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Estimation of Power Transmission Distribution Factors for Predictive Control in Sub-transmission Areas

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Abstract—Transmission system operators target to maximize renewable power injection into the grids while preserving power lines’ safety. Recently, decentralised approaches are developed using Power Transmission Distribution Factors (PTDFs), and consider model-based control strategies for local sub-transmission area. However, the computation of the models is complex due to the time-varying nature of the parameters. In order to answer these shortcomings, the present paper proposes a real-time parameter estimation using past observations of the power flow on a finite time window. Simulations results validate the proposed approach.

Index Terms—Predictive control, congestion management, PTDF.

I. INTRODUCTION

The energy transition of power grids faces an increase of generations from renewable energies connected to the power grids. The transmission system operators’ (TSO) objective is to maximize the injections on the grid for such generations while preserving the safety of the power lines. This objective is handled by power congestion management and the related computational methods are important for the guarantees they can offer regarding the equipment damage and cascading failures implications [1].

Due to the large size of power grids, controlling the grid as a whole is both complex and sensitive. The recent decentralized approaches [2] focus on controlling some predefined sub-transmission areas, alternatively called zones, associated to a particular geographical area. These areas are known to face power congestion issues due to high generations from renewable sources. The local controls to tackle power congestion, acting on levers like curtailment/storage/topological changes, have been shown to exhibit the expected impact on the congestion despite the interconnection uncertainty with the rest of the grid. This work is building on a series of recent

works [2] [3] [4], all based on a Model Predictive Control (MPC) [5] approach.

In the existing works, a discrete-time prediction model associated to each area uses a linearization of the power flow equations based on Power Transfer Distribution Factors [6] (PTDFs). These parameters for power flow equations evolve in time and can be represented by the percentage of increase of power flow a given line would receive due to an increase of generation at a given bus. The use of time-invariant PTDFs is an advantage for the MPC’s computation. As a drawback, the computation of these time-invariant PTDFs requires the topology of the whole power grid to get just the values concerning the sub-transmission areas, and thus a time-consuming computation done at an off-line stage. In order to avoid time-invariant PTDFs computed off-line by means of optimal power flow analysis, exploiting tools like [7], the current work aims at estimating the parameters of the power flow equations dynamically at each time step and with the ultimate goal of including them in the prediction-based control design.

As a first attempt in this endeavor, the method we propose is based on *parameter estimation* using past observations on a finite time window. More precisely, the power flows and the power generations recorded over time will be used to fit the parameters in a classical least squares framework [8] [9]. A discussion on the impact of the observation window and the computation will be done. A biased estimation is expected to be retrieved whenever the past horizon considered is too long as the distant past data might not be relevant for the current mode of functioning. At the same time, if the past horizon is too short, inaccuracy can occur and complementary tools need to be found for regularization.

II. THE PRINCIPLES OF CONGESTION MANAGEMENT BASED ON MODEL PREDICTIVE CONTROL

In this section, we recall in a concise manner all the elements needed in order to construct a mathematical model for the evolution of the power flows within a selected sub-transmission area, described by a subset of buses, generators, and batteries along with the power lines in the selected zone to be controlled. For more details on the construction of the state space model, the interested reader is referred to [4]. Main sets of topological information: \mathcal{Z} : set of buses in the zone; $\mathcal{Z}^G \subset \mathcal{Z}$: set of buses with a generator in the zone; $\mathcal{Z}^B \subset \mathcal{Z}$: set of buses with a battery in the zone; \mathcal{L} : set of power lines in the zone.

Parameter: n^N, n^G, n^B, n^L : cardinality of $\mathcal{Z}, \mathcal{Z}^G, \mathcal{Z}^B, \mathcal{L}$ respectively; T : sampling time of a time step; τ_C : delay of curtailment power control, equal for all generators; τ_B : delay of battery power control, equal for all batteries, $1 \leq \tau_B < \tau_C$; c_b : battery coefficient of power injection of battery b ; P_g^G : generation capacity of generator g , so the maximal reachable generation power (installed capacity).

Time-varying parameter: $\psi_{g,l}^G(t)$: PTDF between line l and generator g at time t ; $\psi_{b,l}^B(t)$: PTDF between line l and battery b at time t ; $\psi_{n,l}^N(t)$: PTDF between line l and bus n at time t .

State variable: $F_l(t)$: power flow of line l ; $P_g^C(t)$: curtailment power of generator g , which is the limitation of generation capacity; $P_b^B(t)$: battery power of battery b ; $E_b^B(t)$: battery energy of battery b ; $P_g^G(t)$: generation power of generator g at t ; $z_g^G(t)$: generation power of generator g at $t+1$, equivalently $P_g^G(t+1)$, but obtained at time t ; $P_g^A(t)$: available power of generator g at time t .

Disturbance: $\delta P_n^T(t)$: transit power disturbance at time t , describing the unknown power variation at bus n due to power flows transiting between the zone and the rest of the network; $\delta P_g^A(t)$: available power disturbance at time t of generator g .

Control: $\delta P_g^C(t)$: curtailment power control decided at time t on generator g ; $\delta P_b^B(t)$: battery power control decided at time t on battery b .

The system dynamics of the zone model is defined, $\forall l \in \mathcal{Z}^L, \forall g \in \mathcal{Z}^G, \forall b \in \mathcal{Z}^B, \forall n \in \mathcal{Z}^N$ as:

$$\begin{aligned} F_l(t+1) &= F_l(t) + \sum_{b \in \mathcal{Z}^B} \psi_{l,b}^B(t) \cdot \delta P_b^B(t - \tau_B) + \\ &\sum_{g \in \mathcal{Z}^G} \psi_{l,g}^G(t) \cdot [z_g^G(t) - P_g^C(t)] + \sum_{n \in \mathcal{Z}^N} \psi_{l,n}^N(t) \cdot \delta P_n^T(t) \\ P_g^C(t+1) &= P_g^C(t) + \delta P_g^C(t - \tau_C) \\ P_b^B(t+1) &= P_b^B(t) + \delta P_b^B(t - \tau_B) \\ E_b^B(t+1) &= E_b^B(t) - T \cdot c_b^B \cdot [P_b^B(t) + \delta P_b^B(t - \tau_B)] \\ P_g^G(t+1) &= z_g^G(t) \\ P_g^A(t+1) &= P_g^A(t) + \delta P_g^A(t) \end{aligned} \quad (\text{I})$$

$$z_g^G(t) = \min \left(\frac{P_g^A(t) + \delta P_g^A(t)}{P_g^G - P_g^C(t) - \delta P_g^C(t - \tau_C)} \right) \quad (\text{II})$$

In practice, the curtailment control delay τ_C is 45 seconds, while the battery control delay τ_B is 5 seconds. To determine the appropriate curtailment control at present, the

controller must anticipate the future impact of the control. Model Predictive Control (MPC) is chosen for this purpose, as it can foresee the outcomes over a prediction horizon, enabling informed decision-making and control actions. More details about the control strategy are in [2] [4], where the model-based predictive control for congestion management was presented.

III. REAL-TIME PTDF ESTIMATION

The previous works of [3] and [4] builds on the working hypothesis that the power flow equation of (I) exhibits PTDFs with constant values computed offline based on MATPOWER library [7], i.e.,

$$\psi_{l,g}^G(t) = \psi_{l,g}^{G,mat}, \quad \psi_{l,b}^B(t) = \psi_{l,b}^{B,mat}, \quad \psi_{l,n}^N(t) = \psi_{l,n}^{N,mat} \quad (\text{III})$$

In the present work, we revisit this hypothesis. The PTDF are now time-varying, and the estimation of these PTDF is data-based: a collection of data estimates the values of the PTDF using regression analysis. The ultimate goal for the control design point of view is to use these time-varying PTDFs for the prediction mechanism. Ideally (from the robustness of the controller), the reconstructed PTDFs are equal to the values computed offline. If this ideal situation differs from the real system operation conditions, then one can represent more accurately the current state of functioning using the estimated PTDFs values, thus leading to better controls, which exploits most of the information available.

Indeed, a possible cause of errors in the [3] and [4] is that the offline computation of PTDF from [7] assumes the generation of only one generator is changing at a time, but the superposition principle might not be applicable for several generators simultaneously. This work aims at estimating the time-varying PTDFs : $\psi_{l,g}^G(t), \psi_{l,b}^B(t), \psi_{l,n}^N(t)$ from (I).

A. Receding-horizon parameter estimation and constrained least squares formulation

The quality of the PTDFs' estimation has an important impact on the model-based prediction capabilities and consequently on the optimal control. For this reason, at each time step t of the simulation, the estimation of the PTDFs will be updated. The estimation is based on a receding horizon strategy and two approaches are considered and detailed next:

- a variable-length horizon starting from the beginning of the simulation and including all the available historical data;
- a fixed horizon, the data considered for the estimation corresponds to a fixed number of previous steps of the simulation, 10 and 30 steps are used in the numerical examples.

The estimation of the time-varying PTDFs is formulated in terms of an optimisation problem. Let us introduce the Power flow variations δF_l and generation power variations δP_g^G :

$$\delta F_l(t) = F_l(t+1) - F_l(t), \quad \delta P_g^G(t) = z_g^G(t) - P_g^G(t) \quad (\text{IV})$$

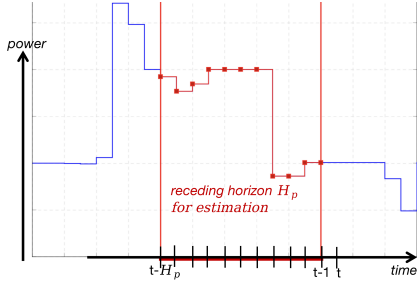


Fig. 1. past horizon H_p , made of past observations usable at time t

thus, the power flow equation of (I) for a given line $l \in \mathcal{L}$ can be written as:

$$\begin{aligned} \delta F_l(t) &= \sum_{b \in \mathcal{Z}^B} \delta P_b^B(t-d) \cdot \psi_{l,b}^B(t) \\ &+ \sum_{g \in \mathcal{Z}^G} \delta P_g^G(t) \cdot \psi_{l,g}^G(t) \\ &+ \sum_{n \in \mathcal{Z}^N} \delta P_n^T(t) \cdot \psi_{l,n}^N(t) \end{aligned} \quad (\text{V})$$

The aim is to formulate Eq. (V) in matrix form, and the following notations of column vectors are instrumental:

$$\begin{aligned} \delta F(t) &= \begin{bmatrix} \delta F_1(t) \\ \vdots \\ \delta F_l(t) \\ \vdots \\ \delta F_{n^L}(t) \end{bmatrix} \in \mathbb{R}^{n^L}, \quad \delta P^B(t) = \begin{bmatrix} \delta P_1^B(t) \\ \vdots \\ \delta P_{n^B}(t) \end{bmatrix} \in \mathbb{R}^{n^B}, \\ \delta P^G(t) &= \begin{bmatrix} \delta P_1^G(t) \\ \vdots \\ \delta P_{n^G}(t) \end{bmatrix} \in \mathbb{R}^{n^G}, \quad \delta P^T(t) = \begin{bmatrix} \delta P_1^T(t) \\ \vdots \\ \delta P_{n^N}(t) \end{bmatrix} \in \mathbb{R}^{n^N}. \end{aligned}$$

Let $\Psi^B(t) = [\psi_{l,b}^B(t)] \in \mathbb{R}^{n^L \times n^B}$, $\Psi^G(t) = [\psi_{l,g}^G(t)] \in \mathbb{R}^{n^L \times n^G}$, $\Psi^N(t) = [\psi_{l,n}^N(t)] \in \mathbb{R}^{n^L \times n^N}$, and the block matrix $\Psi(t) = [\Psi^B(t) \ \Psi^G(t) \ \Psi^N(t)] \in \mathbb{R}^{n^L \times (n^B + n^G + n^N)}$. Each block corresponds to some PTDF values (batteries, generators or buses) at time t .

Consequently, the power flow equation (V) can be formulated in a compact form, by considering all lines l :

$$\delta F(t) = \Psi(t) \cdot \begin{bmatrix} \delta P^B(t - \tau_B) \\ \delta P^G(t) \\ \delta P^T(t) \end{bmatrix} \quad (\text{VI})$$

Considering the available information on the past H_p -step horizon from $t - H_p$ to $t - 1$ as illustrated in Fig. 1, lead to the definition of the matrices of observation:

$$\begin{aligned} \Delta F(t) &= \begin{bmatrix} \delta F(t - H_p) \cdot \delta F(t - 1) \end{bmatrix} \in \mathbb{R}^{n^L \times H_p}, \\ \Delta P^B(t - \tau_B) &= \begin{bmatrix} \delta P^B(t - H_p - \tau_B) \cdot \delta P^B(t - 1 - \tau_B) \end{bmatrix} \in \mathbb{R}^{n^B \times H_p}, \\ \Delta P^G(t) &= \begin{bmatrix} \delta P^G(t - H_p) \cdot \delta P^G(t - 1) \end{bmatrix} \in \mathbb{R}^{n^G \times H_p}, \\ \Delta P^T(t) &= \begin{bmatrix} \delta P^T(t - H_p) \cdot \delta P^T(t - 1) \end{bmatrix} \in \mathbb{R}^{n^N \times H_p}, \end{aligned}$$

and the block matrix $\Delta P(t) = \begin{bmatrix} \Delta P^B(t - \tau_B) \\ \Delta P^G(t) \\ \Delta P^T(t) \end{bmatrix}$ collecting the past observations from $t - H_p$ up to $t - 1$.

A working assumption of pseudo-constant PTDFs over the observation horizon will be made in order to exploit the structure of the dependencies at the computation stage. It should

be noticed that the receding-horizon principle employed here, re-enforces the time-varying PTDFs nature of the estimation. Practically, at each sampling instant a linear regression will be employed to estimate point-wise (at time t) the PTDFs $\Psi(t)$:

$$\Delta F(t) = \Psi(t) \cdot \Delta P(t) + \mathcal{S}(t) \quad (\text{VII})$$

where $\mathcal{S}(t) \in \mathbb{R}^{H \times n^L}$ corresponds to the slack matrix of the residuals between the left-hand side and the remaining terms on the right-hand side. $\Delta F(t)$ and $\Delta P(t)$ are collections of available data, namely past observations. We want to select the PTDFs in $\Psi(t)$, such that the Frobenius-norm of the slack matrix $\mathcal{S}(t)$ is the smallest, noted $\|\cdot\|_F$.

Thus, the optimization problem \mathcal{O}_1 is defined as:

$$\mathcal{O}_1(t) = \min_{\Psi(t)} \|\Delta F(t) - \Psi(t) \cdot \Delta P(t)\|_F \quad (\text{VIII})$$

Additionally, the PTDFs represent a percentage of generation distribution over a line, which is directed but flow is allowed in both directions, so the percentage can also be negative. To impose a percentage-like solution for the coefficients, a constraint is added by defining the all-ones matrix $\mathcal{J} \in \mathbb{R}^{(n^B + n^G + n^N) \times n^L}$:

$$-J \leq \Psi \leq J \quad (\text{IX})$$

With these main principles, several versions of estimations can be obtained according to the length of the observation window and the weight on the information within the respective window. One can enumerate some intuitive choices: use all the available data (with obvious storage complexity); use the data from a finite window; impose a forgetting factor on the data coming from a long observation window, that last point is not investigated here.

B. Topological aspect of the solution

Based on the physical balance, the PTDFs are defined in relationship with the following property: for any given bus, the generation must be entirely distributed on the lines connected with the bus. This topological property can be translated through constraints on the PTDFs estimates: the sum of the PTDFs of all lines connected to this bus is equal to 100%.

The above property should be valid for any type of PTDF coefficient, either ψ^B , ψ^G or ψ^N :

$$\sum_{l \in \mathcal{L} \text{ s.t. } n = \text{fromBus of } l} \psi_{l,n} - \sum_{l \in \mathcal{L} \text{ s.t. } n = \text{toBus of } l} \psi_{l,n} = 100\% \quad (\text{X})$$

As lines are directed edges, the direction partitions the sums into two categories: incoming with a positive sign, and outgoing with an opposite sign. The respective constraints (X) need to be accounted for in the optimization problem constructing the PTDF estimates at each sampling time.

C. Numerical results with all past data collected

A comparison of results based on the methods used for estimation is presented next. It should be noted that for the simulations, no information regarding the transit power disturbances ΔP^N is available and thus the hypothesis $\Delta P^N = 0$ is adopted. In real-world scenarios, such information may be

available and thus enable the estimation of the PTDF Ψ^N , which is not possible here.

Four methods were considered for illustration, the first one, the baseline using the grid properties and three others are based on data observation:

- MATPOWER’s computation of the PTDF, based on the topology and the properties of the power network
- Use of the explicit unconstrained optimum via Matlab’s built-in pseudoinverse function
- Solving a Least Squares optimization problem, Eq.(VIII)-(IX) encoded with Yalmip [10] and solved with Gurobi [11].
- The optimization framework with additionally the topological constraints, i.e. Eq. (VIII)-(IX)-(X).

Along the simulations, the estimation at each step can use all the past observations. Gradually the number of past observations grows and structurally helps in estimating accurately the PTDFs using the optimisation-based procedure. It can be observed that results based on pseudoinverse and least squares are equal. Three types of behavior are apparent. Firstly, MATPOWER’s method and the two Least Squares methods result in the same PTDFs. A clear example in this sense is the PTDF estimation between branch 1 and the battery, cf. Fig. 2. The battery remains inactive for the first 400 seconds, offering no observations for PTDF estimation. It becomes operational afterward, contributing to the estimation process. The PTDF converge rapidly towards MATPOWER’s PTDFs.

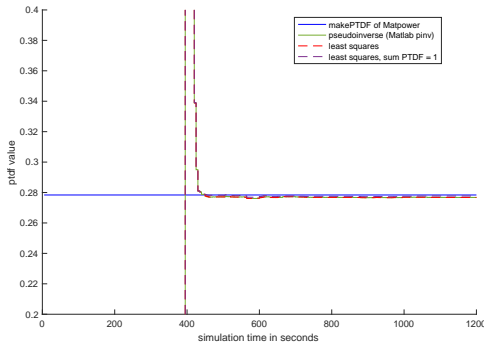


Fig. 2. PTDF estimate between branch 1 and the battery, located at bus 6

A second interesting behavior can be observed between the two Least Squares methods: with and without topology constraint. Convergence to the same PTDF is obtained, but the limit values are different from MATPOWER’s value. For example, the PTDF estimation of branch 5 with generator 3, Fig. 3.

Third, a case where each method converges towards a different PTDF value. For example, the PTDF estimate of branch 7 with generator 4, cf. Fig. 4, where the Least Squares method without topology constraint converges to -43%, MATPOWER’s PTDF to -45% and the Least Squares method with topology constraint converges to -46%.

In summary, when the two Least Squares methods converge to distinct values, the one incorporating topology constraints is usually closer to MATPOWER’s PTDF.

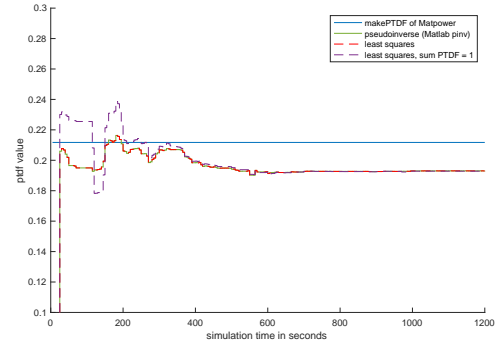


Fig. 3. PTDF estimate between branch 5 and generator 3 located on bus 5

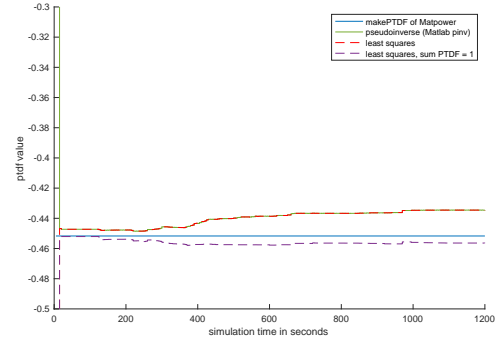


Fig. 4. PTDF estimate of branch 7 with generator 4, all-step horizon

D. Numerical results for estimation over a finite receding horizon window of past collected data

Previous tests assumed that all past observations are collected and stored during the simulation. In the context of a receding-horizon policy, this equates to a dynamically expanding receding horizon estimation, where the past observation window gradually grows. In practical scenarios with limited storage, we relax this structural property and adjust the estimation to a fixed window for the receding horizon.

The goal is to evaluate the impact of the size of horizon on the PTDF estimation. The PTDF estimation between branch 7 and generator 4 is chosen as example. Two cases are illustrated: a 30-step and a 10-step window, with Fig. 5 and 6 respectively; to be compared with the case of all past observations of Fig.4. For the 30-step window case, the Least Squares method without the topology constraint behaves similarly to the estimation using all past observations depicted in Fig. 4. The convergence is similar, with a slight variation in the PTDF asymptotic value. The addition of the topology constraint leads to estimations with higher fluctuations.

For the 10-step window case, the shortness of the observation window implies locally a lack of excitation within the observation window and leads to singularities in the computation of the PTDF values.

To tackle the high variability of the estimates, we introduce a regularized version of the Least Squares method with topology constraint. The PTDF estimation variables are decomposed into two parts: the default time-invariant PTDF

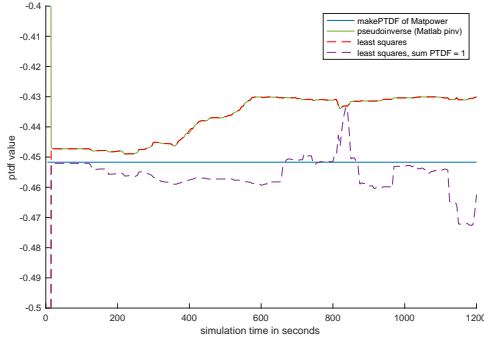


Fig. 5. PTDF estimate of branch 7 with generator 4, 30-step horizon

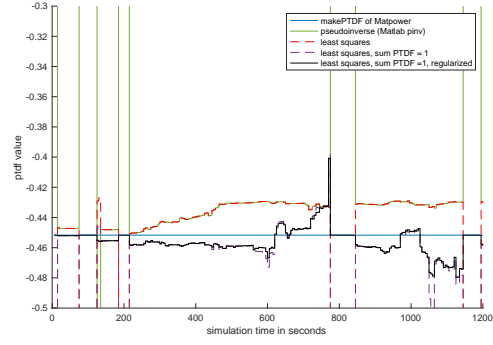


Fig. 7. PTDF estimate: branch 7, gen 4, 10-step horizon, regularization

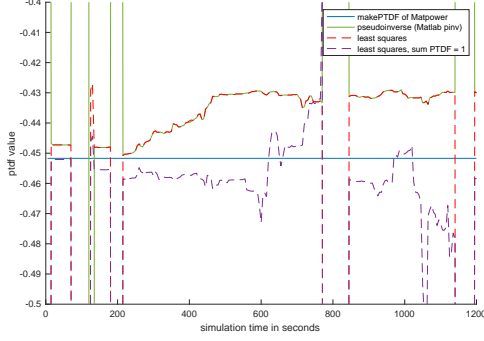


Fig. 6. PTDF estimate of branch 7 with generator 4, 10-step horizon

Ψ^{mat} of Eq.(III), plus a new variable $\Delta\Psi(t)$ that represents the variation of the PTDF with respect to Ψ^{mat} .

$$\Psi(t) = \Psi^{mat} + \Delta\Psi(t) \quad (XI)$$

where Ψ^{mat} is the matrix similar to $\Psi(t)$ but with the entries replaced by the elements in Eq.(III). Since the time-invariant PTDFs Ψ^{mat} respect the topology constraint in Eq.(X), the variation variables $\Delta\Psi(t)$ should not change this property:

$$\sum_{l \in \mathcal{L} \text{ s.t. } n=\text{fromBus of } l} \delta\psi_{l,n}(t) - \sum_{l \in \mathcal{L} \text{ s.t. } n=\text{toBus of } l} \delta\psi_{l,n}(t) = 0 \quad (XII)$$

where $\delta\psi_{l,n}(t)$ are the entries of $\Delta\Psi(t)$. Thus, the optimisation problem becomes:

$$\mathcal{O}_2(t) = \min_{\Delta\Psi(t)} \|\Delta F(t) - \Psi(t) \cdot \Delta P(t)\|_F + c \|\Delta\Psi(t)\|_F \quad (XIII)$$

The first term focuses on accurately determining the PTDFs through precise linear regression. The second term introduces flexibility in PTDF variations while preventing unrealistic values. The parameter c serves as a weighting factor and is set to 1 in these simulations.

In experiments, the case with a 30-step horizon and regularization exhibited no difference compared to previous methods, but significantly improved the behavior of the 10-step horizon case. The PTDF estimation of branch 7 with generator 4 is depicted on Fig. 7. One can observe that whenever the PTDF estimation was previously reaching extreme values, e.g., at 200 or 800 seconds of simulation, the regularization mechanism leads to a selection of the time-invariant PTDF

values. The additional term in the cost function does not influence significantly the PTDF estimation the rest of the time.

IV. CONCLUSION

The paper investigated different methods for the real-time estimation of the Power Transmission Distribution Factors (PTDFs). These factors represent the parameters of the dynamical model used by the controller for prediction purposes. The numerical examples show a good convergence whenever the persistent excitation is ensured despite the fact that the controller used a time-invariant prediction model. As future research direction, the goal is to determine the efficiency of the controllers based on a time-varying dynamical model employing the different PTDF estimations.

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