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Formation Control of Cooperative-Competitive Robot Manipulators with Inter-agent Constraints

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Abstract: This paper addresses the distributed formation control problem of networked robot manipulators in end-effector coordinates. We propose a distributed bipartite formation controller that guarantees collision avoidance and connectivity maintenance for cooperative and competitive manipulators. In the considered setting, two groups of manipulators are formed and reach bipartite consensus. On the other hand, the end-effectors in the same group achieve formation. We design a gradient-based control law using barrier-Lyapunov functions to ensure that end-effectors do not collide and stay in their sensors' range. Moreover, we establish asymptotic stability of the bipartite formation manifold. Finally, we illustrate our theoretical results via numerical simulations.

Keywords: Formation consensus, signed networks, robotic manipulators, barrier-Lyapunov functions

1. INTRODUCTION

In formation control, a group of robots has to undertake a task, such as reaching a consensus point or tracking targets, while keeping a geometric pattern between agents. This problem has been extensively studied in the literature. However, most works consider only cooperative agents, in which case the resulting network can be modeled by a graph containing only positive weighted edges. Yet, robot manipulators are inherently nonlinear; in many scenarios, some agents may be in competition. Apart from applications involving robot manipulators, competitive interactions may be found in several scenarios, *e.g.*, in the context of herding control in Chipade and Panagou (2020); Grover et al. (2022); social-networks theory in Altafini (2013), and aerospace applications in Zhao et al. (2022).

The presence of both cooperative and competitive interactions in a network may be modeled using *signed networks*, in which the edges have both positive and negative weights—see Altafini (2013). The signed network is called to be *structurally balanced*, if the nodes may be split into two disjoint subsets, where agents cooperative with each other are in the same subset and agents competitive with each other are in different ones. For these kinds of networks, the attainable goal, in general, is *bipartite consensus*, in which all the agents converge to the same state in modulus but opposite in signs. See, *e.g.*, Altafini (2013); Du et al. (2018), and Valcher and Misra (2014).

The preceding references, however, only consider generic first-, second-, or higher-order linear models, which are less suited to modeling a network of robot manipulators. The latter is most often modeled by the Euler-Lagrange equations, which are inherently nonlinear, and the cooperative control for such systems is well-studied in the literature. The synchronization of multiple robot manipulators in joint coordinates is addressed, *e.g.*, in Nijmeijer and Rodríguez-Angeles (2003) and the formation of manipulators in end-effector coordinates is studied in Sakurama (2021); Wu et al. (2022). Yet, most of the references in the literature only consider networks with cooperative interactions. For signed networks, the bipartite consensus of networked robot manipulators is addressed, *e.g.*, in Liu et al. (2019); Hu et al. (2019); Zhang et al. (2023), while the leader-follower bipartite consensus is studied in Liu et al. (2022); Liang and Huang (2020); Li et al. (2022), all in joint coordinates. In end-effector coordinates the bipartite formation-control problem is considered in Pan et al. (2023).

This paper addresses the bipartite formation-control problem of end-effectors while ensuring inter-agent collision avoidance and maintenance of information exchange. These inter-agent constraints are commonly addressed using artificial potential functions—see *e.g.*, Dimarogonas and Kyriakopoulos (2008); Panagou et al. (2013) for cooperative networks. For signed networks, the authors in Fan et al. (2014) address the bipartite flocking problem with

collision avoidance and connectivity maintenance, using artificial potential functions. In Şekercioglu et al. (2023), a controller based on the gradient of a barrier-Lyapunov function is proposed for the problem of constrained bipartite formation over undirected signed networks of simple integrators. Nonetheless, all the references mentioned above consider only first and second-order integrators.

In this paper, we consider the distributed bipartite formation-control problem of robot manipulators' end-effectors under relative distance constraints. Robot manipulators are modeled by the Euler-Lagrange equations. The contributions are twofold: firstly, contrary to Nijmeijer and Rodríguez-Angeles (2003); Sakurama (2021); Wu et al. (2022), we consider networks containing both cooperative and competitive robot manipulators. The resulting graph is signed, structurally balanced, and undirected. Then, we consider inter-agent distance constraints on the end effectors, such as collision avoidance and connectivity maintenance using barrier-Lyapunov functions. Relative to Fan et al. (2014), which focuses on bipartite flocking with collision avoidance, in our work, a minimal safety distance is ensured between any two agents. Moreover, contrary to Şekercioglu et al. (2023) and Fan et al. (2014), we consider Euler-Lagrange systems, not simple integrators. Our control design and analysis rely on the edge-based formulation for signed networks—see Du et al. (2018), which allows to recast the problem into one of stabilization of the origin in error coordinates. We establish asymptotic stability of the bipartite formation manifold using Lyapunov's direct method. To the best of our knowledge, similar results are not available in the literature for robot manipulators containing competitive interactions.

2. MODEL AND PROBLEM FORMULATION

2.1 Agents' dynamics

Consider a network of N n -degrees-of-freedom robot manipulators modeled by the Euler-Lagrange equations.

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + \frac{\partial}{\partial q_i}U_i(q_i) = \tau_i, \quad i \leq N, \quad (1)$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n$ are the generalized joint position, velocity, and acceleration respectively, $M_i(q_i) \in \mathbb{R}^{n \times n}$ is the inertia matrix, $U : \mathbb{R}^n \rightarrow \mathbb{R}$ is the potential energy function and $\tau_i \in \mathbb{R}^p$ is the control input. As it is customary, we assume the following.

Assumption 1. The following properties hold.

1. There exist \underline{c}_i and $\bar{c}_i > 0$ such that, $\underline{c}_i I \leq M_i(q_i) \leq \bar{c}_i I$ for all $q_i \in \mathbb{R}^n$.
2. The matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric.
3. The Coriolis matrix $C_i(q_i, \dot{q}_i)$ is uniformly bounded in q_i . Moreover $|C_i(q_i, \dot{q}_i)\dot{q}_i| \leq k_{c_i}|\dot{q}_i|^2$ for $k_{c_i} > 0$.

2.2 Problem statement

The bipartite formation-control problem of the manipulators' end-effectors consists in ensuring that the positions of the end-effectors, in cooperation with each other, achieve a desired geometric shape around a consensus value. Meanwhile, the end-effector positions of the competing agents converge on another spatial configuration,

which is symmetrical. Let $x_i \in \mathbb{R}^p$ be the position of the i th manipulator's end-effector in the task space. The position of the end effector x_i can be mapped to its generalized joint coordinates using a nonlinear forward kinematics mapping—see Murray et al. (1994)

$$x_i = x_{i_0} + h_i(q_i), \quad (2)$$

where x_{i_0} is the position of the manipulator's base and $h_i : \mathbb{R}^n \rightarrow \mathbb{R}^p$ is the mapping from joint-space to the task space. Differentiating (2) with respect to time, we obtain the relation between the task-space and joint velocities—see Murray et al. (1994)

$$\dot{x}_i = J_i(q_i)\dot{q}_i, \quad J_i(q_i) := \frac{\partial h_i(q_i)}{\partial q_i}\dot{q}_i, \quad (3)$$

with $J_i(q_i) \in \mathbb{R}^{n \times p}$ the Jacobian matrix of the forward kinematics. The shape of the end-effector formation is defined by the relative biases b_i and b_j with respect to the two consensus points. Therefore, we can mathematically define the bipartite formation control objective as

$$\lim_{t \rightarrow \infty} \bar{x}_i(t) - \text{sgn}(a_{ij})\bar{x}_j(t) \rightarrow 0, \quad i, j \leq N, \quad (4)$$

where $\bar{x}_i := x_i - b_i$, and $a_{ij} \in \mathbb{R}$ is the adjacency weight between the two agents. Moreover, for agents to achieve the objective in (4), we assume the following.

Assumption 2. The systems described in (1), which are interconnected via inputs τ_i , form a structurally balanced (see below), undirected, and connected signed graph.

We recall that a signed graph is *structurally balanced* if it may be split into two disjoint sets of vertices \mathcal{V}_1 and \mathcal{V}_2 , where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ such that for every $i, j \in \mathcal{V}_p, p \in \{1, 2\}$, if $a_{ij} \geq 0$, while for every $i \in \mathcal{V}_p, j \in \mathcal{V}_q$, with $p, q \in \{1, 2\}, p \neq q$, if $a_{ij} \leq 0$. Otherwise, it is *structurally unbalanced*—cf. Altafini (2013).

In the case, where all agents in the network are cooperative, all \bar{x}_i converge to the same value. However, in this case, given that some robot manipulators are competitive, there appear to be two competing groups of agents. Therefore, all \bar{x}_i reach two symmetrical values, resulting in all end-effectors adopting a desired formation around two symmetrical consensus points. For the purposes of control design and analysis, this is equivalent to making synchronization errors converge to zero. These errors correspond to the edges of the graph and are defined as

$$\bar{e}_k := \bar{x}_i - \text{sgn}(a_{ij})\bar{x}_j, \quad k \leq M, \quad (5)$$

where k denotes the index of the interconnection between the i th and j th end-effectors.

In addition, to ensure the safety of manipulators and the success of their task, the controller τ_i must ensure that the end-effectors do not collide and that they remain within the range of their sensors. This amounts to ensuring that for any pair of communicating nodes ν_i and $\nu_j \in \mathcal{V}$, let $\delta_k := x_i - x_j$, and let $R_k > 0$ and $\Delta_k > 0$ be defined as the maximum and minimum distances with $k \leq M$, the following sets are invariant.

$$\mathcal{I}_r := \{\delta_k \in \mathbb{R}^n : |\delta_k| < R_k, \quad \forall i, j \in \mathcal{V}_l, \quad l \in \{1, 2\}\} \quad (6a)$$

$$\mathcal{I}_c := \{\delta_k \in \mathbb{R}^n : |\delta_k| > \Delta_k \quad \forall k \leq M\}, \quad (6b)$$

where \mathcal{I}_r is the set of proximity constraints and \mathcal{I}_c is the set of collision-avoidance constraints. Under these conditions, it is required to design a distributed bipartite formation control law of the form

$$\tau_i = f(\bar{e}_k, q_i, \dot{q}_i),$$

to achieve bipartite formation of end-effectors, such that,

$$\lim_{t \rightarrow \infty} \bar{e}_k(t) = 0, \quad \lim_{t \rightarrow \infty} \dot{q}_i(t) = 0, \quad k \leq M, i \leq N, \quad (7)$$

and the manipulators' end-effector's trajectories satisfy the proximity and collision-avoidance constraints, that is, $\delta_i(t) \in \mathcal{I}$ for all $t \geq 0$, with $\mathcal{I} := \mathcal{I}_r \cap \mathcal{I}_c$.

3. MAIN RESULT

In this section, we first present our control approach. We deal with the considered problem as the stabilization of the origin in edge coordinates, which correspond to the synchronization errors in (5)—see Mesbahi and Egerstedt (2010); Du et al. (2018). Then, in order to respect the inter-agent distance constraints, the control input is designed based on the gradient of a so-called barrier-Lyapunov function—cf. Panagou et al. (2013), Mesbahi and Egerstedt (2010), Restrepo-Ochoa (2021). Finally, we analyze the stability of the bipartite formation manifold.

3.1 Control design

A *barrier-Lyapunov function* (BLF) is defined in the following—cf. Panagou et al. (2013), Mesbahi and Egerstedt (2010), Restrepo-Ochoa (2021).

Definition 1. Consider the system $\dot{x} = f(x)$ and let \mathcal{I} be an open set containing the origin. A BLF is a positive definite \mathcal{C}^1 function $W : \mathcal{I} \rightarrow \mathbb{R}_{\geq 0}$, $x \mapsto W(x)$, satisfies $\nabla W(x)f(x) \leq 0$, where $\nabla W(x) := \partial W / \partial x$, and has the property that $W(x) \rightarrow \infty$, and $\nabla W(x) \rightarrow \infty$ as $x \rightarrow \partial \mathcal{I}$.

We introduce a barrier-Lyapunov function expressed in terms of the synchronization errors \bar{e}_k for the purposes of this paper. To that end, for a couple of cooperative agents, we express \bar{e}_k in (5) as

$$\bar{e}_k = \bar{x}_i - \bar{x}_j = \delta_k - \bar{b}_k, \quad i, j \in \mathcal{V}_p, \quad (8)$$

and for a couple of competitive agents, we have

$$\bar{e}_k = \bar{x}_i + \bar{x}_j = \delta_k - \bar{b}_k + 2x_j, \quad i \in \mathcal{V}_p, j \in \mathcal{V}_q, \quad (9)$$

where $p, q \in \{1, 2\}$, $p \neq q$ and $\bar{b}_k = b_i - \text{sgn}(a_{ij})b_j$. Then, in terms of synchronization errors, the constraint sets in (6) can be restated as follows

$$\mathcal{I}_r = \{\bar{e}_k \in \mathbb{R}^n : |\bar{e}_{x_k} + \alpha_k| < R_k, \forall k \in \mathcal{E}_m\}, \quad (10a)$$

$$\mathcal{I}_c = \{\bar{e}_k \in \mathbb{R}^n : \Delta_k < |\bar{e}_{x_k} + \alpha_k|, \forall k \leq M\}, \quad (10b)$$

where α_k is defined as

$$\alpha_k := \delta_k - \bar{e}_k. \quad (11)$$

Here, \mathcal{E}_m contains the indices of $m < M$ cooperative edges (edges with strictly positive weights) since the connectivity constraints are only for cooperative agents. Notice that, in the case of two cooperative agents, we have $\alpha_k = \bar{b}_k$, so it is constant. Now, we define the BLF $W_k : \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ for each $k \leq M$,

$$W_k(\bar{e}_k) = \frac{1}{2}[|\bar{e}_k|^2 + B_k(\bar{e}_k)], \quad (12)$$

where $B_k(\bar{e}_k)$ ¹ is the sum of two functions satisfying Definition 1, encoding the constraints in (10), respectively, $B_k(\bar{e}_k) = \frac{1}{2}(1 + \sigma_k)B_{r_k}(\bar{e}_k) + B_{c_k}(\bar{e}_k)$, where $B_{c_k}(\bar{e}_k) \rightarrow \infty$

¹ A particular choice for $B_k(\bar{e}_k)$ is given in Section 4.

as $|\bar{e}_k| \rightarrow \Delta_k$ and $B_{r_k}(\bar{e}_k) \rightarrow \infty$ as $|\bar{e}_k| \rightarrow R_k$ for all k . In the latter, since the connectivity constraints are only for cooperative agents, $\sigma_k = 1$ if $k \in \mathcal{E}_m$, and $\sigma_k = -1$ otherwise. Furthermore, $B_k(\bar{e}_k)$ is non-negative and satisfies $B_k(0) = 0$. Additionally, it tends to infinity as $|\bar{e}_k| \rightarrow \Delta_k$ for all edges and as $|\bar{e}_k| \rightarrow R_k$ for $k \in \mathcal{E}_m$. However, considering the constraints defined in (10), the barrier function has to be adjusted to ensure that the solution lies within the interior of the constraint sets in (10) and to guarantee the system's convergence to the desired point. To achieve this, we employ the concept of the *gradient recentered barrier function*—cf. Wills and Heath (2002). Let $\widehat{W}_k : \mathcal{I}_c \times \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ for competitive interactions and $\widehat{W}_k : \mathcal{I}_c \cap \mathcal{I}_r \times \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ for cooperative ones be defined as

$$\widehat{W}_k(\alpha_k, \bar{e}_k) := W_k(\bar{e}_k + \alpha_k) - W_k(\alpha_k) - \frac{\partial W_k}{\partial s}(\alpha_k)\bar{e}_k, \quad (13)$$

which satisfies $\widehat{W}_k(\alpha_k, 0) = 0$, $\nabla_{\bar{e}_k} \widehat{W}_k(\alpha_k, 0) = 0$, where $\nabla_{\bar{e}_k} \widehat{W}_k = \frac{\partial \widehat{W}_k}{\partial \bar{e}_k}$, and $\widehat{W}_k(\alpha_k, \bar{e}_k) \rightarrow \infty$ as $|\delta_k| \rightarrow \Delta_k$ for $k \leq M$, and as $|\delta_k| \rightarrow R_k$ for all $k \in \mathcal{E}_m$. Moreover, $\widehat{W}_k(\alpha_k, \bar{e}_k)$ satisfies $\frac{\kappa_1}{2}\bar{e}_k^2 \leq \widehat{W}_k(\alpha_k, \bar{e}_k) \leq \kappa_2[\nabla_{\bar{e}_k} \widehat{W}_k]^2$.

Then, we introduce the BLF-gradient-based bipartite formation control law given by

$$\tau_i = -k_{1_i} J_i(q_i)^\top \left[\sum_{k=1}^M [E_s]_{ik} \nabla_{\bar{e}_k} \widehat{W}_k + \sum_{k=1}^M [\mathbb{E}]_{ik} \nabla_{\alpha_k} \widehat{W} \right] - k_{2_i} \dot{q}_i + \frac{\partial}{\partial q_i} U_i(q_i), \quad (14)$$

where $k_{1_i} > 0$, $k_{2_i} > 0$ for all $i \leq N$,

$$\mathbb{E} = E - E_s, \quad (15)$$

with E the incidence matrix of the network if all interactions were cooperative², and E_s the incidence matrix of the considered signed network. We recall that E_s describes the interaction topology of the network and is defined as follows for a structurally balanced signed network.

$$[E_s]_{ik} := \begin{cases} +1, & \text{if } v_i \text{ is the initial node of the edge } \varepsilon_k; \\ -1, & \text{if } v_i, v_j \text{ are cooperative such that } \\ & v_i, v_j \in \mathcal{V}_l, l \in \{1, 2\} \text{ and } v_i \text{ is the} \\ & \text{terminal node of the edge } \varepsilon_k; \\ +1, & \text{if } v_i, v_j \text{ are competitive such that} \\ & v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q, p, q \in \{1, 2\}, p \neq q \text{ and} \\ & v_i \text{ is the terminal node of the edge } \varepsilon_k; \\ 0, & \text{otherwise,} \end{cases}$$

where $\varepsilon_k = \{v_i, v_j\}$, $k \leq M$, $i, j \leq N$ are arbitrarily oriented edges and \mathcal{V}_1 and \mathcal{V}_2 are the two disjoint sets of vertices. The first two terms of the control law in (14) are needed to ensure the bipartite formation of end-effectors while respecting the inter-agent constraints imposed on the task space. The second term is needed specifically because of the use of the gradient recentered barrier function and the presence of competitive interactions between agents. The third term is needed to control the joint velocity. It consists of a damping term to stabilize the joint velocities at zero. The last term is to compensate for the gravitational force.

² A structurally balanced signed graph may be transformed into a cooperative one using the gauge transformation—see Altafini (2013); Du et al. (2018).

3.2 Asymptotic stability

We analyze the stability of the bipartite formation manifold for the closed-loop system (1) interconnected by the control law (14). To that end, using the definition of the incidence matrix, we represent the synchronization errors in (5) and α_k defined in (11), in vector form

$$\bar{e} = E_s^\top \bar{x}, \quad (16a)$$

$$\alpha = E^\top x - E_s^\top \bar{x}. \quad (16b)$$

Then, after (13), we define

$$\tilde{W}(\alpha, \bar{e}) = \sum_{k=1}^M \widehat{W}_k(\alpha_k, \bar{e}_k), \quad (17)$$

to write the closed-loop system (1)–(14) in the compact form

$$\begin{aligned} \ddot{q} = & -M(q)^{-1} \left[C(q, \dot{q})\dot{q} + K_1 J(q)^\top [E_s \otimes I_n] \nabla_{\bar{e}} \tilde{W}(\alpha, \bar{e}) \right. \\ & \left. + K_1 J(q)^\top [\mathbb{E} \otimes I_n] \nabla_{\alpha} \tilde{W}(\alpha, \bar{e}) + [K_2 \otimes I_n] \dot{q} \right], \quad (18) \end{aligned}$$

where $q = [q_i]$, $M(q) = \text{blkdiag}[M_i(q_i)]$, $C(q, \dot{q}) = \text{blkdiag}[C_i(q_i, \dot{q}_i)]$, $K_1 = \text{diag}(k_{1i})$, $K_2 = \text{diag}(k_{2i})$ and $J(q)^\top = \text{blkdiag}[J_i(q_i)^\top]$, for all $i \leq N$.

On the other hand, after Assumption 2, the resulting graph of the considered network is undirected and connected, so it contains a spanning tree. Then, the closed-loop system can be expressed only in terms of the errors corresponding to the spanning tree—see Du et al. (2018), Mesbahi and Egerstedt (2010). In order to obtain the closed-loop equations in spanning-tree coordinates, we recall that the incidence matrix of the considered network can be partitioned into

$$E_s = [E_{t_s} \quad E_{c_s}], \quad (19)$$

where $E_{t_s} \in \mathbb{R}^{N \times N-1}$ is the incidence matrix representing the edges corresponding to the spanning tree, and $E_{c_s} \in \mathbb{R}^{N \times M-(N-1)}$ is the incidence matrix representing the remaining edges. Following the same reasoning, the synchronization errors can also be expressed as

$$\bar{e} = [\bar{e}_t^\top \quad \bar{e}_c^\top]^\top,$$

where \bar{e}_t corresponds to the spanning-tree errors. Moreover, for a structurally balanced signed graph, there exists a matrix R_s such that

$$E_s = E_{t_s} R_s, \quad (20)$$

where $R_s := [I_{N-1} \quad T_s]$ and $T_s := (E_{t_s}^\top E_{t_s})^{-1} E_{t_s}^\top E_{c_s}$ —see Proposition 1 in Şekercioğlu et al. (2023). Thus, we can also express all synchronization errors in terms of the spanning-tree errors as

$$\bar{e} = (E_{t_s} R_s)^\top \bar{x} = R_s^\top \bar{e}_t. \quad (21)$$

Then, to express the control law in spanning-tree coordinates, we introduce $\tilde{W}(\alpha, \bar{e}_t) := \tilde{W}(\alpha, R_s^\top \bar{e}_t)$. That is, in view of (21), $\tilde{W}(\alpha, \bar{e}_t)$ denotes the same quantity as the right-hand-side of (17), but in spanning-tree coordinates. In spanning-tree edge coordinates, Eq. (18) becomes

$$\begin{aligned} \ddot{q} = & -M(q)^{-1} \left[C(q, \dot{q})\dot{q} + K_1 J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W}(\alpha, \bar{e}_t) \right. \\ & \left. + K_1 J(q)^\top [\mathbb{E} \otimes I_n] \nabla_{\alpha} \tilde{W}(\alpha, \bar{e}_t) + [K_2 \otimes I_n] \dot{q} \right]. \quad (22) \end{aligned}$$

Remark 1. Notice that, using (20), the gradient-based control term reads $\nabla_{\bar{e}_t} \tilde{W} \equiv \frac{\partial \tilde{W}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{e}_t} = \nabla_{\bar{e}} \tilde{W}^\top R_s^\top$.

Then, we have the following.

Proposition 2. Consider N robot manipulators modeled by (1) and satisfying the Assumptions 1 and 2, in closed-loop with the distributed control law (14), with $k_{1i}, k_{2i} > 0$, for all $i \leq N$. Then, for any given formation shape reachable by the end-effectors, the set $\{(\bar{e}, \dot{q}) = (0, 0)\}$ is asymptotically stable for any initial conditions such that, for any $k \leq M$, $\delta_k \in \mathcal{I} = \mathcal{I}_r \cap \mathcal{I}_c$ —see (6)—and $|\alpha_k(0)| > \Delta_k$. In addition, the set \mathcal{I} is forward invariant, i.e., the constraints hold for all $t \geq 0$.

Sketch of proof: We first express the collision-avoidance and connectivity constraints in terms of the spanning-tree error coordinates, as below.

$$\mathcal{I}_{r_i} := \{\bar{e}_{t_k} \in \mathbb{R}^n : |r_{s_k}^\top \bar{e}_{t_k} + \alpha_k| < R_k, \forall k \in \mathcal{E}_m\}, \quad (23)$$

$$\mathcal{I}_{c_t} := \{\bar{e}_{t_k} \in \mathbb{R}^n : \Delta_k < |r_{s_k}^\top \bar{e}_{t_k} + \alpha_k|, \forall k \leq M\}, \quad (24)$$

and r_{s_k} is the k th column of R_s .

Next, we consider the Lyapunov function candidate

$$V(\alpha, \bar{e}_t, \dot{q}) = \tilde{W}(\alpha, \bar{e}_t) + \frac{1}{2} \dot{q}^\top M(q) \dot{q},$$

where $M(q) = M(q)^\top$ and its derivative satisfies

$$\dot{V} = -\dot{q}^\top [K_2 \otimes I_n] \dot{q} \leq 0.$$

The rest of the proof consists in establishing asymptotic stability of the origin $\{(\bar{e}_t, \dot{q}) = (0, 0)\}$, using Barbashin-Krasovskii's theorem—cf. Barbashin and Krasovskii (1952); Khalil (1992), and forward invariance of the set $\mathcal{I}_t := \mathcal{I}_{c_t} \cap \mathcal{I}_{r_i}$, for the trajectories of (22), similar to the proof in (Şekercioğlu et al., 2023, Proposition 2).

4. SIMULATION RESULTS

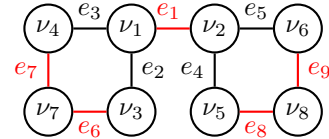


Fig. 1. An undirected signed network of 8 robot manipulators. The black lines represent cooperative edges, and the red lines represent the competitive edges.

We provide a numerical example to show the performance of our control law in (14). For that, we consider a system of 8 two-link robot manipulators interconnected over a structurally balanced undirected signed network, modeled by a graph as the one depicted in Figure 1. For the corresponding graph, we define the orientation of the nine edges as $e_1 = \nu_1 - \nu_2$, $e_2 = \nu_1 - \nu_3$, $e_3 = \nu_1 - \nu_4$, $e_4 = \nu_2 - \nu_5$, $e_5 = \nu_2 - \nu_6$, $e_6 = \nu_3 + \nu_7$, $e_7 = \nu_4 + \nu_7$, $e_8 = \nu_5 + \nu_8$ and $e_9 = \nu_6 + \nu_8$. The set of nodes may be split into two disjoint subgroups as $\mathcal{V}_1 = \{\nu_1, \nu_3, \nu_4, \nu_8\}$ and $\mathcal{V}_2 = \{\nu_2, \nu_5, \nu_6, \nu_7\}$, so the network is structurally balanced. From (19), the edges $e_i, i \leq 5$ correspond to the edges of the spanning tree, and the remaining edges, e_6, e_7, e_8 and e_9 , correspond to the cycles. The corresponding incidence matrix is given by

$$E_s = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Each manipulator is modeled by the Euler-Lagrange equations in (1), with inertia and Coriolis matrices given by

$$M_i(q_i) = \begin{bmatrix} \alpha_i + 2\beta_i \cos(q_{2_i}) & \delta_i + \beta_i \cos(q_{2_i}) \\ \delta_i + \beta_i \cos(q_{2_i}) & \delta_i \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = \delta_i \begin{bmatrix} -\sin(q_{2_i})\dot{q}_{2_i} & -\sin(q_{2_i})(\dot{q}_{1_i} + \dot{q}_{2_i}) \\ -\sin(q_{2_i})\dot{q}_{1_i} & 0 \end{bmatrix},$$

where $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2 (m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$ with l_{1_i} , l_{2_i} and m_{1_i} , m_{2_i} are the length and the mass of links 1 and 2. The physical parameters are $m_1 = 1.2\text{kg}$, $m_2 = 1\text{kg}$, and $l_1 = l_2 = 1\text{m}$ for all $i \leq N$. The kinematic model for each manipulator is given by

$$x_i = \begin{bmatrix} l_1 \cos(q_{i_1}) + l_2 \cos(q_{1_i} + q_{2_i}) \\ l_1 \sin(q_{i_1}) + l_2 \sin(q_{1_i} + q_{2_i}) \end{bmatrix} + x_{i_0},$$

and the Jacobian matrix

$$J_i(q_i) = \begin{bmatrix} -l_1 \sin(q_{i_1}) - l_2 \sin(q_{1_i} + q_{2_i}) & -l_2 \sin(q_{1_i} + q_{2_i}) \\ l_1 \cos(q_{i_1}) + l_2 \cos(q_{1_i} + q_{2_i}) & l_2 \cos(q_{1_i} + q_{2_i}) \end{bmatrix}.$$

Consider the system (1) with the bipartite formation control law (14), where $k_{1_i} = 15$, $k_{2_i} = 10$ for all $i \leq N$ and the barrier-Lyapunov function in (13), with $B_{r_k}(s) = \ln\left(\frac{R_k^2}{R_k^2 - |s|^2}\right)$, $B_{c_k}(s) = \ln\left(\frac{|s|^2}{|s|^2 - \Delta_k^2}\right)$. The bases of eight robot manipulators are located at $x_{1_0} = [0.8, 1]^\top$, $x_{2_0} = [2, -0.5]^\top$, $x_{3_0} = [-0.5, -1]^\top$, $x_{4_0} = [-3, 0.5]^\top$, $x_{5_0} = [-1, -2]^\top$, $x_{6_0} = [3, -1]^\top$, $x_{7_0} = [1, -3]^\top$, $x_{8_0} = [-2.5, -0.5]^\top$. The initial conditions for each agent

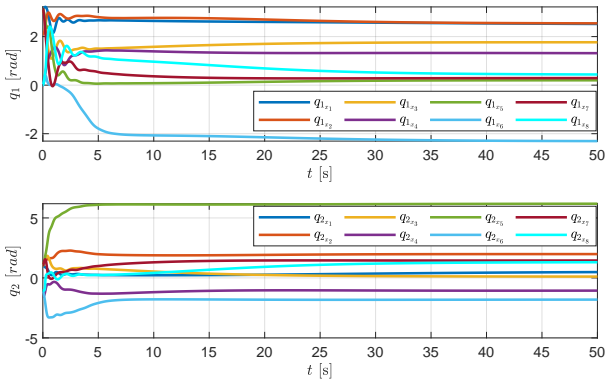


Fig. 2. Bipartite formation of system (1) with control input (14) on joint trajectories.

are $q_1(0) = [\pi, \pi/3]^\top$, $q_2(0) = [2\pi/3, \pi/3]^\top$, $q_3(0) = [\pi, \pi/3]^\top$, $q_4(0) = [0, \pi/2]^\top$, $q_5(0) = [\pi, \pi/3]^\top$, $q_6(0) = [0, -\pi/3]^\top$, $q_7(0) = [\pi, \pi/3]^\top$, $q_8(0) = [0, -\pi/3]^\top$, $\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = \dot{q}_4(0) = \dot{q}_5(0) = \dot{q}_6(0) = \dot{q}_7(0) = \dot{q}_8(0) = [0, 0]^\top$, with $q = [q_1, q_2]^\top$ and $\dot{q} = [\dot{q}_1, \dot{q}_2]^\top$ and the relative displacements of the end-effectors are $b_1 = [0.4, 0.4]^\top$, $b_2 = [-0.4, 0.4]^\top$, $b_3 = [0.4, -0.4]^\top$, $b_4 = [-0.4, 0.4]^\top$, $b_5 = [-0.4, -0.4]^\top$, $b_6 = [0.4, 0.4]^\top$, $b_7 = [0.4, -0.4]^\top$, $b_8 = [-0.4, -0.4]^\top$, with $b = [b_x, b_y]^\top$.

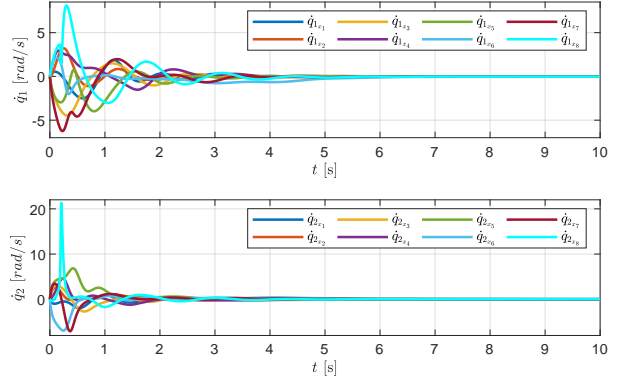


Fig. 3. Bipartite formation of system (1) with control input (14) on joint velocities.

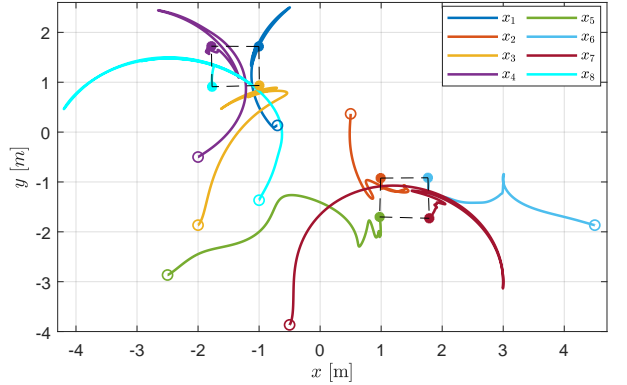


Fig. 4. Evolution of the manipulators' end-effector from the initial positions (o) to the final positions (*). Each group of end-effectors forms a triangle around the symmetric consensus points.

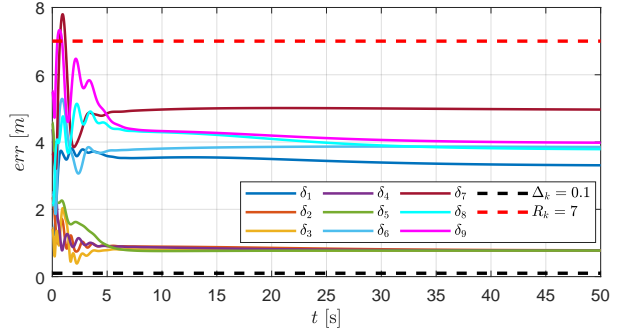


Fig. 5. Trajectories of the norm of inter-agent distances with control input (14). The black dashed line is the minimum distance constraint for all end-effectors, and the red dashed line is the maximum distance constraint for cooperative end-effectors.

The constraint sets are $\Delta_k = 0.1$ for all $k \leq M$ and $R_k = 7$ for all $k \in \mathcal{E}_m$.

The joint positions and velocities are depicted in Figures 2 and 3, respectively, and all velocities converge to zero. The paths of each end-effector up to bipartite formation are depicted in Figure 4, and their final configuration is depicted in Figure 6. Moreover, it is clear from Figure 5 that collision avoidance and connectivity maintenance among the manipulators' end-effectors are both guaranteed.

5. CONCLUSION

We addressed the problem of constrained bipartite formation of cooperative-competitive robot manipulators'

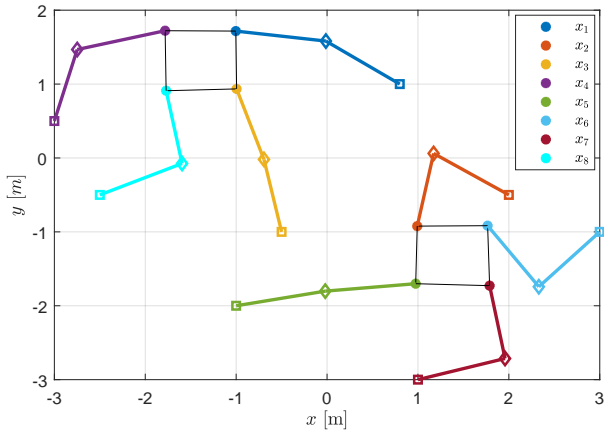


Fig. 6. Final positions of the manipulators and their end-effector.

end-effectors, modeled by Euler-Lagrange equations. We considered a structurally balanced and undirected signed graph. We presented a bipartite formation control law based on the gradient of a barrier-Lyapunov function that guarantees that end-effectors do not collide and stay within their sensing regions. We established the asymptotic stability of the bipartite formation manifold. Further research aims to extend these results to directed signed networks.

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