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TD1: Teleportation of polarized single photon state

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1 Introduction

Discrete variable quantum teleportation is a quantum information processing technique that allows for the teleportation of a discrete quantum state from one location to another. In other words, it allows for the transfer of a quantum state from one particle to another without physically transmitting the particle itself. Discrete variable teleportation is based on the principles of quantum entanglement. To perform discrete variable teleportation, a pair of entangled particles is generated and shared between two parties, often referred to as Alice and Bob. Alice then performs a measurement on a particle that she wishes to teleport, and sends the result of this measurement to Bob. Bob can then use this information, along with the entanglement shared between the two particles, to reconstruct the original quantum state of the particle being teleported. Note the important point. There is a transmission over a classical channel to reconstruct the state of interest and thus, the transmission of information never exceeds the speed of light.

We first proceed to the complete description of the teleportation protocol step-by-step. Then, we discuss the teleportation of the internal state of an atom trapped in a cavity, discussing briefly concrete experimental realizations of gates. In the third section, we mention the methods in the literature discussing how to enhance Bell measurement, and detail one of the methods. In the last section, we ask you to comment one of the possible applications of teleportation in quantum protocols. It is an invitation to learn how to search information in the literature.

2 Teleportation protocol

2.1 Teleportation of a polarized single photon state

The single photon state to teleport is present in the spatial port A:

$$|\psi\rangle = \alpha |H\rangle_A + \beta |V\rangle_A, \quad (1)$$

where $\alpha, \beta \in \mathbb{C}^2$ such that $|\alpha|^2 + |\beta|^2 = 1$ and are unknown. An EPR is prepared in spatial port B,C:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle_{BC} + |VV\rangle_{BC}). \quad (2)$$

(0) How do we prepare such a state? How we can prepare the other three EPR states? Write the associated quantum circuit. The state presents in A will be teleported to the port C.

(1) Write the wavefunction of the three photon states, and also in the Bell state basis.

(2) The spatial port a,b are recombined into a beam-splitter \hat{U} . Write the resulting wavefunction. What are the state leading to bunching and coincidence events?

(3) A Bell-measurement is performed at the spatial port a, b , namely two polarizing beam-splitters are placed at the spatial port a, b , followed by single photon detectors. The final output ports are noted 1, 2, 3, 4. What is the resulting state, with which probability? How to reconstruct the state of interest? What is the overall probability of success of the teleportation scheme? Is there a violation of the non-cloning theorem?

(4) If we have photon number resolving detectors, what would be the overall probability of success?

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(5) In practice, due to the difficulty of the realization of single photon sources, weak coherent states are used. Such a state can be written as: $|\psi\rangle = |0\rangle + \gamma(\alpha|H\rangle + \beta|V\rangle) + O(\gamma^2)$. What is the influence of the γ factor? What is the influence of the vacuum component? What is the influence of the two-photon component?

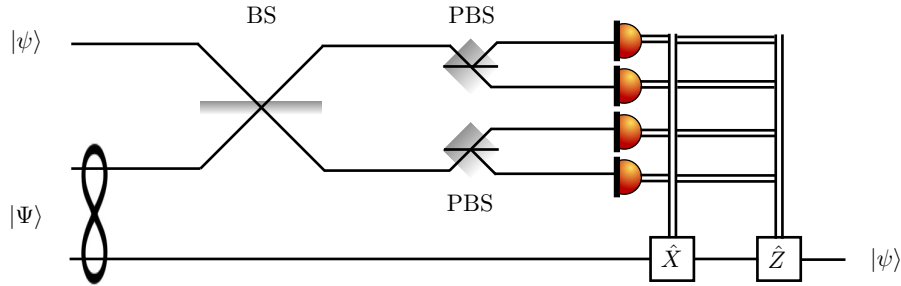


Figure 1: Teleportation scheme of polarized single photon state. BS stands for balanced beam-splitter, and PBS for polarizing beam-splitter.

2.2 Teleportation of the internal state of a atoms trapped in cavity

In this section, we discuss how to teleport the internal state of an atom trapped in a cavity from [1]. We discuss also the experimental preparation of a Bell's pair, composed of a polarized qubit and two energy levels of an atom. The tools developed are crucial for developing a quantum memory, which is a device that is used to store and manipulate quantum states.

- Write the different steps of the protocols.
- Identify the different parameters and find their numerical values.
- What is a dark state? What are the forbidden energy transition? Why?
- Identify each term of the Hamiltonian.

3 Enhancing the Bell measurement

3.1 Different methods

Only two Bell states can be distinguished by linear optics methods, the success of the teleportation protocol is thus bounded by 1/2, and return degenerate results for the other two. Solutions are possible:

- the use of additional degree of freedom: create a hyperentangled state. The Kwiat-Weinfurter scheme for Bell-state analysis [2] and [3] is not enough for distinguishing the 16 Bell's state, require a second copy to unambiguously distinguished all states. However, only one-shot measurement for teleportation, hence can not be used.
- the use of ancilla states [4].
- the use of non-linear interactions [5], atoms-photons, or photons-photons.

3.2 Ancilla state and PNRs

We will follow [6], which presented a scheme that reaches a success probability of 75 %, using balanced beam-splitters, PNRs, and untangled single photons as ancilla.

We start from Bell states in the dual rail encoding, which can be written as:

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|1001\rangle \pm |0110\rangle) \quad (3)$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|1010\rangle \pm |0101\rangle), \quad (4)$$

where $|10\rangle$ denotes the presence (resp. the absence) of a single photon in spatial mode A, B while $|01\rangle$ denotes the absence (resp. the presence) of a single photon in spatial mode A, B . Note importantly that contrary to the first case where the information was encoded into the degree of freedom of single photons, the dual rail encoding

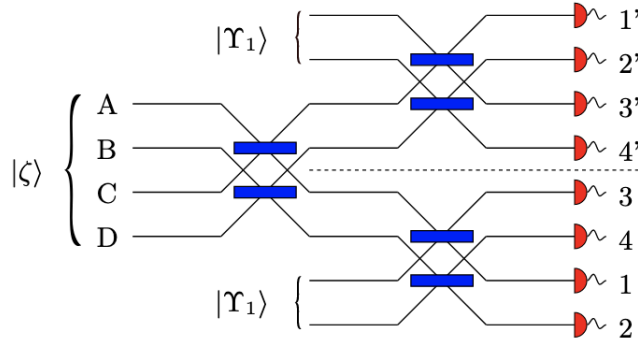


Figure 2: From [6] Optical scheme to distinguish the four Bell's states with an overall 75% efficiency.

is particle sensitive. Photon loss is a logical error, while it was not for the polarization degree of freedom. The spatial modes are labeled A, B, C, D .

(1) How the Bell states are transformed after a balanced beam splitter? How many Bell states it is possible to distinguish with PNRs detectors? What is the overall probability of success if we start from an even distribution for the four Bell states?

(2) Show that the remaining problem is to distinguish the states $|\alpha\rangle = |11\rangle_{xy}$ and $|\beta^\pm\rangle = \frac{1}{\sqrt{2}}(|20\rangle_{xy} \pm |02\rangle_{xy})$, where x, y must be specified.

(3) Ancilla photons $|\Gamma\rangle = \frac{1}{\sqrt{2}}(|20\rangle_{xy} + |02\rangle_{xy})$ are used to distinguish the $|\beta^\pm\rangle$ states. How such a state can be produced experimentally?

(4) By using the optical circuit depicted in Fig.2, write the transformation of the state $|\alpha\rangle |\Gamma\rangle$ and $|\beta\rangle |\Gamma\rangle$. Can the four states be distinguished now? What is the overall probability of success now? Even if you have to learn to be patient until performing the calculation successfully, you can also write a code using qutip to check your result.

(5) Come back to the teleportation. Do you think that, now that the four Bell's states can be distinguished, is that a really better Bell's measurement?

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