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# Extending the LOS Coverage of Vehicular Networks based on Roadside Units and Vehicle Relays 

Chang-Sik Choi and François Baccelli


#### Abstract

This paper investigates the benefits of employing vehicle relays by analyzing the increment of the line-ofsight (LOS) coverage based on vehicle relays to enable highspeed communications between roadside user devices, vehicles, and roadside infrastructure. We characterize a unique spatial relationship between roadside units (RSUs) and vehicles by employing the Cox point processes. Then, the LOS coverage from these RSUs is modeled by a Boolean model on the Cox point process. Assuming vehicle relays provide an additional LOS coverage when they are within the RSU LOS coverage, we quantify the growth of the LOS coverage by separately deriving the mean area fractions of the RSU LOS coverage and RSU-plus-relay LOS coverage, respectively. We explicitly provide the gain in the LOS coverage as an integral formula and show that relays increase the LOS coverage area by nearly 50 percent.


## I. Introduction

## A. Motivation and Related Work

Modern vehicles are envisioned to feature various applications including cooperative driving, sensor data sharing, vehicle and pedestrian positioning, and Internet-of-Things (IoT) data sharing [1], [2], made possible by vehicular networks. In these applications, line-of-sight (LOS) signals play essential roles. For instance, in positioning applications, vehicles and pedestrians calculate their locations by exploiting the time differences of LOS signal arrivals from base stations or other vehicles [3]. Similarly, a large amount of roadside Internet-of-Things (IoT) sensor data can be transmitted over LOS mmWave vehicle-to-all (V2X) communications [4]. Consequently, the fundamental limits of these applications are dictated by the network's LOS coverage, which is the result of the distribution of transmitters, receivers, and obstacles are distributed on a plane.

To serve a large number of network users and IoT devices, roadside units (RSUs) have been considered. However, due to the number of obstacles, the LOS signals from RSUs can be blocked and this undermines the operation of LOSdependent applications in vehicular networks. To ensure the LOS coverage, many studies, including [5], have investigated various technologies. Among these, employing vehicle relays has been considered an effective and practical way to amplify the LOS coverage area by leveraging RSUs [6], [7],
[8]. Conditional on vehicles connecting to the RSUs [9], [10], vehicle relays can opportunistically provide the LOS coverage to network users who were not in the LOS from the existing RSUs. For example, [11] evaluated the increment of the LOS coverage area; in a similar way, [12] analyzed the reliability and latency of vehicular networks with vehicle relays.

Leveraging stochastic geometry [13], this paper aims to quantify the increment of the LOS coverage area when vehicle relays are employed by RSUs. In the context of stochastic geometry, the LOS coverage of random networks was analyzed by [14]. Specifically, [15] used random sets centered on the Poisson point process to represent the LOS coverage area. Yet, these papers did not address the spatial correlation between RSUs and vehicle relays. In this paper, we focus on the fact that RSUs and vehicles are spatially correlated in space [16], as are their LOS coverage areas. Specifically, we use a Poisson line process to model a road layout and Cox point processes to model the locations of RSUs and relays, respectively [17], [18], [19], [16], [20], [21] . The LOS coverage area is then captured by rectangles of random sizes, centered on these Cox point processes. To the best of the authors' knowledge, the LOS coverage area created by the spatially correlated RSUs and relays has never been analyzed.

## B. Theoretical Contributions

This paper concerns the LOS coverage area that RSUs and their relays jointly produce on roads. We start by modeling the road structure as an isotropic and stationary Poisson line process. Conditional on these lines, RSUs and vehicles are modeled by Poisson point processes. Therefore, RSUs and vehicles form Cox point processes. This Cox-based conditional architecture accurately accounts for the geometric relationship between roads, RSUs, and vehicle relays. Then, expressing the LOS coverage area from the users' perspective, we define the LOS coverage area as the location in which network users are in LOS with regards to (w.r.t) any transmitter. We model the LOS coverage as random rectangles centered on the RSU Cox point process. We then assume that the vehicle relays can provide an additional LOS coverage to network users only when vehicles are in the


Fig. 1. The propose network simulated over a $4 \mathrm{~km} \times 3 \mathrm{~km}$ rectangle, centered at the origin. White strips are the streets where network users are located at. The colored polygons represent the LOS-unavailable surfaces such as building interiors. We use $\lambda_{l}=4 / \mathrm{km}, \mu=5 / \mathrm{km}, \mu_{v}=20 / \mathrm{km}$.

RSUs' LOS coverage. we compute the mean area fraction of the RSU LOS coverage and then the mean area fraction of the RSU-plus-relay LOS coverage. By comparing them, we explicitly show that LOS coverage increases up to nearly 50 percent.

## II. System Model

## A. Spatial Model

We model the set of roads as an isotropic Poisson line process $\Phi_{l}[22]$. The Poisson line process is generated by a Poisson point process $\Xi$ of a constant intensity $\lambda_{l} / \pi$ on the cylinder set $\mathbf{C}=\mathbb{R} \times[0, \pi)$ where each point $\left(r_{i}, \theta_{i}\right)$ of $\Xi$ corresponds to an undirected line $l\left(r_{i}, \theta_{i}\right)$ on the Euclidean plane $\mathbb{R}^{2}$. Here, $r_{i}$ is the distance from the origin to the line $l\left(r_{i}, \theta_{i}\right)$ and $\theta_{i}$ is the angle between the line $l\left(r_{i}, \theta_{i}\right)$ and the $x$-axis, measured in the counterclockwise direction. These lines form a Poisson line process $\Phi_{l}$.

Conditional on $\Phi_{l}$, the locations of RSUs are modeled as a Poisson point process $\phi_{l(r, \theta)}$ of intensity $\mu$ on each line of $\Phi_{l}$. Similarly, conditional on $\Phi_{l}$, the vehicle locations are modeled as an independent Poisson point process $\psi_{l(r, \theta)}$ of intensity $\mu_{v}$ on each line of $\Phi_{l}$. Therefore, the locations of RSUs and the vehicles form Cox point processes denoted by $\Phi$ and $\Phi_{v}$, respectively [19], [16]. Fig. 1 shows the RSUs and vehicles jointly created. Since RSUs and vehicles are jointly populated, the Cox point processes accurately represent the fact that RSUs and vehicles are located on roads. We assume that $\mu_{v} \gg \mu$, namely the density of vehicles is much greater than the density of RSUs. In this paper, we assume the vehicles move along their lines at a constant speed. A different mobility model may be considered to address a different distribution of the RSUs and vehicles; however, other mobility models are out of the scope of this paper.
To model the LOS coverage for network users, we assume that the streets have a finite width $\eta$ around the roads and that the RSUs or relays provide the LOS coverage for users on the streets. Fig. 1 shows the streets as the white strips.

## B. Blockage, LOS Coverage, and Relay Selection

To model the blockage of LOS in vehicular networks, we use a random geometric model based on the widely used blockage model [23]. Specifically, we assume that the LOSblocking obstacles are located at distances $W$ and $V$ from any given transmitter and that $W$ and $V$ are i.i.d. exponential random variable with mean $\gamma$. The variables $W$ and $V$ are referred to as the LOS distances.

Based on this random geometric blockage model, we define the LOS segment of an RSU as the line segment on which vehicles of the same roads are accessible to the LOS of the RSU. Specifically, for the RSU $X_{i} \in \phi_{l(r, \theta)}$, its LOS segment $S_{X_{i}}$ is defined by

$$
S_{X_{i}}=\left\{\begin{array}{l}
\left\{x \in \operatorname{left}\left(l(r, \theta) ; X_{i}\right) \text { s.t. }\left\|x-X_{i}\right\|<W_{i}\right\}  \tag{1}\\
\left\{x \in \operatorname{right}\left(l(r, \theta) ; X_{i}\right) \text { s.t. }\left\|x-X_{i}\right\|<V_{i}\right\}
\end{array},\right.
$$

where left $\left(l(r, \theta) ; X_{i}\right)$ is the left side of $l(r, \theta)$ w.r.t. $X_{i}$ and $\operatorname{right}\left(l(r, \theta) ; X_{i}\right)$ is the right side of $l(r, \theta)$ w.r.t. $X_{i}$. Fig. 2 shows the LOS segment of the RSU as the shaded onedimensional segment along the line $l(r, \theta)$.

We define the LOS coverage of RSU $X_{i}$ as the area where any user in that area is capable of establishing the LOS connection with the RSU. The LOS coverage of RSU $X_{i}$ is defined as follows:

$$
\begin{equation*}
C_{X_{i}}=S_{X_{i}} \times[-\eta / 2, \eta / 2] \tag{2}
\end{equation*}
$$

The LOS coverage of $\operatorname{RSU} X_{i}$ is the two-dimensional random rectangle given by the orthogonal product of the onedimensional segments $S_{X}$ and $[-\eta / 2, \eta / 2]$. Fig. 2 describes the LOS coverage of RSU $X$ as the colored rectangle.

Finally, the RSU LOS coverage-the area where any network user is in LOS w.r.t. at least one RSU-is defined as follows:

$$
\begin{equation*}
C_{\mathrm{RSU}}=\bigcup_{X_{i} \in \Phi} C_{X_{i}}=\bigcup_{X_{i} \in \Phi}\left(S_{X_{i}} \times[-\eta / 2, \eta / 2]\right) \tag{3}
\end{equation*}
$$

where we use the union of the LOS coverage of each RSU to define the LOS coverage of all RSUs, because when RSUs are close to each other, their respective LOS coverage areas overlap in space.

In this paper, we consider a case where each RSU selects one vehicle inside its LOS segment as a relay. In other words, only vehicles in $C_{\text {RSU }}$-the RSU LOS coverageare considered as potential relay candidates. In practice, the proposed relay selection principle can be justified in V2X systems, where vehicles in LOS w.r.t. at least one RSU can extend the LOS coverage, such as in mmWave or THz communications. In addition, for tractability, we ignore the cases where the same vehicle is selected by more than two RSUs and where there is no vehicle in the RSU LOS coverage area. These two conditions are met when the number of vehicles is greater than the number of RSUs.

Then, we also assume there exist LOS-blocking obstacles from the selected relays at random distances.

In summary, the locations of relays, their LOS segments, and the relay LOS coverage are defined as follows:

$$
\begin{align*}
\Phi^{\prime} & =\sum_{X_{i} \in \Phi} \operatorname{Uniform}\left[\Phi_{v}\left(S_{X_{i}}\right)\right]=\sum_{Y_{i}} \delta_{Y_{i}},  \tag{4}\\
S_{Y_{i}} & =\left\{\begin{array}{l}
\left\{x \in \operatorname{left}(l(r, \theta) ; X) \text { s.t. }\left\|x-Y_{i}\right\|<W_{i}^{\prime}\right\} \\
\left\{x \in \operatorname{right}(l(r, \theta) ; X) \text { s.t. }\left\|x-Y_{i}\right\|<V_{i}^{\prime}\right\}
\end{array}\right.  \tag{5}\\
C_{\text {Relay }} & =\bigcup_{Y_{i} \in \Phi^{\prime}} C_{Y_{i}}=\bigcup_{Y_{i} \in \Phi^{\prime}}\left(S_{Y_{i}} \times[-\eta / 2, \eta / 2]\right), \tag{6}
\end{align*}
$$

respectively. Above, $\operatorname{Unif}\left[\Phi_{v}\left(S_{X_{i}}\right)\right]$ denotes a point uniformly selected out of the point process $\Phi_{v}\left(S_{X_{i}}\right)$ and $W_{i}^{\prime}$ and $V_{i}^{\prime}$ are i.i.d. exponential random variables. When $\mu_{v} \gg \mu$, we use the fact that the uniform selection of one point out of the Poisson point process in the segment $S_{X_{i}}$ coincides with a point uniformly distributed on the segment $S_{X_{i}}$ [13].

As a result, the locations of relays are given by

$$
\begin{equation*}
\Phi^{\prime}=\sum_{X_{i} \in \Phi} \operatorname{Unif}\left(S_{X_{i}}\right), \tag{7}
\end{equation*}
$$

where $\operatorname{Unif}\left(S_{X_{i}}\right)$ is a point uniformly and independently selected in the segment $S_{X_{i}}$, with a slight abuse of notation. In other words, the locations of the selected relays are given by i.i.d. uniformly distributed points within each of the LOS segments.

Finally, we define the RSU-plus-relay coverage as the area where the network users are in LOS w.r.t. at least one LOS provider, either an RSU or a relay. Using Eqs. (4) and (7), we define the relay LOS coverage $C_{\text {Relay }}$ and the RSU-plusrelay LOS coverage $C_{\mathrm{RSU}+\text { relay }}$ as follows:

$$
\begin{align*}
C_{\text {Relay }} & =\bigcup_{X_{i} \in \Phi}\left(S_{\mathrm{Unif}\left(S_{X_{i}}\right)} \times[-\eta / 2, \eta / 2]\right),  \tag{8}\\
C_{\mathrm{RSU}+\mathrm{relay}} & =C_{\mathrm{RSU}} \cup C_{\mathrm{Relay}} \\
& =\bigcup_{X_{i} \in \Phi}\left(\left(S_{X_{i}} \cup S_{\mathrm{Unif}\left(S_{X_{i}}\right)}\right) \times[-\eta / 2, \eta / 2]\right), \tag{9}
\end{align*}
$$

where $S_{\mathrm{Unif}\left(S_{X_{i}}\right)}$ is the segment centered on the selected relay. Fig. 3 illustrates the RSU-plus-relay coverage. It shows that because of the spatial correlation between RSUs and relays, the LOS coverages of RSUs and relays overlap. At the intersections, the LOS coverage provided by transmitters from different roads overlaps as well.

## C. Performance Metric

To compute the area of the LOS coverage, we use the mean area fraction of the coverage set [13]. The mean area fraction of random sets is the ratio of the average area of the set to the entire plane; as a result, it takes a value between


Fig. 2. Illustration of the LOS segment of $X$ and its LOS coverage.


Fig. 3. Illustration of the LOS coverage from RSU and relay
zero and one. It is close to one when the area of the random sets is as big as the entire plane. Similarly, it is close to zero if the area of the random sets is very small compared to the entire plane.

The area fraction of the RSU LOS coverage is

$$
\begin{equation*}
\nu\left(C_{\mathrm{RSU}}\right)=\mathbf{E}\left[\int_{C_{0}} \mathbb{1}_{C_{\mathrm{RSU}} \cap C_{0}}(x) \mathrm{d} x\right] \tag{10}
\end{equation*}
$$

where $C_{0}$ is a unit square $\mathbb{R}^{2}$ and $B_{0}(r)$ is the ball of radius $r$ centered at the origin.

Similarly, the mean area fraction of RSU-plus-relay LOS coverage is

$$
\begin{align*}
\nu\left(C_{\mathrm{RSU}+\mathrm{relay}}\right) & =\nu\left(C_{\mathrm{RSU}} \cup C_{\text {Relay }}\right) \\
& =\mathbf{E}\left[\int_{C_{0}} \mathbb{1}_{\left(C_{\mathrm{RSU}} \cup C_{\text {Relay }}\right) \cap C_{0}}(x) \mathrm{d} x\right] \\
& =\lim _{r \rightarrow \infty} \frac{\int_{B_{0}(r) \cap\left(C_{\mathrm{RSU}} \cup C_{\text {Relay }}\right)} \mathbb{1}_{x} \mathrm{~d} x}{\int_{B_{0}(r)} \mathbb{1}_{x} \mathrm{~d} x}, \tag{11}
\end{align*}
$$

where $\nu\left(C_{\mathrm{RSU}}\right) \leq \nu\left(C_{\mathrm{RSU}+\text { relay }}\right)$.
In the proposed model, the LOS coverage corresponds to the area where LOS-dependent applications such as mmWave communications or vehicle positioning can be operational. Therefore, the increment of the mean area fraction of the LOS coverage area relates to the increment of the number of network users that can use such applications, thanks to the newly added relays. In this paper, we will assess the benefits of employing vehicle relays, by identifying the difference between $\nu\left(C_{\mathrm{RSU}+\text { relay }}\right)$ and $\nu\left(C_{\mathrm{RSU}}\right)$.

## III. Main Results

In this section, we evaluate the mean area fraction of the RSU LOS coverage and then that of the RSU-plus-relay LOS coverage.

## A. Stationarity of the LOS Coverage

Proposition 1. The RSU coverage $C_{R S U}$, the vehicle relay coverage $C_{\text {Relay }}$, and the $R S U$-plus-relay coverage $C_{R S U+r e l a y}$ are stationary particle models.

Proof: The RSU coverage $C_{\mathrm{RSU}}$ is the union of i.i.d. random sets centered on the RSU point process. Since the RSU point process is stationary [16], $C_{\text {RSU }}$ is a stationary particle model [13].

Since $\Phi^{\prime}$ is stationary and $C_{\text {Relay }}$ is the union of i.i.d. random sets, $C_{\text {Relay }}$ is a stationary particle model. Similarly, we conclude that the RSU-plus-relay coverage $C_{\text {RSU+relay }}$ is also a stationary particle model.

## B. RSU LOS Coverage

Theorem 1. The mean area fraction of RSU coverage is

$$
\begin{equation*}
\nu\left(C_{R S U}\right)=1-\exp \left(-\lambda_{l} \eta\left(1-e^{-2 \mu \gamma}\right)\right) \tag{12}
\end{equation*}
$$

Proof: From the stationarity of RSU coverage, the mean area fraction is equal to the probability that the origin is contained in the RSU coverage region [22]. Therefore, we have $\nu\left(C_{\mathrm{RSU}}\right) \equiv \mathbf{P}\left(0 \in C_{\mathrm{RSU}}\right)=1-\mathbf{P}\left(0 \notin C_{\mathrm{RSU}}\right)$.

Let $0_{i}$ denote the location on the line $l\left(r_{i}, \theta_{i}\right)$ closest to the origin. Then, for a given line $l\left(r_{i}, \theta_{i}\right)$ the typical point at the origin is not contained in the LOS coverage of the RSUs on the line if (i) $W_{j}<\left|X_{j}-0_{i}\right|$ for all the RSUs $X_{j}$ on the right side of $0_{i}$ and (ii) $V_{j}<\left|X_{j}-0_{i}\right|$ for all the RSUs $X_{j}$ on the left side of $0_{i}$.

In addition, for any line $l\left(r_{i}, \theta_{i}\right)$, the LOS coverage of RSUs on those lines does not contain the origin if and only if the distances from the origin to the line is greater than $\eta / 2$. Then, we use the facts that that the lines of the Poisson line process are independent, that $W_{j}$ and $V_{j}$ are i.i.d., and that the locations of RSUs on each line is a Poisson point process of intensity $\mu$. The mean area fraction of the RSU LOS coverage is given by

$$
\begin{align*}
& \mathbf{E}_{\Xi}\left[\prod_{r_{i}, \theta_{i} \in \Xi}^{\left|r_{i}\right|<\eta / 2} \mathbf{E}\left[\prod_{X_{j} \in \phi_{l\left(r_{i}, \theta_{i}\right)}} \mathbf{E}\left[\mathbb{1}_{W<\left|X_{j}-0_{i}\right|} \mid \Phi_{l}, \phi\right]\right]\right] \\
& =\mathbf{E}_{\Xi}\left[\prod_{r_{i}, \theta_{i} \in \Xi}^{\left|r_{i}\right|<\eta / 2} \mathbf{E}\left[\prod_{T_{j} \in \phi} \mathbf{E}\left[\mathbb{1}_{W<\left|T_{j}\right|} \mid \Phi_{l}, \phi\right]\right]\right] \tag{14}
\end{align*}
$$

where we use the Poisson property of $\phi_{l\left(r_{i}, \theta_{i}\right)}$ to obtain

$$
\sum_{X_{j} \in \phi_{l\left(r_{i}, \theta_{i}\right)}} \delta_{X_{j}-0_{i}} \equiv \sum_{T_{j} \in \phi} \delta_{T_{j}}
$$

where $\phi$ is a Poisson point process of intensity $\mu$ on the real axis.

Therefore, we have

$$
\begin{align*}
\mathbf{P}\left(0 \notin C_{\mathrm{RSU}}\right) & =\mathbf{E}_{\Xi}\left[\prod_{r_{i}, \theta_{i} \in \Xi}^{\left|r_{i}\right|<\eta / 2} \mathbf{E}\left[\left.\prod_{T_{j} \in \phi} 1-e^{-\frac{\left|T_{j}\right|}{\gamma}} \right\rvert\, \Phi_{l}\right]\right] \\
& =\mathbf{E}_{\Xi}\left[\prod_{r_{i}, \theta_{i} \in \Xi}^{\left|r_{i}\right|<\eta / 2} \exp (-2 \mu \gamma)\right] \\
& =\exp \left(-\lambda_{l} \eta\left(1-e^{-2 \mu \gamma}\right)\right) \tag{15}
\end{align*}
$$

where we use the total independence property of the Poisson point processes $\phi$ and $\Xi$, and their probability generating functionals [13].

Fig. 4 shows the mean area fraction of the RSU coverage as $\gamma$-the average LOS distance-increases. The figures show that the derived formula accurately matches the simulation results for both $\eta=100$ and $\eta=200$. We learn that as $\gamma$ increases, $\nu\left(C_{\mathrm{RSU}}\right)$ also increases.

## C. RSU-plus-relay LOS Coverage

This section presents the main result of the paper: the mean area fraction of the coverage created by RSUs and their associated vehicle relays.

Theorem 2. The mean area fraction of the RSU-plus-relay coverage is given by Eq. (13).

Proof: The mean area fraction of the RSU-plus-relay coverage is given by the probability that the origin is contained by the RSU-plus-relay coverage. Let $\mathbb{1}_{A}$ denote an indicator function that takes one if the event $A$ occurs and zero if the event $A$ does not occur. The mean area fraction of the RSU-plus-relay coverage is

$$
\begin{equation*}
\nu\left(C_{\mathrm{RSU}+\mathrm{relay}}\right)=\mathbf{P}\left(0 \in C_{\mathrm{RSU}+\text { relay }}\right)=\mathbf{E}\left[\mathbb{1}_{0 \in C_{\mathrm{RSU}+\text { relay }}}\right] \tag{16}
\end{equation*}
$$

Conditional on $\Phi_{l}$, we have

$$
\begin{align*}
& \mathbf{E}\left[\mathbb{1}_{0 \notin C_{\mathrm{RSU} \text { +relay }}}\right] \\
& =\mathbf{E}\left[\prod_{r_{i} \in \Phi_{l}} \mathbf{E}\left[\prod_{t_{j} \in \phi} \mathbf{E}\left[\mathbb{1}_{0 \notin C_{\mathrm{RSU}} \cup C_{\text {Relay }}} \mid \Phi_{l}, \phi\right]\right]\right] \tag{17}
\end{align*}
$$

where we use the same technique as in the proof of Theorem 1. Furthermore, conditional on the LOS coverage distances of RSUs, namely $W_{j}$ and $V_{j}$, the mean area fraction of the RSU-plus-relay coverage is given by

$$
\mathbf{E}\left[\prod_{r_{i} \in \Phi_{l}} \mathbf{E}\left[\prod_{t_{j} \in \phi} \mathbf{E}\left[\mathbf{E}\left[\mathbb{1}_{0 \notin C_{\mathrm{RSU}} \cup C_{\text {Relay }}} \mid \Phi_{l}, \phi, W, V\right]\right]\right]\right]
$$

where we drop the subscripts of the variables $W$ and $V$. Then, using the property of the indicator functions, we have

$$
\begin{equation*}
\mathbb{1}_{0 \notin C_{\mathrm{RSU}} \cup C_{\mathrm{Relay}}}=\mathbb{1}_{0 \notin C_{\mathrm{RSU}}}+\mathbb{1}_{0 \notin C_{\mathrm{Relay}} \cap C_{\mathrm{RSU}}^{c}} \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
1-\exp \left(-\lambda_{l} \eta\left(1-\exp \left(-\mu \int_{-\infty}^{\infty}\left\{e^{-\frac{|x|}{\gamma}}-\int_{0}^{\infty} \int_{0}^{\infty} \int_{-w}^{v} \frac{1-e^{-\frac{|x+y|}{\gamma}}}{w+v} \frac{e^{-\frac{w+v}{\gamma}}}{\gamma^{2}} \mathrm{~d} y \mathrm{~d} w \mathrm{~d} v\right\} \mathrm{d} x\right)\right)\right) . \tag{13}
\end{equation*}
$$



Fig. 4. The mean area fraction of the RSU LOS coverage with $\mu=4 / \mathrm{km}$ and $\eta=100$ meters.

The first term is a measurable function of the random variables $\Phi_{l}, \phi, W$, and $V$. Therefore, the mean area fraction is

$$
\begin{aligned}
& \mathbf{E}\left[\prod _ { \phi _ { i } \in \Phi _ { l } } \mathbf { E } \left[\prod _ { t _ { j } \in \phi } \mathbf { E } \left[\mathbb{1}_{0 \notin C_{\mathrm{RSU}}}\right.\right.\right. \\
&+\underbrace{\mathbf{E}\left[\mathbb{1}_{0 \notin C_{\mathrm{Relay}} \cap C_{\mathrm{RSU}}^{c}} \mid \Phi_{l}, \phi, W, V\right]}_{\text {(a) }}]]
\end{aligned}
$$

The origin is not contained in $C_{\text {Relay }} \cap C_{\mathrm{RSU}}^{c}$ if neither the LOS segments centered on the left side of $0_{i}$ nor the LOS segments on the right side of $0_{i}$ contains the origin, for all line $l\left(r_{i}, \theta_{i}\right)$. Let $Y_{j}$ denote the location of the selected relay of the $j$-th RSU. As in the proof of Theorem 1, we have

$$
\begin{align*}
(\mathrm{a}) & =\left\{\begin{array}{l}
\mathbf{E}\left[\mathbb{1}_{W<\left|Y_{j}\right|} \mid \Phi_{l}, \phi, W, V\right], Y_{j} \in \operatorname{right}\left(l\left(r_{i}, \theta_{i}\right) ; 0_{i}\right) \\
\mathbf{E}\left[\mathbb{1}_{V<\left|Y_{j}\right|} \mid \Phi_{l}, \phi, W, V\right], Y_{j} \in \operatorname{left}\left(l\left(r_{i}, \theta_{i}\right) ; 0_{i}\right)
\end{array}\right. \\
& =\mathbf{E}_{Y_{j}}\left[\left.1-e^{-\frac{\left|Y_{j}\right|}{\gamma}} \right\rvert\, \Phi_{l}, \phi, W, V\right] \\
& =\int_{-w}^{v} \frac{1-\exp \left(-\frac{\left|t_{j}+y\right|}{\gamma}\right)}{w+v} \mathrm{~d} y, \tag{19}
\end{align*}
$$

where we use the facts that (i) the probability density function of the exponential random variable is $f(x)=$ $1-\exp (-x / \gamma)$ and (ii) the location of the relay $Y_{j}$ is uniformly distributed within $\left[t_{j}-w, t_{j}+v\right]$, where $t_{j}$ is the location of the $j$-th RSU and $w$ and $v$ are the LOS distance from the RSU. Here $w$ and $v$ denote the realizations of the i.i.d. exponential random variables.


Fig. 5. The mean area fractions of the LOS coverage of the proposed network where $\mu=4 / \mathrm{km}$ and $\eta=100$ meters.

As a result, by deconditioning w.r.t. $W$ and $V$, the mean area fraction of RSU-plus-relay coverage is

$$
\begin{aligned}
& \mathbf{E}\left[\prod _ { r _ { i } \in \Phi _ { l } } \mathbf { E } \left[\prod _ { t _ { j } \in \phi } \left(1-e^{-\frac{\left|t_{j}\right|}{\gamma}}\right.\right.\right. \\
& \left.\left.\left.+\int_{0}^{\infty} \int_{0}^{\infty}\left(\int_{-w}^{v} \frac{1-e^{-\frac{\left|t_{j}+y\right|}{\gamma}}}{w+v}\right) \frac{e^{-\frac{w+v}{\gamma}}}{\gamma^{2}} \mathrm{~d} y \mathrm{~d} w \mathrm{~d} v\right)\right]\right]
\end{aligned}
$$

We obtain the final result by using the probability generating functional of the Poisson point process.

Theorem 2 characterizes the increased amount of LOS coverage due to the relays. Figs. 5-7 show the simulated mean area fractions of the RSU coverage and RSU-plusrelay coverage.

Fig. 5 displays the increased amount of LOS coverage for $\eta=100$ meters. Fig. 6 shows the LOS coverage for $\eta=100$. Fig. 7 shows the increment of the LOS coverage when the widths of streets are 25,50 , or 100 meters. When the LOS distance $\gamma$ is 66 meters and the width of streets $\eta$ is 25 meters, we have $\nu\left(C_{\mathrm{RSU}+\text { relay }}\right) / \nu\left(C_{\mathrm{RSU}}\right)=1.42$. We have a nearly $40 \%$ increase of the LOS coverage area, thanks to the newly introduced relay. Similarly, for $\gamma=66$ meters and $\eta=$ 50 meters, $\nu\left(C_{\mathrm{RSU}+\text { relay }}\right) / \nu\left(C_{\mathrm{RSU}}\right)=1.39$ and the increase of the LOS coverage area is $39 \%$. For $\gamma=66$ meters and $\eta=100$ meters, $\nu\left(C_{\mathrm{RSU}+\text { relay }}\right) / \nu\left(C_{\mathrm{RSU}}\right)=1.36$, i.e., we have $36 \%$ increment of the LOS coverage. We identify that the benefit obtained from the relays is greater on narrower streets.


Fig. 6. The mean area fractions of the LOS coverage of the proposed network where $\mu=2 / \mathrm{km}$ and $\eta=100$ meters.


Fig. 7. The mean area fractions of the LOS coverage of the proposed network where $\lambda_{l}=5 / \mathrm{km}$ and $\mu=2 / \mathrm{km}$.

## IV. CONCLUSION

The availability of LOS signals around road structure dictates the capacity of communications between vehicles, roadside IoT devices, and RSUs. In this paper, we investigate that RSUs employ vehicle relays to increase the chance that roadside IoT devices and network users are in the LOS coverage. By assuming vehicle relays provide the LOS coverage to network users only when the relays are in the LOS from RSUs, we derive the mean area fraction of the RSU LOS coverage and then the mean area fraction of the RSU-plus-relay LOS coverage. We show that the derived formula matches the simulation results and that vehicle relays will increase the chances that roadside IoT devices and users are in LOS for high capacity communications. The present analysis of the LOS coverage increment not only displays the benefit of using relays but also assesses the viability of the modern V2X applications handling a large number of IoT devices and users densely deployed around road infrastructure.

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