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# TD2: Source and detection in quantum communications

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## 1 Homodyne detection

Homodyne detection is a technique used to measure the amplitude and phase of a light wave. It operates by interfering the light wave with a reference beam that projects the state into its mode, and then measuring the difference in the photocurrent in the two arms. This method is applicable to both classical and quantum states.

The state to be measured  $\hat{\rho}$  is combined into a balanced beam-splitter with a local oscillator, a coherent state with a large photon number and a well defined phase. We note  $\hat{a}_{in}^\dagger$  and  $\hat{a}_{LO}^\dagger$  the creation operators in the spatial mode of the state of interest and the local oscillator respectively. The local oscillator is approximated by a classical field of amplitude  $\alpha$  with phase  $\theta$ , so that  $\hat{a}_{LO}^\dagger = \alpha_{LO}$ . The two outputs modes are sent to two photodiodes. The detection is represented into Fig.1.

(1) Show that the measured photocurrent is:

$$\langle \hat{I}_{1,2} \rangle = \frac{1}{2} \langle (\hat{a}_{in}^\dagger \pm \alpha_{LO}^*) (\hat{a}_{in} \pm \alpha_{LO}) \rangle = \frac{1}{2} \langle (\hat{a}_{in}^\dagger \hat{a}_{in} + |\alpha_{LO}|^2 \pm \hat{a}_{in}^\dagger \alpha_{LO} \pm \hat{a}_{in} \alpha_{LO}^*) \rangle. \quad (1)$$

(2) Then show the expression of the subtraction of the photocurrent:

$$\langle \hat{I}_1 - \hat{I}_2 \rangle = |\alpha_{LO}| \langle \hat{a}_{in}^\dagger e^{i\phi_{LO}} + \hat{a}_{in} e^{-i\phi_{LO}} \rangle = |\alpha_{LO}| \langle \hat{x}(\phi) \rangle, \quad (2)$$

where the average value is taken with respect to the initial state  $\hat{\rho}$ . Repeating the measurement, we obtain the probability distribution  $p(x, \phi_{LO})$  with mean value  $\langle \hat{x}(\phi_{LO}) \rangle$ . We point out that the characteristic function is the Fourier transform of the probability distribution:

$$\begin{aligned} p(x, \phi_{LO}) &= \int dz e^{-izx} \langle e^{iz\hat{x}(\phi_{LO})} \rangle \\ &= \int dz e^{-izx} \langle \hat{D}(iz e^{-i\phi_{LO}}) \rangle. \end{aligned} \quad (3)$$

where the displacement operator has been defined as  $\hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$ . After integration, we obtain:

$$p(x, \phi_{LO}) = \int dp W_{\hat{\rho}}(x \cos(\phi_{LO}) + p \sin(\phi_{LO}), -x \sin(\phi_{LO}) + p \cos(\phi_{LO})), \quad (4)$$

which corresponds to the marginals of the Wigner distribution, defined as:

$$W_{\hat{\rho}}(x, p) = \int dx' e^{2ix'p} \langle x - x' | \hat{\rho} | x + x' \rangle \quad (5)$$

We remind that there is a one-to-one correspondance between the Wigner distribution and the density matrix. It emphasized the importance of the Wigner distribution alternative representation of a quantum state because the Wigner distribution can be directly accessed experimentally.

### Examples of calculation of the Wigner distribution

For the vacuum state, the wavefunction is:

$$|0\rangle = \frac{1}{\pi^{1/4}} \int_{-\infty}^{\infty} e^{-x^2/2} dx |x\rangle \quad (6)$$

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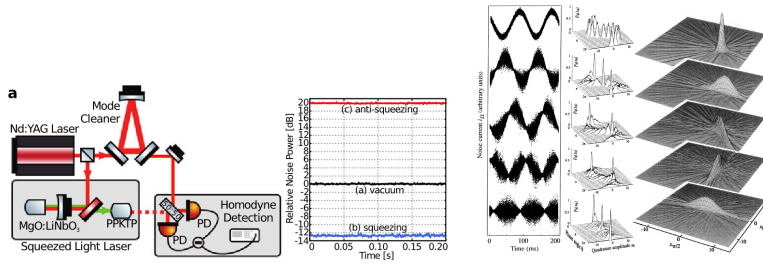


Figure 1: Generation of squeezed state of light and measurement with homodyne detection. 12.7 dB squeezing was generated by parametric down-conversion in a monolithic PPKTP cavity using a frequency-doubled 1064 nm continuous-wave laser beam as pump. (see Ref first course).

(3) Calculate the Wigner distribution of the state, and the marginals along the  $x$ , and  $p$  direction.

For a squeezed state, the quadrature wavefunction is:

$$\psi(x) = \left(\frac{S}{\pi}\right)^{1/4} e^{-sx^2/2} \quad (7)$$

(4) Calculate the Wigner distribution of the state, and the marginals along the  $x$ , and  $p$  direction. Also calculate the variances of squeezed state, directly from the Wigner distribution (and show how to obtain them from the marginals).

### Losses

(5) In practice, due to losses, we do not measure the true value of squeezing. The losses of an optical element are modeled by a beam-splitter operation of transmission  $T$ . The output destruction mode operator  $\hat{a}_{out} = t\hat{a}_{in} - r\hat{a}_v$ . Then show that  $\hat{X}_{out} = t\hat{X}_{in} - r\hat{X}_v$ . Since the quadrature observable of the signal and vacuum states are uncorrelated and since  $\langle \Delta \hat{X}_v^2 \rangle = 1/2$ , then show that:

$$\langle \Delta \hat{X}_{out}^2 \rangle = T \langle \Delta \hat{X}_{in}^2 \rangle + \frac{1}{2}(1 - T) \quad (8)$$

Let us consider that the measured squeezing is 0.9 dB. The efficiency of the photodetectors is 80%, of the beam-splitter of the homodyne detection 93.3 %. What is the overall efficiency? How to deduce the  $\langle \Delta \hat{X}_{in}^2 \rangle$ , and then the associated value in dB?

## 2 Measurement of the characteristics of photodetectors

(6) Express the count rate on the detectors  $N_m$ , as a function of the rate of incident photons  $R$ , the dark count rate  $D$ , the dead time  $\tau$  per count, and the quantum efficiency.

**Solution:** The rate of detectable photons is  $\eta R$ .

The rate of measured counts  $N_m =$  measured photons + measured dark counts

Dead time per second  $N_m \tau$

Measured photon rate: Rate of detectable photons- lost photon rate due to dead time  
 Lost photon rate due to dead time: lost dark count due to dead time  $D$  is  $N_m \tau$ . Measured dark count rate  $D_m = D -$  lost dark counts due to dead time,  $D_m = D(1 - N_m \tau)$ .

Finally, the count rate is  $N_m = \eta R(1 - N_m \tau) + D(1 - N_m \tau)$ , so that:

$$N_m = \frac{\eta R + D}{1 + \tau(\eta R + D)} \quad (9)$$