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Formation Control of Cooperative-Competitive Robot Manipulators with Inter-agent Constraints

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Abstract: This paper addresses the distributed formation control problem of networked robot manipulators in end-effector coordinates. We propose a distributed bipartite formation controller that guarantees collision avoidance and maximum distance maintenance for cooperative and competitive manipulators' end-effectors. In the considered setting, two groups of manipulators are formed and reach bipartite consensus. On the other hand, the end-effectors in the same group achieve formation. We design a gradient-based control law using barrier-Lyapunov functions to ensure that the constraints on the end-effectors are satisfied. Moreover, we establish asymptotic stability of the bipartite formation manifold. Finally, we illustrate our theoretical results via numerical simulations.

Keywords: Formation consensus, signed networks, robotic manipulators, barrier-Lyapunov functions

1. INTRODUCTION

In formation control, a group of robots has to undertake a task, such as reaching a consensus point or tracking targets, while keeping a geometric pattern between agents. This problem has been extensively studied in the literature, but most works consider only cooperative agents, in which case the resulting network can be modeled by a graph containing only positive weighted edges. Yet, in many scenarios some agents may be in competition. For instance, in the context of herding control (Chipade and Panagou, 2020); (Grover et al., 2022); of social-networks theory (Altafani, 2013), and of aerospace applications (Zhao et al., 2022).

The presence of both cooperative and competitive interactions in a network may be modeled using *signed networks*, in which the edges have both positive and negative weights—see (Altafani, 2013). For so-called *structurally balanced* networks¹ the attainable goal, in general, is *bipartite consensus*, which consists in all the agents converging to the same state in modulus but opposite in signs. See, *e.g.*, (Altafani, 2013); (Du et al., 2018), and (Valcher and Misra, 2014).

Now, the above-mentioned references focus on generic first-, second-, or higher-order linear models, which are

¹ A signed network is called structurally balanced if the nodes may be split into two disjoint subsets, where cooperative agents are in the same subset and competitive agents are in different ones.

less suited to model the dynamics of robot manipulators. The latter are most often modeled by the Euler-Lagrange equations, which are inherently nonlinear. Now, multi-agent Euler-Lagrange systems are well-studied in the literature, but mostly for cooperative systems. The synchronization of multiple robot manipulators in joint coordinates is addressed, *e.g.*, in (Nijmeijer and Rodríguez-Angeles, 2003) and the formation of manipulators in end-effector coordinates is studied in (Sakurama, 2021); (Wu et al., 2022). For signed networks, the bipartite consensus of networked robot manipulators is addressed, *e.g.*, in (Liu et al., 2019); (Hu et al., 2019); (Zhang et al., 2023), while the leader-follower bipartite consensus is studied in (Liu et al., 2022); (Liang and Huang, 2020); (Li et al., 2022), all in joint coordinates. In end-effector coordinates the bipartite formation-control problem is considered in (Pan et al., 2023).

This paper addresses the bipartite formation-control problem of end-effectors for robot manipulators while ensuring inter-agent end-effector collision avoidance and maximum distance maintenance for task requirements. These inter-agent constraints are commonly addressed using artificial potential functions—see *e.g.*, (Dimarogonas and Kyriakopoulos, 2008); (Panagou et al., 2013) for cooperative networks. For signed networks, (Fan et al., 2014) addresses the bipartite flocking problem with collision avoidance and connectivity maintenance, using artificial potential functions. In (Şekercioglu et al., 2023), a controller based

on the gradient of a barrier-Lyapunov function is proposed for the problem of constrained bipartite formation over undirected signed networks of simple integrators. Nonetheless, all the references mentioned above consider only first and second-order integrators.

In this paper, we consider robot manipulators modeled by the Euler-Lagrange equations. The contributions are twofold: firstly, in contrast with (Nijmeijer and Rodríguez-Angeles, 2003); (Sakurama, 2021); (Wu et al., 2022), we consider networks containing both cooperative and competitive robot manipulators. The resulting graph is signed, structurally balanced, and undirected. Such a scenario is motivated by applications in industrial robotics where manipulators in the same workspace are assigned symmetric tasks or are working on opposite surfaces of a product. By putting signs on the edges, one group of manipulators is forced to perform the task on one side and the other group on the symmetric side. Then, we address the bipartite formation problem of end-effectors under collision avoidance and maximum distance maintenance constraints using barrier-Lyapunov functions. Relative to (Fan et al., 2014), which focuses on bipartite flocking with collision avoidance, in our work, a minimal safety distance is ensured between any two agents. Moreover, in contrast to (Şekercioğlu et al., 2023) and (Fan et al., 2014), we consider Euler-Lagrange systems, not simple integrators. Our control design and analysis rely on the edge-based formulation for signed networks—see (Du et al., 2018), which allows to recast the problem into one of stabilization of the origin in error coordinates. We establish asymptotic stability of the bipartite formation manifold using Lyapunov’s direct method. To the best of our knowledge, similar results are not available in the literature for robot manipulators containing competitive interactions.

2. MODEL AND PROBLEM FORMULATION

2.1 Agents’ dynamics

Consider a network of N n -degrees-of-freedom robot manipulators modeled by the Euler-Lagrange equations.

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + \frac{\partial}{\partial q_i}U_i(q_i) = \tau_i, \quad i \leq N, \quad (1)$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n$ are the generalized joint position, velocity, and acceleration respectively, $M_i(q_i) \in \mathbb{R}^{n \times n}$ is the inertia matrix, $U : \mathbb{R}^n \rightarrow \mathbb{R}$ is the potential energy function and $\tau_i \in \mathbb{R}^p$ is the control input. As it is customary, we assume the following.

Assumption 1. The following properties hold.

1. There exist \underline{c}_i and $\bar{c}_i > 0$ such that, $\underline{c}_i I \leq M_i(q_i) \leq \bar{c}_i I$ for all $q_i \in \mathbb{R}^n$.
2. The matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric.
3. The Coriolis matrix $C_i(q_i, \dot{q}_i)$ is uniformly bounded in q_i . Moreover $|C_i(q_i, \dot{q}_i)\dot{q}_i| \leq k_{c_i}|\dot{q}_i|^2$ for $k_{c_i} > 0$.

2.2 Problem statement

The bipartite formation-control problem of the manipulators’ end-effectors consists in ensuring that the positions of the end-effectors, in cooperation with each other, achieve a desired geometric shape around a consensus

value. Meanwhile, the end-effector positions of the competing agents converge on another spatial configuration, which is symmetrical. Let $x_i \in \mathbb{R}^p$ be the position of the i th manipulator’s end-effector in the task space. The position of the end effector x_i can be mapped to its generalized joint coordinates using a nonlinear forward kinematics mapping—see (Murray et al., 1994)

$$x_i = x_{i_0} + h_i(q_i), \quad (2)$$

where x_{i_0} is the position of the manipulator’s base and $h_i : \mathbb{R}^n \rightarrow \mathbb{R}^p$ is the mapping from joint-space to the task space. Differentiating (2) with respect to time, we obtain the relation between the task space and joint velocities

$$\dot{x}_i = J_i(q_i)\dot{q}_i, \quad J_i(q_i) := \frac{\partial h_i(q_i)}{\partial q_i}\dot{q}_i, \quad (3)$$

with $J_i(q_i) \in \mathbb{R}^{n \times p}$ the Jacobian matrix of the forward kinematics. The shape of the end-effector formation is defined by the relative biases b_i and b_j with respect to the two consensus points. Therefore, we can mathematically define the bipartite formation control objective as

$$\lim_{t \rightarrow \infty} \bar{x}_i(t) - \text{sgn}(a_{ij})\bar{x}_j(t) \rightarrow 0, \quad i, j \leq N, \quad (4)$$

where $\bar{x}_i := x_i - b_i$, and $a_{ij} \in \mathbb{R}$ is the adjacency weight between the two agents. Moreover, for agents to achieve the objective in (4), we assume the following.

Assumption 2. The systems described in (1), which are interconnected via inputs τ_i , form a structurally balanced², undirected, and connected signed graph.

Remark 1. If we consider a structurally unbalanced undirected graph, all agents converge to the same state (the origin), which does not respect the imposed constraints. Thus, we only consider structurally balanced graphs.

In the case that all the agents in the network are cooperative, all of the \bar{x}_i converge to the same value. However, in this case, given that some robot manipulators are competitive, there appear to be two competing groups of agents. Therefore, all \bar{x}_i reach two symmetrical values, resulting in all end-effectors adopting a desired formation around two symmetrical consensus points. For the purposes of control design and analysis, this is equivalent to making synchronization errors converge to zero. These errors correspond to the edges of the graph and are defined as

$$\bar{e}_k := \bar{x}_i - \text{sgn}(a_{ij})\bar{x}_j, \quad k \leq M, \quad (5)$$

where k denotes the index of the interconnection between the i th and j th end-effectors.

In addition, to ensure the safety of the manipulators and the success of their task, the controller τ_i must ensure that the end-effectors do not collide and that they remain within the range of their sensors. This amounts to ensuring that for any pair of communicating nodes ν_i and $\nu_j \in \mathcal{V}$, given $R_k > 0$ and $\Delta_k > 0$ with $k \leq M$, and setting $\delta_k := x_i - x_j$, the following sets be invariant:

$$\mathcal{I}_r := \{\delta_k \in \mathbb{R}^n : |\delta_k| < R_k, \quad k \in \mathcal{E}_m\} \quad (6a)$$

$$\mathcal{I}_c := \{\delta_k \in \mathbb{R}^n : |\delta_k| > \Delta_k, \quad k \leq M\}. \quad (6b)$$

Here, \mathcal{E}_m contains the indices of $m < M$ cooperative edges (edges with strictly positive weights) since the maximum

² We recall that a signed graph is *structurally balanced* if it may be split into two disjoint sets of vertices \mathcal{V}_1 and \mathcal{V}_2 , where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ such that for every $i, j \in \mathcal{V}_p, p \in \{1, 2\}$, if $a_{ij} \geq 0$, while for every $i \in \mathcal{V}_p, j \in \mathcal{V}_q$, with $p, q \in \{1, 2\}, p \neq q$, if $a_{ij} \leq 0$. Otherwise, it is *structurally unbalanced* (Altafini, 2013).

distance constraints are only for cooperative agents, \mathcal{I}_r is the set of proximity constraints and \mathcal{I}_c is the set of collision-avoidance constraints. Under these conditions, it is required to design a distributed bipartite formation control law of the form

$$\tau_i = f_i(\bar{e}_k, q_i, \dot{q}_i),$$

to achieve bipartite formation of the end-effectors, *i.e.*, such that,

$$\lim_{t \rightarrow \infty} \bar{e}_k(t) = 0, \quad \lim_{t \rightarrow \infty} \dot{q}_i(t) = 0, \quad k \leq M, i \leq N, \quad (7)$$

and the manipulators' end-effector's trajectories satisfy the proximity and collision-avoidance constraints, that is, $\delta(t) \in \mathcal{I}$ for all $t \geq 0$, where $\delta := [\delta_1 \ \delta_2 \ \cdots \ \delta_M]^\top$, $\mathcal{I} := \mathcal{I}_r \cap \mathcal{I}_c$ for cooperative agents and $\mathcal{I} := \mathcal{I}_c$ for competitive agents.

3. MAIN RESULT

To solve the problem posed above, we recast it as one of stabilization of the origin in edge coordinates, which correspond to the synchronization errors in (5)—cf. (Mesbahi and Egerstedt, 2010); (Du et al., 2018). Then, in order to respect the inter-agent distance constraints, the control input is designed based on the gradient of a so-called barrier-Lyapunov function—cf. (Panagou et al., 2013); (Mesbahi and Egerstedt, 2010); (Restrepo-Ochoa, 2021). Finally, we analyze the stability of the bipartite formation manifold.

3.1 Control design

A *barrier-Lyapunov function* (BLF) is defined as follows—cf. (Panagou et al., 2013), (Mesbahi and Egerstedt, 2010), (Restrepo-Ochoa, 2021).

Definition 1. Consider the system $\dot{x} = f(x)$ and let \mathcal{I} be an open set containing the origin. A BLF is a positive definite \mathcal{C}^1 function $W : \mathcal{I} \rightarrow \mathbb{R}_{\geq 0}$, $x \mapsto W(x)$, satisfies $\nabla W(x)f(x) \leq 0$, where $\nabla W(x) := \partial V / \partial x$, and has the property that $W(x) \rightarrow \infty$, and $\nabla W(x) \rightarrow \infty$ as $x \rightarrow \partial \mathcal{I}$.

We introduce a barrier-Lyapunov function expressed in terms of the synchronization errors \bar{e}_k for the purposes of this paper. To that end, for a couple of cooperative agents, we express \bar{e}_k in (5) as

$$\bar{e}_k = \bar{x}_i - \bar{x}_j = \delta_k - \bar{b}_k, \quad i, j \in \mathcal{V}_p, \quad (8)$$

and for a couple of competitive agents, we have

$$\bar{e}_k = \bar{x}_i + \bar{x}_j = \delta_k - \bar{b}_k + 2x_j, \quad i \in \mathcal{V}_p, j \in \mathcal{V}_q, \quad (9)$$

where $p, q \in \{1, 2\}$, $p \neq q$ and $\bar{b}_k = b_i - \text{sgn}(a_{ij})b_j$. Then, in terms of synchronization errors, the constraint sets in (6) can be redefined as follows

$$\mathcal{I}_r = \{\bar{e}_k \in \mathbb{R}^n : |\bar{e}_{x_k} + \alpha_k| < R_k, \ k \in \mathcal{E}_m\}, \quad (10a)$$

$$\mathcal{I}_c = \{\bar{e}_k \in \mathbb{R}^n : \Delta_k < |\bar{e}_{x_k} + \alpha_k|, \ k \leq M\}, \quad (10b)$$

where α_k is defined as

$$\alpha_k := \delta_k - \bar{e}_k. \quad (11)$$

Notice that, in the case of two cooperative agents, we have $\alpha_k = \bar{b}_k$, which is constant. Then, we define the BLF $W_k : \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ for each $k \leq M$,

$$W_k(\bar{e}_k) = \frac{1}{2}[|\bar{e}_k|^2 + B_k(\bar{e}_k)], \quad (12)$$

where³ $B_k(\bar{e}_k)$ is the sum of two functions satisfying Definition 1, encoding the constraints in (10), respectively, $B_k(\bar{e}_k) = \frac{1}{2}(1 + \sigma_k)B_{r_k}(\bar{e}_k) + B_{c_k}(\bar{e}_k)$, where $B_{c_k}(\bar{e}_k) \rightarrow \infty$ as $|\bar{e}_k| \rightarrow \Delta_k$ and $B_{r_k}(\bar{e}_k) \rightarrow \infty$ as $|\bar{e}_k| \rightarrow R_k$ for all k . In the latter, since the maximum distance constraints are imposed only on cooperative agents, $\sigma_k = 1$ if $k \in \mathcal{E}_m$, and $\sigma_k = -1$ otherwise. Furthermore, $B_k(\bar{e}_k)$ is non-negative and satisfies $B_k(0) = 0$. Additionally, it tends to infinity as $|\bar{e}_k| \rightarrow \Delta_k$ for all edges and as $|\bar{e}_k| \rightarrow R_k$ for $k \in \mathcal{E}_m$. However, considering the constraints defined in (10), the barrier function has to be adjusted to ensure that the solution lies within the interior of the constraint sets in (10) and to guarantee the system's convergence to the desired point—cf. (Wills and Heath, 2002). We define

$$\widehat{W}_k(\alpha_k, \bar{e}_k) := W_k(\bar{e}_k + \alpha_k) - W_k(\alpha_k) - \frac{\partial W_k}{\partial s}(\alpha_k)\bar{e}_k. \quad (13)$$

This function satisfies $\widehat{W}_k(\alpha_k, 0) = 0$, $\nabla_{\bar{e}_k} \widehat{W}_k(\alpha_k, 0) = 0$, where $\nabla_{\bar{e}_k} \widehat{W}_k = \frac{\partial \widehat{W}_k}{\partial \bar{e}_k}$, and $\widehat{W}_k(\alpha_k, \bar{e}_k) \rightarrow \infty$ as $|\delta_k| \rightarrow \Delta_k$ for all $k \leq M$, and as $|\delta_k| \rightarrow R_k$ for all $k \in \mathcal{E}_m$. The gradient of the BLF vanishes at the origin and at another isolated saddle point e_k^* , separated from the origin. Then, we define the disjoint set $\mathcal{W}_k := \{0\} \cup \{e_k^*\}$, which corresponds to the critical points of $\widehat{W}_k(\alpha_k, \bar{e}_k)$.

Then, we introduce the BLF-gradient-based bipartite formation control law given by

$$\tau_i = -k_{1_i} J_i(q_i)^\top \left[\sum_{k=1}^M [E_s]_{ik} \nabla_{\bar{e}_k} \widehat{W}_k + \sum_{k=1}^M [\mathbb{E}]_{ik} \nabla_{\alpha_k} \widehat{W} \right] - k_{2_i} \dot{q}_i + \frac{\partial}{\partial q_i} U_i(q_i), \quad (14)$$

where $k_{1_i} > 0$, $k_{2_i} > 0$ for all $i \leq N$,

$$\mathbb{E} = E - E_s, \quad (15)$$

where E denotes the incidence matrix of the network if all interactions were cooperative⁴, and E_s the incidence matrix of the considered signed network. For a structurally balanced signed network, E_s is defined as follows:

$$[E_s]_{ik} := \begin{cases} +1, & \text{if } v_i \text{ is the initial node of the edge } \varepsilon_k; \\ -1, & \text{if } v_i, v_j \text{ are cooperative such that} \\ & v_i, v_j \in \mathcal{V}_l, l \in \{1, 2\} \text{ and } v_i \text{ is the} \\ & \text{terminal node of the edge } \varepsilon_k; \\ +1, & \text{if } v_i, v_j \text{ are competitive such that} \\ & v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q, p, q \in \{1, 2\}, p \neq q \text{ and} \\ & v_i \text{ is the terminal node of the edge } \varepsilon_k; \\ 0, & \text{otherwise,} \end{cases}$$

where $\varepsilon_k = \{v_i, v_j\}$, $k \leq M$, $i, j \leq N$ are arbitrarily oriented edges and \mathcal{V}_1 and \mathcal{V}_2 are the two disjoint sets of vertices.

The first two terms of the control law in (14) are needed to ensure the bipartite formation of end-effectors while respecting the inter-agent constraints imposed on the task space. The second term is needed specifically because of the use of the gradient recentered barrier function and the presence of competitive interactions between agents. The third term is needed to control the joint velocity. It consists

³ A particular choice for $B_k(\bar{e}_k)$ is given in Section 4.

⁴ A structurally balanced signed graph may be transformed into a cooperative one using the gauge transformation—see (Altafini, 2013; Du et al., 2018).

of a damping term to stabilize the joint velocities at zero. The last term is to compensate for the gravitational force.

3.2 Asymptotic stability

We analyze the stability of the bipartite formation manifold for the closed-loop system (1) interconnected by the control law (14). To that end, using the definition of the incidence matrix, we represent the synchronization errors in (5), and α_k in (11), in the vector form

$$\bar{e} = [E_s^\top \otimes I_n] \bar{x}, \quad (16a)$$

$$\alpha = [E^\top \otimes I_n] x - [E_s^\top \otimes I_n] \bar{x}. \quad (16b)$$

Then, after (13), we define

$$\bar{W}(\alpha, \bar{e}) = \sum_{k=1}^M \widehat{W}_k(\alpha_k, \bar{e}_k), \quad (17)$$

and we write the closed-loop system (1)–(14) in the compact form

$$\begin{aligned} \dot{q} = & -M(q)^{-1} \left[C(q, \dot{q}) \dot{q} + K_1 J(q)^\top [E_s \otimes I_n] \nabla_{\bar{e}} \bar{W}(\alpha, \bar{e}) \right. \\ & \left. + K_1 J(q)^\top [\mathbb{E} \otimes I_n] \nabla_{\alpha} \bar{W}(\alpha, \bar{e}) + [K_2 \otimes I_n] \dot{q} \right], \quad (18) \end{aligned}$$

where $q = [q_i]$, $M(q) = \text{blkdiag}[M_i(q_i)]$, $C(q, \dot{q}) = \text{blkdiag}[C_i(q_i, \dot{q}_i)]$, $K_1 = \text{diag}(k_{1_i})$, $K_2 = \text{diag}(k_{2_i})$ and $J(q)^\top = \text{blkdiag}[J_i(q_i)^\top]$, for all $i \leq N$.

On the other hand, after Assumption 2, the resulting graph of the considered network is undirected and connected, so it contains a spanning tree. Then, the closed-loop system can be expressed only in terms of the errors corresponding to the spanning tree. In order to obtain the closed-loop equations in spanning-tree coordinates, we recall that the incidence matrix of the considered network can be partitioned into

$$E_s = [E_{t_s} \quad E_{c_s}], \quad (19)$$

where $E_{t_s} \in \mathbb{R}^{N \times N-1}$ is the incidence matrix representing the edges corresponding to the spanning tree, and $E_{c_s} \in \mathbb{R}^{N \times M-(N-1)}$ is the incidence matrix representing the remaining edges. Following the same reasoning, the synchronization errors can also be expressed as $\bar{e} = [\bar{e}_t^\top \quad \bar{e}_c^\top]^\top$, where \bar{e}_t corresponds to the spanning-tree errors. Moreover, for a structurally balanced signed graph, there exists a matrix R_s such that

$$E_s = E_{t_s} R_s, \quad (20)$$

where $R_s := [I_{N-1} \quad T_s]$ and $T_s := (E_{t_s}^\top E_{t_s})^{-1} E_{t_s}^\top E_{c_s}$ —see Proposition 1 in (Şekercioglu et al., 2023). Thus, we can also express all synchronization errors in terms of the spanning-tree errors as

$$\bar{e} = [(E_{t_s} R_s)^\top \otimes I_n] \bar{x} = [R_s^\top \otimes I_n] \bar{e}_t. \quad (21)$$

Next, we express the control law in spanning-tree coordinates, for which we introduce $\tilde{W}(\alpha, \bar{e}_t) := \bar{W}(\alpha, [R_s^\top \otimes I_n] \bar{e}_t)$. That is, in view of (21), $\tilde{W}(\alpha, \bar{e}_t)$ denotes the same quantity as the right-hand-side of (17), but in spanning-tree coordinates. Thus, Eq. (18) becomes

$$\begin{aligned} \ddot{q} = & -M(q)^{-1} \left[C(q, \dot{q}) \dot{q} + K_1 J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W}(\alpha, \bar{e}_t) \right. \\ & \left. + K_1 J(q)^\top [\mathbb{E} \otimes I_n] \nabla_{\alpha} \tilde{W}(\alpha, \bar{e}_t) + [K_2 \otimes I_n] \dot{q} \right]. \quad (22) \end{aligned}$$

Remark 2. Notice that, using (20), the gradient-based control term reads $\nabla_{\bar{e}_t} \tilde{W} \equiv \frac{\partial \tilde{W}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{e}_t} = \nabla_{\bar{e}} \tilde{W}^\top [R_s^\top \otimes I_n]$.

Then, we have the following.

Proposition 3. Consider N robot manipulators modeled by (1) and satisfying the Assumptions 1 and 2, in closed-loop with the distributed control law (14), with $k_{1_i}, k_{2_i} > 0$, for all $i \leq N$ and \widehat{W}_k as defined in (13). Then, for any given formation shape reachable by the end-effectors, the set $\{(\bar{e}, \dot{q}) = (0, 0)\}$ is asymptotically stable for almost all initial conditions such that $(\bar{e}(0), \dot{q}(0)) \in \mathcal{I} \times \mathbb{R}^{nN}$ and $|\alpha_k(0)| > \Delta_k$ for any $k \leq M$.

Sketch of proof: The proof may be constructed using the Lyapunov function candidate

$$V(\alpha, \bar{e}_t, \dot{q}) = \tilde{W}(\alpha, \bar{e}_t) + \frac{1}{2} \dot{q}^\top M(q) \dot{q},$$

where $M(q) = M(q)^\top$, whose derivative satisfies

$$\dot{V} = -\dot{q}^\top [K_2 \otimes I_n] \dot{q} \leq 0.$$

Next, using LaSalle's invariance principle, it follows that $\nabla_{\bar{e}_t} \tilde{W} = 0$, which holds if and only if $\bar{e}_t \in \mathcal{W}^t$, where $\mathcal{W}^t = \{0, e_t^*\}$, and e_t^* is a saddle point. Since e_t^* is a saddle point, the set of initial conditions that converge to $(e_t^*, 0)$ has zero Lebesgue measure. Thus, almost all initial conditions converge to the origin. Asymptotic stability follows.

Furthermore, forward invariance of the set $\mathcal{I}_t \times \mathbb{R}^{nN}$, where $\mathcal{I}_t := \mathcal{I}_{c_t} \cap \mathcal{I}_{r_t}$ for cooperative agents, $\mathcal{I}_t := \mathcal{I}_{c_t}$ for competitive agents, while

$$\mathcal{I}_{r_t} := \{\bar{e}_{t_k} \in \mathbb{R}^n : |r_{s_k}^\top \bar{e}_{t_k} + \alpha_k| < R_k, k \in \mathcal{E}_m\}, \quad (23)$$

$$\mathcal{I}_{c_t} := \{\bar{e}_{t_k} \in \mathbb{R}^n : \Delta_k < |r_{s_k}^\top \bar{e}_{t_k} + \alpha_k|, k \leq M\}, \quad (24)$$

and r_{s_k} is the k th column of R_s , follows by observing that V is non-increasing along the trajectories of (22) and $V \rightarrow \infty$ as $|r_{s_k}^\top \bar{e}_{t_k} + \alpha_k| \rightarrow \Delta_k$, $k \leq M$ or $|r_{s_k}^\top \bar{e}_{t_k} + \alpha_k| \rightarrow R_k$, $k \in \mathcal{E}_m$. Thus, we conclude that the origin is asymptotically stable for almost all trajectories starting in $\mathcal{I}_t \times \mathbb{R}^{nN}$.

4. SIMULATION RESULTS

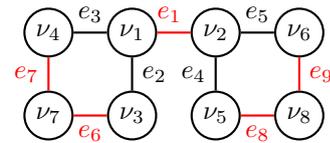


Fig. 1. An undirected signed network of 8 robot manipulators. The black lines represent cooperative edges, and the red lines represent the competitive edges.

We provide a numerical example to show the performance of our control law in (14). For that, we consider a system of 8 two-link robot manipulators interconnected over a structurally balanced undirected signed network, modeled by a graph as the one depicted in Figure 1. For the corresponding graph, we define the orientation of the nine edges as $e_1 = \nu_1 - \nu_2$, $e_2 = \nu_1 - \nu_3$, $e_3 = \nu_1 - \nu_4$, $e_4 = \nu_2 - \nu_5$, $e_5 = \nu_2 - \nu_6$, $e_6 = \nu_3 - \nu_7$, $e_7 = \nu_4 - \nu_7$, $e_8 = \nu_5 - \nu_8$ and $e_9 = \nu_6 - \nu_8$. The set of nodes may be split into two disjoint subgroups as $\mathcal{V}_1 = \{\nu_1, \nu_3, \nu_4, \nu_8\}$ and $\mathcal{V}_2 = \{\nu_2, \nu_5, \nu_6, \nu_7\}$, so the network is structurally balanced. From (19), the edges e_i , $i \leq 5$ correspond to the edges of the spanning tree, and the remaining edges, e_6, e_7, e_8 and e_9 , correspond to the cycles.

The corresponding incidence matrix is given by

$$E_s = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Each manipulator is modeled by the Euler-Lagrange equations in (1), with inertia and Coriolis matrices given by

$$M_i(q_i) = \begin{bmatrix} \alpha_i + 2\beta_i \cos(q_{2_i}) & \delta_i + \beta_i \cos(q_{2_i}) \\ \delta_i + \beta_i \cos(q_{2_i}) & \delta_i \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = \delta_i \begin{bmatrix} -\sin(q_{2_i})\dot{q}_{2_i} & -\sin(q_{2_i})(\dot{q}_{1_i} + \dot{q}_{2_i}) \\ -\sin(q_{2_i})\dot{q}_{1_i} & 0 \end{bmatrix},$$

where $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2 (m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$ with l_{1_i}, l_{2_i} and m_{1_i}, m_{2_i} are the length and the mass of links 1 and 2. The physical parameters are $m_1 = 1.2\text{kg}$, $m_2 = 1\text{kg}$, and $l_1 = l_2 = 1\text{m}$ for all $i \leq N$. The kinematic model for each manipulator is given by

$$x_i = \begin{bmatrix} l_1 \cos(q_{1_i}) + l_2 \cos(q_{1_i} + q_{2_i}) \\ l_1 \sin(q_{1_i}) + l_2 \sin(q_{1_i} + q_{2_i}) \end{bmatrix} + x_{i_0},$$

and the Jacobian matrix

$$J_i(q_i) = \begin{bmatrix} -l_1 \sin(q_{1_i}) - l_2 \sin(q_{1_i} + q_{2_i}) & -l_2 \sin(q_{1_i} + q_{2_i}) \\ l_1 \cos(q_{1_i}) + l_2 \cos(q_{1_i} + q_{2_i}) & l_2 \cos(q_{1_i} + q_{2_i}) \end{bmatrix}.$$

Consider the system (1) with the bipartite formation control law (14), where $k_{1_i} = 15$, $k_{2_i} = 10$ for all $i \leq N$ and the barrier-Lyapunov function in (13), with $B_{r_k}(s) = \ln\left(\frac{R_k^2}{R_k^2 - |s|^2}\right)$, $B_{c_k}(s) = \ln\left(\frac{|s|^2}{|s|^2 - \Delta_k^2}\right)$. The bases of eight robot manipulators are located at $x_{1_0} = [0.8, 1]^\top$, $x_{2_0} = [2, -0.5]^\top$, $x_{3_0} = [-0.5, -1]^\top$, $x_{4_0} = [-3, 0.5]^\top$, $x_{5_0} = [-1, -2]^\top$, $x_{6_0} = [3, -1]^\top$, $x_{7_0} = [1, -3]^\top$, $x_{8_0} = [-2.5, -0.5]^\top$.

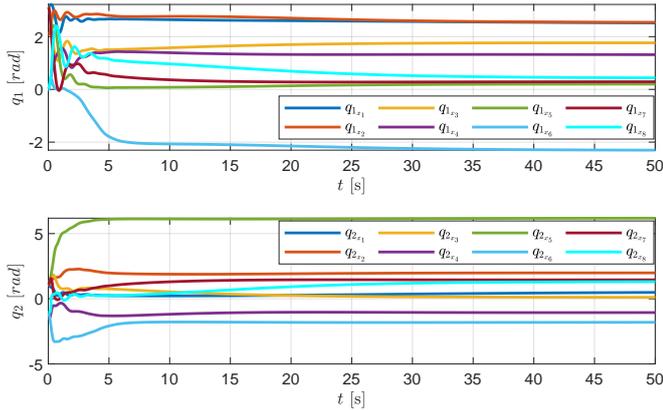


Fig. 2. Bipartite formation of system (1) with control input (14) on joint trajectories.

The initial conditions for each agent are $q_1(0) = [\pi, \pi/3]^\top$, $q_2(0) = [2\pi/3, \pi/3]^\top$, $q_3(0) = [\pi, \pi/3]^\top$, $q_4(0) = [0, \pi/2]^\top$, $q_5(0) = [\pi, \pi/3]^\top$, $q_6(0) = [0, -\pi/3]^\top$, $q_7(0) = [\pi, \pi/3]^\top$, $q_8(0) = [0, -\pi/3]^\top$, $\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = \dot{q}_4(0) = \dot{q}_5(0) = \dot{q}_6(0) = \dot{q}_7(0) = \dot{q}_8(0) = [0, 0]^\top$, with $q = [q_1, q_2]^\top$ and $\dot{q} = [\dot{q}_1, \dot{q}_2]^\top$ and the relative displacements of the end-effectors are $b_1 = [0.4, 0.4]^\top$,

$b_2 = [-0.4, 0.4]^\top$, $b_3 = [0.4, -0.4]^\top$, $b_4 = [-0.4, 0.4]^\top$, $b_5 = [-0.4, -0.4]^\top$, $b_6 = [0.4, 0.4]^\top$, $b_7 = [0.4, -0.4]^\top$, $b_8 = [-0.4, -0.4]^\top$, with $b = [b_x, b_y]^\top$. The constraint sets are $\Delta_k = 0.1$ for all $k \leq M$ and $R_k = 7$ for all $k \in \mathcal{E}_m$.

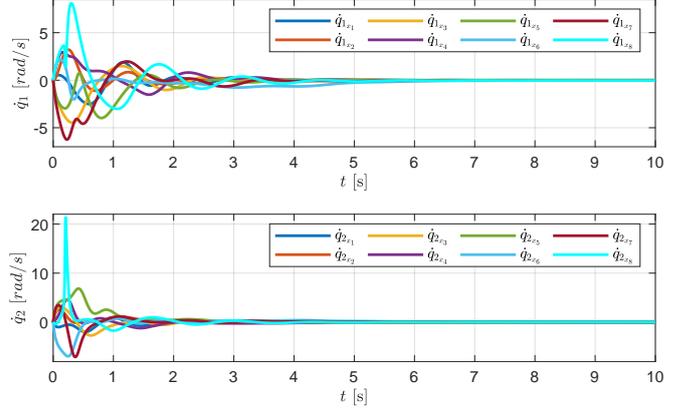


Fig. 3. Bipartite formation of system (1) with control input (14) on joint velocities.

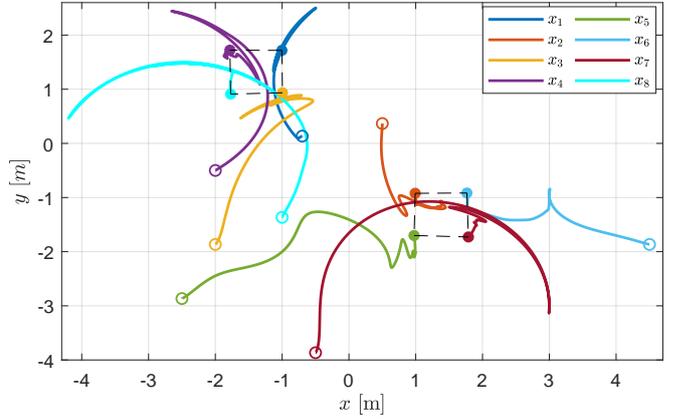


Fig. 4. Evolution of the manipulators' end-effector from the initial positions (o) to the final positions (*). Each group of end-effectors forms a triangle around the symmetric consensus points.

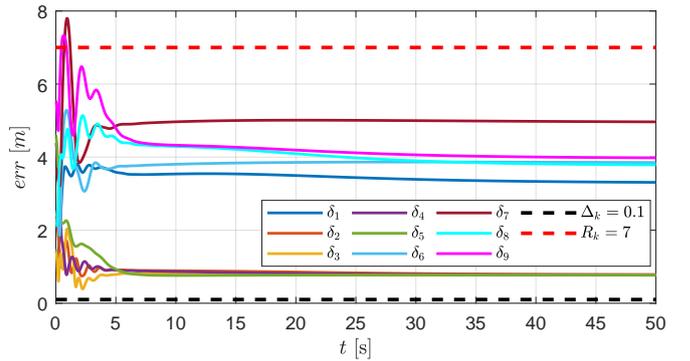


Fig. 5. Trajectories of the norm of inter-agent distances with control input (14). The black dashed line is the minimum distance constraint for all end-effectors, and the red dashed line is the maximum distance constraint for cooperative end-effectors.

The joint positions and velocities are depicted in Figures 2 and 3, respectively, and all velocities converge to zero. The paths of each end-effector up to bipartite formation are depicted in Figure 4, and their final configuration is depicted in Figure 6. Moreover, it is clear from Figure

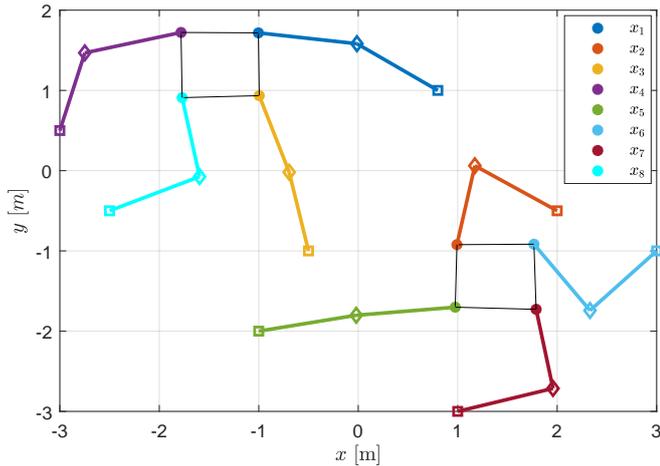


Fig. 6. Final positions of the manipulators and their end-effector.

5 that collision avoidance and maximum distance maintenance among the manipulators' end-effectors are both guaranteed.

5. CONCLUSION

We addressed the problem of constrained bipartite formation of cooperative-competitive robot manipulators' end-effectors, modeled by Euler-Lagrange equations. We considered a structurally balanced and undirected signed graph. We presented a bipartite formation control law based on the gradient of a barrier-Lyapunov function that guarantees that end-effectors do not collide and stay within their sensing regions. Further research aims to extend these results to directed signed networks.

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