

Pest detection from a biology-informed inverse problem and pheromone sensors

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- 1/3 of crops destroyed by pest insects.
- Early detection, a challenge for an efficient action before infestation.
- Communication between the insects by means of species-specific pheromone.
- New innovative pheromone sensors under development.

• Sex pheromone



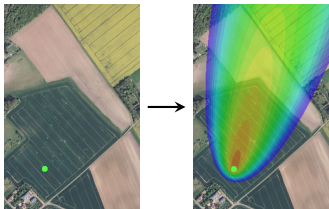
Early detection of pest insects (before infestation).



Targeted biocontrol strategy and prophylactic approaches.

Different mathematical problems

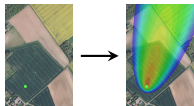
Direct problem



Knowing : the localization of the insects and environmental parameters,
predict : the propagation of the pheromone.

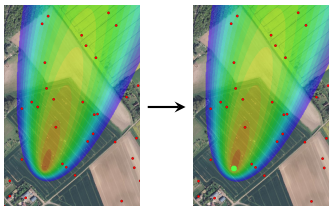
Different mathematical problems

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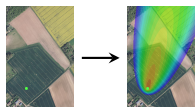
Inverse problem



Knowing : the localization and the data of the sensors and environmental parameters,
predict : the quantity of pheromone emitted by the insects, and thus the localization of the insects.

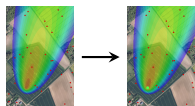
Different mathematical problems

Direct problem



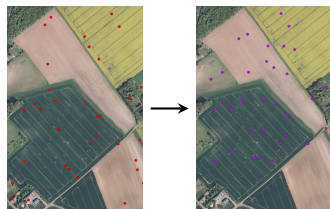
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predict : the propagation of the pheromone.

Inverse problem



Knowing : the localization and the data of the sensors and environmental parameters,
predict : the quantity of pheromone emitted by the insects, and thus the localization of the insects.

Optimal design of experiments



Knowing : the environmental parameters,
predict : the optimal placement of the sensor in order to reduce the uncertainty in the risk map.

Direct Problem

Environmental parameters :

- wind field $\vec{u}(x, y, t)$,
- diffusion tensor $\mathbf{K}(x, y, t)$,
- loss coefficient depending on the vegetation $\tau_{loss}(x, y)$.

State variable:
pheromone concentration c .

2D model of the pheromone propagation [1]:

$$\frac{\partial c}{\partial t} - \underbrace{\nabla \cdot (\mathbf{K} \nabla c)}_{\text{diffusion}} + \underbrace{\nabla \cdot (\vec{u} c)}_{\text{wind}} + \underbrace{\tau_{loss} c}_{\text{deposition}} = s$$

Quantity of pheromone emitted by the insects $s(x, y, t)$.

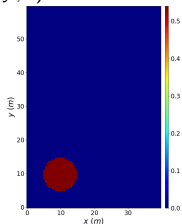


Figure: Uniform density of insects located in a circle with constant emission rate leading to s^{circ} , a constant quantity of pheromone over a circular support.

Pheromone concentration c given a source term s .

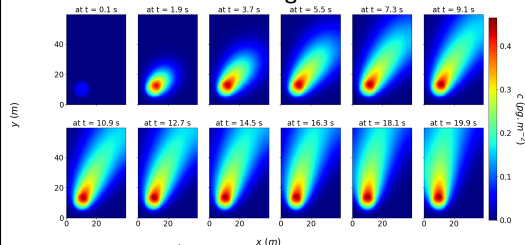


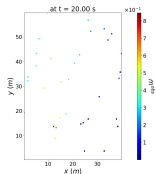
Figure: Concentration $c(s^{circ})$, solution of the pheromone propagation model with the circulare s^{circ} .

Inverse problem

Sensors

Observations m^{obs}

Figure: Noisy synthetic observations m^{obs} generated from the target s_t (see dashed lines on the figure on the right), at a given time.



Optimisation problem [2] :

$$\begin{cases} \text{find } s_a(x, y, t) \text{ such that} \\ s_a(x, y, t) = \arg \min_{s(x, y, t)} (j_{obs}(s) + j_{reg}(s)) \end{cases}$$

with $j_{obs}(s) = \|m(c(s)) - m^{obs}\|_2^2$
and j_{reg} a regularization term.

Prediction of the observed variable $m(c(s))$.

Direct model

Sensors

Control variable:
quantity of pheromone emitted s .

Optimal source term s_a :
quantity of pheromone emitted
enabling the **direct model** to
reproduce at best the **observations** m^{obs} .

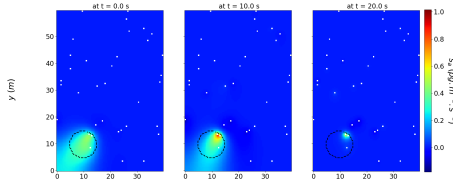


Figure: Optimal source term s_a and the target s_t (dashed line, constant over the circular support and used to generate m^{obs} , see figure on the left) during a synthetic experiment.

Inverse problem: resolution method

Minimization problem solved using gradient descent method.

Requires the computation of the gradient of the cost function $\nabla j(\mathbf{s})$.

$$\langle \nabla j(\mathbf{s}), \delta \mathbf{s} \rangle = \langle \nabla j_{obs}(\mathbf{s}), \delta \mathbf{s} \rangle + \langle \nabla j_{reg}(\mathbf{s}), \delta \mathbf{s} \rangle$$

- Adjoint model [3] associated to the pheromone propagation model:

$$\partial_t c^* + \nabla \cdot (\mathbf{K}^T \nabla c^*) + \nabla \cdot (\vec{u} c^*) - (\nabla \cdot \vec{u}) c^* - \tau_{loss} c^* = \left(\frac{dm}{dc}(c(\mathbf{s})) \right)^* \cdot 2 (m(c(\mathbf{s})) - m^{obs})$$

with the final condition $c^*(t = T) = 0$.

Expression of the gradient: $\nabla j_{obs}(\mathbf{s}) = -c^*$

⇒ 1 resolution of the direct model and 1 resolution of the adjoint model.

The gradient is the retro-propagation with diffusion of the gap between the data and the output of the direct model.

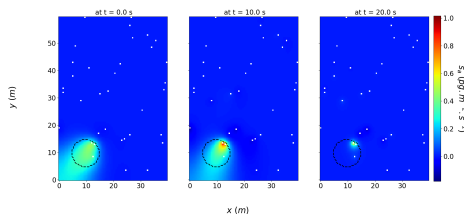


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[3] J.-L. Lions. *Optimal control of systems governed by partial differential equations*, 1971.

Biology-informed regularization term

Knowing:

the pheromone propagation,
data of the pheromone sensors,
the biological behaviour of the insects,

estimer :

(a better) localization of the insects.

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Improving the inference by including biological information in the regularization term.

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Improving the inference by including biological information in the regularization term.

- $j_{reg}(\mathbf{s}) = \|\mathcal{M}(\mathbf{s}) - \sigma\|_{L^2}^2$ with $\mathcal{M}(\mathbf{s}) = \sigma$ a model describing biological information, e.g.:
 - $\mathcal{M}(\mathbf{s}) = \sigma$ given by a population dynamic model,
 - Tikhonov regularization term [4] $\mathcal{M}(\mathbf{s}) = \mathbf{s}$ et $\sigma = \mathbf{s}_b$ with \mathbf{s}_b a background value of \mathbf{s} based on biological information.

[4] B. Kaltenbacher et al. *Iterative regularization methods for nonlinear ill-posed problems*, 2008.

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- LASSO regularization term [5] $j_{reg}(s) = \|s\|_{L^1}$ to ensure sparsity.

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- LASSO regularization term [5] $j_{reg}(s) = \|s\|_{L^1}$ to ensure sparsity.
- Logarithmic barrier [6] $j_{reg}(s) = \int_0^T \int_{\Omega_{exc}} -\epsilon \log(\epsilon - s(x, y, t)^2) dx dy dt$ in order to exclude pheromone emission in the zones Ω_{exc} .

[4] B. Kaltenbacher et al. *Iterative regularization methods for nonlinear ill-posed problems*, 2008.

[5] R. Tibshirani. *Regression shrinkage and selection via LASSO*, 1996.

[6] Y. Nesterov. *Lectures on convex optimization*, 2018.

Population dynamic-informed regularization term

Let p be the insects density, satisfying the population dynamic PDE [7] :

$$\partial_t p + \nabla \left(\sum_i F_i(p) \right) + \sum_i R_i(p) = 0$$

Population dynamic-informed regularization term

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$$\partial_t p + \nabla \left(\sum_i F_i(p) \right) + \sum_i R_i(p) = 0$$

with :

- $F_d(p) = -\sigma \nabla p$: motion of the individuals in a random direction,
- $F_a(p) = \vec{v}_i p$: motion of the individuals in a given direction, e.g.
 - toward the preferred habitat,
 - $\vec{v}_i = -\sigma_{qs} \nabla p$ to model population pressure,
 - $\vec{v}_i = \chi \nabla c(s)$ to model attraction toward the source of pheromone (chemotaxis),
- $R_l(p) = \frac{r}{\kappa} p(\kappa - p)$ to model logistic growth,
- $R_{Ae}(p) = \frac{r}{\kappa_+ \kappa_-} p(\kappa_+ - p)(p - \kappa_-)$ to model Allee effect,
- non-local terms, e.g. $\vec{v}_i = \vec{v}_i(c(s))$ with $c(s) \approx g_{gp} * s$.

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$s = pq$, with q the quantity of pheromone emitted per insect

$$\Rightarrow \partial_t \left(\frac{s}{q} \right) + \nabla \left(\sum_i F_i \left(\frac{s}{q} \right) \right) + \sum_i R_i \left(\frac{s}{q} \right) = 0$$

$$\text{and } j_{reg}(s) = \left\| \partial_t \left(\frac{s}{q} \right) + \nabla \left(\sum_i F_i \left(\frac{s}{q} \right) \right) + \sum_i R_i \left(\frac{s}{q} \right) \right\|_{L^2}^2.$$

Biology-informed regularization terms comparison

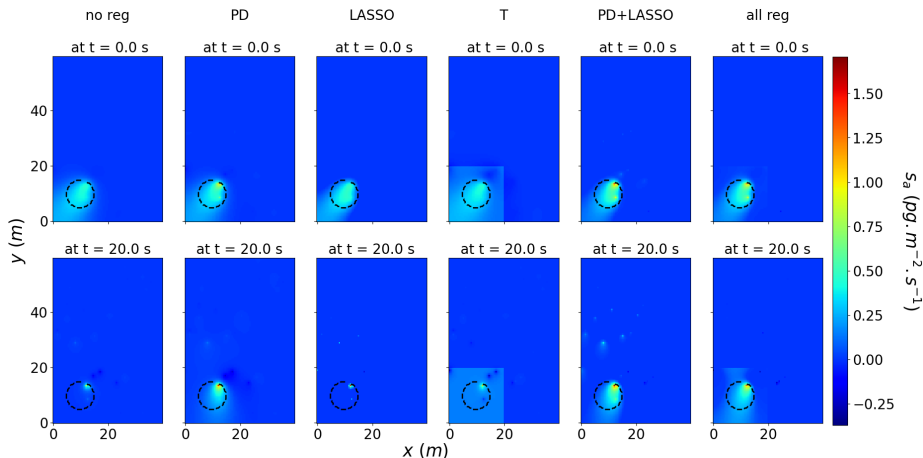


Figure: Optimal source term s_a obtained with different biology-informed regularization terms s_{reg} and the target s_T (dashed line, constant over the circular support and used to generate m^{obs}).

From left to right: without regularization (no reg), with a population dynamic-informed regularization term $j_{reg}(s) \propto \|\partial_t s\|_2^2$ (PD), with a LASSO regularization term (LASSO), with the Tikhonov regularization term (T), with a combination of population dynamic-informed and LASSO regularization terms (PD+LASSO) and with a combination of population dynamic-informed, LASSO and Tikhonov regularization terms (all reg).

Biology-informed regularization terms comparison

- Total error $RxMSE_{tot}(t) = \sqrt{\frac{1}{|\Omega|} \int_{\Omega} (s_a(X, t) - s_t(X, t))^2 dX}$,
- error over the target support $RxMSE_{supp}(t; s) = \sqrt{\frac{1}{|\Omega_{supp}|} \int_{\Omega_{supp}} (s_a(X, t) - s_t(X, t))^2 dX}$.

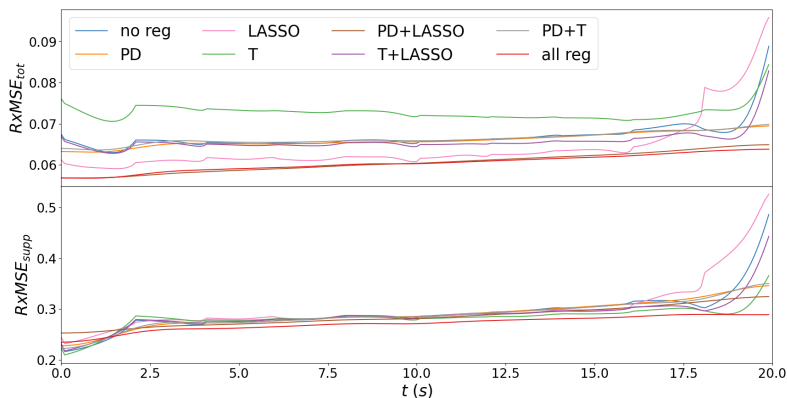


Figure: Root Mean Square Errors between the target s_t and the optimal source term s_a over the whole domain and over s_t support (resp. $RxMSE_{tot}$ and $RxMSE_{supp}$) vs the time for three different regularization strategies: without regularization (no reg), with a population dynamic-informed regularization term $J_{reg}(s) \propto \|\partial_t s\|_2^2$ (PD), with a LASSO regularization term (LASSO), with the Tikhonov regularization term (T) and with combinations of these regularization terms.

Mono-sensor adjoint state

Decomposition of the observations into mono-sensor observations: $m = \sum_i m_i$.

Mono-sensor adjoint state c_i^* related to the i^{th} sensor, solution of the mono-sensor adjoint model:

$$\partial_t c_i^* + \nabla \cdot (\mathbf{K}^T \nabla c_i^*) + \nabla(\bar{u} c_i^*) - (\nabla \cdot \bar{u}) c_i^* - \tau_{\text{loss}} c_i^* = \left(\frac{dm_i}{dc} (c(s)) \right)^* \cdot 2\mathbf{R}^{-1} (m_i(c(s)) - m_i^{\text{obs}})$$

with the final condition $c_i^*(t = T) = 0$.

Link between c^* and c_i^* : $c^*(s) = \sum_i c_i^*(s)$

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Let $(s_k)_{k \in \mathbb{N}}$ be the sequence of s given by the gradient-descent algorithm, the optimum s_a can be expressed as:

$$s_a = s_0 - \sum_k \alpha_k \nabla j(s_k)$$

Mono-sensor adjoint state

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$$\begin{aligned} s_a &= s_0 - \sum_k \alpha_k \nabla j(s_k) \\ &= s_0 - \sum_k \alpha_k \left(\nabla j_{\text{reg}}(s_k) - c^*(s_k) \right) \end{aligned}$$

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c_i^* : the contribution of a sensor to the gradient of the cost function, and thus to the decrease of the cost function.

Mono-sensor adjoint state

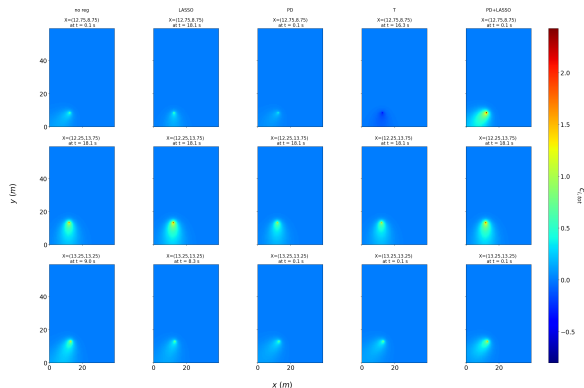
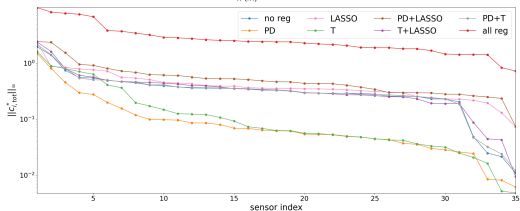


Figure: Comparison of

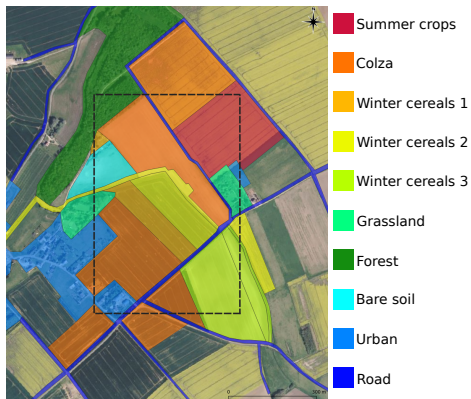
$c_{i,tot}^* = \sum_k \alpha_k c_{i,tot}^*(s_k)$, the total contribution of each sensors to the minimization of j : without regularization (no reg), with a population dynamic-informed regularization term $j_{reg}(s) \propto \|\partial_t s\|_2^2$ (PD), with a LASSO regularization term (LASSO), with the Tikhonov regularization term (T) and with combinations of these regularization terms. On the top three rows: $c_{i,tot}^*$ maps for the 3 sensors of higher $\|c_{i,tot}^*\|_\infty$ at the time where $\|c_{i,tot}^*\|_\infty$ is reached.

On the bottom row: $\|c_{i,tot}^*\|_\infty$ for every sensors, sorted from the higher to the lower $\|c_{i,tot}^*\|_\infty$

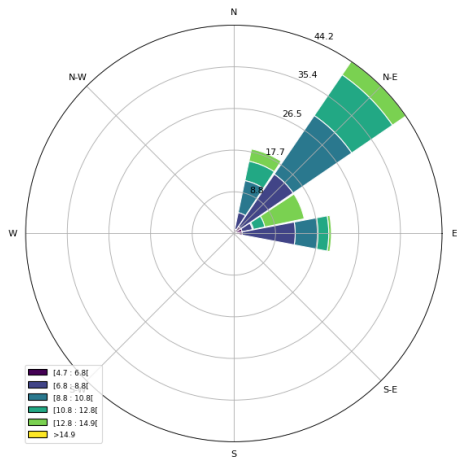


- Application to progressively more realistic test cases.
- Sensitivity analysis of the pheromone propagation model to uncertainty in the environmental parameters.
- What is the optimal placement of the sensors to improve the inference of the quantity of pheromone emitted? \Rightarrow Optimal design of experiment
- Metamodel (reduced order model) to solve the direct model and estimate the cost function (and its gradient) in a limited computation time and/or with limited computation resources.
- Application and evaluation of the performance of the inference to real data and in conditions less and less controlled.
- Continuous integration and development of the solvers and methods in a git repository.

Work in progress: application to realistic test cases



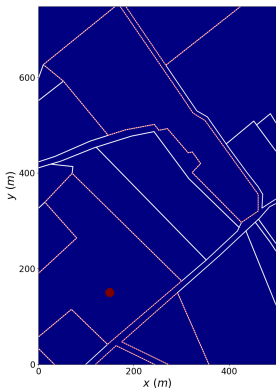
(a) Map with the land occupation data



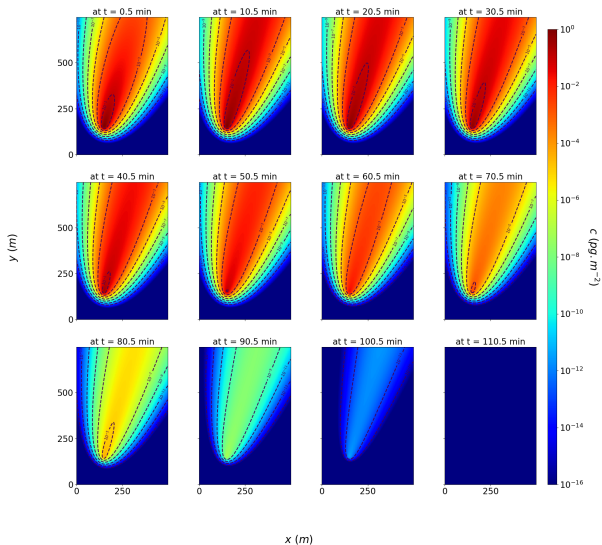
(b) Wind rose of MétéoFrance (AROME model) data

Figure: Realistic test case (Loir-et-Cher, France) between the 16/11/2022 12:00 and the 17/11/2022 12:00

Work in progress: application to realistic test cases



(a) Map of the target support (in red) with the landscape decomposition (in white lines and red dashed lines for the colza field)



(b) Map of the pheromone concentration $c(s_t)$ at different time

Figure: Results of the pheromone propagation model for the realistic test case based on the fall armyworm behaviour

Work in progress: sensitivity analysis of the pheromone propagation model to uncertainty in the environmental parameters

2D model of the pheromone propagation:

$$\frac{\partial c}{\partial t} - \underbrace{\nabla \cdot (\mathbf{K} \nabla c)}_{\text{diffusion}} + \underbrace{\nabla \cdot (\vec{u} c)}_{\text{wind}} + \underbrace{\tau_{loss} c}_{\text{deposition}} = s$$

⇒ the estimate of c depends environmental parameters, especially on the wind velocity \vec{u} and on the loss coefficient τ_{loss} .

τ_{loss} depends on the landscape decomposition and on the soil occupation but the value τ_{loss} wrt the vegetation are uncertainty.

\vec{u} computed by large scale meteo model with large spatio-temporal sampling ($\approx 1.3km$ and $1h$)
⇒ the scales smaller are unresolved.

Analyse of the global sensitivity of c to uncertainty in the value of τ_{loss} wrt the vegetation and in multi-scale structures of \vec{u} .