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# Localization and quantification of geometry damage and their size estimation using frequency shift coefficient and damage library

A. Dubey<sup>1,2</sup>, V. Denis<sup>1</sup>, F. Hennebelle<sup>2</sup>, R. Serra<sup>1</sup>

<sup>1</sup> INSA Centre Val de Loire, Laboratoire de Mécanique Gabriel Lamé EA 7494,  
3 Rue de la Chocolaterie, 41034 Blois, France  
e-mail: [anurag.dubey@insa-cvl.fr](mailto:anurag.dubey@insa-cvl.fr)

<sup>2</sup> Université Bourgogne Franche Comté, DRIVE EA 1859,  
F58000 Nevers, France

## Abstract

The research presents a novel strategy to locate and quantify damage, as well as estimate their geometry damage sizes. The strategy is implemented to estimate the size of geometry damage (width and depth) based on the damage library and frequency changes between healthy and damaged Euler-Bernoulli cantilever beams. Firstly, a geometry damage library is built by a concept of correlation of 2D and 3D finite element (FE) models using a frequency shift coefficient (FSC). Secondly, FSC-based algorithm is implemented to localize and quantify damage along the beam structure. FSC algorithm is employed as a particle swarm optimization (PSO) solver to determine the best-fit minimized values that represent damage position and severity. Finally, the severity of damage is used to estimate the size of the damage from a damage library. The strategy is demonstrated by testing the experimental beam and estimating damage parameters from a real geometry damaged case.

## 1 Introduction

Structural health monitoring (SHM) aims to maintain the integrity of an entire structure by performing a variety of activities such as monitoring, inspection and damage detection. Over the years, vibration-based structural health monitoring (VBSHM) has become popular as a structural diagnostic tool due to its ability to monitor and diagnose damage in structures [1, 2]. Several reviews on VBSHM have been provided in Doebling *et al.* [3] and Montalvao *et al.* [4] based on advances in damage detection methods and SHM with a concentration on structures and dynamic parameters. According to Rytter [5], damage detection and assessment are divided into four levels: (i) detection (ii) localization (iii) quantification (iv) prediction of the remaining life of the structure. Damage identification based on natural frequencies can be reliable due to the sensitivity of frequency changes between healthy and damaged states. Several vibration methods have been given in the literature for detection, localization and quantification of damage by considering the frequency changes. Salawu [6] discovered that natural frequency is a sensitive parameter for damage detection. Messina *et al.* [7] introduced damage location assurance criterion (DLAC) and later included multiple damage location assurance criterion (MDLAC) [8] for damage identification. Silva and Gomes [8] first introduced the frequency shift coefficient (FSC) to detect the damage position and estimate the crack length. Dubey *et al.* [10, 11] presented an FSC-based damage identification algorithm and its efficiency with modeling uncertainties on natural frequency by testing the damaged structure. The authors efficiently tested the FSC minimization algorithm on an experimental beam with double damage. Additionally, Dubey *et al.* [12, 13] introduced a concept of correlation of 2D and 3D FE models to build a damage library using FSC minimization, and further detailed work was presented to identify properties of geometry damage (rectangular and hole) with experimental results [14]. Serra and Lopez [15] evaluated many traditional indicators for

their effectiveness and their ability to identify and locate damage in various damaged cases.

Yet, few studies have introduced a relationship between natural frequency change and damage geometry. In this research, a VBSHM technique based on frequency changes is proposed to estimate geometry damage parameters, such as position, severity and sizes. In this context, the focus is on the development of a damage library to estimate geometry damage parameters. The proposed strategy uses the FSC minimization technique to correlate damage location, elemental stiffness reduction, and damage geometry using the correlation of 2D and 3D FE models of the structure. A numerical damage library is built for the cantilever beam structure through the correlation between the 2D and 3D FE models. The proposed strategy is demonstrated by performing an experimental test and identifying the geometry properties of damage (position, severity, and size) in a real beam to assess the effectiveness of the strategy.

The rest of the paper is organized as follows: Section 2 recalls the dynamic of a cantilever beam structure in bending vibration, and FE models (2D and 3D) are introduced to extract the dynamic properties of a structure; Section 3 presents the proposed a new strategy to identify the damage, as well as a technique to minimize FSC as a function of position and severity; Finally, Section 4, an experimental beam is tested to determine the position, severity and size of the real geometry damage.

## 2 Cantilever beam vibration background and finite element models

The equation of motion for the flexural vibration of a cantilever Euler-Bernoulli beam can be expressed as:

$$M(x) \frac{\partial^2 w(x, t)}{\partial t^2} + C(x) \frac{\partial w(x, t)}{\partial t} + \frac{\partial^2}{\partial x^2} \left( EI(x) \frac{\partial^2 w(x, t)}{\partial x^2} \right) = F(x, t), \quad (1)$$

where  $M(x)$  is the mass per unit length,  $C(x)$  is the viscous damping coefficient,  $EI(x)$  is the bending stiffness,  $w(x, t)$  is the transverse deviation,  $x$  is the coordinate along the main direction of the beam,  $t$  is the time, and  $F(x, t)$  is the external force per unit length of the beam. When discretized, for example using the finite element method, the equation of motion for each degree of freedom can be given as a matrix:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\}, \quad (2)$$

where  $[M]$ ,  $[C]$ , and  $[K]$  are the mass, damping, and stiffness matrices, respectively. In this case,  $\{\ddot{X}\}$ ,  $\{\dot{X}\}$ , and  $\{X\}$  are vectors containing acceleration, velocity and displacement in all degree of freedom of the model. The eigenvalue equation for the  $i$ th vibration mode of a dynamic structure with no external force or damping, is expressed as follows:

$$([K] - \lambda_i[M])\phi_i = 0, \quad (3)$$

where  $\lambda_i$  and  $\phi_i$  are the  $i$ th eigenvalue and eigenvector, and  $\lambda_i = \omega_i^2$ , where  $\omega_i$  is the  $i$ th natural frequency. The natural frequencies (in Hz) are given by  $f_i = \frac{\omega_i}{2\pi}$ .

### 2.1 2D finite element model of beam

As shown in Figure 1, the cantilever beam is divided into  $N$  equal size elements, and the model includes  $N+1$  nodes. A healthy and damaged 2D FE model (stiffness updating) of the beam is developed in this work. It is considered that the damage is placed at location  $x_l$  in the element with the nodes  $i$  and  $i + 1$  of the beam. The beam is divided into 100 finite elements, and the size of each element is 10 mm. The total number of degrees of freedom (DOF) of the 2D FE model is 202, with each node having two DOF: displacement ( $y$ ) and rotation ( $\theta_z$ ).

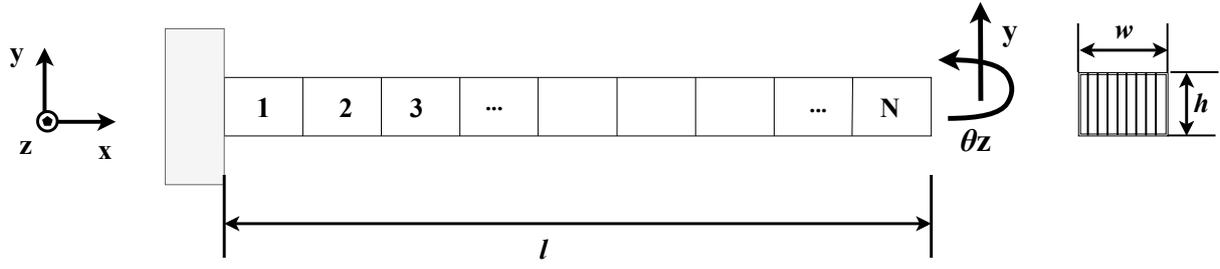


Figure 1: 2D FE model of a cantilever beam and cross-section area.

The severity of the damage is indicated by the stiffness reduction coefficient  $\beta_c$ , which is the ratio of the stiffness reduction to the initial stiffness. The stiffness matrix of the damaged beam is written by the following equation:

$$[K_d] = \sum_{c=1}^n (1 - \beta_c) [K_c], \quad (4)$$

where  $[K_d]$  is the global stiffness matrix for a damaged beam,  $n$  is the number of elements,  $[K_c]$  is the stiffness matrix of element  $c$ , and  $\beta_c$  is a bending stiffness reduction parameter for the damaged structure ranging from 0 to 1. A value of  $\beta_c = 0$  indicates healthy element.

## 2.2 3D finite element model of the beam

3D FE models of the healthy and damaged cantilever beam structure are developed using COMSOL Multiphysics software. The beams are modeled by a *fine* mesh using tetrahedral elements (6 DOF per node), and the mesh density is controlled by COMSOL. Typically, converged damage models have 22,000 DDLs. A high mesh density is used near the damaged area. Figure 2 shows a zoomed view of the damaged rectangular geometry and its physical parameters (width, depth, and height). The natural frequencies of healthy and damaged 3D models are estimated using COMSOL Multiphysics own solver.

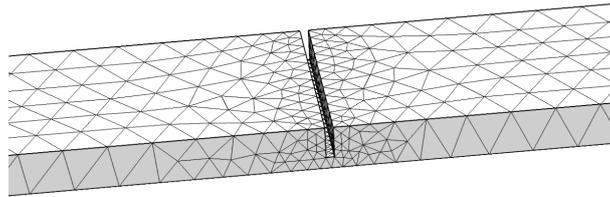


Figure 2: 3D finite element mesh of the beam with zoomed damage

## 3 Strategy and objective function for damage identification

The novel objective of this study is to build a damage library using an existing severity indicator to estimate the properties of rectangular geometry damage [12–14]. The strategy for identifying damage properties is described in three main steps:

**Step 1:** In the beginning, the experimental beam is used to identify a set of healthy beam frequencies. The healthy 2D and 3D FE models are developed, and Young's modulus of the FE models is updated using the maximization function to closely match the frequencies with the healthy experimental beam. Then, the damage is represented as a rectangular discontinuity in the 3D FE model, considering position, width and depth of the damage. At the same time, the damage is simulated in the 2D FE model as a local elemental reduction of bending stiffness. Numerical correlation is performed between 2D and 3D FE models by FSC

minimization, i.e. for each set of physical parameters, a reduction in bending stiffness is performed in the 2D model. At the end of this step, a damage library is built, with 3D rectangular damage providing the width and depth, and a 2D model providing the bending stiffness reduction value (%) of the severity at a position specific to the damage. Note that the damage library is numerically validated once built.

**Step 2:** FSC is used to update the 2D FE model of a damaged beam to identify the position and severity of the damage. Four beam models with similar characteristics (healthy beam, damaged beam, healthy 2D model and damaged 2D model with stiffness updating) are used for a given FSC update procedure. During this procedure, the stiffness reduction coefficient (or severity) and the location of the reduction are updated. The minimum value of the FSC criterion gives the desired location and severity. In practice, the FSC minimization is calculated using Particle Swarm Optimization (PSO) [16] solver in MATLAB global optimization toolbox.

**Step 3:** The damage severity from Step 2 is searched in the damage library for the identification of physical geometry parameters. In the case of rectangular damage, the severity is linked to a (potentially non-unique) set of width and depth. It should be noted that curve fitting may be required if the library is not accurate enough, and tolerance can be applied. The non-uniqueness of the solution from the rectangular damage library as well as the results are discussed in the experimental part.

### 3.1 Frequency shift coefficient

The frequency shift coefficient (FSC) is used for the minimization procedure to determine the position and severity (%) of the damage and to build a damage library. Silva and Gomes [9] were first proposed FSC for damage identification. The FSC equation is written as follows:

$$\text{FSC} = \sqrt{\frac{1}{m} \left| \sum_{i=1}^m \left( \frac{(R_i)_X - (R_i)_A}{(R_i)_X} \right) \right|} \quad \text{et} \quad R_i = \frac{f_i^d}{f_i^h} \quad (5)$$

where  $R_i$  represents the ratio of the frequencies of the damaged beam ( $f_i^d$ ) to those of the healthy beam ( $f_i^h$ ),  $m$  is the total number of modes,  $D$  denotes the damaged case,  $X$  refers to the tested case,  $A$  denotes to the reference, i.e. the healthy FE model, and  $i$  denotes the frequency indices. For the FSC to be calculated, at least two vibration frequencies must be considered.

## 4 Identification of damage on an experimental beam

The efficiency of the proposed strategy based on the FSC approach is examined by testing an experimental cantilever beam. Figure 3 shows a damaged steel beam with two accelerometers (length  $L = 1000$  mm, width  $W = 25$  mm, and thickness  $T = 5.3$  mm). The material characteristics of the steel beam are considered with the standard values (Young's modulus  $E = 210$  GPa,  $\rho = 7850$  kg/m<sup>3</sup>,  $\nu = 0.33$ ). A rectangular saw cut is created on the damaged beam with a depth of 1.35 mm and a width of 1.00 mm. The hammer impact test is performed, and vibration signals are recorded with an LMS multi-analyzer (Siemens LMS software, Plano, TX, USA) to extract natural frequencies from healthy and damaged beams. The hammer generated the input excitation, while the monoaxial accelerometer (Brüel and Kjaer-DeltaTron Type 4507) recorded the output data. The experimental device is used to extract natural frequencies from cantilever beams (healthy and damaged). The frequency response functions of healthy and damaged cantilever beams are used to obtain the natural frequencies using the circle fit method and the application of the EasyMod package [17]. The first seven measured natural frequencies of healthy and damaged beams are listed in Table 1.

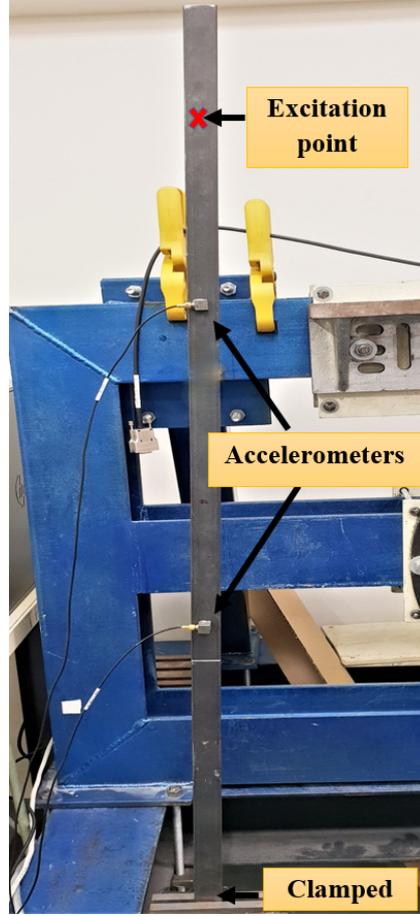


Figure 3: Experimental damaged cantilever beam and illustration of measurement input and output points.

Table 1: Experimentally identified natural frequencies of healthy and damaged beams.

Experimental beam	Natural frequencies (Hz)						
	1	2	3	4	5	6	7
$f_{\text{healthy}}$	4.23	26.34	73.26	142.34	240.13	355.67	499.67
$f_{\text{damaged}}$	4.21	26.35	73.14	141.87	240.02	357.80	498.78

## 4.1 Results

FSC is used to evaluate and compare the real damaged beam case (see Figure 3) to the numerical healthy and damaged 2D FE models. The mass of the accelerometers is not taken into account, as the localized mass in the FE models when identifying natural frequencies and damage in a beam. The FE model is used to update based on the natural frequencies of the experimental beam with the accelerometers. The damage is first located and quantified, and then its size is extracted from the damage library according to the estimated severity. Step 2 is used to calculate FSC, where position and severity are estimated as shown in Figure 4(a). The colorbar represents the FSC value, while the white circle represents (obtained by particleswarm() function in MATLAB Global Optimization Toolbox) the position and severity based on FSC minimization. In the real case, the damage is estimated with the value of 27.75% at the position of 0.238 m from the clamped part of the beam, where the FSC minimum value is 0.0494. It is important to note that the estimation of the position of the damage (0.238 m) is obtained close to the real position (0.245 m). The FSC minimization problem is solved in 5-10 seconds. Finally, the determined damage severity (27.75%) is searched in the damage library with a tolerance of 1% (see Step 3) to extract the width and depth of the damage. As shown in Figure 4(b), multiple damage sizes correspond to a single severity. For demonstration

purpose, we assume that the width is equal to the real width value (1.0 mm). It is noted that the depth is obtained by the value of 1.408 mm for a damage width of 1 mm (see Figure 4(b)). This estimated value is close to the real depth value (1.350 mm). This means that the damage library is suitable to provide the physical properties of geometry type damage. Therefore, the proposed strategy is an efficient identification tool to estimate the characteristics of geometry damage in a real problem.

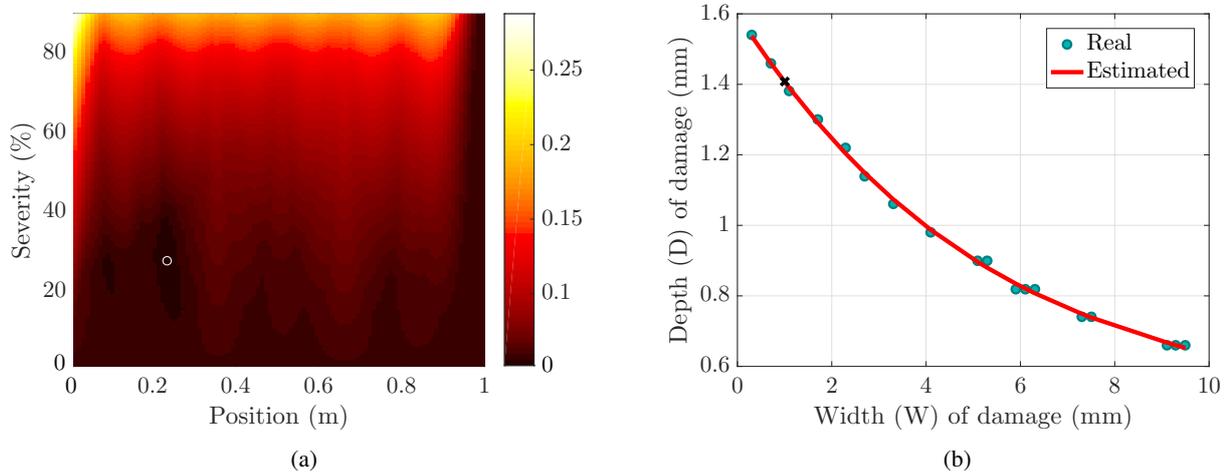


Figure 4: (a) FSC as a function of position and severity where white circle represents 27.75% estimated damage severity at 0.238 m position; (b) width 1.0 mm and depth 1.408 mm for a severity of 27.75% extracted from the damage library (blue dots: raw extraction, red line: cubic fit, black cross: retrieved parameters).

## 5 Conclusion

In this research, a strategy based on bending stiffness reduction is developed to identify damage properties (position, severity and size) in beams. The approach uses only natural frequencies and FE models of the structure. The method is tested on a steel cantilever beam. In this context, minimization of the frequency shift coefficient is used in two ways. First, a damage library is built by correlating the 2D and 3D FE models of the beams. The library is designed for rectangular damage which combines depth and width of the 3D FE model with bending stiffness reduction of the 2D FE model. The damage library does not influence by the position of the beam. Second, a FE model of the damaged beam is updated to determine the position and severity of the damage. At last, the damage library is used to determine more about the damage geometry size. The method has been successfully tested and estimated geometry damage properties (position, severity and size) on an experimental cantilever beam in which saw-cut type damage was incorporated. In the future, this strategy will be validated by testing it with other types of structures and other damage geometries such as hole, oblique or triangular cut.

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