C}_\mathrm{J} \equiv \widetilde{w}_\mathrm{J}.n_\mathrm{J}$	$-\frac{\widetilde{w}_\mathrm{J}t_0}{n_\mathrm{B}^2H_\mathrm{J}}Q_\mathrm{F}$	$-V_\mathrm{J}^{(2)}Q_\mathrm{F}/\sigma$	$-V_\mathrm{J}^{(2)}Q_\mathrm{F}/A$	-	$V_\mathrm{J}^{(2)}Q_\mathrm{F}/\chi_\mathrm{E}$	$V_\mathrm{J}^{(2)}Q_\mathrm{F}/Q$
$C_\mathrm{E} = p'_\mathrm{E}.Q_\mathrm{E}$	-	-	-	-	$p'_\mathrm{E}Q_\mathrm{L}$	C_E/Q
$C_\mathrm{U} = c_\mathrm{u}.Q$	-	-	-	-	-	c_u

4. Juicing tours optimization

4.1 The optimal tour productivity

From Table 1, we have that

$$\frac{\partial C_\Sigma}{\partial n_\mathrm{B}} = \tilde{p}\,\frac{Q_\mathrm{F}\,\dot{t}}{2\mathrm{H}} - \frac{\widetilde{w}_\mathrm{J}\,Q_\mathrm{F}\,t_0}{\mathrm{H_J}\,n_\mathrm{B}^2}$$

Cost minimizing behaviour with respect to n_B amounts to giving it the value that satisfies the first-order optimality condition for minimization, i.e.

$$\frac{\partial C_\Sigma}{\partial n_\mathrm{B}} = 0$$

The solution is easily obtained as:

$$n_\mathrm{B}^* = \sqrt{\frac{2\mathrm{H}}{\mathrm{H_J}}\frac{\widetilde{w}_\mathrm{J}.\,t_0}{\tilde{p}.\,\dot{t}}} \tag{4.1}$$

Put in words, the optimal tour productivity is equal to the square root of the ratio of twice the period duration times the tour base time t_0 times the augmented daily cost per juicer duty $\widetilde{w}_\mathrm{J}$, on the numerator side, to the product of the juicer duty duration $\mathrm{H_J}$, times the swapping time per-scooter $\dot{t}$, times the composite in-field daily cost per scooter $\tilde{p}$, on the denominator side.

Thus the optimal productivity depends only on technical conditions $t_0, \dot{t}$, operational set-ups $\mathrm{H}, \mathrm{H_J}$ and resource costs $\tilde{w}_\mathrm{J}, \tilde{p}$. It involves neither the battery features nor the scooters attributes.

4.2 The optimal tour time

The tour productivity n_B is the key factor of the tour time $\mathrm{T_J}$. Based on its formula (2.5), its optimal value stems from n_B^* as follows:

$$T_\mathrm{J}^* = t_0 + \dot{t}.n_\mathrm{B}^* = t_0 + \sqrt{\frac{2\mathrm{H}}{\mathrm{H_J}}\frac{w_\mathrm{J}}{\tilde{p}}t_0\dot{t}} \tag{4.2}$$

4.3 The optimal number of juicer duties

From n_B^* stems the optimal daily number of juicer duties n_J^* owing to (2.8):

$$n_\mathrm{J}^* = Q_\mathrm{F}\left(\frac{\dot{t}}{\mathrm{H_J}} + \sqrt{\frac{\dot{t}.t_0.\tilde{p}}{2\mathrm{H}.\mathrm{H_J}.\tilde{w}_\mathrm{J}}}\right) \tag{4.3}$$

It is proportional to the daily number of refills Q_F, times a coefficient that only involves technical conditions $t_0, \dot{t}$, operational set-ups $\mathrm{H}, \mathrm{H_J}$ and resource costs $\tilde{w}_\mathrm{J}, \tilde{p}$. Again, the battery and scooter features exert no direct influence.

4.4 The optimal unit cost of battery swapping

At optimum, the unit cost of battery swapping V_J defined in (3.11) takes on the following value V_J^* such that:

$V_\mathrm{J}^* - V_\mathrm{J}^0 = T_\mathrm{J}^*\left(\frac{\tilde{p}}{2\mathrm{H}} + \frac{\tilde{w}_\mathrm{J}}{H_\mathrm{J}.n_\mathrm{B}^*}\right) = t_0\frac{\tilde{p}}{2\mathrm{H}} + \sqrt{\frac{\tilde{p}.\tilde{w}_\mathrm{J}}{2\mathrm{H}.\mathrm{H_J}}t_0\dot{t}} + \frac{\tilde{w}_\mathrm{J}}{H_\mathrm{J}}\left(\dot{t} + \frac{t_0}{n_\mathrm{B}^*}\right)$

$= t_0\frac{\tilde{p}}{2\mathrm{H}} + \sqrt{\frac{\tilde{p}.\tilde{w}_\mathrm{J}}{2\mathrm{H}.\mathrm{H_J}}t_0\dot{t}} + \frac{\tilde{w}_\mathrm{J}}{H_\mathrm{J}}\dot{t} + \sqrt{\frac{\tilde{p}.\tilde{w}_\mathrm{J}}{2\mathrm{H}.\mathrm{H_J}}t_0\dot{t}} = \left(\sqrt{\frac{\dot{t}.\tilde{w}_\mathrm{J}}{\mathrm{H_J}}} + \sqrt{\frac{t_0.\tilde{p}}{2\mathrm{H}}}\right)^2$, yielding that

$$V_\mathrm{J}^* = V_\mathrm{J}^0 + \left(\sqrt{\frac{\dot{t}.\tilde{w}_\mathrm{J}}{\mathrm{H_J}}} + \sqrt{\frac{t_0.\tilde{p}}{2\mathrm{H}}}\right)^2 \tag{4.4}$$

In this formula, the term $\dot{t}.\tilde{w}_\mathrm{J}/\mathrm{H_J}$ is the juicer-based cost of the elementary time $\dot{t}$, whereas the other term $t_0.\tilde{p}/2\mathrm{H}$ conveys the influences of both the base tour time and the composite in-field daily cost per scooter.

5. Battery optimization

5.1 The optimal depth of discharge

From Table 1 we get the sensitivity coefficient of production costs to target DoD σ:

$$\frac{\partial C_\Sigma}{\partial \sigma} = -\frac{Q_\mathrm{F}}{\sigma} V_\mathrm{J} - \frac{Q_\mathrm{F}}{\sigma} p_\mathrm{B} \frac{\mathrm{R_B} + \sigma \dot{\mathrm{R}}_\mathrm{B}}{\mathrm{R_B^2}}$$

We restate it as

$$\frac{\partial C_\Sigma}{\partial \sigma} = -\frac{Q_\mathrm{F}}{\sigma} (V_\mathrm{J} + p_\mathrm{B} \frac{\mathrm{R_B} + \sigma \dot{\mathrm{R}}_\mathrm{B}}{\mathrm{R_B^2}}) \tag{5.1}$$

Let us define function $\psi_B : \sigma \mapsto -(\mathrm{R_B} + \sigma \dot{\mathrm{R}}_\mathrm{B})/\mathrm{R_B^2}$ that stems from the wearing law. Using function ψ_B, the sensitivity coefficient of production costs to σ restates as:

$$\frac{\partial C_\Sigma}{\partial \sigma} = \frac{Q_\mathrm{F}}{\sigma} p_\mathrm{B} \left(\psi_B(\sigma) - \frac{V_\mathrm{J}}{p_\mathrm{B}} \right) \tag{5.2}$$

From its definition, DoD σ takes its values in $[0,1]$. Let us restrict its range by introducing an upper bound $\bar{\sigma} \leq 1$ to avoid the undesirable effects of excess discharge.

The operator's behavior of cost minimization with respect to $\sigma \in [0, \bar{\sigma}]$ is expressed as the following mathematical program:

$$\min_\sigma C_\Sigma \text{ w.r.t. } \sigma \in [0, \bar{\sigma}]. \tag{5.3}$$

Its first-order optimality condition is therefore

$$\frac{\partial C_\Sigma}{\partial \sigma} . \sigma . (\bar{\sigma} - \sigma) = 0 \tag{5.4a}$$

$$\sigma \in]0, \bar{\sigma}[\ \Rightarrow \ \frac{\partial C_\Sigma}{\partial \sigma} = 0 \tag{5.4b}$$

$$\sigma = \bar{\sigma} \ \Rightarrow \ \frac{\partial C_\Sigma}{\partial \sigma} \leq 0 \tag{5.4c}$$

$$\sigma = 0 \ \Rightarrow \ \frac{\partial C_\Sigma}{\partial \sigma} \geq 0 \tag{5.4d}$$

The null value can be dismissed if $\psi_B(0) = 0$, making $\frac{\partial C_\Sigma}{\partial \sigma} \to -\infty$ when $\sigma \to 0^+$. Let us assume more generally that $\psi_B(0) < V_\mathrm{J}/p_\mathrm{B}$. Then, owing to (5.2) the optimality condition can be restated as:

$$\left(\psi_B(\sigma) - \frac{V_\mathrm{J}}{p_\mathrm{B}} \right) . (\bar{\sigma} - \sigma) = 0 \tag{5.5a}$$

$$\sigma \in]0, \bar{\sigma}[\ \Rightarrow \ \psi_B(\sigma) = \frac{V_\mathrm{J}}{p_\mathrm{B}} \tag{5.5b}$$

$$\sigma = \bar{\sigma} \Rightarrow \ \psi_B(\sigma) \leq \frac{V_\mathrm{J}}{p_\mathrm{B}} \tag{5.5c}$$

We shall call "Partial Discharge" the case when the optimal σ is strictly less than $\bar{\sigma}$, and "Full Discharge" the case when it is bound to $\bar{\sigma}$ because $\psi_B(\bar{\sigma}) \leq V_\mathrm{J}/p_\mathrm{B}$. Postulating that the ψ_B

function is nonnegative and increasing, we can then consider the inverse function $\psi_{\mathrm{B}}^{(-1)}$ which is well-defined, nonnegative and increasing. Using this inverse function the optimal DoD policy can be put as follows:

$$\sigma^* = \min\left\{\bar{\sigma}, \psi_{\mathrm{B}}^{(-1)}\left(\frac{V_{\mathrm{J}}}{p_{\mathrm{B}}}\right)\right\} \tag{5.6}$$

Proposition 1: existence and uniqueness of an optimal DoD. Assume that function ψ_B is continuous and increasing and that $V_{\mathrm{J}}/p_{\mathrm{B}} > \psi_B(0)$. (i) If $V_{\mathrm{J}}/p_{\mathrm{B}} < \psi_B(\bar{\sigma})$ then there exists an optimal DoD $\sigma^* = \psi_{\mathrm{B}}^{(-1)}(V_{\mathrm{J}}/p_{\mathrm{B}})$ in $]0, \bar{\sigma}[$: it is unique if ψ_B is strictly increasing at σ^*. (ii) If $V_{\mathrm{J}}/p_{\mathrm{B}} \geq \psi_B(\bar{\sigma})$ then $\bar{\sigma}$ is the optimal DoD.

The proof is easy and its details are provided in Appendix A.

Thus, given wearing law R_B hence function ψ_B, the ratio of the per-refill cost V_{J} and the battery price p_{B} is the key variable driving the DoD policy.

Another way to look at the Partial Discharge condition, $\psi_B(\sigma) = V_{\mathrm{J}}/p_{\mathrm{B}}$, is to involve the lifetime energy intensity function $\mathrm{E}_B(\sigma) \equiv \sigma.\mathrm{R}_{\mathrm{B}}(\sigma)$ and replace ψ_B with $-\dot{\mathrm{E}}_{\mathrm{B}}/\mathrm{R}_{\mathrm{B}}^2$, yielding that

$$-\frac{\dot{\mathrm{E}}_{\mathrm{B}}}{\mathrm{R}_{\mathrm{B}}} = \frac{V_{\mathrm{J}}.\mathrm{R}_{\mathrm{B}}}{p_{\mathrm{B}}} \tag{5.7}$$

As $\mathrm{R}_{\mathrm{B}} = \mathrm{E}_B/\sigma$, in the left hand side we recognize the elasticity $\epsilon_{\mathrm{E}:\sigma}$ of lifetime energy intensity E_B to DoD: then $-\epsilon_{\mathrm{E}:\sigma} = V_{\mathrm{J}}.\mathrm{R}_{\mathrm{B}}/p_{\mathrm{B}}$ meaning that, at optimum target DoD below $\bar{\sigma}$,

$$V_{\mathrm{J}}.\mathrm{R}_{\mathrm{B}} = -\epsilon_{\mathrm{E}:\sigma}.p_{\mathrm{B}} \tag{5.8}$$

Put in words, under Partial Discharge the optimal number of refills times the per-refill swapping cost, $V_{\mathrm{J}}.\mathrm{R}_{\mathrm{B}}$ that is the battery lifetime swapping cost, balances $|\epsilon_{\mathrm{E}:\sigma}|$ times the battery price p_{B}.

5.2 The Constant Elasticity technology of battery lifetime energy intensity

Let us now introduce a specific technology function R_{B} by postulating that the E_{B} function of lifetime energy intensity is a constant elasticity function of σ as follows:

$$\mathrm{E}_{\mathrm{B}}(\sigma) = \mathrm{E}_1\sigma^{-\alpha} \tag{5.9a}$$

Parameter $\mathrm{E}_1 = \mathrm{E}_{\mathrm{B}}(1) > 0$ stands for the minimum lifetime energy intensity. Parameter $\alpha > 0$ makes E_{B} a decreasing function of σ.Thus

$$\mathrm{R}_{\mathrm{B}}(\sigma) = \sigma^{-1}\mathrm{E}_{\mathrm{B}}(\sigma) = \mathrm{E}_1\sigma^{-\alpha-1} \tag{5.9b}$$

Then, $\dot{\mathrm{E}}_{\mathrm{B}} = -\alpha\mathrm{E}_1\sigma^{-\alpha-1} = -\alpha\mathrm{R}_{\mathrm{B}}$, so that

$$\psi_B(\sigma) = -\frac{\dot{\mathrm{E}}_{\mathrm{B}}}{\mathrm{R}_{\mathrm{B}}^2} = \frac{\alpha}{\mathrm{R}_{\mathrm{B}}} = \alpha\frac{\sigma^{\alpha+1}}{\mathrm{E}_1} \tag{5.10a}$$

Thus ψ_B is an increasing function of σ, with inverse function such that $\alpha\sigma^{\alpha+1} = \mathrm{E}_1 y$, hence $\mathrm{R}_{\mathrm{B}}(\sigma) = \alpha/y$ and

$$\psi_{\mathrm{B}}^{(-1)}(y) = \left(\frac{\mathrm{E}_1 y}{\alpha}\right)^{\frac{1}{\alpha+1}} \tag{5.10b}$$

For the value of $\psi_{\mathrm{B}}^{(-1)}(y)$ to be less than $\bar{\sigma}$ it requires that

$$y < \psi_B(\bar{\sigma}) = \alpha \frac{\bar{\sigma}^{\alpha+1}}{\mathrm{E}_1}$$

If $y < \psi_B(\bar{\sigma})$, the lifetime effective intensity is obtained as $\mathrm{E}_{\mathrm{B}} \circ \psi_{\mathrm{B}}^{(-1)}(y) = \mathrm{E}_1(\frac{\mathrm{E}_1 y}{\alpha})^{\frac{-\alpha}{\alpha+1}}$, i.e.

$$\mathrm{E}_{\mathrm{B}}(\sigma) = \mathrm{E}_1{}^{\frac{1}{\alpha+1}} (\frac{\alpha}{y})^{\frac{\alpha}{\alpha+1}} \tag{5.11}$$

As for DoD optimization, depending on whether $V_{\mathrm{J}}/p_{\mathrm{B}} < \psi_B(\bar{\sigma})$ or not, the optimal DoD is less than $\bar{\sigma}$ i.e. partial discharge or equal to $\bar{\sigma}$ i.e. full discharge. The following table provides the associated outcomes under the specific battery technology with constant elasticity lifetime energy intensity:

Condition	σ^*	$\mathrm{R}_{\mathrm{B}}(\sigma^*)$	$\mathrm{E}_{\mathrm{B}}(\sigma^*)$
$V_{\mathrm{J}}/p_{\mathrm{B}} < \psi_B(\bar{\sigma})$ Partial discharge	$(\frac{\mathrm{E}_1 V_{\mathrm{J}}}{\alpha\, p_{\mathrm{B}}})^{\frac{1}{\alpha+1}}$	$\alpha \frac{p_{\mathrm{B}}}{V_{\mathrm{J}}}$	$\mathrm{E}_1{}^{\frac{1}{\alpha+1}} (\alpha \frac{p_{\mathrm{B}}}{V_{\mathrm{J}}})^{\frac{\alpha}{\alpha+1}}$
$V_{\mathrm{J}}/p_{\mathrm{B}} \geq \psi_B(\bar{\sigma})$ Full discharge	$\bar{\sigma}$	$\mathrm{E}_1 \bar{\sigma}^{-\alpha-1}$	$\mathrm{E}_1 \bar{\sigma}^{-\alpha}$

(5.12a,b,c)

A specific property of the constant elasticity wearing law under Partial discharge stems from (5.8): the per-refill share of battery price, $p_{\mathrm{B}}/\mathrm{R}_{\mathrm{B}}$, is proportional to V_{J}.

5.3 The optimal battery energy capacity

The nominal battery energy capacity A does determine the battery price p_{B} and in turn the optimal target DoD σ^*. Let us now take a full view of its influence on the service production costs and its outreach for cost minimization. From Table 1 and the definition of the per-refill swapping cost V_{J}, the sensitivity coefficient of production costs to BEC A is:

$$\frac{\partial C_\Sigma}{\partial A} = -\frac{Q_{\mathrm{F}}}{A}.V_{\mathrm{J}} + f_{\mathrm{S}}.\dot{\mathrm{P}}_{\mathrm{S}|A} + \frac{Q_{\mathrm{F}}}{\mathrm{R}_{\mathrm{B}}}\left(\dot{\mathrm{P}}_{\mathrm{B}} - \frac{\mathrm{P}_{\mathrm{B}}}{A}\right) \tag{5.13}$$

Indeed, the influence of A on production costs pertains to (i) juicing logistics and field implementation, with sensitivity coefficient $-V_{\mathrm{J}}Q_{\mathrm{F}}/A$, (ii) scooters' inventory with sensitivity coefficient $f_{\mathrm{S}}.\dot{\mathrm{P}}_{\mathrm{S}|A}$, (iii) batteries' inventory with a twofold sensitivity of plus $f_{\mathrm{B}}.\dot{\mathrm{P}}_{\mathrm{B}}$ minus the indirect influence of A via the number of refills.

Let us denote the number of refills over a scooter lifetime as $r_{\mathrm{S}} \equiv \chi_{\mathrm{E}}.L_{\mathrm{S}}/(\mathrm{A}.\sigma)$. The ratio of battery refill number and scooter refill number, $\rho \equiv \frac{R_{\mathrm{B}}}{r_{\mathrm{S}}} = \frac{\mathrm{A}.\sigma.R_{\mathrm{B}}}{\chi_{\mathrm{E}}.L_{\mathrm{S}}}$, is also the ratio of lifetime energy flows between one battery and one scooter. As $Q_{\mathrm{F}} = Q_{\mathrm{L}}.\chi_{\mathrm{E}}/(\mathrm{A}.\sigma)$ and $f_{\mathrm{S}} = Q_{\mathrm{L}}/L_{\mathrm{S}}$, the sensitivity coefficient states as:

$$\frac{\partial C_\Sigma}{\partial A} = -\frac{Q_{\mathrm{L}}.\chi_{\mathrm{E}}}{\mathrm{A}.\sigma}\left\{\frac{V_{\mathrm{J}}}{\mathrm{A}} + \frac{\mathrm{P}_{\mathrm{B}}/A - \dot{\mathrm{P}}_{\mathrm{B}}}{\mathrm{R}_{\mathrm{B}}} - \frac{\dot{\mathrm{P}}_{\mathrm{S}|A}}{r_{\mathrm{S}}}\right\} \tag{5.14}$$

Cost minimizing behaviour with respect to A amounts to giving it the value that satisfies the first-order optimality condition for minimization, i.e.

$$\frac{\partial C_\Sigma}{\partial A} = 0$$

It is equivalent to

$$\frac{\dot{\mathrm{P}}_\mathrm{B}}{\mathrm{R}_\mathrm{B}} + \frac{\dot{P}_{\mathrm{S}|A}}{r_\mathrm{S}} = \frac{1}{\mathrm{A}}\left(V_\mathrm{J} + \frac{\mathrm{P}_\mathrm{B}}{\mathrm{R}_\mathrm{B}}\right)$$

Put in words, an optimal BEC balances the marginal per-refill price of the mobility equipment (including the battery and the scooter body) with the holistic per-swapping cost per unit of energy capacity.

Using the elasticities $\epsilon_{\mathrm{B}|A}$ and $\epsilon_{\mathrm{S}|A}$ of battery and scooter prices to A, the condition becomes

$$\epsilon_{\mathrm{B}|A}\frac{\mathrm{P}_\mathrm{B}}{\mathrm{R}_\mathrm{B}} + \epsilon_{\mathrm{S}|A}\frac{\mathrm{P}_\mathrm{S}}{r_\mathrm{S}} = V_\mathrm{J} + \frac{\mathrm{P}_\mathrm{B}}{\mathrm{R}_\mathrm{B}} \tag{5.15}$$

Another statement of the optimal BEC is then

$$r_\mathrm{S} = \frac{\epsilon_{\mathrm{S}|A}\mathrm{P}_\mathrm{S}}{V_\mathrm{J} + (1-\epsilon_{\mathrm{B}|A})\frac{\mathrm{P}_\mathrm{B}}{\mathrm{R}_\mathrm{B}}} \tag{5.16}$$

With respect to BEC, from r_S and $\epsilon_{\mathrm{S}|A}\mathrm{P}_\mathrm{S} = A.\dot{\mathrm{P}}_{\mathrm{S}|A}$ we can also state the optimal BEC as follows:

$$A^2 = \frac{\chi_\mathrm{E}.L_\mathrm{S}}{\sigma.\dot{\mathrm{P}}_{\mathrm{S}|A}}\left(V_\mathrm{J} + (1-\epsilon_{\mathrm{B}|A})\frac{\mathrm{P}_\mathrm{B}}{\mathrm{R}_\mathrm{B}}\right) \tag{5.17}$$

As the Right Hand Side is a function of A, the condition is a Fixed Point Problem in A.

Proposition 2: existence of an optimal BEC. Assume that (i) $\dot{\mathrm{P}}_{\mathrm{S}|A}$ is continuous and takes its values in a bounded interval $\left[\dot{\mathrm{P}}_{\mathrm{S}|A}^{min}, \dot{\mathrm{P}}_{\mathrm{S}|A}^{max}\right]$ with $\dot{\mathrm{P}}_{\mathrm{S}|A}^{min} > 0$, (ii) P_B and $\epsilon_{\mathrm{B}|A}$ are continuous and $\epsilon_{\mathrm{B}|A} \in [0,1]$. Then, given σ, there is at least one BEC A_σ^* that satisfies the optimality condition.

The proof is given in Appendix A.

5.4 Optimal capacity under linear battery price

Under linear battery price $P_\mathrm{B} = A.\dot{P}_\mathrm{B}$, A is related to ratio $y \equiv V_\mathrm{J}/P_\mathrm{B}$ as follows:

$$A = \frac{V_\mathrm{J}}{y\dot{P}_\mathrm{B}}$$

Partial discharge is optimal if $y \leq \psi_B(\bar{\sigma})$ hence, denoting $\bar{\psi}_B \equiv \psi_B(\bar{\sigma})$, if

$$A \geq \underline{A} \equiv \frac{V_\mathrm{J}}{\dot{P}_\mathrm{B}\bar{\psi}_B} \tag{5.18}$$

Given V_J, capacity $\underline{A}$ is the threshold value of the Partial discharge regime, hence also the ceiling value of the Full discharge domain.

Now, linear battery price has elasticity $\epsilon_{\mathrm{B}|A} = 1$ and the BEC optimality condition (5.17) becomes

$$A^2 \dot{P}_{\mathrm{S}|A} = \frac{\chi_{\mathrm{E}}.L_{\mathrm{S}}}{\sigma} V_{\mathrm{J}} \tag{5.19}$$

hence

$$P_{\mathrm{S}} = \frac{r_{\mathrm{S}} V_{\mathrm{J}}}{\epsilon_{\mathrm{S}|A}}$$

The scooter price must be equal to the swapping cost over the scooter lifetime, divided by the elasticity of scooter price to battery capacity.

Under linear affine scooter price $\mathrm{P_S} = P_{\mathrm{S0}} + A.\dot{P}_{\mathrm{S|A}}$ and given the target DoD σ, equation (5.19) provides an easy rule to determine the optimal battery capacity as follows:

$$A^* = \sqrt{\frac{\chi_{\mathrm{E}}.L_{\mathrm{S}}}{\sigma} \frac{V_{\mathrm{J}}}{\dot{P}_{\mathrm{S}|A}}} \tag{5.20}$$

While it is in principle more valuable to optimize the (A, σ) pair jointly, in practice it may well arise that full discharge $\sigma = \bar{\sigma}$ is optimal: then the previous formula applied to $\sigma = \bar{\sigma}$ suffices to optimize the capacity.

Let us now utilize the affine linear specification of prices to investigate some general conditions for Partial or Full discharge to apply, depending on battery capacity. Full discharge involves $\sigma = \bar{\sigma}$ and $A \leq \underline{A}$, hence $A^2 \leq \underline{A}^2$: combining with (5.19) yields

$\frac{\chi_{\mathrm{E}}.L_{\mathrm{S}}}{\bar{\sigma}\dot{P}_{\mathrm{S}|A}} V_{\mathrm{J}} \leq \frac{V_{\mathrm{J}}^2}{(\dot{P}_{\mathrm{B}}\bar{\Psi}_B)^2}$, hence

$$V_{\mathrm{J}} \geq \hat{V}_{\mathrm{J}} \equiv \frac{\chi_{\mathrm{E}}.L_{\mathrm{S}}}{\bar{\sigma}.\dot{P}_{\mathrm{S}|A}} (\dot{P}_{\mathrm{B}}\bar{\Psi}_B)^2 \tag{5.21a}$$

The per-refill swapping cost $\hat{V}_{\mathrm{J}}$ is a threshold value for Full discharge to possibly occur. It implies that values of V_{J} below $\hat{V}_{\mathrm{J}}$ only give rise to Partial discharge in DoD optimization. But it does not mean that only Full discharge could be the optimal policy for values of V_{J} above $\hat{V}_{\mathrm{J}}$.

Given V_{J}, we can also put the Full discharge condition as an upper limit on the scooter energy flow during its lifetime:

$$\chi_{\mathrm{E}}.L_{\mathrm{S}} \leq \bar{E}_{\mathrm{S}} \equiv \frac{\bar{\sigma}.\dot{P}_{\mathrm{S}|A}}{(\dot{P}_{\mathrm{B}}\bar{\Psi}_B)^2} V_{\mathrm{J}} \tag{5.21b}$$

Or as an upper limit on just the scooter lifespan:

$$L_{\mathrm{S}} \leq \bar{L}_{\mathrm{S}} \equiv \frac{\bar{\sigma}.\dot{P}_{\mathrm{S}|A}}{\chi_{\mathrm{E}}(\dot{P}_{\mathrm{B}}\bar{\Psi}_B)^2} V_{\mathrm{J}} \tag{5.21c}$$

Variables above their Full discharge ceiling give rise to slack lifespan $L_{\mathrm{S}} - \bar{L}_{\mathrm{S}}$, slack lifetime energy $\chi_{\mathrm{E}}.L_{\mathrm{S}} - \bar{E}_{\mathrm{S}}$, slack capacity $A - \underline{A}$ that enable for Partial discharge to hold, i.e. for higher battery lifetime energy intensity than just $\mathrm{E_B}(\bar{\sigma})$.

5.5 The joint optimization of DoD and battery capacity

To optimize the (A, σ) pair jointly, let us then bring together the optimality conditions that pertain to DoD σ and capacity A respectively: from (5.6) and (5.15),

$$\sigma^* = \min\left\{\bar{\sigma}, \psi_{\mathrm{B}}^{(-1)}\left(\frac{V_{\mathrm{J}}}{\mathrm{P_B}}\right)\right\} \tag{5.22a}$$

$$\epsilon_{\mathrm{B}|A}\frac{\mathrm{P_B}}{\mathrm{R_B}} + \epsilon_{\mathrm{S}|A}\frac{\mathrm{P_S}}{r_{\mathrm{S}}} = V_{\mathrm{J}} + \frac{\mathrm{P_B}}{\mathrm{R_B}} \tag{5.22b}$$

Two cases may arise depending on whether $V_{\mathrm{J}}/\mathrm{P_B}(A)$ is less or greater than $\psi_B(\bar{\sigma})$, i.e. Partial vs. Full Discharge. Under Full Discharge the joint optimization reduces to optimization with respect to BEC only, yielding solution $A^*_{\bar{\sigma}}$. If furthermore $V_{\mathrm{J}}/\mathrm{P_B}(\mathrm{A}^*_{\bar{\sigma}}) \geq \psi_{\mathrm{B}}(\bar{\sigma})$ then both optimality conditions are satisfied jointly. Under Partial Discharge, we may take σ as a function of A, namely $\sigma^*_A = \psi_{\mathrm{B}}^{(-1)}(V_{\mathrm{J}}/\mathrm{P_B})$, so that the joint optimization reduces to a modified fixed point problem in A only as follows:

$$A^2\sigma^*_A = \frac{\chi_{\mathrm{E}}.L_{\mathrm{S}}}{\dot{\mathrm{P}}_{\mathrm{S}|A}}\left(V_{\mathrm{J}} + (1-\epsilon_{\mathrm{B}|A})\frac{\mathrm{P_B}}{\mathrm{R_B}}\right) \tag{5.23}$$

Proposition 3: existence and uniqueness of joint optimal BEC-DoD. Assume that (i) $\mathrm{P_B}$ increases with A and (ii) $(1-\epsilon_{\mathrm{B}|A})\mathrm{P_B}$ decreases with A while remaining nonnegative, (iii) $\dot{\mathrm{P}}_{\mathrm{S}|A}$ is nonnegative and (iv) $\dot{\mathrm{P}}_{\mathrm{S}|A}A^2$ increases indefinitely with A. Then the Partial Discharge optimality condition in A only has one unique solution A^*, with associated DoD $\sigma^*_{A^*} \equiv \psi_{\mathrm{B}}^{(-1)}(V_{\mathrm{J}}/\mathrm{P_B}(A^*))$. If $V_{\mathrm{J}}/\mathrm{P_B}(A^*) < \psi_{\mathrm{B}}(\bar{\sigma})$, then $\sigma^*_{A^*} \in]0, \bar{\sigma}[$ and the pair $(A^*, \sigma^*_{A^*})$ is the unique optimal pair, meaning that Partial Discharge is optimal. Otherwise, if $V_{\mathrm{J}}/\mathrm{P_B}(A^*) \geq \psi_{\mathrm{B}}(\bar{\sigma})$ then Full Discharge is optimal and the pair $(\mathrm{A}^*_{\bar{\sigma}}, \bar{\sigma})$ is the unique optimal pair.

The proof is given in Appendix A.

5.6 Optimal BEC and DoD under constant elasticity prices

Let us assume here that the battery price is a constant elasticity function of battery capacity, $\mathrm{P_B} = z_{\mathrm{B}}A^{\beta}$, and similarly that the scooter price has constant elasticity with respect to A, say $\mathrm{P_S} = z_{\mathrm{S}}A^{\eta}$.

Then the optimality condition (5.15) on BEC becomes

$$\beta\frac{\mathrm{P_B}}{\mathrm{R_B}} + \eta\frac{\mathrm{P_S}}{r_{\mathrm{S}}} = V_{\mathrm{J}} + \frac{\mathrm{P_B}}{\mathrm{R_B}} \tag{5.24}$$

Under Partial discharge and the constant elasticity battery technology, $\frac{\mathrm{P_B}}{\mathrm{R_B}} = \frac{V_{\mathrm{J}}}{\alpha}$ owing to (5.8). Substituting in (5.24) yields that

$$\frac{\mathrm{P_S}}{r_{\mathrm{S}}} = V_{\mathrm{J}}\frac{\alpha + 1 - \beta}{\eta\,\alpha}$$

Thus the per-refill scooter price is proportional to the unit swapping cost, as is the per-refill battery price.

Recalling that $r_{\mathrm{S}} = \frac{\chi_{\mathrm{E}}.L_{\mathrm{S}}}{\mathrm{A}.\sigma}$ and σ from (5.12a), we get

$$\frac{z_S A^{\eta+1}}{\chi_E . L_S}\left(\frac{E_1 V_J}{\alpha\, z_B A^{\beta}}\right)^{\frac{1}{\alpha+1}} = V_J \frac{\alpha+1-\beta}{\eta\,\alpha}$$

$$\left(\frac{z_S}{\chi_E . L_S}\right)^{\alpha+1} A^{(\eta+1)(\alpha+1)-\beta} \frac{E_1 V_J}{\alpha\, z_B} = \left(V_J \frac{\alpha+1-\beta}{\eta\,\alpha}\right)^{\alpha+1}$$

$$A^{(\eta+1)(\alpha+1)-\beta} = \frac{z_B}{E_1}\left(\frac{V_J}{\alpha}\right)^{\alpha}\left(\frac{\chi_E . L_S}{z_S}\frac{\alpha+1-\beta}{\eta\,\alpha}\right)^{\alpha+1}$$

Thus, in the constant elasticity model under Partial Discharge, the optimal BEC comes out in a straightforward way:

$$A^* = \left(\frac{z_B}{E_1}\right)^{\frac{1}{(\eta+1)(\alpha+1)-\beta}}\left(\frac{V_J}{\alpha}\right)^{\frac{\alpha}{(\eta+1)(\alpha+1)-\beta}}\left(\frac{\chi_E . L_S}{z_S}\frac{\alpha+1-\beta}{\eta\,\alpha}\right)^{\frac{\alpha+1}{(\eta+1)(\alpha+1)-\beta}} \tag{5.25}$$

If $(\eta+1)(\alpha+1) > \beta$ then A^* is an increasing function of V_J, χ_E, L_S and z_B, but a decreasing function of E_1 and z_S.

The associated optimal DoD comes from (5.12a):

$$(\sigma^*)^{\alpha+1} = \frac{E_1 V_J}{\alpha\, z_B} A^{*-\beta}$$

$$\sigma^{*(\alpha+1)} = \left(\frac{E_1}{z_B}\right)^{1+\frac{\beta}{(\eta+1)(\alpha+1)-\beta}}\left(\frac{V_J}{\alpha}\right)^{1-\frac{\alpha\beta}{(\eta+1)(\alpha+1)-\beta}}\left(\frac{z_S}{\chi_E . L_S}\frac{\eta\,\alpha}{\alpha+1-\beta}\right)^{\frac{(\alpha+1)\beta}{(\eta+1)(\alpha+1)-\beta}}$$

Hence

$$\sigma^* = \left(\frac{E_1}{z_B}\right)^{\frac{\eta+1}{(\eta+1)(\alpha+1)-\beta}}\left(\frac{V_J}{\alpha}\right)^{\frac{\eta+1-\beta}{(\eta+1)(\alpha+1)-\beta}}\left(\frac{z_S}{\chi_E . L_S}\frac{\eta\,\alpha}{\alpha+1-\beta}\right)^{\frac{\beta}{(\eta+1)(\alpha+1)-\beta}} \tag{5.26}$$

If $(\eta+1)(\alpha+1) > \beta$ then σ^* is an increasing function of E_1 and z_S, but a decreasing function of χ_E, L_S and z_B. If furthermore $\eta+1 > \beta$ then it is an increasing function of V_J.

Partial Discharge is optimal if the resulting σ^* is less than the upper bound $\bar{\sigma}$. When this condition is not met, then Full Discharge applies: the target DoD is set to $\bar{\sigma}$ and yields a battery lifetime number of refills of $\bar{R}_B \equiv R_B(\bar{\sigma})$. The optimality condition on A, say (5.24), becomes

$\beta \frac{z_B A^{\beta}}{\bar{R}_B} + \eta \frac{z_S A^{\eta}}{r_S} = V_J + \frac{z_B A^{\beta}}{\bar{R}_B}$, hence

$$\eta \frac{z_S A^{\eta+1}}{\chi_E . L_S}\bar{\sigma} = V_J + (1-\beta)\frac{z_B A^{\beta}}{\bar{R}_B} \tag{5.27}$$

This equation is a fixed point problem in A only, to which there is a solution that is unique (see Appendix A).

To sum up, the model with constant elasticities gives us insight into the influences of V_J and other factors onto the cost optimization of battery parameters A and σ. Among the other factors are the scooter parameters L_S and χ_E that will be studied in the next Section. In the numerical study, we shall recourse to not only the Constant Elasticity battery technology and prices but also to Affine Linear battery technology and prices: the formulas for battery optimization under the AL set of assumptions are given in Appendix B.

6. Scooter optimization

The optimization of service production costs with respect to battery energy capacity involves the scooter price together with the battery price and the swapping cost per refill: this stems from the model influence tree, see Figure 2. We will now study the role of scooter lifespan L_S energy consumption rate χ_E in cost optimization through the scooter price $p_\mathrm{S} = \mathrm{P_S}(L_\mathrm{S}, \chi_\mathrm{E}, A, \xi)$: we shall denote the local sensitivity coefficient of that price to L_S as $\dot{P}_{\mathrm{S}|L} \equiv \frac{\partial}{\partial L_\mathrm{S}} P_\mathrm{S}$ and that to χ_E as $\dot{P}_{\mathrm{S}|\chi} \equiv \frac{\partial}{\partial \chi_\mathrm{E}} P_\mathrm{S}$. We shall state the specific cost optimality conditions and associate a specific unit cost of service production to each scooter attribute.

6.1 The optimal lifespan

From Table 1, the sensitivity coefficient of production costs to L_S is

$$\frac{\partial C_\Sigma}{\partial L_\mathrm{S}} = f_\mathrm{S}\left(\dot{P}_{\mathrm{S}|L} - \frac{p_\mathrm{S}}{L_\mathrm{S}}\right) + \widetilde{N}\,\frac{\partial \Gamma_\mathrm{N}}{\partial L_\mathrm{S}} \tag{6.1}$$

Indeed, the influence of L_S on production costs pertains to (i) scooter inventory and (ii) scooter maintenance as stated in Γ_N.

The service supplier's behaviour of cost minimization with respect to L_S consists in giving it the value that satisfies the first-order optimality condition, namely

$$\frac{\partial C_\Sigma}{\partial L_\mathrm{S}} = 0$$

It is equivalent to

$$\dot{P}_{\mathrm{S}|L} = \frac{p_\mathrm{S}}{L_\mathrm{S}} - \frac{\widetilde{N}}{f_\mathrm{S}}\,\frac{\partial \Gamma_\mathrm{N}}{\partial L_\mathrm{S}} \tag{6.2}$$

Recalling our assumption that $\partial\Gamma_\mathrm{N}/\partial L_\mathrm{S} \leq 0$, the condition states that marginal scooter cost will be in excess of average cost. The ratio $\widetilde{N}/f_\mathrm{S}$ between scooter inventory and its daily renewal flow measures the average scooter lifetime in number of days, according to Little's law. Product $\frac{\widetilde{N}}{f_\mathrm{S}}\,\frac{\partial \Gamma_\mathrm{N}}{\partial L_\mathrm{S}}$ states the marginal influence of L_S on the maintenance costs over the scooter lifetime.

Under scooter price with constant elasticity $\epsilon_{\mathrm{S}|L}$ to lifespan i.e. $\mathrm{P_S} = P_{\mathrm{S}0}(\frac{L_\mathrm{S}}{L_{\mathrm{S}0}})^{\epsilon_{\mathrm{S}|L}}$, eqn. (6.2) implies that

$$(\epsilon_{\mathrm{S}|L} - 1)\frac{\mathrm{P_S}}{L_\mathrm{S}} = -\frac{\widetilde{N}}{f_\mathrm{S}}\,\frac{\partial \Gamma_\mathrm{N}}{\partial L_\mathrm{S}} \tag{6.3}$$

As $f_\mathrm{S} = Q_\mathrm{L}/L_\mathrm{S}$ the optimum lifespan would satisfy that

$$(\epsilon_{\mathrm{S}|L} - 1)\frac{P_{\mathrm{S}0}}{(L_{\mathrm{S}0})^{\epsilon_{\mathrm{S}|L}}} L_S^{\epsilon_{\mathrm{S}|L}-2} = -\frac{\widetilde{N}}{Q_\mathrm{L}}\,\frac{\partial \Gamma_\mathrm{N}}{\partial L_\mathrm{S}}$$

i.e.

$$L_S^* = \left(\frac{(\epsilon_{\mathrm{S}|L} - 1)P_{\mathrm{S}0}}{L_{S0}^{\epsilon_{\mathrm{S}|L}}\left(-\frac{\partial \Gamma_\mathrm{N}}{\partial L_\mathrm{S}}\right)}\frac{Q_\mathrm{L}}{\widetilde{N}}\right)^{\frac{1}{2-\epsilon_{\mathrm{S}|L}}} \tag{6.4}$$

Wherein $Q_\mathrm{L}/\widetilde{N}$ is the daily commercial traffic per scooter.

6.2 The optimal energy consumption factor

The sensitivity of production costs to χ_{E} is more complex: from Figure 2,

$$\frac{\partial C_{\Sigma}}{\partial \chi_{\mathrm{E}}} = f_{\mathrm{S}}.\dot{P}_{\mathrm{S}|\chi} + \frac{\partial C_{\Sigma}}{\partial Q_{\mathrm{F}}}\frac{\partial Q_{\mathrm{F}}}{\partial \chi_{\mathrm{E}}} + \frac{\partial C_{\mathrm{E}}}{\partial \chi_{\mathrm{E}}}$$

We know that $\frac{\partial C_{\Sigma}}{\partial Q_{\mathrm{F}}} = V_{\mathrm{J}} + \frac{p_{\mathrm{B}}}{R_{\mathrm{B}}}$. As $\frac{\partial Q_{\mathrm{F}}}{\partial \chi_{\mathrm{E}}} = \frac{Q_{\mathrm{L}}}{\sigma A}$, it comes out that

$$\frac{\partial C_{\Sigma}}{\partial \chi_{\mathrm{E}}} = \frac{Q_{\mathrm{L}}}{L_{\mathrm{S}}}\dot{P}_{\mathrm{S}|\chi} + \frac{Q_{\mathrm{L}}}{\sigma A}(V_{\mathrm{J}} + \frac{p_{\mathrm{B}}}{R_{\mathrm{B}}}) + p'_{\mathrm{E}}Q_{\mathrm{L}} \tag{6.5}$$

The supplier behaviour of cost minimization with respect to χ_{E} consists in assigning to it the value that satisfies the first-order optimality condition,

$$\frac{\partial C_{\Sigma}}{\partial \chi_{\mathrm{E}}} = 0$$

It is equivalent to

$$\dot{P}_{\mathrm{S}|\chi} = -L_{\mathrm{S}}\left(p'_{\mathrm{E}} + \frac{1}{\sigma A}(V_{\mathrm{J}} + \frac{p_{\mathrm{B}}}{R_{\mathrm{B}}})\right) \tag{6.6}$$

At optimum, the marginal value of the energy consumption factor is proportional to the scooter lifetime length, with a coefficient that includes the effective cost per unit of fed-in energy p'_{E} plus the holistic cost per swapping divided by the refill productivity σA.

6.3 The holistic cost of fed-in-scooter energy

We may extend the lifecycle perspective to the scooter body as well. The energy related part of the scooter price, say ΔP_{S}, is amortized over r_{S} refills each of σA energy flow. Thus the scooter-specific cost per unit of energy flow is

$$p_{\mathrm{S}}^{(\mathrm{E})} \equiv \frac{\Delta P_{\mathrm{S}}}{r_{\mathrm{S}}\sigma A} = \frac{\Delta P_{\mathrm{S}}}{\chi_{\mathrm{E}}L_{\mathrm{S}}}$$

By gathering the respective costs, we obtain the overall service cost per unit of fed-in energy: this holistic energy unit cost amounts to

$$\hat{p}_E \equiv p'_{\mathrm{E}} + \frac{1}{\sigma A}(V_{\mathrm{J}} + \frac{p_{\mathrm{B}}}{R_{\mathrm{B}}} + \frac{\Delta P_{\mathrm{S}}}{r_{\mathrm{S}}}) \tag{6.7}$$

Put in words, the marginal service cost per energy unit fed in the scooter includes the energy provision price augmented by charging losses, plus the cost of carrying the energy up to the scooter and those of using the battery and the scooter.

Under optimal capacity and linear prices (cf. Appendix B, eqn. (B5)), then $\Delta P_{\mathrm{S}} = r_{\mathrm{S}}V_{\mathrm{J}}$ so that the holistic energy unit cost becomes

$$\hat{p}_E = p'_{\mathrm{E}} + \frac{1}{\sigma A}(2V_{\mathrm{J}} + \frac{p_{\mathrm{B}}}{R_{\mathrm{B}}}) \tag{6.8}$$

6.4 On the service cost per unit of ride length

The production cost function C_Σ constituted in Section 3 can be restated as follows:

$$C_\Sigma = C_\mathrm{U} + C_\mathrm{E} + C_\mathrm{FJ} + C_\mathrm{S} + C_\mathrm{B}$$

$$= c_\mathrm{u} Q + p'_\mathrm{E} Q_\mathrm{E} + C^0_\mathrm{F} + (p_\mathrm{F} + \gamma_\mathrm{N})\tilde{N} + \gamma_\mathrm{B}\tilde{B} + \gamma_\mathrm{J} n_\mathrm{J} + w_\mathrm{J} n_\mathrm{J} + p_\mathrm{S} f_\mathrm{S} + p_\mathrm{B} f_\mathrm{B}$$

$$C_\Sigma = C^0_\mathrm{F} + (p_\mathrm{F} + \gamma_\mathrm{N} + \gamma_\mathrm{B})N + Q\left(c_\mathrm{u} + \ell_R \left\{\frac{\mathrm{P_S}}{L_\mathrm{S}} + \chi_E \left[p'_\mathrm{E} + (A\sigma)^{-1}\left\{\frac{\mathrm{P_B}}{\mathrm{R_B}} + V_\mathrm{J}\right\}\right]\right\}\right) \tag{6.9}$$

The holistic cost per energy unit encompasses the term factored by χ_E plus the battery-related part of the scooter price. The term factored by ℓ_R is a service cost per unit of ride length: it encompasses the swapping process, energy charging, the scooter and battery inventories and part of the field implementation costs – notably $Q_\mathrm{F} V_\mathrm{J}$ involves the field costs for that part of the fleet that is waiting for swapping. Let us denote it as:

$$c_\ell \equiv \frac{\mathrm{P_S}}{L_\mathrm{S}} + \chi_E \left[p'_\mathrm{E} + (A\sigma)^{-1}\left\{\frac{\mathrm{P_B}}{\mathrm{R_B}} + V_\mathrm{J}\right\}\right] \tag{6.10}$$

We may further extend the assignment of production costs to ride lengths by adding to c_ℓ:

- The commercial costs per ride length unit, i.e. c_u/ℓ_R,
- The other costs i.e. fixed field cost C^0_F and fleet-related costs $(p_\mathrm{F} + \gamma_\mathrm{N} + \gamma_\mathrm{B})N$, divided by the service commercial traffic i.e. $Q\ell_R$.

This gives a holistic service cost per unit of ride length:

$$\hat{c}_\ell \equiv c_\ell + \frac{c_\mathrm{u}}{\ell_R} + \frac{C^0_\mathrm{F} + (p_\mathrm{F} + \gamma_\mathrm{N} + \gamma_\mathrm{B})N}{Q\ell_R} \tag{6.11}$$

It is the unit cost that corresponds to the L_S scooter lifespan, in the same way that the holistic unit cost of energy corresponds to the χ_E scooter energy consumption rate.

7. Numerical study

To illustrate the theory, we shall investigate the potential value of the per-refill cost and beyond it of battery price, scooter price, inventory costs and the service total cost per unit of fed-in energy. After establishing plausible ranges for the different model parameters (§7.1), we will study juicing logistics and quantify the per-refill cost (§7.2). Turning to battery wearing, we will model wearing laws on the basis of empirical data and prepare for DoD optimization (§7.3). Then we can address the optimization of both DoD and battery energy capacity A, by parametric analysis according to the scooter price sensitivity to A, $\dot{P}_{\mathrm{S|A}}$ (§7.4). After that, we will draw consequences on the component lifetimes in number of refills (§7.5), on the scooter lifespan (§7.6), on the component prices and their per-refill values (§7.7) and on the S3 cost per unit of fed-in energy (§7.8).

7.1 Setting up the parameters

Let us put forward numerical values and ranges for the model parameters.

Field costs. Our interest lies in variable costs, hence in the per-vehicle daily cost $\tilde{p}$: it is a complex parameter that involves regulatory fee p_{F} together with telecommunication, smart payment, maintenance, insurance and other administration costs $\gamma_{\mathrm{N}} + \gamma_{\mathrm{B}}$. Regarding regulatory fees, a twofold per-scooter fee of \$50 per year plus \$1 per day was reported by Griswold (2019) for Louisville; another two-fold combination of \$1 per day plus \$.25 per ride is mentioned about Portland by Button et al. (2020). As the latter combination applied to 5 rides per day, all in all we shall consider a regulatory per-scooter fee of say €1.5/day. The telecommunication cost may amount to €2 per month per scooter, i.e. about €.065 per day. An insurance cost of \$.05 per ride (Griswold, 2019) is turned in a €.25 per day. As for maintenance, per ride costs of \$.50 and \$.32 were reported for Bird's 1st and 2nd generations with associated average ride length of about 2km (BCG 2020): taking a unit cost at €.17/km, we will multiply it by the daily commercial traffic per scooter, at either 7 km (ITF, 2020), or 12 or 20 km to aim for better productivity, yielding €1.0 / 1.7 / 3.4 as base / middle / high values. As it pertains to e-scooters including batteries, the corresponding parameter is $\gamma_{\mathrm{N}} + \gamma_{\mathrm{B}}$. Assuming that $\gamma_{\mathrm{B}} = \gamma_{\mathrm{N}}/2$, its low / middle / high values are €.33 / .57 / 1.1. All in all, we will take €3.1 / 4.1 / 6.3 as low / middle / high values for $\tilde{p}$.

Juicing issues. Under French conditions typical of 2019 (Insee, 2021), the daily cost of a juicer duty may be estimated by assuming a gross salary of €1800/month (2nd decile in wage distribution of full-time workers), plus 65% of social contributions, times 3 employees per duty, and 30 days in the month: we thus obtain a daily cost w_{J} of almost €300. As the wages are relatively low, in France they may be exempted from social contributions, yielding a 40% abatement. The service operator could further reduce its costs by reducing the number of employees per duty measured in "full time equivalent" (fte): our initial estimate of 3 could be reduced to 2 fte, possibly by employing part-time workers rather than full-time ones. This could yield a further reduction of say 30%. To sum up, applying a 40% drop followed by a 30% drop starting from €300/day for w_{J}, would reduce it to 42% of €300, i.e. €126/day. To include the juicer vehicle, we shall add an amount of €10 to €20 per day. Concerning the duty-based daily expense for depot administration and service management, γ_{J}, let us make a rough estimate: 10m² per duty as resting space in depot, at floor rental price of say €1/m² per day, makes €10/day: doubling it for management expenses yields γ_{J} at €30/day. All in all, we obtain a range from €165 to €347 per day for $\tilde{w}_{\mathrm{J}}$.

Assuming average charging and waiting time t_{R} of 12h, the fixed part $V_{\mathrm{J}}^{0} \equiv \gamma_{\mathrm{B}} t_{\mathrm{R}}/\mathrm{H}$ of the unit swapping cost will take low / middle / high values of €.17 /.28 / .57.

The time parameters t_{0C}, t_{SB} and t_D are also important for juicer duties and tour productivity. The range for base cycle time t_{0C} is 15-90 min: within this rather broad interval, local conditions will be decisive to narrow down a specific estimate. As for depot time t_D, a lower bound could be 15 minutes for a very short break of 10 min added to 3 min of battery manipulation and 2 min of vehicle parking: driving manoeuvres, door opening and closing etc. An upper bound of 30 min would enable for a more proper break of 20 min, combined to 2 min of vehicle manoeuvres and 8 min of battery manipulation – this task should indeed be kept short, by suitable depot layout and logistical organization of the depot process. Coming to the inter-swap time t_{SB}, which encompasses not only the time to change batteries at a plugging spot ([1]) but also the time to go from one such spot to the next one, we shall consider low / middle / high values of 2' / 3' / 5'.

Energy and battery issues. An average energy price of €.10/kWh (excluding VAT) applied to firms in France as of 2020 (Datalab, 2021). Yet there is a rising trend in French prices because the share of renewable sources is growing in the national electricity mix. Let us then consider a range from €.10/kWh to €.40/kWh for p_E. By involving an efficiency factor θ_E of about 90% (Ellingsen et al. 2016), we obtain a range from €.11/kWh to €.44/kWh for p'_E.

Coming to battery price p_B, a range of $100-500 was reported by Strobel (2021), encompassing batteries for all kinds of kicked e-scooters from low-end models with battery capacity less that .1kWh, to mid-tier with BEC of .2-.3kWh, and up to high-end with BEC at 1.2kWh enabling for 80+km range. For shared e-scooters with swappable batteries we consider the .3 – 1.0 kWh range of energy capacity and the price range of €200-400 per battery. On assuming battery price linear with respect to energy capacity, we obtain price sensitivity to capacity $\dot{P}_{B|A}$ of €286/kWh of battery capacity. Knowing that in 2020 the price of LIB cells had fallen down to €120/kWh for NMC and €100/kWh for LFP (Henze, 2020), the factor of 286/110≈2.6 would account for the manufacturing of e-scooter swappable batteries from cell components. Yet the operator is likely to obtain bulk prices with reduction of say 40%, making a low value of €170/kWh for $\dot{P}_{B|A}$. Concerning the lifetime number of refills R_B and the associated lifetime energy intensity E_B, we shall consider two different chemistries of Lithium-Ion Batteies, hence two alternative pairs of parameters (E_1, α) for the Constant Elasticity technology as well as two alternative pairs of parameters (E_0, b) for the Affine Linear technology: the details are given in §7.3.

Scooter-related parameters. Kamps (2018) inferred a bulk price of $300 for a 1st generation shared e-scooter and conceived a target price of $800 for a Next generation model that came later to existence with much sturdier design and swappable batteries. BCG (2020) reported Bird's per scooter acquisition costs of $375 for 1st generation in 2018 and $630 for 2nd generation in 2019. These figures are fairly consistent with public unit prices reported by Strobel (2021): about $300 for an entry-range adult model, mid-tier models between $500-1500 and high-end models above $1500, all including batteries. Considering an $800-1,800 range of upper mid-tier and lower high-end models, with corresponding battery energy capacity ranging from .3 to 1.0 kWh, adding connectivity device at say $80 (Kamps, 2018) but subtracting the swappable battery from the e-scooter price, applying bulk reduction factor of 40% and converting to euros at 1:1 rate, we obtain a price range of €528-888 for a shared e-scooter ready for swappable battery but without it. The range variation of Δp_S=€360 can be related to three factors: (i) sturdier design hence increased lifespan L_S, (ii) larger battery hence increased A, and (iii) improved equipment – motors, braking system, handles and controls,

[1] Voi (2020) claims that 30 seconds would be enough for a rider to change batteries at a plugging spot

tyres and lights (Strobel, 2021). Let us assume respective shares of 60%, 10% and 30% of range variations.

The influence of battery capacity may then be estimated as $\dot{P}_{S|A} = \frac{10\%.\Delta p_S}{1.0-0.3} = €51$/kWh. We take it as low-end estimate and also consider values in €/kWh of 102 and 153 that would respectively attribute 20% and 30% of Δp_S to ΔA.

As for scooter lifespan L_S, in the Introduction (§1.3) we mentioned reference values that are still under construction: taking 14,000 km as high-end and 4,000 km as low-end, the sensitivity coefficient of p_S to L_S may be estimated as $\dot{P}_{S|L} = \frac{60\%.\Delta p_S}{14{,}000-4{,}000} = €.022$/km. But setting the high-end lifespan to 7,000 km (ITF, 2020) would yield a $\dot{P}_{S|L}$ of €.07/km: we shall consider values of 0.02 / 0.05 / 0.07 in € per km for $\dot{P}_{S|L}$.

Regarding the energy consumption rate χ_E, an average value of 0.014kWh/km was reported in Ademe (2016): let us take 0.015 as mid-tier and 0.020 as high-end. The rate is expected to influence scooter price p_S in a decreasing way. To our knowledge, no regenerative braking has yet been implemented on commercially-available scooters. Such set-up and others such as special tyre specification could entail a 30% reduction in our initial estimate: let us then consider a [.010, .020] kWh/km interval for χ_E.

Table 2 summarizes our set of numerical assumptions. Parameter t_0 of base time per tour adds up the tour base run time t_{0C} and the depot time t_D.

Tab. 2: Parameters and their ranges.

Notation	Meaning	Unit	Low value	Mid-tier	High value
$\tilde{p}$	Per-vehicle cost	€/day	2.8	3.5	5.2
$\widetilde{w}_J$	Augmented juicer duty cost	€/day	165	223	347
V_J^0	Fixed part of unit swap cost	€	.13	.28	.37
t_0	Base time per tour	Min	30	70	110
$\dot{t} = t_{SB}$	Juicer inter-swap time	Min	2	3	5
p'_E	Effective Energy price	€/kWh	.11	.20	.44
p_B	Battery price	€	100	200	400
$\dot{P}_{B\|A}$	Sensitivity to capacity	€/kWh	170	286	350
p_S	Scooter price (body only)	€	250	528 *A=.3kWh*	888 *A=1.0kWh*
$\dot{P}_{S\|A}$	Sensitivity to capacity	€/kWh	51	102	153
$\dot{P}_{S\|L}$	Sensitivity to lifespan	€/km	.02	.05	.07
L_S	Scooter lifespan	Km	4,000	7,000	14,000
χ_E	Energy consumption	kWh/km	.010	.015	.020

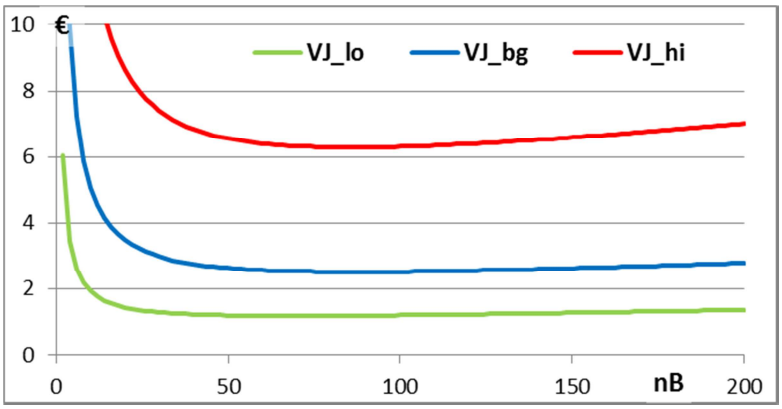
7.2 Juicing logistics and the per-refill cost

Prior to any optimization, let us study the variables in juicing logistics. Assuming an ex-ante tour productivity n_B of 30, by combining the values at each level we obtain tour times T_J of 90, 160 or 260 minutes respectively. Furthermore, involving the low, best guess and high values of the economic parameters $\tilde{p}$ and w_J, we obtain per-refill costs V_J of 1.3, 3.0 and 7.4 euros respectively. The higher value compares to the €5 fee paid to independent juicers in the early times of S3 services.

Coming now to optimization, the three set-up levels yield refill tour productivity n_B^* of 69, 87 and 85 respectively: the values are fairly consistent and quite high. The related tour times T_J^* would be of 168, 332 and 536 minutes respectively – i.e. all significantly high, from 40% to 110% of an 8 h duty day. The related per-refill costs, V_J^*, would be of 1.18, 2.52 and 6.3 euros respectively: the variations mirror essentially those of the $\dot{t}.\tilde{w}_J/H_J$ part which dominates the other part $t_0.\tilde{p}/2H$ by two orders of magnitude.

Thus the optimization enables for some savings of 10% to 20% compared to base case at n_B =30. Admittedly, the outreach of optimization depends strongly on the base case specification (figure 4): setting n_B =20 yields per-refill costs V_J of 1.45, 3.5 and 8.6 euros respectively, with related savings of 20% to 30%. For the following steps of numerical exploration, we shall consider a range of 1 to 6 euros for the per-refill cost V_J, together with an ambitious target of €.5. Table 3 gathers the results and also points to the ratio of juicer duties and refills, n_J/Q_F. As a point of reference, the Tier service that involves its riders to swap the batteries by themselves rewards each swap by one free unlock worth €1 plus a free ride worth €.2 times the ride length in km, say €.8 for a 4 km ride, totalling €1.8 per swap (Scammell, 2020).

Fig. 4: Per-refill swapping cost according to tour productivity n_B**.**



Tab. 3: From prior beliefs to optimized values in juicing logistics.

	Prior to optimization			With optimization		
	Low	Median	High	Low	Median	High
n_B	30	30	30	69	87	85
T_J (min)	90	160	260	168	332	536
V_J (€)	1.3	3.0	7.4	1.18	2.52	6.3
n_J/Q_F	.006	.011	.018	.0047	.0071	.012

7.3 Battery wearing laws and DoD optimization

The wearing law $\sigma \mapsto R_B$ and the associated functions E_B and ψ_B play a key role in battery optimization. We introduced two alternative mathematical specifications for technologies (E_B, R_B, ψ_B), namely Constant Elasticity (CE) in Section 5 vs. Affine Linear (AL) in Appendix B:

- CE function $E_B = E_1 \sigma^{-\alpha}$ with elasticity $-\alpha$ and minimum lifetime energy intensity E_1,
- AL function $E_B = E_0(1 - \sigma/b)$ with linear coefficient $-E_0/b$ and maximum lifetime energy intensity E_0.

At total discharge with $\sigma = 1$, consistency is expected between values of E_1 on the CE side and $E_0(1 - 1/b)$ on the AF side; they should also coincide with $R_B(1)$. But at $\sigma = 0$ value E_0 is finite on the AL side whereas $E_1\sigma^{-\alpha} \to \infty$ on the CE side, in sharp contrast.

Both specifications need be tailored for any given chemistry of electric battery. We referred to "expert-say-that" data supplied by the Battery University ([2]) about two chemistries of Lithium-Ion Batteries (LIB), namely NMC for Nickel-Manganese-Cobalt and LFP for Lithium-Fer(Iron)-Phosphate as the chemical components of the cathode part in the electric cells. Table 4 provides the original R_B data and the associated predictions for the two chemistries. Figure 5 depicts both the wearing functions $R_B(\sigma)$ (left part) and the associated $E_B(\sigma)$ (right part).

Under CE the respective minimum intensity parameters E_1 are about 300 for NMC vs. 620 for LFP i.e. more than double; the respective elasticity parameters α are about .25 for NMC vs. .44 or .68 for LFP – depending on taking the first data point at $\sigma = .1$ in or out of the regression.

Under AL the respective maximum intensity parameters E_0 are about 530 for NMC vs. 1771 for LFP i.e. thrice. The respective b parameters are close to 2 for NMC and to $\sqrt{2}$ for LFP.

Tab. 4. Estimation of wearing laws and lifetime energy intensity laws.

NMC chemistry						
Sigma	R_B_Data	R_B_AL	R_B_CE	E_B_Data	E_B_AL	E_B_CE
0,1	6 000	5 035	5 459	600	504	546
0,2	2 000	2 385	2 279	400	477	456
0,4	1 000	1 060	952	400	424	381
0,6	600	618	571	360	371	343
0,8	400	398	397	320	318	318
1	300	265	300	300	265	300

LFP chemistry						
Sigma	R_B_Data	R_B_AL	R_B_CE	E_B_Data	E_B_AL	E_B_CE
0,1	15 000	16 454	15 975	1 500	1 645	1 597
0,2	9 000	7 599	5 888	1 800	1 520	1 178
0,4	3 000	3 171	2 170	1 200	1 269	868
0,6	1 500	1 696	1 210	900	1 017	726
0,8	900	958	800	720	766	640
1	600	515	580	600	515	580

[2] https://batteryuniversity.com/article/bu-808-how-to-prolong-lithium-based-batteries

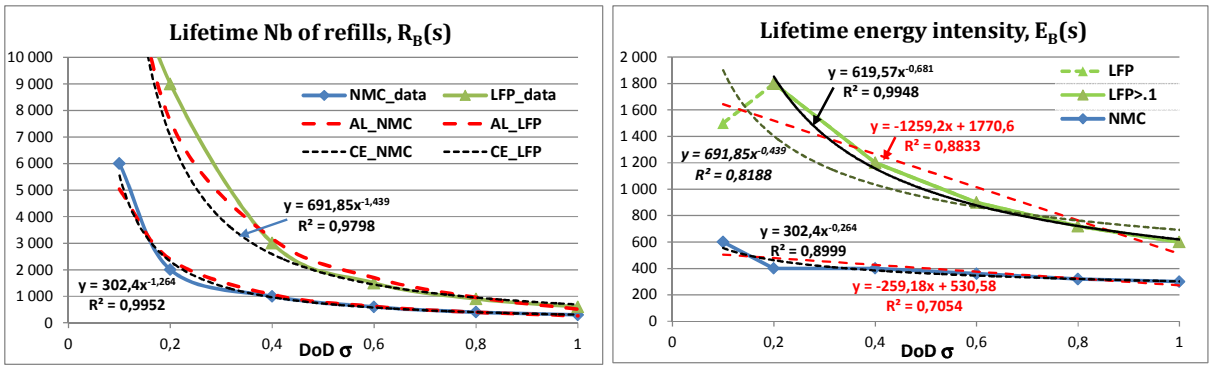


Fig. 5: CE and AL functions R_B ***(left) and*** E_B ***(right) depending on the LIB chemistry.***

For a given chemistry, the matching $E_1 \approx E_0(1 - 1/b)$ at $\sigma = 1$ is expected to hold between CE and AL, all the more so as both estimations come from the same dataset: the relative difference is limited to 10% for NMC and to 15% for LFP, which looks tolerable.

Note that while the LFP seems to dominate NMC, it is in fact less used in electric mobility because its energy density per mass unit is lower by one third: the storage of 1 kWh requires about 6 kg of NMC LIB but 8 kg of LFP LIB (Battery University, 2019).

Turning to the associated functions ψ_B and $\psi_B^{(-1)}$, it comes out that the functional specification, CE or AL, is much more determinant than the LIB chemistry, NMC or LFP. Given the specification, the $\psi_B^{(-1)}$ functions of the two chemistries are fairly close. Figure 6 shows that under AL the respective laws $\psi_B^{(-1)}$ of NMC and LFP differ by at most .1 on the [0, .006] range of abscissas. Under CE the laws $\psi_B^{(-1)}$ are even closer to each other. But both CE functions increase much more quickly than their AL counterparts. Looking at the value y at which function $\psi_B^{(-1)}$ reaches 1, i.e. the threshold value $\psi_B(1)$ from which Full discharge becomes the best strategy for DoD management, the CE functions predict values of $y = \psi_B(1) = \alpha/E_1$ around .0008 (.00087 for NMC vs. .00076 for LFP) while the AL functions predict theoretical values of $\psi_B(1) = b/((b-1)^2 E_0)$ that amount to .0038 for NMC vs. .0048 for LFP: the one fourth relative difference between the two chemistries under AL is minor compared to the factor 4 (for NMC) or 6 (for LFP) between the AL and CE outcomes.

Let us summarize that CE laws predict threshold $\psi_B(1)$ of about 0.8‰ vs. about 4‰ according to AL laws. These values represent ratios of about 1/1200 or 1/250 for comparison to ratio V_J/P_B that is key to DoD optimization. Thus, an intermediary value of 2‰ i.e. 1/500 gives a gross order of magnitude for the split between Partial discharge policies (below threshold) and Full discharge policies (above threshold). Yet the factor of 10 between the respective thresholds of the theoretical laws calls for further statistical analysis of battery wearing.

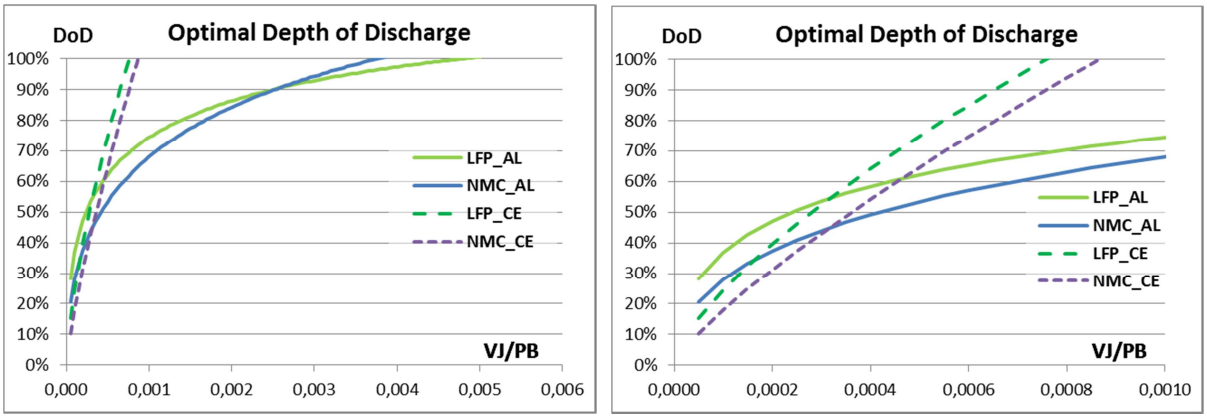


Fig. 6: The CE and AL $\psi_B^{(-1)}$ functions, depending on the LIB chemistry.

In the next sub-sections we will rely upon AL technologies only: this is because our empirical ground for scooter and battery prices with respect to battery energy capacity is scarce, so that affine linear price functions look more robust than constant elasticity functions.

7.4 Optimal battery energy capacity

While DoD optimization only involves the cost ratio V_J/P_B, that of battery energy capacity A depends essentially on the sensitivities of battery and scooter prices to A, with respective sensitivity coefficients $\dot{P}_B$ and $\dot{P}_{S|A}$. As for $\dot{P}_B$ we take a median estimate of €250/kWh.

From §5.4 we know that Partial discharge is optimal when A is greater than its specific threshold value

$$\underline{A} \equiv \frac{V_J}{\dot{P}_B.\psi_B(1)}$$

Under AL values $\psi_B(1)$ of 4‰, the value €250/kWh of $\dot{P}_B$ makes $\dot{P}_B.\psi_B(1) \approx$€1/kWh, thereby enabling for a quick numerical equivalency between $\underline{A}$ in kWh and V_J in euros. Under $V_J \geq$ €2 then Partial Discharge would require BEC beyond 2kWh – which is impractical with respect to battery mass and scooter weight.

Let us then focus on the scooter price sensitivity to capacity, $\dot{P}_{S|A}$. When Full discharge is optimal, under linear battery price and affine linear scooter price the optimal BEC satisfies (5.19) i.e.:

$$A^* = \sqrt{\frac{\chi_E.L_S}{\bar{\sigma}}\frac{V_J}{\dot{P}_{S|A}}}$$

Thus under Full discharge we expect A^* to increase with V_J, χ_E and L_S (proportionally to their square roots) and to decrease with $\dot{P}_{S|A}$ (proportionally to $(\dot{P}_{S|A})^{-.5}$).

As there is little evidence about it from market prices and no specific study available, we shall study its specific variations over a large range of 5 to 500 euros per kWh of battery capacity, within which the 50-150 sub-range looks more plausible.

Figure 7 depicts the variations of A^* (left part) and the implications on σ^* (right part) according to $\dot{P}_{S|A}$, assuming also $\dot{P}_B =$ €250/kWh, χ_E =.015 kWh/km and L_S =10,000 km, considering the two battery chemistries and different values of V_J, namely .5, 1, 3 and 6 euros.

It turns out that the two chemistries yield very similar optimal capacities but specific optimal DoDs: Partial discharge is optimal for LFP on larger ranges than for NMC.

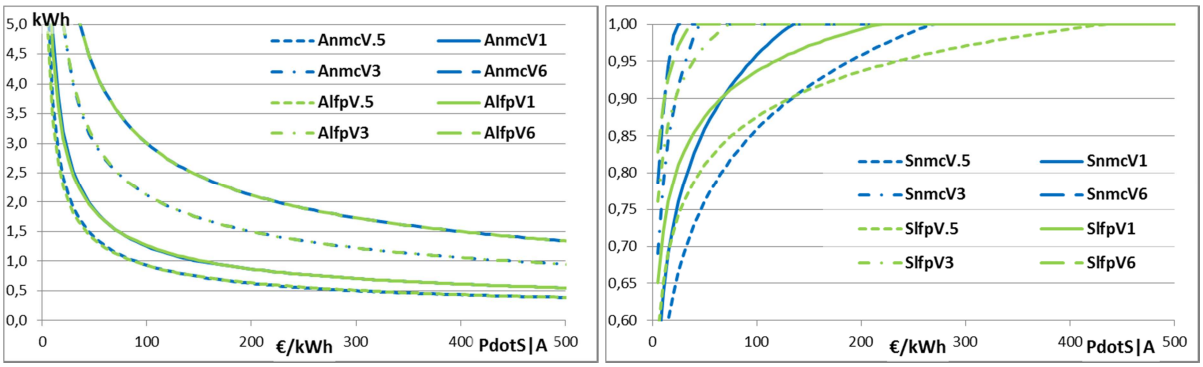


Fig. 7. Optimal capacity (left part) and DoD (right part) according to $\dot{P}_{S|A}$.

Given $\dot{P}_{S|A}$, both the optimal capacity and the optimal DoD increase with V_J: higher per-refill swapping cost triggers the development of the $A.\sigma$ amount of energy per refill, so as to reduce swapping frequency per scooter as well as per battery.

Given V_J, increasing $\dot{P}_{S|A}$ decreases the optimal capacity, from high values of several kWh down to lower ones that keep above .5 kWh. The mass requirements make the values above 2 kWh unsuitable for implementation. At $V_J = €3$ the best guess range of [50, 150] yields optimal capacities decreasing from 5 down to 2.5 kWh – quite impractical masses. The corresponding ranges of variations are from 3 down to 1.5 kWh under $V_J = €1$ and from 2 down to 1 kWh under $V_J = €.5$ that would make an ambitious target.

All in all, we may infer that optimal battery energy capacity would lie between 1 and 3 kWh. The lower bound is akin to the capacity of batteries embedded in e-scooters but not swappable (Link, 2020). It is larger than the .7 kWh of Tier's swappable batteries as of 2021 (swapping by service users) and much higher than the .3 kWh capacity of Lime's swappable batteries (company-operated tours).

Let then take the other view around: given capacity A, Partial discharge is cost-optimal if V_J is less than the "ceiling function" $\psi_B(\bar{\sigma}).P_B$ which amounts to $\psi_B(\bar{\sigma}).\dot{P}_B A$ under linear battery price. Under $\dot{P}_B.\psi_B(1) \approx €1$/kWh, capacity A of .3 or .7 kWh requires swapping cost V_J of less than €.3 or €.7, respectively, for Partial discharge to be cost-optimal. Both conditions look unlikely for the service implementations as of 2021.

7.5 Component lifetimes in number of refills

Given V_J and under cost-optimization of both BEC and DoD, the lifetime number of refills per battery, R_B, and per scooter, r_S, vary with $\dot{P}_{S|A}$ in opposite ways, increasing for scooter bodies but decreasing for batteries (figure 8). Even if the optimal capacity and DoD are similar between the two battery chemistries, the battery lifetimes would differ greatly: the LFP one would be twice that of NMC.

Given $\dot{P}_{S|A}$, increasing unit swapping cost V_J would decrease both r_S and R_B, i.e. reduce the frequency of refilling per scooter as well as per battery.

Figure 8 shows that low per-refill swapping cost V_J and/or low sensitivity of scooter price to capacity can make Partial discharge the most economical policy: in turn, Partial discharge enables for higher lifetime energy intensity i.e. for better use of the battery in a circular economy perspective.

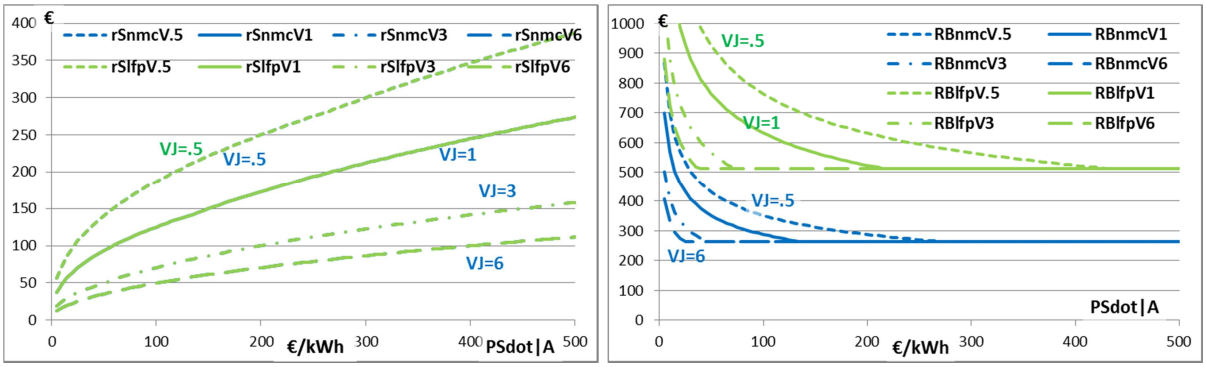


Fig. 8. Lifetime refill number of scooters (left) and batteries (right) according to $\dot{P}_{S|A}$.

Regarding scooter bodies, the lifetime number of refills involves primarily χ_E, L_S and the optimal A^*, together with the optimal σ^*. It does not depend on the battery chemistry (yet the lower energy density of LFP would induce a higher $\dot{P}_{S|A}$, all other things equal). When full discharge is optimal, r_S varies with A^* in a simple way. The variations of r_S according to $\dot{P}_{S|A}$ exhibit an increasing trend which is sub-linear: the reason is that higher $\dot{P}_{S|A}$ yield lower A^*, hence requiring more frequent refills over a given lifespan, thereby mitigating part of the increasing influence of $\dot{P}_{S|A}$.

On comparing the respective lifetime refill numbers between the batteries and the scooter bodies, we will get some idea of which kind of component has the longer technical lifetime in an S3 system. The idea is quite rough because we should compare $T_B \equiv \tilde{B}/f_B$ and $T_S \equiv \tilde{N}/f_S$ to put the issue clearly. On assuming that $\tilde{B}$ and $\tilde{N}$ are about the same, say $N' \ll N$ in theory or $\tilde{B}/\tilde{N} \leq 150\%$ in practice, then f_B/f_S stands as a proxy for T_S/T_B.

From $f_S = Q_L/L_S$ and $f_B = \frac{\chi_E Q_L}{A\sigma R_B}$, we get the following proxy of T_S/T_B:

$$\frac{f_B}{f_S} = \frac{\chi_E L_S}{A\,\sigma R_B} = \frac{r_S}{R_B}$$

Figure 9 depicts the variations of r_S/R_B according to $\dot{P}_{S|A}$ for both battery chemistries, depending on the unit swapping cost V_J. Given chemistry and $\dot{P}_{S|A}$, the lifetime ratio decreases when V_J increases: this is because r_S decreases with V_J more quickly than does R_B, all the more so under Full discharge hence constant, minimal R_B.

For every chemistry and unit swapping cost, the ratio increases with $\dot{P}_{S|A}$ i.e. as the adaptation of P_S to A becomes more costly. The LFP chemistry would definitely make battery lifetime longer than its scooter counterpart. Under the NMC chemistry, battery lifetime is longer than scooter lifetime over the more plausible range of $\dot{P}_{S|A}$.

References on such lifetime issues are (Link, 2020) which indicated the 150% ratio between the numbers of batteries and scooters in a swapping-based service. The respective lifetimes of batteries and scooters were under debate in 2020: Link (2020) and Bird (2020) advocated that embedded batteries would endure twice as much refills as their embedding scooter bodies, but Voi (2020) claimed the more recent generations of scooters to have larger lifespans up to 5 years – a claim too recent at the time to have been tested in the field.

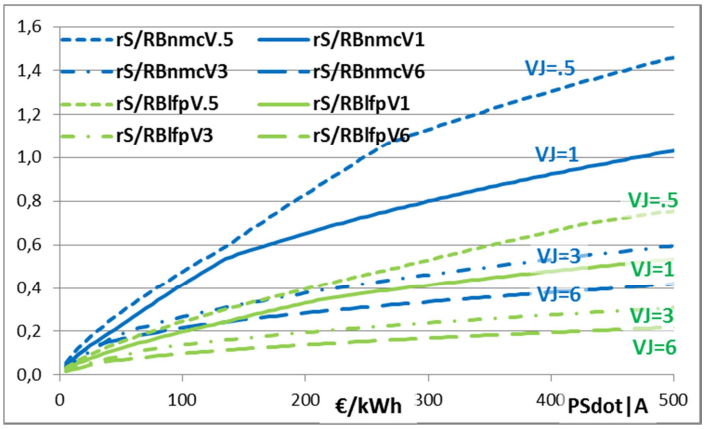


Fig. 9. Ratio r_S/R_B of technical lifetimes with respect to $\dot{P}_{S|A}$, depending on V_J.

For each kind of component the number of refills over the lifetime looks relatively low: down to 300 or 530 for batteries depending on the chemistry and even lower for scooter bodies. The rationale is both technical and economic. For batteries the technical rationale pertains to the chemistry-specific wearing laws and their bottom values, while the economic rationale is that high DoD up to Full discharge is more economical than low DoD. For scooter bodies the technical rationale pertains to the lifespan L_S: we postulated a value of 10,000 km so that larger values twice or n times that would lead to greater lifetime number of refills r_S by a corresponding factor (given χ_E, $\bar{\sigma}$ and A). The economic rationale is the scooter price sensitivity to sturdier design as conveyed by sensitivity coefficient $\dot{P}_{S|A}$.

7.6 On scooter lifespans

On setting up the parameters we established low / middle / high values of γ_N for 2nd generation shared e-scooters with expected lifespan of 7,000 km, based on BCG (2020). This study also provided daily maintenance costs of 1st generation scooters, at about one half more, along with lower lifespan at say 4,000 km. By attributing the change in the per scooter daily maintenance cost between the two generations, $\Delta\gamma_N$, to the change in lifespan, ΔL_S, the ratio $\Delta\gamma_N/\Delta L_S$ constitutes a basis for $\partial\Gamma_N/\partial L_S$. Its low / middle / high values are respectively $-1.\,10^{-4}$ / $-1.9\,10^{-4}$ / $-3.7\,10^{-4}$ in €/km.

Concerning scooter prices, let us infer its elasticity to lifespan on comparing (P_S, L_S) pairs at (€400, 4000km) and (€900, 7000km). The price ratio 9/4 and the lifespan ratio 7/4 yield an elasticity $\epsilon_{S|L}$ of about 1.45. We shall also consider elasticity values of 1.3, 1.5 and 1.6 to enlarge the range of exploration. The following table gives outcomes L_S^* for rounded values of $\partial\Gamma_N/\partial L_S$ and for the different elasticity values, depending on a daily traffic per scooter of either 10 km (left part) or 20 km (right part).

$Q_L/\widetilde{N}$ = 10 km

$-\frac{\partial\Gamma_N}{\partial L_S}$ (€/km) / $\epsilon_{S\|L}$	$1.\,10^{-4}$	$2.\,10^{-4}$	$4.\,10^{-4}$
1.3	2,652	985	366
1.45	4,955	1,405	398
1.5	6,250	1,563	391
1.6	11,023	1,949	344

$Q_L/\widetilde{N}$ = 20 km

$-\frac{\partial\Gamma_N}{\partial L_S}$ (€/km) / $\epsilon_{S\|L}$	$1.\,10^{-4}$	$2.\,10^{-4}$	$4.\,10^{-4}$
1.3	7,139	2,652	985
1.45	17,474	4,955	1,405
1.5	25,000	6,250	1,563
1.6	62,354	11,023	1,949

The table shows that lower sensitivity of maintenance costs to lifespan fosters larger lifespans; only the low maintenance cost values yield plausible optimal lifespans. Larger elasticity yields larger optimal lifespan. Furthermore, doubling the daily traffic per scooter exerts a more than proportional effect on the optimal lifespan.

7.7 Component prices and their per-refill values

Let us now study the component prices that would stem from cost minimization with respect to BEC and DoD. It is assumed again that battery price is a linear function of A with coefficient $\dot{P}_{\mathrm{B}}$ of €250/kWh. As for scooter price we take the median set-up in §7.1 as the base value P_{S0} of the affine linear function $\mathrm{P_S} = P_{\mathrm{S0}} + \dot{P}_{\mathrm{S|A}}.A$. Figure 10 depicts the respective variations of $\mathrm{P_S}$ and $\mathrm{P_B}$ with respect to $\dot{P}_{\mathrm{S|A}}$, given V_{J} and battery chemistry. The component prices vary with $\dot{P}_{\mathrm{S|A}}$ in opposite ways, increasing for scooters but decreasing for batteries.

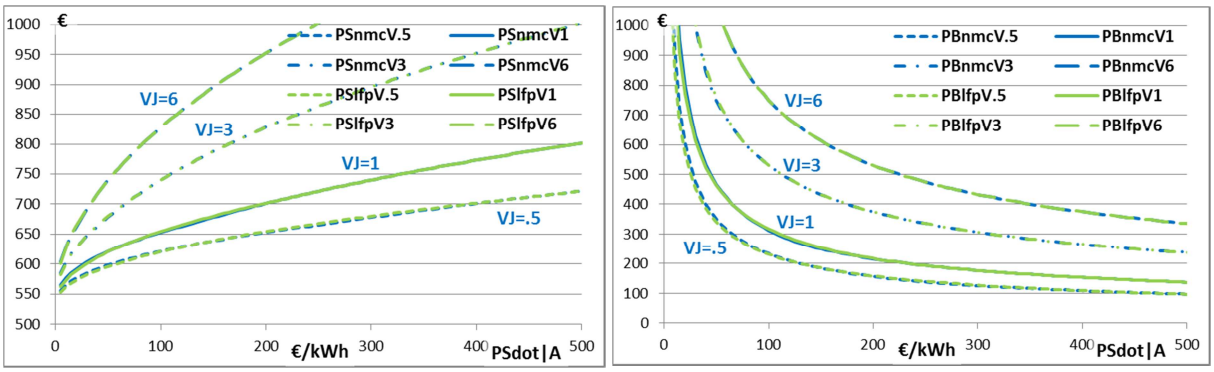


Fig. 10. Scooter price (left) and battery price (right) with respect to $\dot{P}_{\mathrm{S|A}}$.

The inventory costs of scooter bodies, on one hand, and batteries, on the other hand, are easy to estimate on a per refill basis as $P_{\mathrm{S}}/r_{\mathrm{S}}$ and $P_{\mathrm{B}}/R_{\mathrm{B}}$: figure 11 depicts their variations with respect to $\dot{P}_{\mathrm{S|A}}$ for different per-refill swapping costs V_{J} and the two battery chemistries. Given the chemistry and $\dot{P}_{\mathrm{S|A}}$, both per refill inventory costs increase with V_{J}. The scooter per-refill inventory costs are analogous between the two chemistries. Regarding the battery per-refill inventory costs, the LFP chemistry is much more economical than NMC, yielding half price owing to its more advantageous wearing law.

Note that as the refill numbers r_{S} and R_{B} are inversely proportional to the renewal flows f_{S} and f_{B}, the per-refill component costs are proportional to their respective inventory costs at the service level.

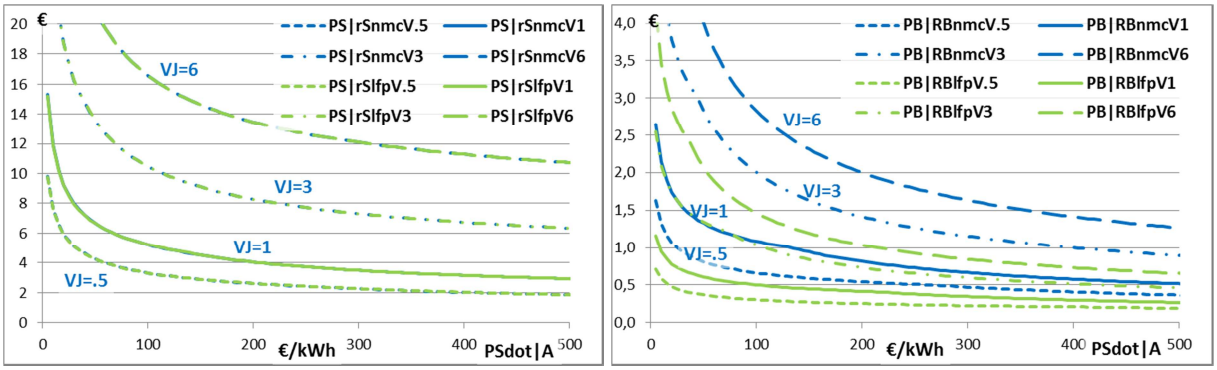


Fig. 11. Per-refill inventory costs of scooters (left) and batteries (right) with respect to $\dot{P}_{\mathrm{S|A}}$.

7.8 On S3 costs per energy unit and scooter optimization

The consumption of one unit of energy entails several costs to the service supplier:

(i) the effective price of fed-in energy p'_{E},

(ii) the swapping cost $V_{\mathrm{J}}/\sigma A$ per unit of energy flow,

(iii) the capacity cost of the battery $P_{\mathrm{B}}/\sigma A R_{\mathrm{B}}$ per unit of energy flow,

(iv) the cost of the scooter battery capacity per unit of energy flow, $(P_{\mathrm{S}} - P_{\mathrm{S0}})/\sigma A r_{\mathrm{S}} = \frac{\dot{P}_{\mathrm{S|A}}}{\sigma r_{\mathrm{S}}}$.

At low V_{J} =€.5 the total energy cost per kWh varies from €1.3 to €3.8 according to $\dot{P}_{\mathrm{S|A}}$, i.e. from 6 to 19 times our middle value of p'_{E}. The last step of energy transport is much more expensive than its out-of-the-grid cost (left part in figure 12). This is confirmed at high V_{J} =€6 (right part in figure 12), given which the total energy cost per kWh varies from €1.2 to €12 according to $\dot{P}_{\mathrm{S|A}}$, i.e. from 6 to 60 times p'_{E}.

Denoting $p^{\#}_{\mathrm{E}} \equiv p'_{\mathrm{E}} + (V_{\mathrm{J}} + P_{\mathrm{B}}/R_{\mathrm{B}})/\sigma A$, from (6.6) the product $L_{\mathrm{S}}.p^{\#}_{\mathrm{E}}$ indicates the marginal value of reducing the energy consumption factor χ_{E}: the .5-7 range of $p^{\#}_{\mathrm{E}}$ in €/kWh times L_{S} =10,000 km gives a range of 5-70 thousand €.km/kWh. From the mid-tier value χ_{E} =.015 kWh/km, a 5% decrease can be valued at €40-560 for a shared e-scooter.

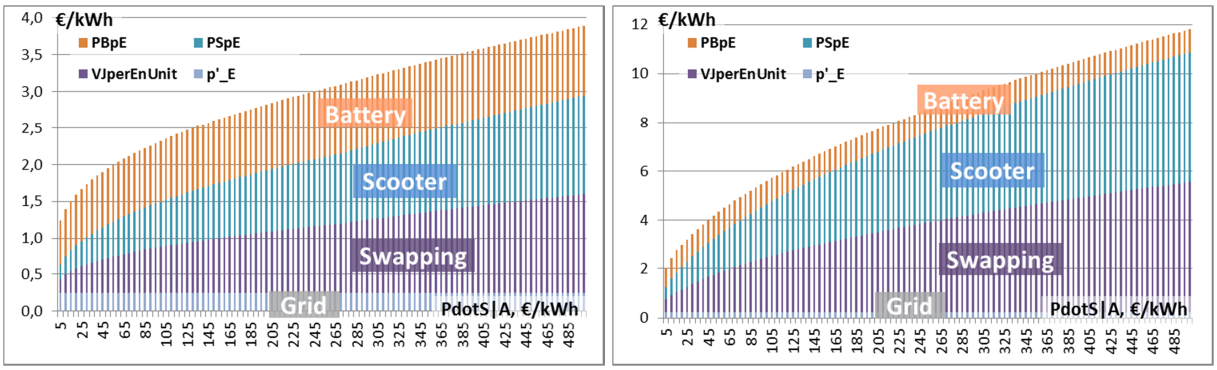


Fig. 12. S3 system costs per unit of energy flow depending on $\dot{P}_{\mathrm{S|A}}$: low vs. high V_{J} .

8. Discussion

Let us now discuss the outreach and limitations of specific parts, say sub-models of the model.

8.1 On the unit swapping cost

By adding up the field implementation costs and the swapping costs, we obtained an affine function of the number of refills, Q_{F}: its coefficient denoted as V_{J} is the unit cost per swapping. The basic V_{J} formula involves the technical conditions of swapping tours, along with costs $\tilde{p}$ per scooter deployed in the field and $\tilde{w}_{\mathrm{J}}$ of augmented juicer wages. From the mathematical optimization of swapping logistics, we obtained more elaborate formulas for V^*_{J} and the associated n^*_{B}, n^*_{J}, T^*_{J}. Beyond the mathematical formulas, the major model property is that the V_{J} variable sums out the sub-models of swapping logistics and field implementation

and constitutes a basic input to the medium and long term management policies concerning target DoD σ, BEC A and scooter body specifications.

It may be conjectured that other processes for swapping, e.g. user-based swapping, are similarly summarized by their specific cost per swapping. Thus, the rest of the management model would still hold for alternative swapping strategies.

8.2 On the swapping strategy by company-operated tours

Our model of an S3 service with swappable batteries only considers a specific process of battery swapping in the form of juicer tours operated by the company. Alternative processes call for specific alternative models. For instance, gig workers could be involved to collect recharged batteries at a company depot, make the swapping tour and bring back the associated depleted batteries: then the term w_J would have a different composition. Alternatively, gig workers could be involved not only to make the swapping tours but also to perform battery charging by their own means; this would lead to specific battery inventory and heterogeneous depot conditions. As for user-based swapping, the depot function would be divided between a number of charging spots, each with specific charging equipment (called a wall or a kiosk): then there would be per-spot costs, specific battery inventory, as well as revenue losses associated to the compensation of the swapping users.

The per-refill time $\dot{t}$ is likely to depend on the specific process of swapping tours: firm economies of operational coordination are expected for company-operated tours. This should be taken into account in a comparison of such processes. Of course, the $\dot{t}$ parameter also depends on the space and time conditions of the service implementation: demand density is a key driver.

On setting up the numerical study we provided some hints of practical composition for the parameter of juicer wages, w_J: it embeds the cost of the purported service vehicle, which we considered lower than the exact juicer wages by one or more order of magnitude. Additional specification would be required to compare cargo e-bikes, on one hand, to vans powered either by fuel or electricity, on the other hand (Bilboe, 2021, Lime, 2020, Voi, 2020). The kind of vehicle will impact not only w_J but also the tour time parameters t_0 and $\dot{t}$, in line with the respective speeds of the vehicles and the right-of-way conditions attributed to bikes and cars, respectively.

Yet an even greater difference pertains to the maximum number of batteries that a tour vehicle may carry: as each battery weighs a couple of kg, assigning two or three tens of them to a cargo-bike looks like a maximum. The weight issue is not covered here and it certainly deserves further research.

8.3 On battery wearing laws and management strategies

We introduced the R_B function and the companion functions E_B and ψ_B as a technology function for battery wearing and lifetime in number of cycles. The mathematical specification is an abstract one. The Affine Linear specification of E_B and its Constant Elasticity counterpart are convenient for model tractability. We estimated each specification for two chemistries of Lithium-Ion batteries, respectively NMC and LFP. The estimation data taken from the Battery University (2019) stem from car batteries that are larger than e-scooter batteries by 2 or 3 orders of magnitude. As the charging conditions in terms of voltage and above all of amperage are quite different, wearing data series should specifically be collected for e-scooter batteries.

The two regression estimations of E_B functions pertain to their respective chemistries and are likely not relevant to alternative chemistries.

We have not considered the progressive reduction of effective capacity along battery life. This certainly deserves further research that will lead to a deeper understanding of the relationship between the target DoD and BEC.

Another issue of battery wearing is specific to swappable ones: that of fixers. In operators and OEMs that did not turn to swappable batteries, some engineers evocated "wire connectors being worn down over time, or enclosure latches" (Eric Barber from Superpedestrian, quoted by Link, 2020). We did not model such wearing which is of another kind than that of battery cells. Neither did our numerical application take into account the effect on battery price of "hinges and locking mechanisms which supplant battery space" (Link, 2020).

8.4 On battery capacity and its price effects

We already mentioned the issue of battery weight that stems from battery capacity in an about proportional way.

The assumption of battery price with respective to battery capacity was taken from electric car makers. It may be less well-suited to e-scooter batteries due to their much smaller sizes.

We specified nothing about the end-of-life treatment of batteries: it may alter the price function, especially if the battery is given a second life in another field of application.

The postulate that the scooter body price, excluding the battery, will depend on the battery capacity may look a heroic assumption. Yet for relatively large scooter batteries, a significant increment in battery capacity induces a significant increase in weight to be distributed over the scooter body, in such way as to limit the inconvenience to the riders. Specific adaptations to the scooter layout are likely to be costly. A heavier battery requires a stronger scooter body and this makes the scooter & battery combo even heavier. This may be inconvenient to potential customers and deter some of them from using the service.([3]) In the model, the scooter price function P_S may include not only the purchase price of the scooter but also the opportunity cost of service characteristics on the demand volume.

The functional specification of the e-scooter price as a linear affine function of the battery capacity can be thought of as a local linear approximation. As the associated local slope, $\dot{P}_{S|A}$, has no obvious reference value, we made it the main dimension of sensitivity analysis in our numerical study and we spanned a very large range for it.

8.5 Scooter body parameters

The second generation of shared e-scooters was purported to be much sturdier than the first one, involving much bulkier bodies and about once more weight: from 11 to 22 kg, including batteries (ITF, 2020). The next generations have gone on the increasing trend, up to weight of 30+ kg as of early 2022, out of which aluminium alloy still contributes more than one half (Okai's models exhibited at the Autonomy meeting in Paris, March 2022). There are even some e-scooter makers such as the Swiss-based Micro-Mobility that aim for everlasting bodies (Hénin, 2022): this corresponds of course to modular design allowing for then progressive replacement of worn parts by spares.

[3] Cf. Mona (2021) comparing the Tier e-scooter to its Lime and Dott counterparts in Paris

The electric engine contributes a significant part of the weight: 4 kg in 2nd generation models (ITF, 2020), and more for dual-motor scooters that are more powerful and improve user experience. Both the sturdier design and more powerful engines enable e-scooters to accommodate bigger people, thereby enlarging the customer basis.

Coming to the energy consumption rates, we have not yet heard of regenerative braking for kicked e-scooters. Such function would enable to reduce the average net consumption rate χ_E, at the expense of additional weight and acquisition cost. While the saving in fed-in energy would not look profitable, in fact an economic comparison should take into account the full value of such investment, notably the reduction in swapping frequency. Yet the effect of regenerating braking on battery wearing is unclear.

In the same vein, some innovators proposed to add photovoltaic panels on top of the scooter baseboard: the expected yield would be 1 mile of additional range per hour of sunny weather (Ridden, 2013).

9. Conclusion

9.1 Summary

The article provides a technical and economic model of service production for a Shared Scooter Service (S3) under free-floating and with swappable batteries. The model has modular architecture with sub-models that pertain to specific functions of service production, namely (i) commercial operations, (ii) energy feeding, (iii) in-field implementation, (iv) juicing logistics, (v) battery inventory, (vi) scooter inventory. The model logical structure, exhibited as an influence tree (figure 2), enables us to address the problem of Cost optimization in a hierarchical way, from short-run factor of tour productivity, to medium-run factors of battery DoD and energy capacity, and up to long-run factors of scooter lifetime length and energy consumption rate.

It comes out that the daily number of refills is a key variable in the economic model as it links the field implementation and swapping logistics functions to the other functions in service production (scooter inventory, battery inventory, energy charging and commercial). The sensitivity coefficient of production costs to the number of refills is a holistic per-swapping cost made up of a unit swapping cost plus a per-refill share of the battery acquisition cost.

The joint optimization of field and swapping costs with respect to swapping tour productivity yields a unit swapping cost that is essentially a CES function of two composite factors.

The juicing strategy also involves setting up the battery DoD that triggers refilling: the optimization of service production costs with respect to the target DoD yields an optimal number of refills over the battery lifetime that is a specific function of the ratio between the unit swapping cost and the battery price.

As for battery energy capacity, it influences the battery price and also the scooter price. Thus, its optimization involves the juicing strategy as well as battery inventory and scooter inventory. On a per-refill basis, the optimal battery energy capacity balances the marginal cost of battery and scooter capacity with the augmented swapping cost.

Turning to scooter optimization, the optimal lifespan balances the marginal cost of acquisition and the average acquisition cost augmented by marginal maintenance gains over the scooter lifetime. The optimal energy consumption factor balances the marginal scooter price divided by the lifespan with the overall energy cost per unit of ride length, including the effective

price of fed-in energy and the holistic per-swapping cost divided by the amount of refilled energy.

Analytical formulas are available for all variables of interest, thereby making the model both theoretical and tractable. A specific battery technology (i.e. a battery wearing function of the DoD, yielding a lifetime number of charging cycles) with constant elasticity was introduced: combined to battery and scooter prices that are constant elasticity functions of battery energy capacity, it allows for straightforward solution of the cost optimization problem. We also introduced an Affine Linear technology which combines conveniently to linear battery price and affine linear scooter price.

9.2 Outreach and limitations

Coming back to the original research question on the value of swappable batteries for shared e-scooters, it can be answered on two levels, theoretical and applied. On the theoretical level, the model provides a clear understanding of the "systemic economy" of swappable batteries by company-operated tours: the swappability enables not only for more efficient juicing logistics, but also for a juicing strategy combining juicing logistics with a specific discharge policy taking into account the battery wearing law, together with a cost-efficient choice of battery capacity that involves both the battery price and the scooter price. This way, swappable batteries link together the juicing strategy and the inventory policies of batteries and scooters. Figure 13 traces out the influence graph of the per-refill swapping cost on the realm of service supply policies.

On the applied level, we substantiated some hints about the per-refill swapping cost: it would lie between €1 and €3 or €6 at most. Thus the €5 bounty paid to gig juicers in the early times of shared e-scooters appears as an upper bound. As for Lithium-Ion chemistries, the better endurance of LFP compared to NMC would enable for saving one half of the battery inventory costs, i.e. about 15% of the holistic energy cost (see figures 11 and 12). Of course, such hints do not constitute empirical outcomes. Econometric studies are in order on the sub-models, from battery wearing laws, to battery prices with respect to chemistry and nominal capacity, to scooter prices with respect to battery capacity and body weight as a proxy of longevity, and up to the longevity of specific e-scooters.

Among the theoretical limitations of the model, let us mention first the absence of a fleet repositioning process (which may induce a major part of service production costs) and second the lack of a scooter wearing function: such function would be essential to understand scooter lifespan in a deeper way. Another direction for further research would be to link this production model to a model of service traffic and revenues, so as to study the conditions of economic viability of an S3 system and to look for its potential areas of relevance, beyond the initial niche of high density urban areas.

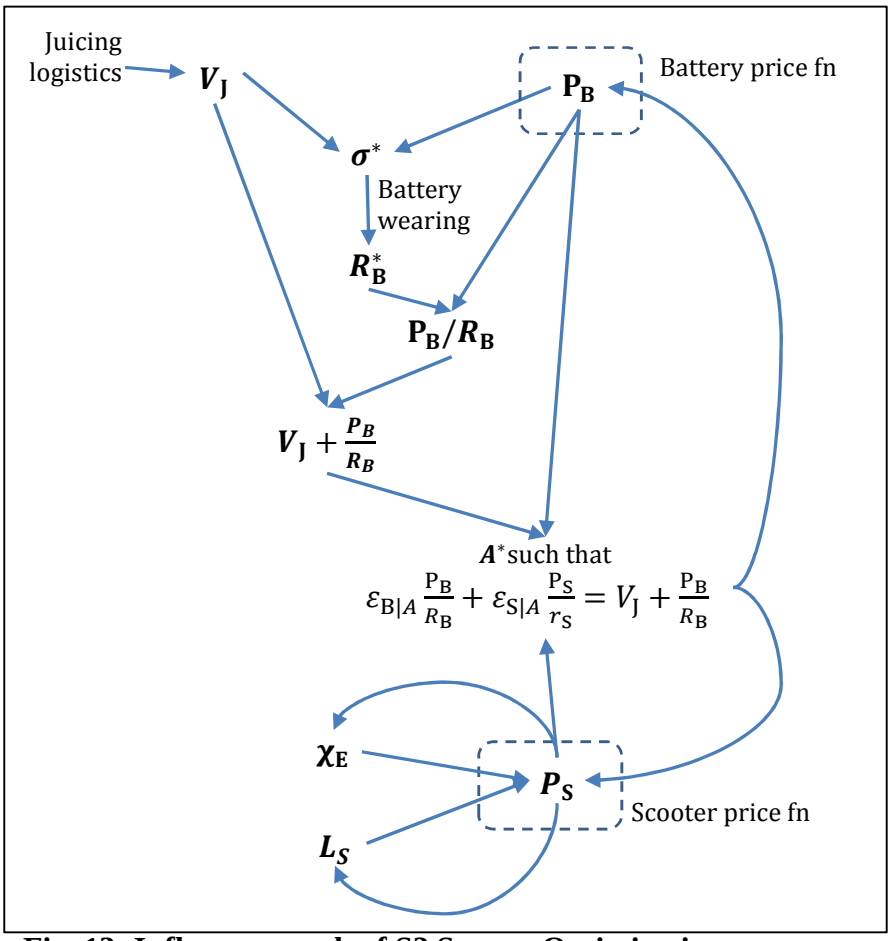


Fig. 13: Influence graph of S3 System Optimization.

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11. Appendix A

Proof of Proposition 1.

Considering the wearing law $\mathrm{R_B}$, we shall assume that the lifetime energy intensity function $\mathrm{E}_B\colon \sigma \mapsto \sigma.\,\mathrm{R_B}(\sigma)$ is decreasing. Thus, its derivative $\dot{\mathrm{E}}_\mathrm{B} = \mathrm{R_B} + \sigma\dot{\mathrm{R}}_\mathrm{B}$ is less than zero.

Function $\psi_B\colon \sigma \mapsto -(\mathrm{R_B} + \sigma\dot{\mathrm{R}}_\mathrm{B})/\mathrm{R_B^2}$ satisfies that $\psi_B(\sigma) = -\dot{\mathrm{E}}_\mathrm{B}\mathrm{R_B^{-2}}$. If E_B is decreasing then function ψ_B is nonnegative. Assuming further that E_B is a concave function, then its derivative is decreasing, so that $-\dot{\mathrm{E}}_\mathrm{B}$ is nonnegative and increasing. The decreasingness of E_B implies that of $\mathrm{R_B}$, which is positive, so that function $\mathrm{R_B^{-2}}$ is positive and increasing. Then, as a product of two nonnegative increasing functions, so is function ψ_B. Then the inverse function $\psi_\mathrm{B}^{(-1)}$ is well-defined and it is also nonnegative and increasing.

Now, given $y \equiv V_\mathrm{I}/p_\mathrm{B}$, either $\psi_B(\bar{\sigma}) \geq y$ or $\psi_B(\bar{\sigma}) < y$. In the latter case, as ψ_B is increasing then no $\sigma \in [0, \bar{\sigma}]$ can satisfy $\psi_B(\sigma) = y$, leading to claim (ii) in the proposition and the optimality of full discharge. In the former case, it holds that $\psi_B(0) < y \leq \psi_B(\bar{\sigma})$. From the continuity postulate, by the Bolzano-Weierstrass theorem there exists at least an intermediary value $\sigma^* \in [0, \bar{\sigma}]$ that satisfies $\psi_B(\sigma^*) = y$. Under global increasingness of ψ_B and its local strict increasingness, this σ^* is unique. Thus the existence and uniqueness are established in both cases.

Proof of Proposition 2.

Proposition 2: existence of an optimal BEC. Assume that (i) $\dot{\mathrm{P}}_{\mathrm{S}|A}$ is continuous and takes its values in a bounded interval $\left[\dot{\mathrm{P}}_{\mathrm{S}|A}^{min}, \dot{\mathrm{P}}_{\mathrm{S}|A}^{max}\right]$ with $\dot{\mathrm{P}}_{\mathrm{S}|A}^{min} > 0$, (ii) $\mathrm{P_B}$ and $\varepsilon_{\mathrm{B}|A}$ are continuous and $\varepsilon_{\mathrm{B}|A} \in [0,1]$. Then, given σ, there is at least one BEC that satisfies the optimality condition.

Proof. Let us define $\mathrm{F}(A) \equiv \frac{V_\mathrm{J}+(1-\varepsilon_{\mathrm{B}|A})\mathrm{P_B}/\mathrm{R_B}}{\dot{\mathrm{P}}_{\mathrm{S}|A}.A^2}$. Then eqn. (5.17) restates as $\mathrm{F}(A) = \frac{\sigma}{\chi_\mathrm{E}.L_\mathrm{S}}$. The continuity of $\dot{\mathrm{P}}_{\mathrm{S}|A}$ with respect to A and its specific range of variations, together with the continuity of $\mathrm{P_B}$ and $\varepsilon_{\mathrm{B}|A}$, ensure the continuity of F on $]0,\infty[$. From the assumption that $\varepsilon_{\mathrm{B}|A} \in [0,1]$, we first get that the numerator in F is greater than V_J hence positive, and second that $\mathrm{P_B}/A$ is bounded when $A \to +\infty$. Under the additional postulate $\dot{\mathrm{P}}_{\mathrm{S}|A} \in \left[\dot{\mathrm{P}}_{\mathrm{S}|A}^{min}, \dot{\mathrm{P}}_{\mathrm{S}|A}^{max}\right]$ with $\dot{\mathrm{P}}_{\mathrm{S}|A}^{min} > 0$, it comes out that $\mathrm{F} \to +\infty$ when $A \to 0^+$ and $\mathrm{F} \to 0^+$ when $A \to +\infty$. Finally, as F ranges from 0^+ to $+\infty$ and is continuous, by the Bolzano-Weierstrass theorem there is at least one value A such that $\mathrm{F}(A) = \frac{\sigma}{\chi_\mathrm{E}.L_\mathrm{S}} > 0$, i.e. an optimal BEC.

Proof of Proposition 3.

Proposition 3: existence and uniqueness of joint optimal BEC-DoD. Assume that (i) $\mathrm{P_B}$ increases with A and (ii) $(1-\varepsilon_{\mathrm{B}|A})\mathrm{P_B}$ decreases with A while remaining nonnegative, (iii) $\dot{\mathrm{P}}_{\mathrm{S}|A}$ is nonnegative and (iv) $\dot{\mathrm{P}}_{\mathrm{S}|A}A^2$ increases indefinitely with A. Then the Partial Discharge optimality condition in A only has one unique solution A^*, with associated DoD $\sigma_{A^*}^* \equiv \psi_\mathrm{B}^{(-1)}(V_\mathrm{J}/\mathrm{P_B}(A^*))$. If $V_\mathrm{J}/\mathrm{P_B}(A^*) < \psi_\mathrm{B}(\bar{\sigma})$, then $\sigma_{A^*}^* \in]0,\bar{\sigma}[$ and the pair $(A^*, \sigma_{A^*}^*)$ is the unique optimal pair, meaning that Partial Discharge is optimal. Otherwise, if $V_\mathrm{J}/\mathrm{P_B}(A^*) \geq \psi_\mathrm{B}(\bar{\sigma})$ then Full Discharge is optimal and the pair $(\mathrm{A}_{\bar{\sigma}}^*, \bar{\sigma})$ is the unique optimal pair.

Proof. Let us address the Partial Discharge case by restating the fixed point problem (5.23) as

$$\dot{\mathrm{P}}_{\mathrm{S}|A}A^2 = \frac{\chi_\mathrm{E}.L_\mathrm{S}}{\sigma_A^*}\left(V_\mathrm{J} + (1-\varepsilon_{\mathrm{B}|A})\frac{\mathrm{P_B}}{\mathrm{R_B}}\right)$$

$$\dot{\mathrm{P}}_{\mathrm{S}|A}A^2 - V_\mathrm{J}\frac{\chi_\mathrm{E}.L_\mathrm{S}}{\sigma_A^*} = \frac{\chi_\mathrm{E}.L_\mathrm{S}}{\mathrm{R_B}\sigma_A^*}(1-\varepsilon_{\mathrm{B}|A})\mathrm{P_B}$$

By Assumption (i), $\mathrm{P_B}$ increases with A hence $V_\mathrm{J}/\mathrm{P_B}$ decreases with A and so does $\sigma_A^* = \psi_\mathrm{B}^{(-1)}(V_\mathrm{J}/\mathrm{P_B})$ since $\psi_\mathrm{B}^{(-1)}$ is increasing: thus $-V_\mathrm{J}\chi_\mathrm{E}.L_\mathrm{S}/\sigma_A^*$ is an increasing function of A. As from (iv) $\dot{\mathrm{P}}_{\mathrm{S}|A}A^2$ is increasing, we get that the LHS is an increasing function of A. Conversely, from (ii) $(1-\varepsilon_{\mathrm{B}|A})\mathrm{P_B}$ is decreasing and nonnegative, while $\mathrm{R_B}\sigma_A^* = \mathrm{E_B}(\sigma_A^*)$ increases with A hence $1/\mathrm{R_B}\sigma_A^*$ decreases with A: thus, as the product of two nonnegative decreasing functions of A, the RHS is a nonnegative decreasing function of A. All in all, the fixed point problem is cast as the intersection between the increasing LHS curve and the decreasing RHS curve. Both curves are continuous. The LHS takes negative values at $A \to 0^+$ but tends to infinity when A does so. Thus, by the Bolzano-Weierstrass theorem there is an intersection point (A^*, y) between the two curves, with nonnegative ordinate y due to the nonnegative RHS. From the increasing and decreasing behaviors this point is unique.

Now, if $V_\mathrm{J}/\mathrm{P_B}(A^*) \leq \psi_\mathrm{B}(\bar{\sigma})$ then the $(A^*, \sigma_{A^*}^*)$ pair satisfies both optimality conditions and it is the only one to do so, meaning that it is an optimal solution and the unique one: thus, Partial Discharge is optimal. But if $V_\mathrm{J}/\mathrm{P_B}(A^*) > \psi_\mathrm{B}(\bar{\sigma})$ then Partial Discharge cannot be optimal: then, Full Discharge is in order and the $(\mathrm{A}_{\bar{\sigma}}^*, \bar{\sigma})$ pair is optimal. Furthermore, due to the stronger assumptions in Proposition 3 than in Proposition 2, $\mathrm{A}_{\bar{\sigma}}^*$ is the unique solution of optimal BEC problem associated to $\bar{\sigma}$, making pair $(\mathrm{A}_{\bar{\sigma}}^*, \bar{\sigma})$ the unique optimal point.

Full discharge optimal BEC under Constant Elasticities

Let us demonstrate the existence and uniqueness of solution to equation (5.27).

Proof: The LHS increases with A from 0 to $+\infty$. Consider first the case $\beta < 1$: then the RHS decreases with A from V_J to $-\infty$: as both the LHS and RHS are continuous, there is an intersection point and it is unique. Consider then the case with $\beta \geq 1$: then the RHS is sub-linear in A while LHS is super-linear, so that LHS will catch up RHS at a point A^* where the LHS derivative is greater than its RHS counterpart: beyond that value the gap between the derivatives will go on increasing, so there won't be any other intersection, ensuring the uniqueness of A^*.

12. Appendix B: affine linear model of battery lifetime energy intensity, battery price and scooter price

12.1 A specific battery technology of affine linear lifetime energy intensity

In Section 5 we introduced a specific law, say technology, of battery wearing with respect to target DoD: the constant elasticity battery wearing technology. Let us now introduce an alternative specification by postulating that the $\mathrm{E_B}$ function of lifetime energy intensity is an affine linear function of σ as follows:

$$\mathrm{E_B}(\sigma) = \mathrm{E_0}.(1 - \sigma/b) \tag{B1a}$$

Parameter $\mathrm{E_0}$ stands for the maximum lifetime energy intensity. The effective intensity depends on σ in a decreasing way, proportional to $1 - \sigma/b$ with parameter $b > 1$ (recall that $\sigma \in]0,1[$).Thus

$$\mathrm{R_B}(\sigma) = \sigma^{-1}\mathrm{E_B}(\sigma) = \mathrm{E_0}.(\sigma^{-1} - b^{-1}) \tag{B1b}$$

Then, $\dot{\mathrm{R}}_\mathrm{B} = -\mathrm{E_0}/\sigma^2$ and $\dot{\mathrm{E}}_\mathrm{B} = -\mathrm{E_0}/b$, so that

$$\psi_B(\sigma) = -\frac{\dot{\mathrm{E}}_\mathrm{B}}{R_\mathrm{B}^2} = \frac{\mathrm{E_0}}{b.R_\mathrm{B}^2} = \frac{(\sigma^{-1} - b^{-1})^{-2}}{b.\mathrm{E_0}} \tag{B2a}$$

Thus ψ_B is an increasing function of σ, with inverse function such that $(\sigma^{-1} - b^{-1})^{-2} = b.\mathrm{E_0}.y$, hence $\sigma^{-1} - b^{-1} = (b.\mathrm{E_0}.y)^{-.5}$ yielding that $\mathrm{R_B}(\sigma) = \sqrt{\mathrm{E_0}/by}$ and

$$\psi_\mathrm{B}^{(-1)}(y) = \frac{b}{1 + \sqrt{b/(\mathrm{E_0}y)}} \tag{B2b}$$

For the value of $\psi_\mathrm{B}^{(-1)}$ to be less than $\bar{\sigma}$, it must hold that

$$y < \psi_B(\bar{\sigma}) = (\bar{\sigma}^{-1} - b^{-1})^{-2}/(b\mathrm{E_0})$$

If $y < \psi_B(\bar{\sigma})$, the lifetime effective intensity is obtained as

$$\mathrm{E_B}(\sigma) = \frac{\mathrm{E_0}}{1 + \sqrt{\mathrm{E_0}y/b}} \tag{B3}$$

Put in words, the effective intensity is the maximum intensity divided by $1 + \sqrt{\mathrm{E_0}y/b}$.

As for DoD optimization, depending on whether $V_\mathrm{J}/p_\mathrm{B} < \psi_B(\bar{\sigma})$ or not, the optimal DoD is less than $\bar{\sigma}$ i.e. partial discharge or equal to $\bar{\sigma}$ i.e. full discharge. The following table provides

the associated outcomes under the specific battery technology with linear lifetime energy intensity:

Condition	σ^*	$\mathrm{R_B}(\sigma^*)$	$\mathrm{E_B}(\sigma^*)$
$V_\mathrm{J}/p_\mathrm{B} < \psi_B(\bar{\sigma})$ Partial discharge	$\dfrac{b}{1+\sqrt{\dfrac{b}{\mathrm{E}_0}\dfrac{p_\mathrm{B}}{V_\mathrm{J}}}}$	$\sqrt{\dfrac{\mathrm{E}_0\, p_\mathrm{B}}{b\, V_\mathrm{J}}}$	$\dfrac{\mathrm{E}_0}{1+\sqrt{\dfrac{\mathrm{E}_0\, V_\mathrm{J}}{b\, p_\mathrm{B}}}}$
$V_\mathrm{J}/p_\mathrm{B} \geq \psi_B(\bar{\sigma})$ Full discharge	$\bar{\sigma}$	$\mathrm{E}_0(\bar{\sigma}^{-1} - b^{-1})$	$\mathrm{E}_0(1-\bar{\sigma}/b)$

(B4a,b,c)

Under partial discharge the optimal number of cycles combines a technical ratio, E_0/b, with an economic ratio, $V_\mathrm{J}/p_\mathrm{B}$.

12.2 Optimal capacity and DoD under linear prices

Let us assume here that not only the battery price is an affine linear function of battery capacity, $\mathrm{P_B} = P_{\mathrm{B}0} + A.\dot{P}_\mathrm{B}$, and also the scooter price, say $\mathrm{P_S} = P_{\mathrm{S}0} + A.\dot{P}_{\mathrm{S|A}}$. Then from (5.16) the optimal number of refills in a scooter lifetime becomes

$$r_\mathrm{S} = \frac{A.\dot{P}_{\mathrm{S|A}}}{V_\mathrm{J} + P_{\mathrm{B}0}/\mathrm{R_B}} = \frac{\mathrm{P_S} - P_{\mathrm{S}0}}{V_\mathrm{J} + P_{\mathrm{B}0}/\mathrm{R_B}} \tag{B5}$$

The lifetime refill number, which is essentially a technical parameter, is entirely determined on economic grounds.

Thus $\frac{\chi_\mathrm{E}.L_\mathrm{S}}{\mathrm{A}.\sigma} = \frac{A}{V_\mathrm{J}+P_{\mathrm{B}0}/\mathrm{R_B}}\dot{P}_{\mathrm{S|A}}$, implying as in (5.18) that

$$A^2\sigma = \chi_\mathrm{E} L_\mathrm{S}\frac{V_\mathrm{J} + P_{\mathrm{B}0}/\mathrm{R_B}}{\dot{P}_{\mathrm{S}|A}} \tag{B6}$$

Under Partial Discharge, σ is itself a function of A via $\mathrm{P_B}$ so that the optimal BEC is characterized by the following condition:

$$A^2\psi_\mathrm{B}^{(-1)}\left(\frac{V_\mathrm{J}}{P_{\mathrm{B}0} + A.\dot{P}_\mathrm{B}}\right) = \frac{\chi_\mathrm{E}L_\mathrm{S}}{\dot{P}_{\mathrm{S}|A}}\left(V_\mathrm{J} + \frac{P_{\mathrm{B}0}}{\mathrm{R_B}\circ\psi_\mathrm{B}^{(-1)}\left(\frac{V_\mathrm{J}}{P_{\mathrm{B}0} + A.\dot{P}_\mathrm{B}}\right)}\right)$$

It is a fixed point problem in A only.

Under the affine linear battery technology, owing to (B4a,c) it gives rise to

$$A^2 b = \left(1+\sqrt{\frac{b}{\mathrm{E}_0}\frac{P_{\mathrm{B}0} + A.\dot{P}_\mathrm{B}}{V_\mathrm{J}}}\right)\frac{\chi_\mathrm{E}L_\mathrm{S}}{\dot{P}_{\mathrm{S}|A}}\left(V_\mathrm{J} + P_{\mathrm{B}0}\sqrt{\frac{b\, V_\mathrm{J}}{\mathrm{E}_0\,(P_{\mathrm{B}0} + A.\dot{P}_\mathrm{B})}}\right)$$

Which is a fifth-order equation in $\sqrt{P_{\mathrm{B}0} + A.\dot{P}_\mathrm{B}}$.

Under null base battery cost $P_{\mathrm{B}0} = 0$ the condition simplifies into the following

$$A^2 = \frac{\chi_\mathrm{E}L_\mathrm{S}V_\mathrm{J}}{b\dot{P}_{\mathrm{S}|A}} + \frac{\chi_\mathrm{E}L_\mathrm{S}}{\dot{P}_{\mathrm{S}|A}}\sqrt{\frac{\dot{P}_\mathrm{B}V_\mathrm{J}}{b\mathrm{E}_0}}A$$

Denoting $v \equiv \frac{\chi_E L_S V_J}{b \dot{P}_{S|A}}$ and $w \equiv \frac{\chi_E L_S}{\dot{P}_{S|A}} \sqrt{\frac{\dot{P}_B V_J}{b E_0}}$, the optimal capacity satisfies the following equation

$$A^2 - w\sqrt{A} - v = 0 \tag{B7}$$

It is a quartic equation in $\sqrt{A}$. The analytical solution is given in the next subsection.

Under Full Discharge i.e. $\sigma = \bar{\sigma}$, the optimal capacity is straightforwardly recovered as

$$A^* = \sqrt{\frac{\chi_E . L_S}{\bar{\sigma} . \dot{P}_{S|A}} V_J} \tag{B8}$$

Under linear battery price, the Full Discharge condition $V_J / p_B \geq \psi_B(\bar{\sigma})$ becomes that

$$V_J \geq A \, \dot{P}_B \, \psi_B(\bar{\sigma})$$

Combining the two conditions, we obtain that

$$V_J \geq \hat{V}_J \equiv \frac{\chi_E . L_S}{\bar{\sigma} . \dot{P}_{S|A}} (\dot{P}_B . \psi_B(\bar{\sigma}))^2 \tag{B9}$$

The per-refill swapping cost $\hat{V}_J$ is a threshold value for Full discharge to possibly occur. It implies that values of V_J below $\hat{V}_J$ give rise to Partial discharge in DoD optimization. But it does not mean that only Full discharge could be the optimal policy for values of V_J above $\hat{V}_J$. In (5.18) we provided a threshold BEC of Partial discharge that is also a ceiling capacity of Full discharge:

$$\underline{A} \equiv \frac{V_J}{\dot{P}_B \psi_B(\bar{\sigma})} \tag{B10}$$

Under the specific battery wearing law with linear lifetime energy intensity, after solving the quartic equation in $\sqrt{A}$, one has to compare the resulting A to $\underline{A}$: if $A < \underline{A}$ then the solution is consistent with partial discharge, otherwise full discharge is optimal and the optimal capacity stems from (B7).

12.3 Optimal battery energy capacity under partial discharge

Under linear scooter and battery price functions and specific battery technology with linear lifetime energy intensity, if Partial Discharge is optimal then the optimal capacity A^* satisfies a characteristic condition (B7) that is a quartic equation in $\sqrt{A}$. Let us recall it:

$$A^2 - w\sqrt{A} - v = 0$$

Wherein the composite parameters $v \equiv \frac{\chi_E L_S V_J}{b \dot{P}_{S|A}}$ and $w \equiv \frac{\chi_E L_S}{\dot{P}_{S|A}} \sqrt{\frac{\dot{P}_B V_J}{b E_0}}$ are positive.

As the fourth-order equation $z^4 - wz - v = 0$ has no third-degree term, it is straightforward to apply the Ferrari solution scheme. Let us look for a parameter λ such that

$$0 = (z^2 + \lambda)^2 - 2\lambda z^2 - \lambda^2 - wz - v$$

$$\Leftrightarrow 0 = (z^2 + \lambda)^2 - 2\lambda \left(z^2 + \frac{w}{2\lambda} z + \frac{\lambda}{2} + \frac{v}{2\lambda} \right)$$

$$\Leftrightarrow 0 = (z^2+\lambda)^2 - 2\lambda\left\{(z+\frac{w}{4\lambda})^2 - (\frac{w}{4\lambda})^2 + \frac{\lambda}{2} + \frac{v}{2\lambda}\right\} \tag{B11}$$

And such that $(\frac{w}{4\lambda})^2 = \frac{\lambda}{2} + \frac{v}{2\lambda}$. Thus, the equation parameterized by λ is both equivalent to the original quartic equation and in the form of a well-known remarkable identity. The condition on λ is equivalent to a cubic equation in canonical form, namely

$$\lambda^3 + v\lambda - \frac{w^2}{8} = 0 \tag{B12}$$

Let us solve this "resolvant cubic" equation by applying Viete's method. We look for y such that $\lambda = y - \frac{v}{3y}$, so that $Y \equiv y^3$ satisfies the following quadratic equation

$$Y^2 - \frac{w^2}{8}Y - \frac{v^3}{27} = 0 \tag{B13}$$

In our instance, as both w and v are real and v is positive, then the discriminant of the quadratic equation is positive:

$$\Delta \equiv (\frac{w^2}{8})^2 + \frac{4}{27}v^3 > 0$$

Letting $\varepsilon \in \{-1, +1\}$, the quadratic equation is solved as

$$Y_\varepsilon = \frac{w^2}{16} + \frac{\varepsilon}{2}\sqrt{\Delta} \tag{B14}$$

As $\frac{1}{2}\sqrt{\Delta} > \frac{w^2}{16}$, it holds that $Y_{+1} > 0$ and $Y_{-1} < 0$. We then recover the associated cubic roots that have the same signs:

$$y_\varepsilon = \sqrt[3]{Y_\varepsilon} = \sqrt[3]{\frac{w^2}{16} + \frac{\varepsilon}{2}\sqrt{\Delta}} \tag{B15}$$

The next step is to recover Ferrari's parameter from y_ε:

$$\lambda_\varepsilon = y_\varepsilon - \frac{v}{3y_\varepsilon} \tag{B16}$$

It turns out that the sign of λ_ε is identical to that of y_ε and Y_ε: the reason is that

$\lambda_\varepsilon > 0 \Leftrightarrow y_\varepsilon^2 > \frac{v}{3} \Leftrightarrow y_\varepsilon^6 > \frac{v^3}{27} \Leftrightarrow Y_\varepsilon^2 > \frac{v^3}{27} \Leftrightarrow \frac{w^2}{8}Y_\varepsilon > 0$ owing to (B12).

As v is positive, by using (B12) once again it comes out that λ must be positive, too. Considering from now on the value λ_{+1} of λ, we can restate the original quartic equation as

$$(z^2+\lambda)^2 - 2\lambda\left(z+\frac{w}{4\lambda}\right)^2 = \left(z^2+\lambda-\sqrt{2\lambda}\left(z+\frac{w}{4\lambda}\right)\right).\left(z^2+\lambda+\sqrt{2\lambda}\left(z+\frac{w}{4\lambda}\right)\right)$$

$$= \left((z-\sqrt{\lambda/2})^2 + \frac{\lambda}{2} - \frac{w}{\sqrt{8\lambda}}\right).\left((z+\sqrt{\lambda/2})^2 + \frac{\lambda}{2} + \frac{w}{\sqrt{8\lambda}}\right)$$

In our instance with positive w, as λ is positive only the first term may be null (looking for real roots only). Yet the existence of a solution still requires that $\frac{\lambda}{2} - \frac{w}{\sqrt{8\lambda}} < 0$ i.e. that $\lambda^3 < \frac{w^2}{2}$:

from (B12) it is equivalent to $-v\lambda + \frac{w^2}{8} < \frac{w^2}{2}$ i.e. $-v\lambda < \frac{3\,w^2}{8}$ which holds true since both v and λ are positive.

Then, any (real) solution of the quartic equation has the following form:

$$z = \sqrt{\frac{1}{2}\lambda} \pm \sqrt{\frac{w}{\sqrt{8\lambda}} - \frac{1}{2}\lambda} \tag{B17}$$

The upper root is obviously positive. For the lower root to be greater than 0, the condition is $\frac{1}{2}\lambda > \frac{w}{\sqrt{8\lambda}} - \frac{1}{2}\lambda$ i.e. $\lambda > \frac{w}{\sqrt{8\lambda}}$ and in turn $\lambda^3 - \frac{w^2}{8} > 0$. But (B12) implies that $\lambda^3 - \frac{w^2}{8} = -v\lambda$ which is negative, yielding negative lower root.

In all, there is one and only one real and positive solution to the quartic equation in $\sqrt{A}$:

$$\sqrt{A} = \sqrt{\frac{1}{2}\lambda} + \sqrt{\frac{w}{\sqrt{8\lambda}} - \frac{1}{2}\lambda} \tag{B18}$$

Wherein $\lambda = \sqrt[3]{Y} - \frac{v}{3\sqrt[3]{Y}}$ and $Y = \frac{w^2}{16} + \frac{1}{2}\sqrt{(\frac{w^2}{8})^2 + \frac{4}{27}v^3}$.

Of course, the existence and uniqueness of the solution could also be demonstrated on simple functional considerations.