Passive stability control of hanging articulated pipes using an additional mass

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Abstract

Depending on their physical characteristics, freely hanging articulated pipes can undergo divergence- or flutter-type instability above a critical velocity of the flow they conveye. The effect of adding a point mass, and more precisely influence of the mass magnitude and location, on the stability of such pipes is investigated by means of a linear analysis. It is found that this additional load can have a stabilizing or destabilizing effect depending on its location along the pipe and that the effect is all the more important as the mass is high. The influence of the mass on the shape of the destabilized linear mode is also discussed.

Keywords: pipe, futter, divergence, passive control

1. Introduction

Pipes conveying fluid are encountered in many industrial applications and their dynamics and stability have motivated numerous studies. A freely hanging pipe is one of many configurations of pipes conveying fluid, it consists of a vertical pipe with a fixed upstream end and a free downstream end where the fluid discharges to the atmosphere. When the velocity of the conveyed flow exceeds a critical value, an hanging pipe initially straight can be destabilized and either static or oscillatory deflection occur, respectively referred to as divergence or flutter. One of the first comprehensive study on the stability of hanging pipes was devoted to articulated pipes that is pipes made of rigid segments connected by flexible joints. In this theoretical and experimental work, Benjamin [1, 2] showed that, depending on their physical characteristics, these

pipes can be destabilized by divergence or by flutter. In contrast, a study by Païdoussis [3] revealed that hanging continuously flexible pipes are only subject to planar flutter.

The effect of adding a mass at the free end of continuously flexible pipes has been considered by Copeland and Moon [4]. Their experiments revealed that, depending of the end mass value, the flexible pipe can exhibit various routes to chaos as the flow velocity is increased, and a great variety of dynamical states. This richness of states, including periodic, quasiperiodic or chaotic motions, planar and three-dimensional, while only periodic planar motions have been reported for a pipe without additional mass, has motivated further studies [5, 6, 7]. These studies have also shown that the stability limit of the hanging flexible pipes is modified by such an end mass. Thus, with a small mass the flutter appears at a smaller critical velocity, that is a small mass is destabilizing, whereas higher masses are stabilizing [4, 5, 7]. Configurations with several additional masses [8, 9] or with a single mass at various locations along the pipe [10] have also been investigated and the pipe stability has been found to depend on both the location and the value of the masses. For a comprehensive review, the reader can refer to the monography of Païdoussis [11]. For articulated pipes, effect of an additional mass has only be tested for horizontal pipes, rather than hanging. Thus, Sugiyama and Noda [12] showed that the critical velocity for a pipe made of two segments is reduced wherever a mass is added on the second segment.

The present study aims to extend these studies to hanging articulated pipes with an arbitrary number of segments, horizontal configuration is also briefly adressed as a limit case. The effect of adding a mass on the stability of the pipe is investigated by means of a linear analysis whose formulation is derived in the next section (Sec. 2) using the Lagrange's formalism as introduced by Benjamin [1]. Analyses were performed for articulated pipes with different characteristics and considering different mass values and positions all along the pipe. Results are presented in Section 3 before to conclude.

2. Formulation of the linear analysis

The articulated pipe under investigation is schematized in Fig. 1. It consists of n identical rigid segments of length L and mass per unit length M_p, connected by n identical flexible joints J_j. The upper segment is connected in O by the joint J_1 to a pipe fixed in the frame $(O, \overrightarrow{x}, \overrightarrow{y}, \overrightarrow{z})$. The pipe conveys a fluid, whose mass contained in one unit length of pipe is M_f, at the velocity U and discharging to the atmosphere at the free end E. The pipe is fitted with an additional mass M_a located on the the segment n_a $(1 \leqslant n_a \leqslant n)$ at a distance L_a $(0 \leqslant L_a \leqslant L))$ from the joint J_a. Without flow, the pipe freely hangs along the vertical axis $(O, \overrightarrow{z})$ under the action of gravity $G\overrightarrow{z}$. The motions of the pipe are constrained within the plane $(O, \overrightarrow{x}, \overrightarrow{z})$ in such a way that they are fully described by the n angles ϕ_j between the segments and $(O, \overrightarrow{z})$, its equilibrium position is defined by $(\phi_i = 0)_{i=1..n}$. It should be pointed out that, as discussed by Benjamin [2], the conditions of stability of this equilibrium position presented in the present study are deduced from a linearized theory in such a way that they are not modified whether or not the motions of the pipe are constrained in a vertical plane.

Following Benjamin [1], the equations of motion of the articulated pipe are derived from the n Lagrange's equations for the open system composed of the n segments and n joints, of the fluid they contain at any times and of the additional mass, that are given by

$$\frac{\mathrm{d}}{\mathrm{d}T}\frac{\partial \mathcal{L}}{\partial \dot{\phi}_i} - \frac{\partial \mathcal{L}}{\partial \phi_i} = -M_f U \left(\dot{\overrightarrow{OE}} + U\overrightarrow{\tau_n} \right) \cdot \frac{\partial \overrightarrow{OE}}{\partial \phi_i} \quad (i = 1..n) \tag{1}$$

where overdot denotes derivative with respect to time T.

In these equations $\mathcal{L} = \mathcal{T} - \mathcal{V}$ is the system Lagrangian, $\mathcal{T} = \mathcal{T}_{p+f} + \mathcal{T}_a$ is kinetic energy of the pipe and the fluid plus that of the additional mass, $\mathcal{V} = \mathcal{V}_{p+f} + \mathcal{V}_a + \mathcal{V}_J$ is the potential energy associated with the restoring forces due to gravity, for the pipe and the conveyed fluid, for the additional mass and

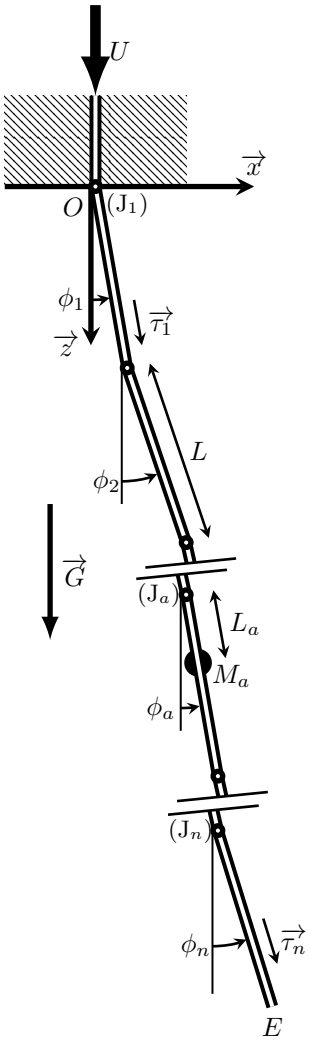

Figure 1: Schematic of the system under consideration.

due to joint stiffness. $\overrightarrow{\tau_i}$ is the unit vector parallel to the segment i and then $\dot{\overrightarrow{OE}} + U\overrightarrow{\tau_n}$ is the fluid velocity at the pipe free end.

For simplicity inner and outer pipe diameters are considered as being small compared to the segment length L and the additional mass is treated as a point mass. With these assumptions and by introducing the constant angle $\phi_0 = 0$

(then $\dot{\phi}_0 = \ddot{\phi}_0 = 0$), the terms of Eq. 1 are given by

$$\mathcal{T}_{p+f} = \frac{(M_p + M_f) L^3}{2} \sum_{i=1}^{n} \left[\frac{\dot{\phi}_i^2}{3} + \dot{\phi}_i \sum_{j=0}^{i-1} \dot{\phi}_j \cos(\phi_i - \phi_j) + \left(\sum_{j=0}^{i-1} \dot{\phi}_j \cos(\phi_i - \phi_j) \right)^2 + \left(\sum_{j=0}^{i-1} \dot{\phi}_j \sin(\phi_i - \phi_j) \right)^2 \right] + M_f L^2 U \sum_{i=1}^{n} \sum_{j=0}^{i-1} \dot{\phi}_j \sin(\phi_i - \phi_j) + n \frac{M_f L U^2}{2}, \quad (2)$$

$$\mathcal{T}_a = \frac{M_a}{2} \sum_{i=0}^{n_a - 1} \left[L^2 \sum_{j=0}^{n_a - 1} \dot{\phi}_i \dot{\phi}_j \cos(\phi_i - \phi_j) + 2 L L_a \dot{\phi}_i \dot{\phi}_{n_a} \cos(\phi_i - \phi_{n_a}) \right] + \frac{M_a L_a^2}{2} \dot{\phi}_{n_a}^2, \quad (3)$$

$$\mathcal{V}_{p+f} = -(M_p + M_f) G L^2 \sum_{i=1}^{n} \left[\frac{\cos \phi_i}{2} + \sum_{j=0}^{i-1} \cos \phi_j \right] + \text{const.}, \quad (4)$$

$$\mathcal{V}_a = -M_a G \left[\sum_{i=0}^{n_a - 1} L \cos \phi_i + L_a \cos \phi_{n_a} \right] + \text{const.} \quad (5)$$

The restoring forces due to the joints of stiffness K are supposed to be linearly dependent on the deformation angle in such a way that

$$\mathcal{V}_J = \frac{K}{2} \sum_{i=1}^{n} (\phi_i - \phi_{i-1})^2 + \text{const.} \quad (6)$$

The n coupled non linear differential equations governing the motion of the articulated pipe are then derived by substituting Eqs. 2-6 into Eq. 1. Next, in order to analyze the linear stability of the straight pipe fitted with an additional mass, these equations are linearized about the equilibrium position $(\phi_i = 0)_{i=1..n}$ and harmonic displacement modes $\phi_i(T) = \varphi_i e^{\Sigma T}$ are considered, where Σ is the complex circular frequency. The resulting linear system is nondimensionalized using the time scale Ω^{-1} where $\Omega = \left(\frac{3K}{(M_p + M_f) L^3} \right)^{\frac{1}{2}}$ is the frequency of the oscillator made of a single horizontal ($G = 0$) segment filled

with the fluid at rest ($U = 0$). It follows the nondimensional linear homogeneous system for the complex amplitude vector $\varphi = (\varphi_i)_{i=1..n}$

$$\mathbf{a}\varphi = 0 \tag{7}$$

where $\mathbf{a}$ can be written as the sum of two $n \times n$ matrices $\mathbf{a} = \mathbf{a}_{p+f} + \mathbf{a}_a$ with $\mathbf{a}_{p+f}$ the matrix for the system without additional mass and $\mathbf{a}_a$ associated with the additional mass. Expressions of the elements $a_{p+f}(i,j)$ and $a_a(i,j)$ are given in Appendix, the former depend on the five nondimensional parameters n, m, g, u and σ, and the latter depend on n_a, m_a, l_a, g and σ, where σ is the nondimensional eigenvalue $\sigma = \Sigma\Omega^{-1}$. The other nondimensional parameters are defined as functions of the physical parameters as follows

$$\begin{aligned} n,\ m = \frac{3M_f}{M_p + M_f},\ g = \frac{3G}{2L\Omega^2},\ u = \frac{U}{L\Omega}, \\ m_a = \frac{3M_a}{(M_p + M_f)\,L},\ d_a = \frac{(n_a - 1)\,L + L_a}{L} = (n_a - 1) + l_a, \end{aligned} \tag{8}$$

thus d_a is the distance nondimensionalized with the segment length L from the fixed pipe end 0 to M_a when the pipe is straight and $l_a = L_a/L$.

The linear stability of the static equilibrium position $(\phi_i = 0)_{i=1..n}$ of a given configuration (n, m, g, n_a, d_a) is then analyzed by calculating the $2n$ complex eigenvalues σ of the linear problem Eq. (7) for a range of the nondimensional velocity u starting at 0. As u is increased, the straight pipe $(\phi_i = 0)_{i=1..n}$ is linearly destabilized as soon as one of the growth rate $\Re(\sigma) > 0$ becomes positive. The critical velocity is given by $\Re(\sigma) = 0$ and the pipe loses stability by divergence if the displacement mode at the threshold is static $\Im(\sigma) = 0$, or by flutter if the displacement is oscillatory $\Im(\sigma) \neq 0$. These two behaviors are illustrated in Fig. 2 showing the trajectories of the eigenvalues σ in the complex plane, as u is varied, for pipes $n = 2$, $m = 1$, $g = 10$ and two configurations, namely without additional ($m_a = 0$) and with an additional mass $m_a = 2$ at the free end $d_a = 2$. For the configuration without additional mass, a purely real eigenvalue crosses first the imaginary axis, at a critical velocity between 5.5

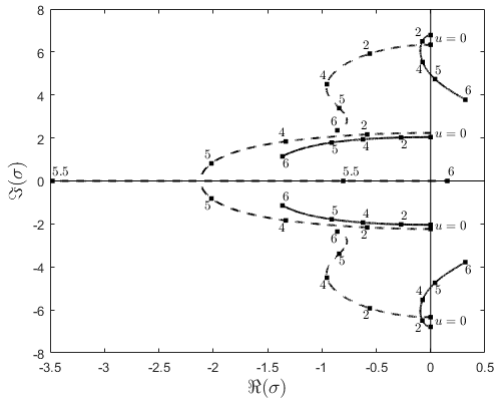

Figure 2: Trajectories of the eigenvalues σ in the complex plane, as u is varied from 0 to 6, for pipes $n = 2$, $m = 1$, $g = 10$ without additional (dashed line) and with an additional mass $m_a = 2$ at the free end $d_a = 2$ (solid line).

and 6, indicating that the pipe loss stability by divergence. Conversely, for the pipe with the additional mass $m_a = 2$ at $d_a = 2$, a pair of complex conjugate eigenvalues cross first the imaginary axis, for a value of u between 4 and 5, indicating a flutter mode at the frequency given by the positive value of $\Im(\sigma)$. For some configurations, the linear displacement modes at the threshold are plotted. They are given by the real part of the solution of Eq. (7), associated with the first eigenvalue σ becoming positive and calculated at the critical velocity, using Matlab built-in functions. It should be noted that such linear modes are of arbitrary amplitude. For the flutter modes, the pipe displacement is shown at several instants of a cycle (see e.g. Fig. 3).

3. Results

3.1. Articulated pipe without additional mass

The linear stability of articulated pipes without additional mass is first analyzed by setting $m_a = 0$ then $\mathbf{a}_a$ is a zero matrix. In this case, the pipe stability is controlled by four nondimensional parameters, namely n, m, g and u where the mass ratio m is by definition between 0 and 3 and g is the ratio of the two restoring forces that is gravity to joint stiffness, $g > 0$ for hanging pipes

and $g = 0$ corresponds to horizontal pipes. As u is increased, pipes with low g and/or low m are destabilized by flutter, otherwise they lose stability by divergence, with a boundary that depends on n (see e.g. Ref. [13]). To investigate these two behaviors, the linear analysis was mainly performed for pipes with $m = 1$, $g = 0.5$ and $m = 1$, $g = 10$, respectively referred to as "low gravity/high stiffness" and "high gravity/low stiffness" pipes. In fact, all pipes without additional mass made of $n = 2$ to 20 segments, as considered in the present study, and characterized by $m = 1$ and $g = 0.5$ lose stability by flutter, whereas it is by divergence if $m = 1$ and $g = 10$. Pipes made of only one segment $n = 1$ are always stable as discussed in Ref. [13].

The critical velocity u_0 at which articulated pipes $m = 1$ and $g = 0.5$ are destabilized, as well as the frequency ω_0 of the resulting flutter mode at u_0, that is the value of $\Im(\sigma)$ at u_0, are plotted as function of n in Fig. 3. The evolution of u_0 with n presents lobes which correspond to discontinuities of the flutter frequency, they are due to changes of the flutter mode. Thus, a zero-neck mode is predicted by the linear analysis for $n = 2$, then a one-neck mode for $n = 3$ and 4, a two-neck mode for $n = 5$, 6 and 7 and so on as illustrated in Fig. 3 for $n = 2$, 3, 5 and 8.

For $m = 1$ and $g = 10$, the loss of stability of the pipes leads to divergence at a critical velocity u_0 plotted in Fig. 4, u_0 weakly increases between $n = 2$ to 4 then is independent on n as $n \geqslant 4$. The shape of the linear divergence modes at u_0 is shown in Fig. 4 for $n = 2$, 3, 5 and 8. The pipe deformation is mainly confined to the free end and closely resembles for all n. It is notable that the last segment is almost vertical, a feature already noted by Benjamin [2] in his experiments who analytically shown that it would be exactly vertical ($\varphi_n = 0$) for pipes with joints without stiffness i.e. when gravity is the only restoring force.

3.2. Effect of an additional mass

When a mass is added on the pipe there are two more nondimensional control parameters in addition to n, m, g and u, namely the additional mass m_a, which

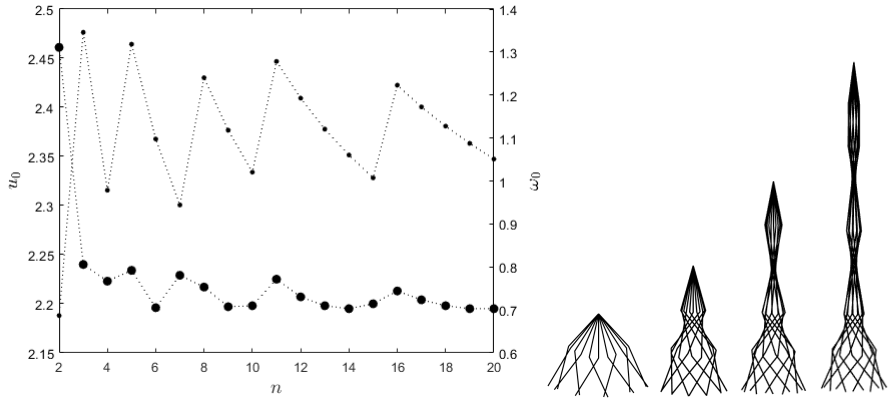

Figure 3: Critical velocity u_0 (•) and flutter frequency ω_0 (•) for pipes $m = 1$ and $g = 0.5$ as function of the number of segments (all pipes are destabilized by flutter) and linear flutter mode at u_0 for $n = 2$, 3, 5 and 8, from left to right.

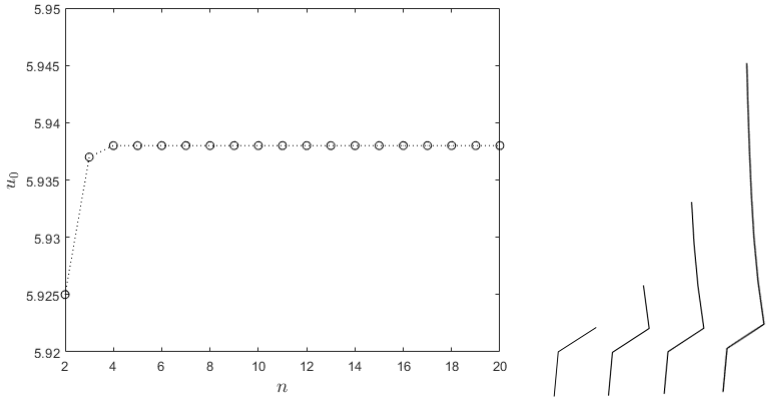

Figure 4: Critical velocity u_0 for pipes $m = 1$ and $g = 10$ as function of the number of segments (all pipes are destabilized by divergence) and linear divergence mode at u_0 for $n = 2$, 3, 5 and 8, from left to right.

was varied up to 12, and its positions d_a. Positions all along the pipe were considered, that is $0 \leqslant d_a \leqslant n$, by steps of 0.2, reduced to 0.05 around the extremes of the critical velocity u_c.

Main results of the present study are presented in Figs. 5 and 8 for "low gravity/high stiffness" pipes and for "high gravity/low stiffness" pipes, respectively. These figures show the evolution of the critical velocity as function of the additional mass location for different configurations. To highlight the effect of adding a mass m_a on the stability of articulated pipes, critical velocities u_c for a pipe fitted with an additional mass are normalized by the critical velocity u_0 for the same pipe (same n, m and g) without additional mass. Normalized critical velocities are presented either at a given additional mass $m_a = 4$ and for different pipes with n between 2 and 14 (Figs. 5(a) and 8(a)), or for the pipe with $n = 6$ and mass m_a varied from 0.5 to 8 (Figs. 5(b) and 8(b)). Results in these figures are qualitatively representative of all tested combinations such that $g > 0$, n up to 20 and $0 < m < 3$. The case $g = 0$ is also briefly discussed.

3.2.1. "Low gravity/high stiffness" pipes

For the configurations $m = 1$, $g = 0.5$ and $n = 2$ to 20 segments, articulated pipes without or with an additional mass m_a, whatever m_a and its position d_a, are found to be destabilized by flutter. Fig. 5 shows that, depending on its location along the pipe, an additional mass can have a stabilizing ($u_c/u_0 > 1$) or destabilizing ($u_c/u_0 < 1$) effect on the hanging articulated pipes. More precisely, the influence of the mass is null or weak when it is located on the upper part of the pipe and becomes more pronounced when the mass is on the last two segments. It is maximal on the last segment where the two extema of u_c/u_0 are observed, u_c/u_0 reaches its maximum around the half of this last segment and its minimum at the free end $d_a = n$. Moreover, as shown in Figure 5(b) for $n = 6$, both stabilizing or destabilizing effects are greater when the mass m_a is larger. For example, the critical velocity u_c of a pipe $n = 6$, $m = 1$ and $g = 0.5$ can be increased by 1%, compared to the critical velocity of the same pipe without additional mass, when it is fitted with an additional mass $m_a = 1$

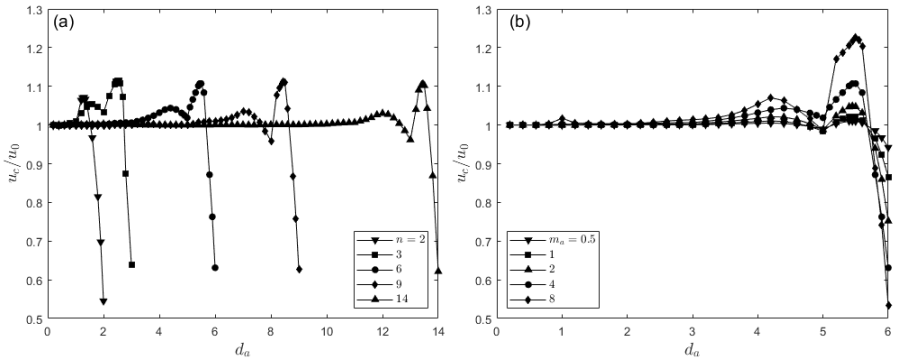

Figure 5: Normalized critical velocity u_c/u_0 as function of the additional mass location d_a for $m = 1$ and $g = 0.5$: $m_a = 4$ and $n = 2$, 3, 6, 9 and 14 (a), $n = 6$ and $m_a = 0.5$, 1, 2, 4 and 8 (b). All pipes are destabilized by flutter.

at $d_a = 5.45$, and increased by 23% when $m_a = 8$ and $d_a = 5.5$. For this later configuration, when m_a is located at the pipe free end $d_a = 6$, the critical velocity can be decreased by 6% to 47% for $m_a = 0.5$ to 8.

Fig. 6 shows the shape of the linear flutter modes for a pipe $n = 6$, $m = 1$ and $g = 0.5$ without or with an additional mass $m_a = 4$ located at the positions corresponding to the extremes of the critical velocity, that is $d_a = 5.5$ and $d_a = 6$. At $d_a = 5.5$, the mass has little influence on the shape of the unstable mode, in contrast, if m_a is added at the free end it strongly influences by imposing a quasi-nodal point at this end, rather than an antinode without mass.

The conclusion that a mass m_a added at the free end of articulated pipes is increasingly destabilizing as m_a is increased, see e.g. Fig. 5(b) for a pipe $n = 6$ and m_a up to 8, differs from the observations for continuously flexible pipes where a small mass at the free end reduces the critical flow velocity for the flutter, whereas the effect of a higher mass is stabilizing [4, 5, 7]. For this reason, the linear stability analysis of articulated pipes was conducted with higher masses m_a, up to 1000, located at the free end and u_c is found to be monotonuously decreasing over the whole interval as seen for pipes $n = 6$, $m = 1$ and $g = 0.5$ in Fig. 7 where m_a is limited to 400 for clarity. The evolution of the corresponding nondimensional flutter frequency ω_c with m_a is also plotted

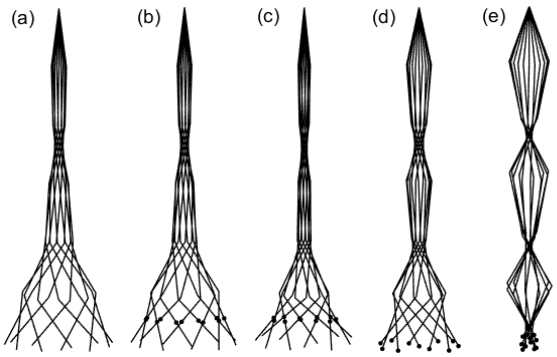

Figure 6: Linear flutter mode at the critical velocity for pipe $n = 6$, $m = 1$ and $g = 0.5$ without additional mass at $u_0 = 2.196$ (a) and with a mass added at $d_a = 5.5$ and $m_a = 0.5$, $u_c = 2.217$ (b), $m_a = 4$, $u_c = 2.431$ (c) and at $d_a = 6$ and $m_a = 0.5$, $u_c = 2.069$ (d), $m_a = 4$, $u_c = 1.386$ (e).

in the figure. The discontinuity that can be observed for m_a between 39 and 40 corresponds to a jump from a flutter mode with 4 nodes to a mode with 5 nodes as displayed in Fig. 7.

3.2.2. "High gravity/low stiffness" pipes

Results for $m = 1$ and $g = 10$ are presented in Fig. 8 recalling that, for these values of the parameters, pipes without additional mass are destabilized by divergence. It appears first that adding a mass affects significantly the stability of these pipes only if it is placed on the last two segments $(n-2) < d_a \leqslant n$, and second that here also the additional mass can be stabililizing or destabililizing depending on its location. The positions for which these two effects are maximal are on the last segment, around its centre for the maximum of u_c/u_0 or at its free end for its minimum, except for the $n = 2$ for which it is maximum at the joint between the two segments. Moreover, Fig. 8(b) shows that both stabilizing or destabilizing effects are increased as m_a is increased.

The main difference with the "low gravity/high stiffness" pipes is that here adding a mass can change not only the critical velocity but also the type of the instability. Thus, when fitted with a mass, pipes $m = 1$ and $g = 10$ can be destabilized by flutter (filled symbols in Fig. 8) or divergence (open symbols),

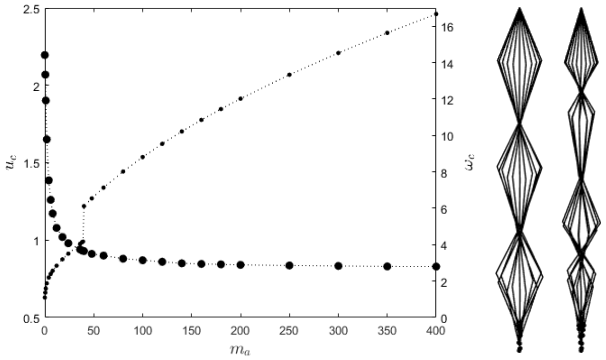

Figure 7: Critical velocity u_c (•) and flutter frequency ω_c (•) for pipes $m = 1$, $g = 0.5$ and $n = 6$ as function of the additional mass m_a located at $d_a = 6$ (all pipes are destabilized by flutter) and linear flutter mode at u_c for $m_a = 39$ (left) and 40 (right).

depending on the position and intensity of this mass, whereas the same pipes without additional mass are always destabilized by divergence.

To further examine this behaviour, stability analyses were conducted up to velocities such that at least one divergence mode and one flutter mode became unstable. The resulting critical velocity u_c for divergence and flutter are plotted as function of the additional m_a in Fig. 9 for pipes $n = 6$, $m = 1$, $g = 10$ fitted with a mass at $d_a = 5.4$ and 6.

When the mass is added at $d_a = 5.4$ (Fig. 9(a)), the two critical velocities, for divergence and for flutter, rapidly converge towards each other as m_a is increased. They intersect for m_a between 2.5 and 3 meaning that the pipe is first destabilized by divergence then by flutter, the corresponding unstable modes are shown in Fig. 10(a-c) for $m_a = 0$, 2 and 4, respectively. Note that the difference between the two critical velocities is less than 0.7% as soon as $m_a \geqslant 2$ and at least up to 100.

Fig. 9(b) shows that when the mass m_a is located at the free end, similar change of the type of the first mode to be destabilized occurs. In this case the two curves intersect between $m_a = 0.1$, where the pipe loses stability by divergence Fig. 10(d), and $m_a = 0.2$ where flutter occurs first Fig. 10(e). If m_a is still

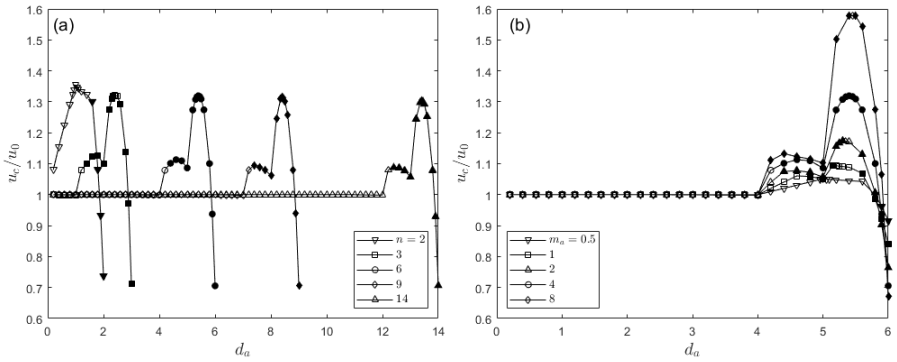

Figure 8: Normalized critical velocity u_c/u_0 as function of the additional mass location d_a for $m = 1$ and $g = 10$: $m_a = 4$ and $n = 2$, 3, 6, 9 and 14 (a), $n = 6$ and $m_a = 0.5$, 1, 2, 4 and 8 (b). Open and filled symbols are for destabilizations by divergence and by flutter, respectively.

increased it imposes a node at the free end to the flutter mode as observed in Fig. 10(f).

3.2.3. Horizontal pipe

Present results for hanging articulated pipes $g > 0$ differ from those reported by Sugiyama and Noda [12] for horizontal pipes with $n = 2$ segments. In this configuration joint stiffness is the only restoring force then $g = 0$ and the pipe is destabilized by flutter, and the authors observed that wherever an additional mass is on the second segment, it is always destabilizing. Some analyses have been conducted for this horizontal configuration by setting $g = 0$ in the expression of the matrix elements, given in Appendix, of the linear system Eq. (7). Results are illustrated in Fig. 11 for pipes with $m = 1$, n between 2 and 14, and an additional mass $m_a = 4$. It can be seen that when the mass is positioned approximately on the first part of the pipe its impact remains limited whereas, on the second part, it is increasingly destabilizing as the mass is moved toward the free end $d_a = n$ in agreement with the results of Sugiyama and Noda. For example, for a pipe $m = 1$ and $n = 6$ fitted with a mass $m_a = 4$, u_c/u_0 evolves non-monotonously between 0.98 and 1.01 for $0 < d_a \leqslant 2.6$ then

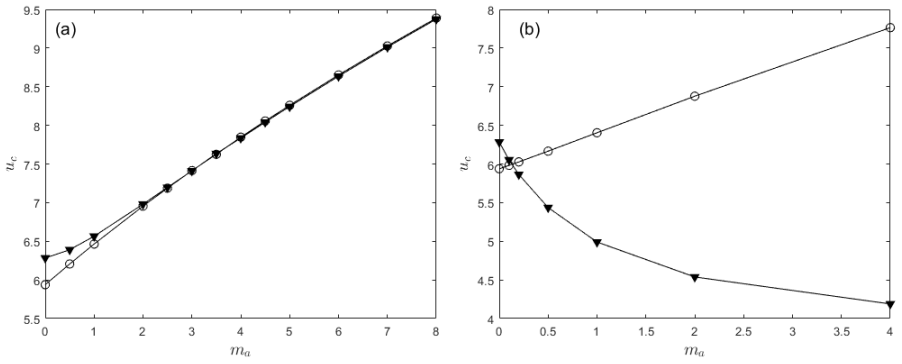

Figure 9: Critical velocity u_c for divergence (open symbols) and flutter (filled symbols) as function of the additional mass m_a for $n = 6$, $m = 1$, $g = 10$ and $d_a = 5.4$ (a), $d_a = 6$ (b).

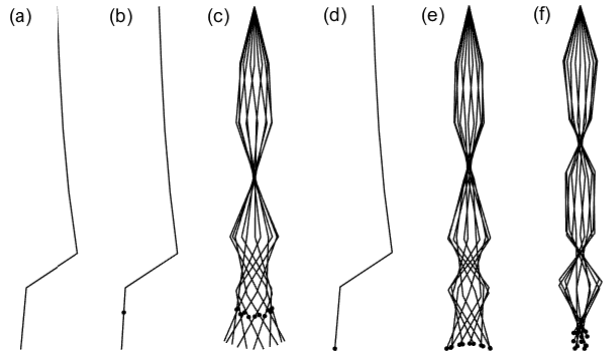

Figure 10: Linearly unstable mode at the critical velocity for pipe $n = 6$, $m = 1$ and $g = 10$ without additional mass at $u_0 = 5.938$ (a) and with a mass added at $d_a = 5.4$ and $m_a = 2$, $u_c = 6.954$ (b), $m_a = 4$, $u_c = 7.835$ (c) and at $d_a = 6$ and $m_a = 0.1$, $u_c = 5.982$ (d), $m_a = 0.5$, $u_c = 5.436$ (e), $m_a = 4$, $u_c = 4.189$ (f).

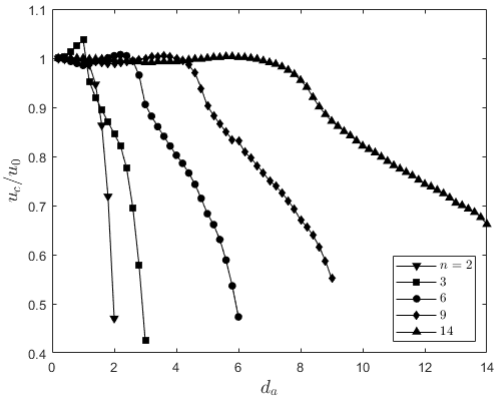

Figure 11: Normalized critical velocity u_c/u_0 as function of the additional mass location d_a for $m = 1$ and $g = 0$, $m_a = 4$ and $n = 2$, 3, 6, 9 and 14. All pipes are destabilized by flutter.

decreases to less than 0.48 at $d_a = 6$ that is a reduction of the critical velocity by more than 52%.

4. Conlusion

The present study was undertaken to investigate the effect of adding a mass on the stability of an articulated pipe conveying a fluid by means of a linear analysis. When the pipe is hanging, an additional mass can be destabilizing or stabilizing depending on its position, compared to the stability of the same pipe without additional mass, both effects increasing with the mass value. More precisely, the critical velocity of the conveyed fluid flow is found for all hanging configurations tested to be maximum when the mass is around the centre of the last segment, and minimum when it is at the free end, that is at the end of the last segment. In contrast, adding a mass on a horizontal articulated pipe has been found to be effective only to reduce the critical flow velocity at which pipe begins to flutter, this stabilizing effect occurs when m_a is located roughly on the second half of the articulated and maximum at the free end. Moreover it has been found that a sufficiently high mass added on the last two segments of "high gravity/low stiffness" articulated pipes can also change the

instability type from static to oscillatory, that is from divergence to flutter. Note that similar observations have previsously been reported for another system of fluid/structure interaction. In fact, Howell *et al.*[14] showed that when a line mass is added parallel to the span of a flexible plate immersed in a flow, it is possible to promote as well as to delay the flow velocity at which plate flutter occurs by changing the location of the mass along the chord. Thus, the present study confirms that an "inertial inhomogeneity" [14] is an effective, and easy to implement, means to passively control the stability of a structure interacting with a fluid flow. Additional information, as the amplitude of the displacement modes or the nature of the bifurcations can be obtained by considering the system of non linear equations Eq. 1 with Eqs. 2-6. Moreover, this non linear system, possibly extended to motions not constrained in a plane, could be used to check if articulated pipes with an additionnal mass can exhibit a wide variety of dynamical states and routes to chaos as observed for a continuously flexible pipe fitted with an end mass [4, 5, 6, 7].

Appendix

The matrix $\mathbf{a}$ of the linear system Eq. (7) is written as the sum of two $n \times n$ matrices, where $n \geqslant 2$ is the number of segments into the pipe, $\mathbf{a} = \mathbf{a}_{p+f} + \mathbf{a}_a$. Elements of $\mathbf{a}_{p+f}$ are given by

for all $n \geqslant 2$

$$a_{p+f}(n,n) = \sigma^2 + mu\sigma + g + 1,$$

$$a_{p+f}(n-1,n) = \frac{3}{2}\sigma^2 + 2mu\sigma + mu^2 - 1,$$

and for $i = 1$ to $n-1$

$$a_{p+f}(i,i) = [4 + 3\,(n-1-i)]\,\sigma^2 + mu\sigma - mu^2 + [3 + 2\,(n-1-i)]\,g + 2,$$

$$a_{p+f}(i+1,i) = [3 + 6\,(n-1-i)]\,\frac{\sigma^2}{2} - 1;$$

if $n > 2$, for $i = 1$ to $n-2$

$$a_{p+f}(i,n) = \frac{3}{2}\sigma^2 + 2mu\sigma + mu^2,$$

$$a_{p+f}(i,i+1) = [9 + 6\,(n-2-i)]\,\frac{\sigma^2}{2} + 2mu\sigma - 1,$$

and for $j = 1$ to i

$$a_{p+f}(i+2,j) = [3 + 6\,(n-2-i)]\,\frac{\sigma^2}{2};$$

if $n > 3$, for $i = 1$ to $n-3$ and $j = i+2$ to $n-1$

$$a_{p+f}(i,j) = [9 + 6\,(n-1-j)]\,\frac{\sigma^2}{2} + 2mu\sigma.$$

Remembering that n_a is the number of the segment on which the additional mass mass is placed $1 \leqslant n_a \leqslant n$, elements of $\mathbf{a}_a$, which is a $n \times n$ symmetric matrix, are given by

for all n_a

$$a_a(n_a,n_a) = m_a\left(\sigma^2 l_a^2 + \frac{2}{3}gl_a\right);$$

if $n_a \geqslant 2$, for $i = 1$ to $n_a - 1$

$$a_a(i,i) = m_a\left(\sigma^2 + \frac{2}{3}g\right)$$

and for $i = 1$ to $n_a - 1$ and $j = i+1$ to n_a

$$a_a(i,j) = a_a(j,i) = m_a\sigma^2\,[1 + (l_a - 1)\,\delta_{j,n_a}]$$

where δ_{j,n_a} is the Kronecker delta;

if $n_a < n$, for i or $j > n_a$

$$a_a(i,j) = 0.$$

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