Gradient reliability-based design optimization of a composite plate through multi-scale design spaces

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International Forum on Aeroelasticity and Structural Dynamics
IFASD 2022
13-17 June 2022, Madrid, Spain

GRADIENT RELIABILITY-BASED DESIGN OPTIMIZATION OF A COMPOSITE PLATE THROUGH MULTI-SCALE DESIGN SPACES

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Keywords: Aeroelastic tailoring, composite structures, reliability-based design, surrogate models

Abstract: The inherent incertitudes linked to the manufacturing processs call for a reliability-based approach to structural optimisation. In this case, one faces optimization in a mesoscopic space where design variables (i.e., the orientations) are many, discrete, and often bear some uncertainty. In some cases, it seems more advantageous to work on some homogeneous macroscopic space (e.g., the laminations parameters) instead of using orientations. Indeed, this allows a reduction of the problem dimension and naturally exploits regular gradients information. Once the optimal design is obtained in the macroscopic space, an inverse problem must be solved to identify a corresponding optimized set of mesoscopic design parameters. This identification is far from trivial in standard optimization frameworks because of non-uniqueness issues and it often relies on meta-heuristic methods.
In the context of reliability-based design optimization, an uncertainty quantification from the mesoscopic to the macroscopic space is necessary, requiring a scaling up to transport the impact of these uncertainties. An optimization carried out in the macroscopic space and accounting for uncertainties now requires solving an inverse problem at each iteration to correctly identify an optimized set of mesoscopic parameters in terms of the statistical description of the macroscopic solution. This expensive search must be reiterated, necessitating highly efficient propagation of uncertainties. To this end, a particular orthonormal basis has been constructed with Fourier chaos expansion. Moreover, due to the modal nature of the aeroelastic quantity of interest (i.e., the flutter velocity), a surrogate model strategy is implemented by combining the Kriging method and a classification.
This approach is applied to a composite plate optimization with uncertain ply angles to promote the wing's flexibility while remaining reliable with respect to the flutter phenomenon. The preliminary results show a good convergence of this optimization approach with a significant improvement in the reliability relative to the deterministic optimized design.

1 INTRODUCTION

Composite materials offer an interesting potential in structural optimization. Nevertheless, many sources of uncertainties, such as the dispersion of material properties and geometrical uncertainties related to the manufacturing process, can affect the response of a structure and, hence, its aeroelastic performance [1], limiting the exploitation of this kind of materials. When

these uncertainties are not considered, reliability of the design process relies on conservative approaches based on safety factors that may often lead to oversizing the structure.

Several methods exist for stacking sequence optimization with constant stiffness. The most popular are stochastic direct search methods which are the most appropriate for composite lay-up design because of their capabilities of handling discrete variables and finding the global optimum of a multi-modal objective function. The genetic algorithm has been the most popular method for optimizing the stacking sequence of a laminated composite despite the high computational cost [2]. Another method is bi-level optimization, where the stiffness is characterized by the lamination parameters or the polar formalism, and the optimization is completed with a gradient-based algorithm. Then the stacking sequence is retrieved with a genetic algorithm [3] [4] [5]. A review of all the methods can be found in Ghiasi et al. [6].

The optimal solution usually activates a subset of the optimization constraints. The constraints can be of different kinds, spanning from a conventional local limit on the strain to more complex ones involving the structure response and its interaction with other physical models. Aeroelastic flutter is an example of the latters. This aeroelastic phenomenon is a dynamic instability, that may onset at different velocity because of the coupled response of the aerodynamic structural dynamics. For this reason, it is dependent on the structural design parameters and can experience discontinuities in the design space. Taking into account uncertainties in an aeroelastic design process is therefore essential to avoid these discontinuity zones when they might lead to an unstable case with uncertainties.

Numerous techniques have been used to model uncertainty in aeroelasticity of composite structures. Monte Carlo Simulation (MCS) is a commonly used technique and was used by Murugan et al. [7] to model the aeroelastic response of a composite rotor blade with uncertain elastic moduli and Poisson's ratio. Manan and Cooper [8] used a non-intrusive Polynomial Chaos Expansion to model the flutter of composite plate wings with uncertain ply orientations, thickness, and longitudinal and shear moduli. Scarth et al. [9] use lamination parameters to represent the ply orientation uncertainty in order to reduce the number of random variables. This work shows the important deviation in the critical flutter velocity, taking into account the uncertainty of orientations. A similar analysis was undertaken by Chassaing et al. [10], in which the polar formalism is used.

The above work details the calculation of metrics in order to quantify the effects of uncertainty on model outputs. Such metrics may be used as an optimization objective or constraint in line with various design strategies. In a Reliability-Based Design Optimization (RBDO), the challenge is to find an optimal *reliable* solution such that the probability that the design does not satisfy a constraint under uncertainty is less than a limit failure probability. Scrath and Cooper [11] investigated the minimization of the probability of the flutter instability, modeled as Gaussian processes, in a simple, composite-plate wing, with random design variables, i.e., the ply orientations. Even if the resulting design is in some sense an *optimized* solution with respect to the onset of the flutter instability, their proposed procedure is far to be representative of the real aircraft design process since the aeroelastic stability has to be taken into account as a constraint instead of an *improvable* objective. Moreover, a genetic algorithm is used to solve the optimization, which can be limited to complex high-dimensional problems. For laminate optimization, gradient-based could be used with the lamination parameters or the polar formalism. In the multidisciplinary optimization field, some works have been done in optimization under uncertainty using gradient-based algorithms and surrogate model strategies to reduce the

time of convergence [12] [13].

This work presents an optimization formalism for RBDO composite laminates with uncertainty on ply orientations and a surrogate model strategy adapted for critical flutter velocity following the bi-level optimization using lamination parameters and a gradient-based algorithm. The approach could be applied to more complex geometries handling several constraints in a decent convergence time.

2 MULTI-SCALE RELIABILITY-BASED DESIGN OPTIMIZATION APPROACH

2.1 Reliability assessment and optimization

Reliability analysis evaluate the structural safety considering the random nature of all phenomena affecting a structural system. The performance function $g(\mathbf{a})$ characterizes the response of the system. The design region is divided into two domains:

$$\text{Failure domain: } F = \{\mathbf{a} \mid g(\mathbf{a}) > 0\}\,. \tag{1}$$

$$\text{Safety domain: } S = \{\mathbf{a} \mid g(\mathbf{a}) \leq 0\}\,. \tag{2}$$

The boundary between failure and safety domains is the limit state surface, which generally is a hypersurface in the n-dimensional space of random variables $\mathbf{a}$. According to this, the failure probability P_f is formulated as:

$$\mathbb{P}\left(g(\mathbf{a}) > 0\right) = \int \cdots \int_{g(\mathbf{a}))>0} \pi_{\mathbf{a}} d\mathbf{a}, \tag{3}$$

where $\pi_{\mathbf{a}}$ is the joint probability density function of the random variables. Except in some particular cases, the integral expression cannot be resolved analytically, because of the nonlinearity of $\pi_{\mathbf{a}}$ and also due to the number of random variables which can be large.

Reliability-based design optimization (RBDO) is an optimization process where the constraints are enforced with acceptable probability under inherent variability in parameters. The goal is to find an optimal *reliable* solution such that the probability that the design does not satisfy a constraint under uncertainty is less than a limit failure probability. It is a active multi-disciplinary research topic and some reviews can be found in Aoues and Chateauneuf [14], Yao et al. [15], Lelievre et al. [16], Acar et al. [17].

In general RBDO problems, both design variables and other system parameters can contain deterministic and/or random quantities. According to that, $\boldsymbol{\theta}$ is defined as the vector of design variables, and $\mathbf{p}$ describes the environmental parameters. With our choice of notations, $\mathbf{a}$ refers to the random variables and $\mathbf{d}$ to the deterministic quantities. These vectors can be written as follows:

$$\boldsymbol{\theta} = \begin{Bmatrix} \boldsymbol{\theta}_a \\ \boldsymbol{\theta}_d \end{Bmatrix}, \quad \mathbf{p} = \begin{Bmatrix} \boldsymbol{p}_a \\ \boldsymbol{p}_d \end{Bmatrix}, \quad \mathbf{a} = \begin{Bmatrix} \boldsymbol{\theta}_a \\ \boldsymbol{p}_a \end{Bmatrix}, \quad \mathbf{d} = \begin{Bmatrix} \boldsymbol{\theta}_d \\ \boldsymbol{p}_d \end{Bmatrix}. \tag{4}$$

Furthermore, we introduce the mean values of the random variables $\boldsymbol{\mu}_a$ which might be needed for a deterministic approach.

The RBDO formulation can be set out in different ways. The double loop approaches consider the reliability constraints within the optimization loop [18]. Mono-level approaches exist where the probabilistic constraint is approximated with deterministic values, converting the double loop into a single loop [19]. Finally, uncoupled approaches solve sequentially deterministic optimization procedures with a reliability analysis at the end of each optimization [20].

In order to have control over the uncertainties on the failure probability, a bi-level approach is considered, and the formulation of the double loop RBDO problem can be formalized in the following Equation 5.

$$\begin{array}{rl} \min\limits_{\boldsymbol{\mu}_{\boldsymbol{\theta_a}},\boldsymbol{\theta}_d} & f\left(\boldsymbol{\mu}_a,\boldsymbol{\theta}_d,\mathbf{p}_d\right), \\ h_i(\boldsymbol{\mu}_a,\boldsymbol{\theta}_d,\mathbf{p}_d)\leq 0 & i=1,...,m \\ \mathbb{P}(g_j(\mathbf{a},\mathbf{d})>0)\leq \mathbb{P}_j^{max} & j=m+1,...,M \end{array} \tag{5}$$

where f is the cost function, h_i are the deterministic constraints, $\mathbb{P}$ are the failure probabilities of the limit state functions g_j, which have to be below the maximum failure probabilities $\mathbb{P}_j^{max}$.

In the following work, the uncertainty is included only on the design variables $\boldsymbol{\theta}$. Therefore the functions f, g or h will be written as function of $\boldsymbol{\Theta}$ or $\boldsymbol{\mu}_{\boldsymbol{\Theta}}$.

2.2 Multi-scale formulation

2.2.1 Uncertain multi-scale design variables

The optimization of composite laminates is a combinatorial problem involving variables such as ply orientations that can take only a discrete set of possibilities. Moreover, the cost function and constraints responses can be highly multimodal in this mesoscopic design space. This type of problem can be solved using metaheuristic algorithms like genetic algorithms [21] [22] or particle swarm optimization [23] [24], which are particularly suited for discrete problems. However, the cost function and constraints can also be expressed with continuous variables in a homogenized (or macroscopic) space using laminations parameters. Using this design space has the advantage of reducing the number of parameters required to express the quantity of interest to a maximum number, regardless of number of discrete variables. Moreover the cost function and constraints response have a linear dependence on the macroscopic parameters and therefore allow the use of a gradient-based method for optimization. This choice however presents further challenges related to the inverse mapping problem, i.e., the efficient identification of a unique set of discrete orientations from their homogenized description. However, the mapping from the mesoscopic to the macroscopic space is explicit.

Due to complex manufacturing processes, the ply orientations can be uncertain. In a probabilistic framework, the uncertainty of the mesoscopic variables is known and modeled with a probability density function π_Θ. However, the uncertainty of the continuous variables can be complex to model due to various correlations which makes it difficult to calculate the failure probability for reliability optimization directly in this space. Nevertheless, the desire is still to exploit the lamination parameters where gradient algorithms can accelerate the convergence of the optimization problem.

2.2.2 Optimization formulation

At each iteration of the optimization process, the probability of failure and its gradient must be computed. Because the uncertainty modeling modeling of the continuous parameters is not straight forward, the developed approach uses two design spaces: the homogenized space (with the lamination parameters) and the mesoscopic space (with the orientations).

The first one is used for the global optimization process to take advantage of a gradient algorithm. The objective function and the constraints are defined in this space. The second space is used to evaluate the failure probability because the dispersion of the discrete design variables is known. Nevertheless, the mapping from the continuous design variables to the discrete ones is not trivial. It is a highly multimodal optimization problem where it is necessary to use metaheuristic optimization methods. In this study, the solution of the inverse problem is performed using a genetic algorithm.

Once the genetic algorithm finds a set of discrete mean variables, it is simple to propagate the associated uncertainty to the homogenized space and thus to the model to calculate the failure probability by the Monte Carlo method. This strategy, exploiting both spaces for the calculation of the failure probability, is described in Figure 1.

The formulation of the reliability-based design optimization taking into account discrete design variables uncertainties can be described by Equation 6.

$$\begin{array}{c} \min\limits_{\boldsymbol{\mu}_{\boldsymbol{\Theta}}} \\ h(\boldsymbol{\mu}_{\mathbf{v}})\leq 0 \\ \mathbb{P}(g(\mathbf{v}=H(\boldsymbol{\Theta}))>0)\leq \mathbb{P}^{max} \\ \Theta\sim\pi_{\boldsymbol{\Theta}|\sigma_{\boldsymbol{\Theta}}} \end{array} \quad f(\boldsymbol{\mu}_{\mathbf{v}}(\boldsymbol{\Theta})), \tag{6}$$

where f is the cost function to minimize, $\boldsymbol{\Theta}$ are the random orientations design variables with their means $\mu_{\boldsymbol{\Theta}}$ to be optimized, $\mathbf{v}$ are the lamination parameters, h is a deterministic nonlinear constraint emerging from the multiscale formulation, g is the limit state function, H is the mapping from the mesoscopic space to the macroscopic space, used to compute $\mathbf{v}$ for a given $\boldsymbol{\theta}$, $\pi_{\Theta|\sigma_{\boldsymbol{\Theta}}}$ is the joint probability density function of discrete design random variables given their variances, $\mathbb{P}$ is the failure probability calculated by the multi scale approach, and $\mathbb{P}^{max}$ is the threshold failure probability.

3 COMPUTATION OF FAILURE PROBABILITY

3.1 Surrogate-based strategy for the inverse problem

The effect of uncertainty of the input parameter $\boldsymbol{\Theta}$ is expressed as:

$$\boldsymbol{\Theta} = \boldsymbol{\mu}_{\boldsymbol{\Theta}} + \sigma_{\Theta} X, \tag{7}$$

where $\boldsymbol{\mu}_{\boldsymbol{\Theta}}$ is the mean in the value Θ, σ_{Θ} is the known standard deviation, in radian, and X is the error, which is modeled as a Gaussian distribution π_X with zero mean and unit variance.

In a variability framework, this inverse problem needs to be applied to the statistical moments. The distance can be minimized with respect to the mean of the mesoscopic and macroscopic parameters as in Equation 8.

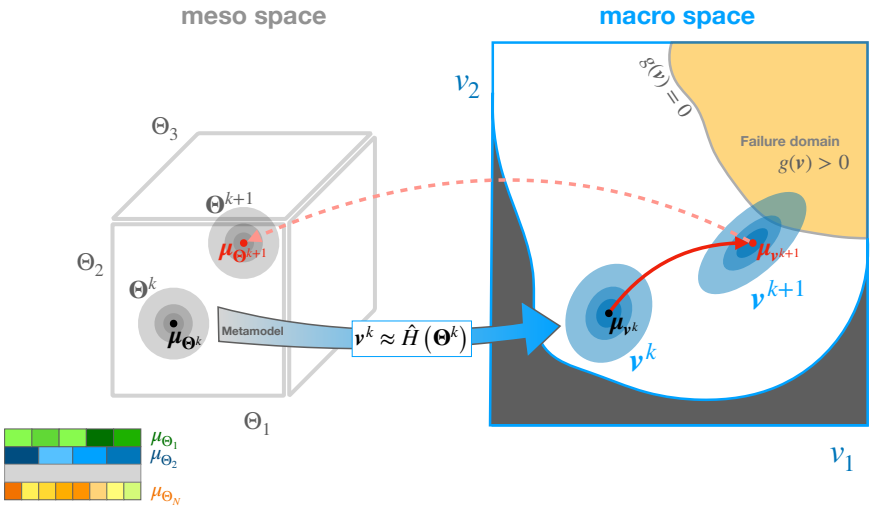


Figure 1: Multi-scale approach for the RBDO

$$\min_{\boldsymbol{\mu}_{\boldsymbol{\Theta}}} \|\boldsymbol{\mu}_{\mathbf{v}^{\text{target}}} - \boldsymbol{\mu}_H(\boldsymbol{\Theta})\|, \tag{8}$$

The variance could also be used to "satisfy" some hypotheses like the orthotropic property of the stacking sequence. However, working with statistical moments means a high number of calls of the mapping function H. Therefore, retrieving a set of discrete variables with respect to the statistics can become costly. To alleviate the cost of this procedure, a surrogate model can approximate the mapping H and access rapidly to the statistics. Moreover, the dimension is acceptable, which motivates the choice to use the polynomial chaos expansion (PCE) [25], which is accurate for these statistical moments with a decent time computation.

Therefore, the quantity of interest H is expanded into a series of polynomials:

$$H(X) = \sum_{i}^{p} h_i \phi_i(X), \tag{9}$$

where ϕ_i are orthogonal polynomials, h_i are the expansion coefficients, p is the order of the polynomials. The deterministic coefficients are defined as:

$$h_i = \frac{\mathbb{E}\left[H\left(\Theta\left(X\right)\right)\phi_i\left(\Theta\left(X\right)\right)\right]}{\mathbb{E}\left[\phi_i\left(\Theta\left(X\right)\right)^2\right]}. \tag{10}$$

Initially, polynomial chaos was created for Gaussian distributions, which uses Hermite polynomials [26], and then generalized to different families of distributions summarised in the Askey scheme [25]. In this work, even if the random parameters are modeled with Gaussian distribution, another basis is used in order to calculate the coefficients in Equation 10 analytically. The Fourier chaos expansion (FCE) is used instead of the PCE.

The development of the FCE is analogous to the development of the PCE but the basis to be orthogonalized is not the monomial basis $\{1, x, x^2, ...\}$; instead, the development begins with the

Fourier basis, that is, the set $\{1, \sin(nx), \cos(nx)\}$, where $n = 1, ..., \infty$. The FCE is first used in [27] to obtain the probability distribution of an airfoil's pitch angle where oscillatory motion is involved. The one-dimensional polynomials $\phi_i(X)$ are constructed using the Gram–Schmidt orthogonalization method [28]. This method has been applied for the critical flutter velocity uncertainty quantification, which used arbitrary polynomial chaos (aPC) for non parametric density probability function [29].

3.2 Surrogate-based strategy for the failure probability

Once the mesoscopic variables are found, the uncertainty quantification can be carried out knowing the distribution π_Θ, and the failure probability in Equation 6 is defined as:

$$\mathbb{P}\left(g(\mathbf{\Theta}) > 0\right) = \int_{g(\mathbf{\Theta}))>0} \pi_{\mathbf{\Theta}} d\boldsymbol{\theta}. \tag{11}$$

In the multi-scale approach, the failure probability is approximated by the Monte Carlo method as:

$$\mathbb{P}\left(g(\mathbf{v} = H(\mathbf{\Theta})) > 0\right) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}_{\mathcal{F}} \left(\mathbf{v^i} = H(\mathbf{\Theta}^i)\right), \tag{12}$$

where $\left\{\mathbf{\Theta}^{(i)}, i = 1, ..., N\right\}$ is a sample of N independent copies of the random vector $\mathbf{\Theta}$, H is the mapping from the mesoscopic space to the macroscopic space, and $\mathbb{I}_{\mathcal{F}}$ is the failure indicator defined as:

$$\mathbb{I}_{\mathcal{F}}(\theta) = \begin{cases} 1 & \text{if } g\left(\mathbf{v} = H\left(\theta\right)\right) > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

The variance of the Monte Carlo estimator is easily proved in [30] to read as follows:

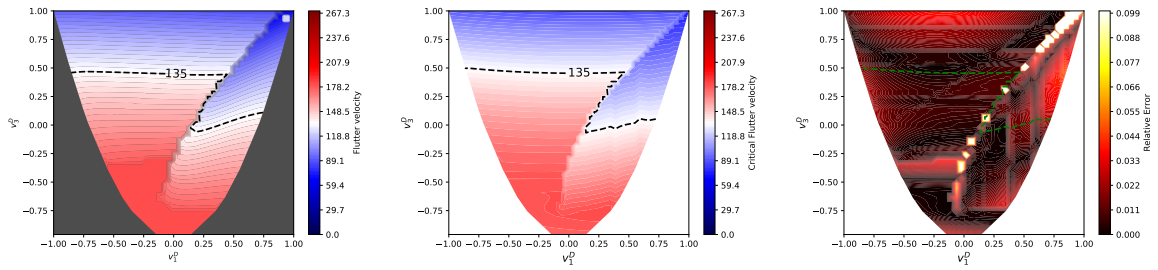
$$\sigma^2 = \frac{1}{N}\mathbb{P}(1 - \mathbb{P}) \tag{14}$$

It can be seen from the above equation that this variance decays with the number of samples used for estimating the failure probability, meaning that the uncertainty in the estimation is purely epistemic (reducible). This is often referred to as the statistical error.

In order to assess the accuracy of a sampling run, when $\mathbb{P} \leq 0.5$ one usually resorts to the coefficient of variation defined as follows:

$$CoV = \frac{\sigma}{\mathbb{P}} = \sqrt{\frac{1 - \mathbb{P}}{N\mathbb{P}}}. \tag{15}$$

The drawback of this formulation is that it requires a higher number of calls to the aeroelastic model g for low failure probabilities. Therefore, a metamodeling strategy for the flutter critical velocity U^{crit} was implemented for the calculation of the failure probability. A coupled



(a) True function. (b) Surrogate model/classifier. (c) Relative error.

Figure 2: Flutter critical velocity U^{crit} in the 2D design space with the design flutter speed $U^{crit}_{lim} = 135m/s$.

eigenvalues problem is performed for a prescribed velocity range in order to identify the critical flutter velocity – i.e. the minimum velocity for which a mode is unstable – for a subset of structural modes. Hence, the global critical velocity is defined as the minimum of the computed ones (Figure 2a).

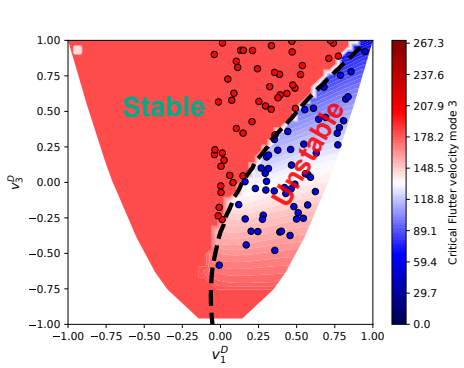
In Figure 2a, we exhibit a discontinuity of the flutter critical velocity in the design space; this discontinuity derives from one of the modes that behaves as a "hump mode" [31], that can abruptly switch its stability status within a small perturbation of the lamination parameters. Some techniques of clustering were applied for the flutter velocity [10] to identify the different modal regimes. A multi-element surrogate model strategy is used to handle sharp and sudden flutter onset for the uncertainty quantification in limit-cycle oscillations [32].

In this work, the idea is to create a metamodel of this velocity for each of the modes and build a classifier for the "hump mode" to know if it is stable or not. The first two modes are quite regular in the design space, while the third one shows the above mentioned discontinuous behaviour (Figure 3a). Gaussian process (or Kriging) is used to build the surrogate models for each mode. The randomness of this approach can also quantify the uncertainty of the interpolation. Indeed, at each point, the Gaussian process interpolation gives not only the mean value of the Gaussian random variable but also its variance.

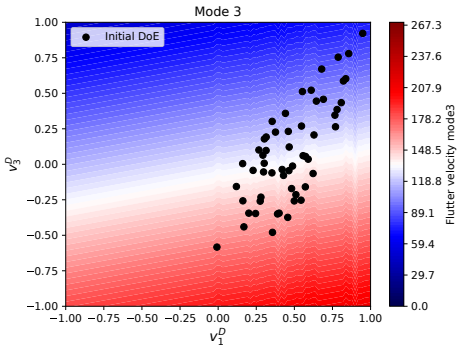
For the first two modes, a quadratic model and a square exponential function are used for, respectively, the mean and the covariance kernel. For the third mode, a classifier is trained by a Gaussian process to identify the stable and unstable areas using the design of experiments shown in Figure 3a. The dashed line represents the boundary created by the classification. Then, the points located in the unstable zone are used to construct the metamodel. A linear model and a square exponential function are used, and the kriging response is shown in Figure 3b. In order to have the minimum critical velocity, the metamodel of the third mode is used only when the optimizer is in the unstable part of this mode thanks to the classifier. The kriging response of the flutter velocity in the design space is shown in Figure 2b. The associated relative error in Figure 2c validates the approach coupling classification and metamodeling.

3.3 Sensitivity of failure probability

Since it is a gradient-based optimization, the failure probability sensitivity is needed. In this case, the joint probability density function of the macroscopic parameters is unknown a priori. But, in many cases, the uncertainty quantification of lamination parameters exhibit a Gaussian trend [33] which can be exploited for the failure probability sensitivity computation. The score



(a) Classifier delimitation between the stabe and unstable region.



(b) Continuous surrogate model response of the unstable region, constructed from the sample (blue dots).

Figure 3: Critical flutter velocity of the third mode.

function approach [34] was brought to the structural reliability community by Wu [35]. The partial derivative of the failure probability with respect to the i-th design variable reads:

$$\frac{\partial \mathbb{P}}{\partial \mu_{\mathrm{v}_i}} = \frac{\partial}{\partial \mu_{\mathrm{v}_i}} \int_{\mathbb{V}} \mathbb{I}_{\mathcal{F}}(\mathbf{v}) \pi_{\mathrm{v}}(\mathbf{v}, \mu_{\mathbf{v}}) d\mathbf{v}, \tag{16}$$

where $\mathbb{I}_{\mathcal{F}}$ is the indicator function defined in Equation 13, and π_{v} is the known-pdf of the continuous parameters. Assuming that the joint PDF π_{v} is continuously differentiable with respect to μ_{v_i} and that the integration range $\mathbb{V}$ does not depend on μ_{v_i}, the partial derivative of the failure probability recasts as follows:

$$\frac{\partial \mathbb{P}}{\partial \mu_{\mathrm{v}_i}} = \int_{\mathbb{V}} \mathbb{I}_{\mathcal{F}}(\mathbf{v}) \frac{\partial \pi_{\mathrm{v}}(\mathbf{v}, \mu_{\mathbf{v}})}{\partial \mu_{\mathrm{v}_i}} d\mathbf{v}. \tag{17}$$

Then, in order to compute this integral as an expectation, it is proposed to use an importance sampling trick:

$$\frac{\partial \mathbb{P}}{\partial \mu_{\mathrm{v}_i}} = \int_{\mathbb{V}} \mathbb{I}_{\mathcal{F}}(\mathbf{v}) \frac{\partial \pi_{\mathrm{v}}(\mathbf{v}, \mu_{\mathbf{v}})}{\partial \mu_{\mathrm{v}_i}} \frac{1}{\pi_{\mathrm{v}}(\mathbf{v}, \mu_{\mathbf{v}})} \pi_{\mathrm{v}}(\mathbf{v}, \mu_{\mathbf{v}}) d\mathbf{v} \tag{18}$$

Thus, the following estimator:

$$\frac{\partial \mathbb{P}}{\partial \mu_{\mathrm{v}_i}}_{MCS} \equiv \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbb{I}_{\mathcal{F}}(\mathbf{v}^{(i)})}{\pi_v(\mathbf{v}^{(i)}, \mu_{\mathrm{v}_i})} \frac{\partial \pi_{\mathrm{v}}(\mathbf{v}^{(i)}, \mu_{\mathrm{v}_i})}{\partial \mu_{\mathrm{v}_i}} \tag{19}$$

is unbiased and asymptotically convergent according to the central limit theorem. The advantage is that the failure probability gradient is estimated using the same sample used for the failure probability estimation in Equation 12.

Depending on the application, when π_{v} is not known directly, statistical tests can be done on a sample to know if it can be fitted with a parametric law. Otherwise finite differences can be used [36].

4 APPLICATION TO THE RELIABILITY-BASED AEROELASTIC DESIGN OPTIMIZATION OF A COMPOSITE PLATE

The multi-scale approach is applied to reliability-based design optimization of a composite plate where the ply orientations design variables are uncertain due to some given dispersion around some "nominal" values. The lamination parameters space is introduced for the global optimization process to use a gradient algorithm, and the orientation space is leveraged for the evaluation of the failure probability because the dispersion of the orientations is known and represented as in Equation 7. The objective is to promote the flexibility of the wing, while remaining reliable with respect to the flutter phenomenon.

4.1 Modeling of the composite

In classical lamination theory [37], the constitutive equation relating applied bending moments to the curvature of a symmetrically laminated plate may be written as

$$\mathbf{M} = \mathbf{D}\kappa, \tag{20}$$

where $\mathbf{M}$ is a vector of resultant out-of-plane moments, and κ is the vector of plate curvatures. The out-of-plane stiffness matrix $\mathbf{D}$ can be written as a linear combinaison of the material invariant $\mathbf{U}$, the lamination parameters $\mathbf{v^D}$ and the thickness h of the plate [37] [38]:

$$\begin{pmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \\ D_{16} \\ D_{26} \end{pmatrix} = \frac{h^3}{12} \begin{bmatrix} 1 & v_1^D & v_3^D & 0 & 0 \\ 1 & -v_1^D & v_3^D & 0 & 0 \\ 0 & 0 & -v_3^D & 1 & 0 \\ 0 & 0 & -v_3^D & 0 & 1 \\ 0 & v_2^D/2 & v_4^D & 0 & 0 \\ 0 & v_2^D/2 & -v_4^D & 0 & 0 \end{bmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{pmatrix}, \tag{21}$$

where the material invariants $\mathbf{U}$ write as:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{pmatrix} = \frac{1}{8} \begin{bmatrix} 3 & 3 & 2 & 4 \\ 4 & -4 & 0 & 0 \\ 1 & 1 & -2 & -4 \\ 1 & 1 & -6 & -4 \\ 1 & 1 & -2 & 4 \end{bmatrix} \begin{pmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \end{pmatrix}, \tag{22}$$

where Q_{ij} are the reduced stiffness components for unidirectional lamina and defined as:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_1}{1-\nu_{12}\nu_{21}}, Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}. \tag{23}$$

In Eq. 23, E_{11}, E_{22}, G_{12} and ν_{12} are the lamina longitudinal, transverse and shear moduli, and Poisson's ratio respectively. Then, out-of-plane lamination parameters are defined as:

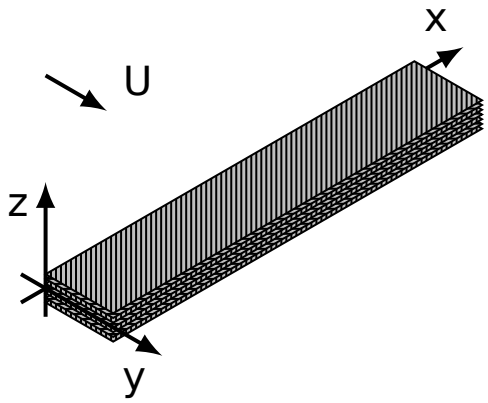


Figure 4: Geometry of the wing.

$$\left[v_1^D, v_2^D, v_3^D, v_4^D\right] = H(\boldsymbol{\theta}) = \frac{12}{h^3}\int_{-h/2}^{h/2} z^2 \left[\cos(2\theta(z)), \sin(2\theta(z)), \cos(4\theta(z)), \sin(4\theta(z))\right] \, dz, \tag{24}$$

where $\theta(z)$ denotes the distribution of the ply orientations throughout the laminate thickness. The laminate has a discrete set of plies with orientations $[\theta_1, \theta_2, ..., \theta_n]$. The lamination parameters are defined as functions of the ply orientations and allow representing a laminate with a finite and small number of variables. They are defined in a convex space and, therefore, can be used as design variables allowing the use of efficient gradient-based optimization. Nevertheless, stacking sequences have to be reconstructed in a second step. To do so, the genetic algorithm optimiser developed by Vicente [39], which followed the formulation devised by Irrisari et al. [40] known as SST, is used to retrieve the stacking sequence at each iteration.

4.2 Aeroelastic RBDO

This approach is applied to a composite flat plate subjected to a flow $U_{lim}^{crit} = 135m/s$ as shown in Figure 4. The objective is to promote the flexibility of the wing, by maximizing the displacement at the tip d_{tip} while remaining reliable with respect to the flutter g. The critical flutter velocity is computed by coupling the finite element method (FEM) and the doublet-lattice method (DLM) on Nastran.

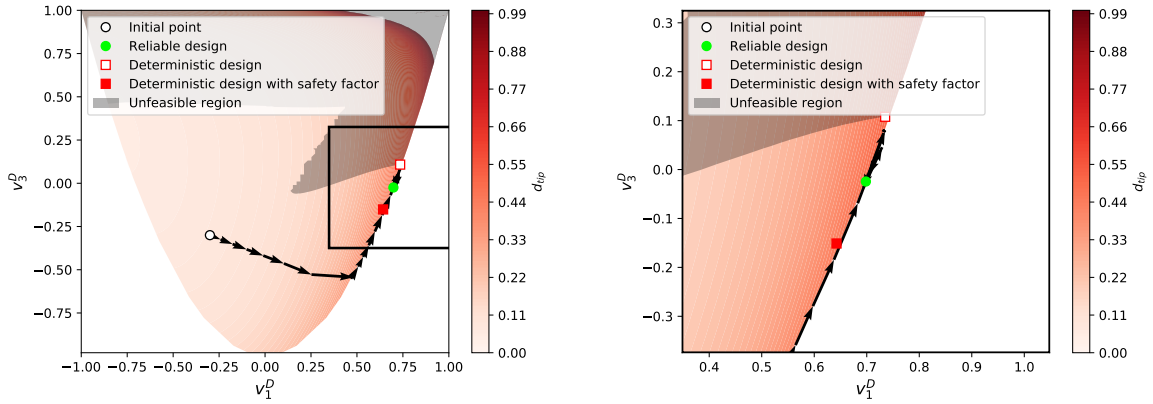
The composite laminate is 2-mm thick, which accounts for a total of 16 plies, each stacked at a specific θ_i orientation with respect to the global coordinate system. The orientations can take only a few discrete between -75° and 90° with a 15° increment and the balance rule is respected. The dimension and material properties are shown in Table 1. Moreover, the lamination parameters v_2^D and v_4^D are set to zero during the optimization process to follow the orthotropic nature of the laminate.

The MMA method [41] is used to solve the optimization problem. The formulation of the latter is described in the following form:

$$\max_{\substack{\boldsymbol{\mu}_\mathbf{v} \\ h_{LP}(\boldsymbol{\mu}_\mathbf{v}) \leq 0 \\ \mathbb{P}\left(U_{lim}^{crit} - \tilde{g}(\mathbf{v}(\boldsymbol{\Theta})) > 0\right) \leq \mathbb{P}^{max}}} \quad d_{tip}\left(\boldsymbol{\mu}_\mathbf{v}\left(\boldsymbol{\Theta}\right)\right) \tag{25}$$

Parameters	Value
Length a (m)	0.3048
Width b (m)	0.0762
E_{11}(GPa)	140
E_{22}(GPa)	10
G_{12}(GPa)	5
ν_{12}	0.3
ρ(kg/m^3)	1600

Table 1: Dimension and material properties.



(a) 2D lamination parameters space.

(b) Zoom around the final design.

Figure 5: Optimization path in the macroscopic space obtained with the probability sensitivity computed with the score function approach and the Fourier Chaos Expansion.

where h_{LP} is the compatibility constraint defined by Miki [38] for an orthotropic laminate, $\mathbb{P}^{max} = 0.01$ is the maximum failure probability, and $\tilde{g} = \tilde{U}^{crit}$ is the surrogate model of the flutter velocity.

The first results were obtained with the hypothesis that the lamination parameters follow a Gaussian distribution modeled at each iteration with the statistics coming from the FCE. Therefore, the score function approach is performed for the gradient probability computation at each iteration. In Figure 5, the optimization path of the RBDO and the zoom around the final design are shown with the shaded area corresponding to the failure domain. The convergence of the RBDO is shown in Figure 6 with the objective d_{tip} and the probability plotted through the 26 iterations. Below is the design evolution in the lamination parameters and orientations spaces. The convergence is a bit slow at the end, but that is a feature of the MMA. In this optimization, the maximum probability constraint is activated. Two deterministic optimizations were also performed with and without a 10% safety factor. The RBDO design shows better performance than a deterministic one with a safety factor. Moreover, a 98% reliability improvement is found compared to the deterministic optimization (Table 2). The uncertainties impact is known during the optimization process, which is an advantage of this RBDO approach.

The same optimizations were run with another initial guess in Figure 7. The deterministic design with a safety factor is blocked near a failure zone. That is a disadvantage of the gradient-based methods, which converge to a local optimum. Nevertheless, with a RBDO formulation,

Type	Stacking sequence	Failure probability
Deterministic	$[15, -30, -15, 30, -15, 30, 15, -30]_s$	0.52
Deterministic with safety factor	$[-30, 30, 0, 30, 0, -30, -30, 30]_s$	0
RBDO	$[0, -30, 30, 30, -30, 30, -30, 0]_s$	0.008

Table 2: Results of the different optimizations of the Figure 5.

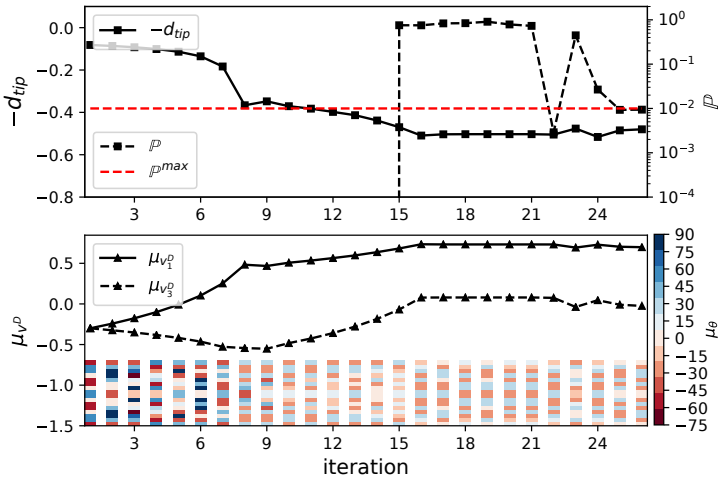


Figure 6: Convergence of the objective f, failure probability $\mathbb{P}$ and the design variables through the optimization with the probability sensitivity computed with the score function approach and the FCE of the Figure 5.

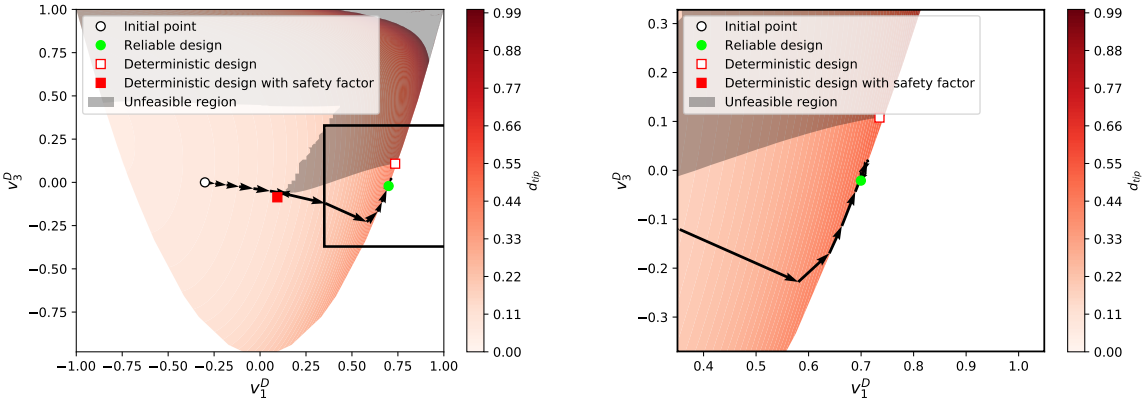
the optimization process seems more robust and different zones of convergence could be found between deterministic and reliable optimization. This phenomenon may be accentuated for more complex applications.

5 CONCLUSION

In this work, a new RBDO approach for composite laminates has been proposed taking into account the uncertainty of orientation design variables. This approach exploits two design spaces at different scales: the orientations space and the lamination parameters space. Moreover, two surrogate strategies have been proposed: one to very efficiently and accurately propagate the effect of ply orientations uncertainties from the mesoscopic (orientations) to the macroscopic (lamination parameters) space, which is used for the inverse problem, and one to accelerate the evaluation of the aeroelastic quantity of interest despite the presence of discontinuities in the response.

This proposed approach has been applied to an aeroelastic optimization of a composite plate. The usage of a gradient-based optimization in the macroscale space translates to a fast convergence to the optimal design, while the uncertainty propagation from the mesoscale allows for a control of the failure probability – reduced of 98% with respect to the equivalent deterministic design. Moreover, the preliminary results show enhanced performances and, sometimes, different zone of convergence in the homogenized design space with respect to a deterministic design where reliability is taken into account via a safety-factor.

Future developments will include the parametrisation of the thickness (i.e. number of plys) of the composite stack and spatially non-uniform distributions of stiffness in the structure.



(a) 2D lamination parameters space.

(b) Zoom around the final design.

Figure 7: Optimization path in the macroscopic space obtained with the probability sensitivity computed with the score function approach and the Fourier Chaos Expansion: for another initial guess.

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