



HAL
open science

On the modeling of elasto-acoustic network of pipes composed of straight waveguides and curved joints

Diego Salam Claro, Vivien Denis, Jean-Mathieu Mencik

► **To cite this version:**

Diego Salam Claro, Vivien Denis, Jean-Mathieu Mencik. On the modeling of elasto-acoustic network of pipes composed of straight waveguides and curved joints. Workshop "Ondes de Lamb : de la théorie à la pratique", Feb 2024, Paris, France. hal-04446992

HAL Id: hal-04446992

<https://hal.science/hal-04446992>

Submitted on 8 Feb 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

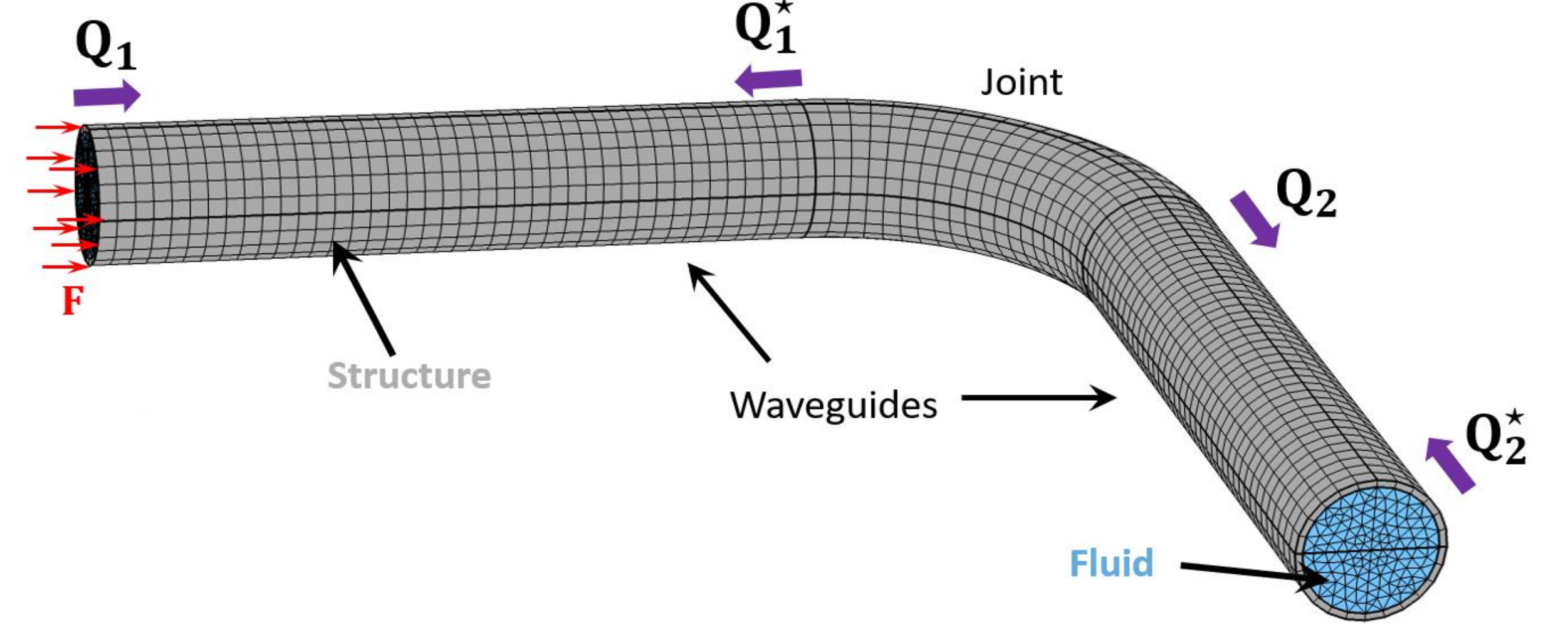
On the modeling of elasto-acoustic network of pipes composed of straight waveguides and curved joints

Diego SALAM CLARO*, Vivien DENIS and Jean-Mathieu MENCIK

INSA Centre Val de Loire, Laboratoire de Mécanique Gabriel Lamé
*Email address: diego.salam_claro@insa-cvl.fr

Objectives

- Modeling elasto-acoustic networks of pipes:
 - Wave modes and dispersion curves
 - Wave scattering at coupling elements
- Using model reduction for coupling elements:
 - Craig-Bampton method with basis enrichment
- Assessing transmission and reflection coefficients for the torsional mode $T(0,1)$ at a curved joint
- Assessing fluid loading effect (transmission and reflection coefficients)



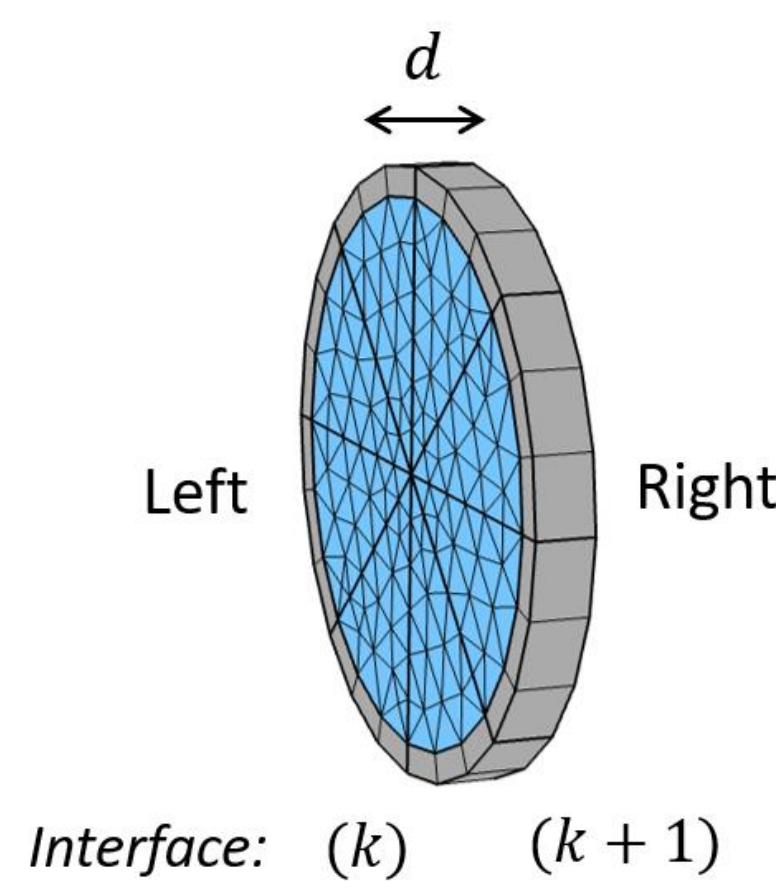
Wave Finite Element (WFE) method

- Periodic systems $\rightarrow N$ identical substructures (length d)
- FE model of a substructure \rightarrow wave mode basis

- Transfer matrix:

$$\begin{bmatrix} \mathbf{q}_L^{(k+1)} \\ -\mathbf{F}_L^{(k+1)} \end{bmatrix} = \mathbf{S} \begin{bmatrix} \mathbf{q}_L^{(k)} \\ -\mathbf{F}_L^{(k)} \end{bmatrix}$$

\mathbf{q} : displacement (solid) and velocity potential (acoustic)
 \mathbf{F} : elastic and acoustic forces



- $\mathbf{S} + \mathbf{S}^{-1}$ transformation $\rightarrow 2n$ eigenvalues μ_j and eigenvectors ϕ_j of \mathbf{S} :
 - n right-going wave modes (μ, Φ) and n left-going wave modes (μ^*, Φ^*)
 - Wavenumber β_j : $\mu_j = \exp(-i\beta_j d)$

Waveshapes: $\Phi = \begin{bmatrix} \Phi_q \\ \Phi_F \end{bmatrix}$ and $\Phi^* = \begin{bmatrix} \Phi_q^* \\ \Phi_F^* \end{bmatrix}$

- Wave mode expansion for a substructure boundary (k)

$$\begin{aligned} \mathbf{q}_L^{(k)} &= \mathbf{q}_R^{(k)} = \Phi_q \mu^{k-1} \mathbf{Q} + \Phi_q^* \mu^{N+k-1} \mathbf{Q}^* \\ -\mathbf{F}_L^{(k)} &= \mathbf{F}_R^{(k)} = \Phi_F \mu^{k-1} \mathbf{Q} + \Phi_F^* \mu^{N+k-1} \mathbf{Q}^* \end{aligned}$$

\mathbf{Q}, \mathbf{Q}^* : wave amplitude vectors at the left and right ends of the pipe

Joint modeling: Craig-Bampton (CB) method with basis enrichment

- Large-sized FE model \rightarrow Reduction is needed!

$$\mathbb{D}^j = -\omega^2 \mathbb{M}^j + i\omega \mathbb{G}^j + \mathbb{K}^j$$

\mathbb{D}^j : dynamic stiffness matrix

- Transformation matrices:

Elastic part:

$$\Gamma_{CB}^E = \begin{bmatrix} \tilde{\mathbf{X}}_{el}^E & \tilde{\mathbf{u}}^E & \mathbf{X}_{st}^E \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

Acoustic part:

$$\Gamma_{CB}^A = \begin{bmatrix} \tilde{\mathbf{X}}_{el}^A & \tilde{\mathbf{u}}^A & \mathbf{X}_{st}^A \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

$$\mathbf{T}_{CB} = \begin{bmatrix} \Gamma_{CB}^E & \mathbf{0} \\ \mathbf{0} & \Gamma_{CB}^A \end{bmatrix}$$

$\mathbf{X}_{st}^{E,A}$: matrix of static modes

$\tilde{\mathbf{X}}_{el}^{E,A}$: matrix of fixed-interface modes

$\tilde{\mathbf{u}}^{E,A}$: enrichment vectors

$$\Rightarrow \tilde{\mathbb{D}}^j = (\mathbf{T}_{CB})^T \mathbb{D}^j \mathbf{T}_{CB} \text{ where } \text{size}(\tilde{\mathbb{D}}^j) \ll \text{size}(\mathbb{D}^j)$$

Scattering matrix

- Scattering matrix of the joint (\mathbb{C}^j) (depends on \mathbb{D}^j and wave modes):

$$\begin{bmatrix} \mathbf{Q}_1^* \\ \mathbf{Q}_2 \end{bmatrix} = \begin{bmatrix} \mathbb{C}_{11}^j & \mathbb{C}_{12}^j \\ \mathbb{C}_{21}^j & \mathbb{C}_{22}^j \end{bmatrix} \begin{bmatrix} \mu^{N_1} \mathbf{Q}_1 \\ \mu^{N_2} \mathbf{Q}_2 \end{bmatrix}$$

N_1 : number of substructures in waveguide 1

N_2 : number of substructures in waveguide 2

\mathbb{C}_{11}^j : transmission coefficients between waveguides 1 and 2

Numerical results

- Material properties:

- Elastic domain: steel
- Acoustic domain: water
- Rayleigh damping

- Geometrical parameters:

- Pipe radius: 0.05 m
- Pipe thickness: 0.005 m
- Substructure length: $d = 2 \times 10^{-3}$ m
- Joint radius: $R_j = 0.20$ m

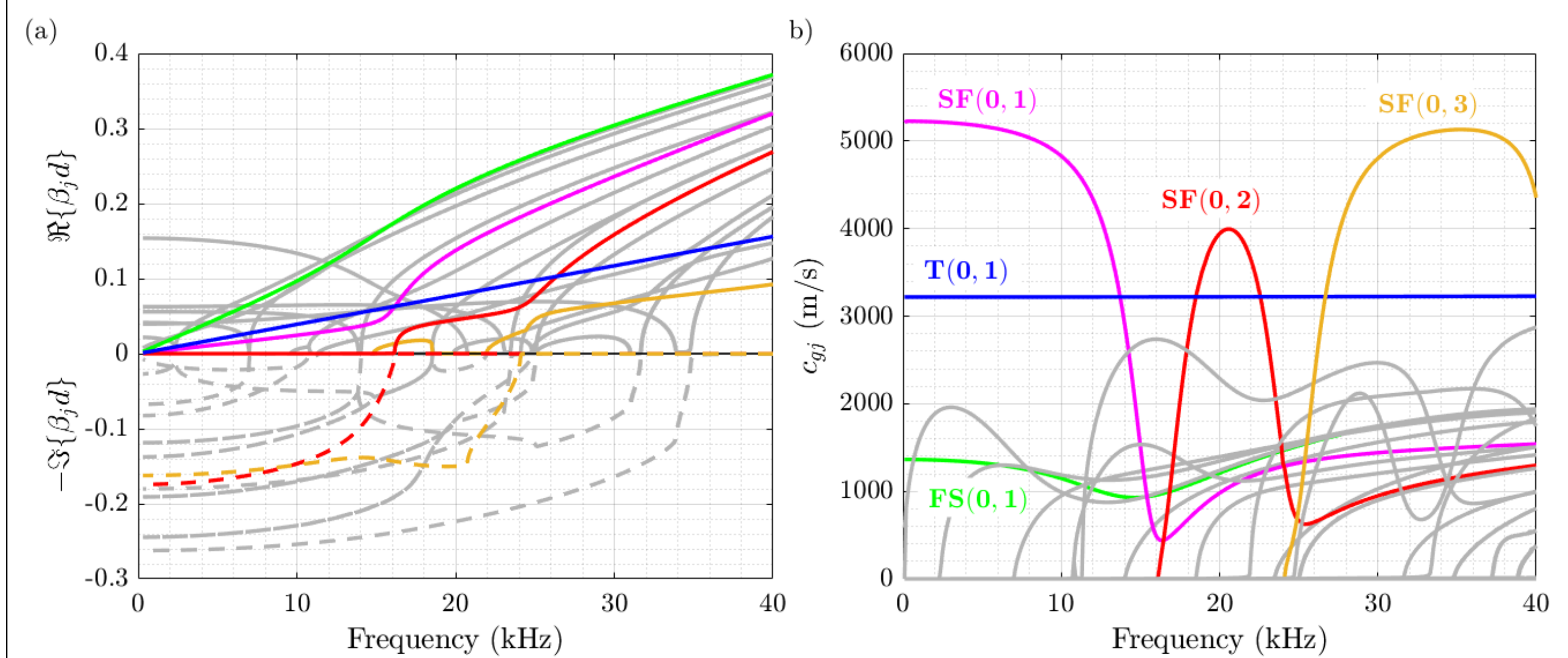
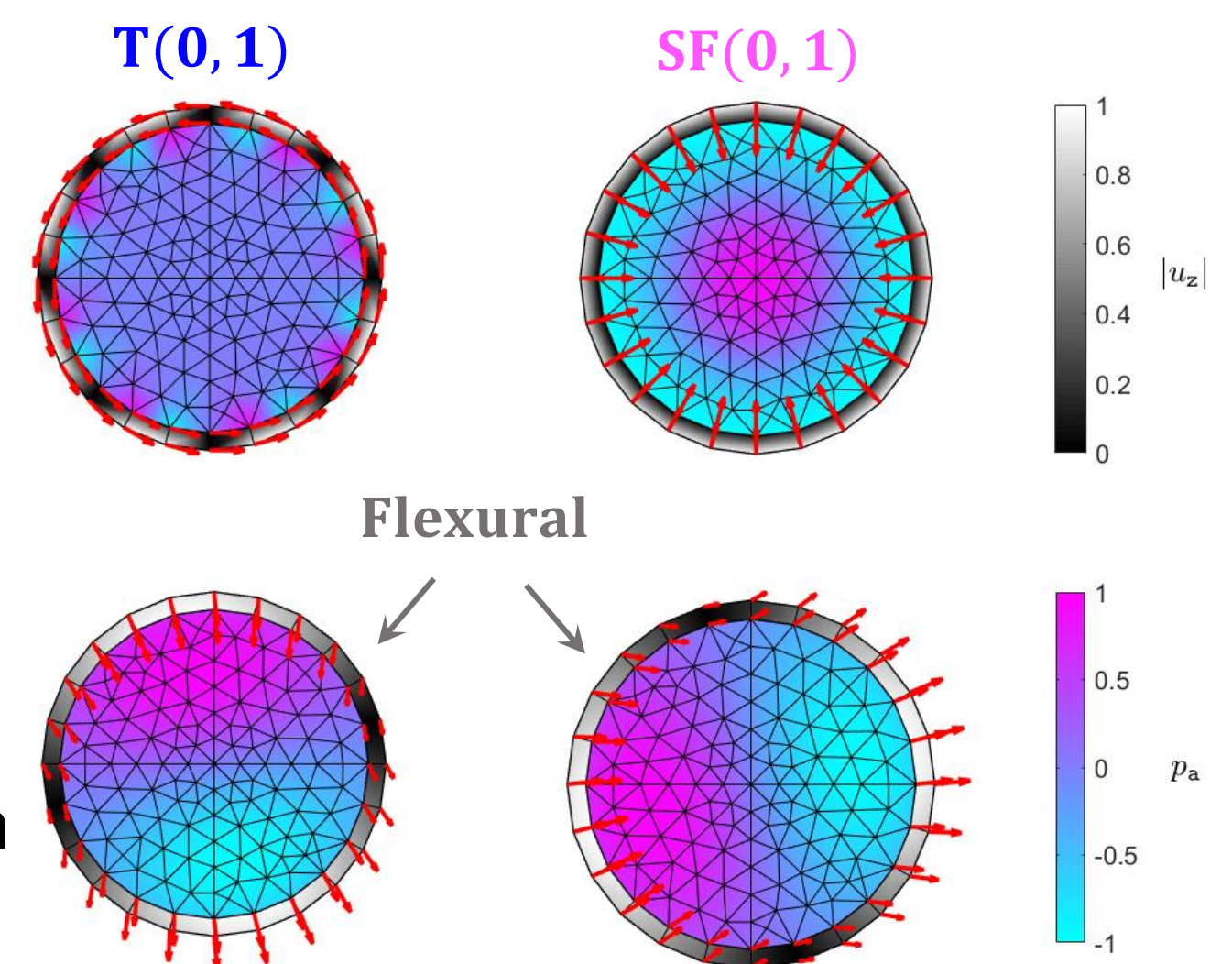


Figure : (a) Dispersion curves (real and imaginary parts of $\beta_j d$ are plotted in continuous and dashed lines, respectively) and (b) group velocities of modes

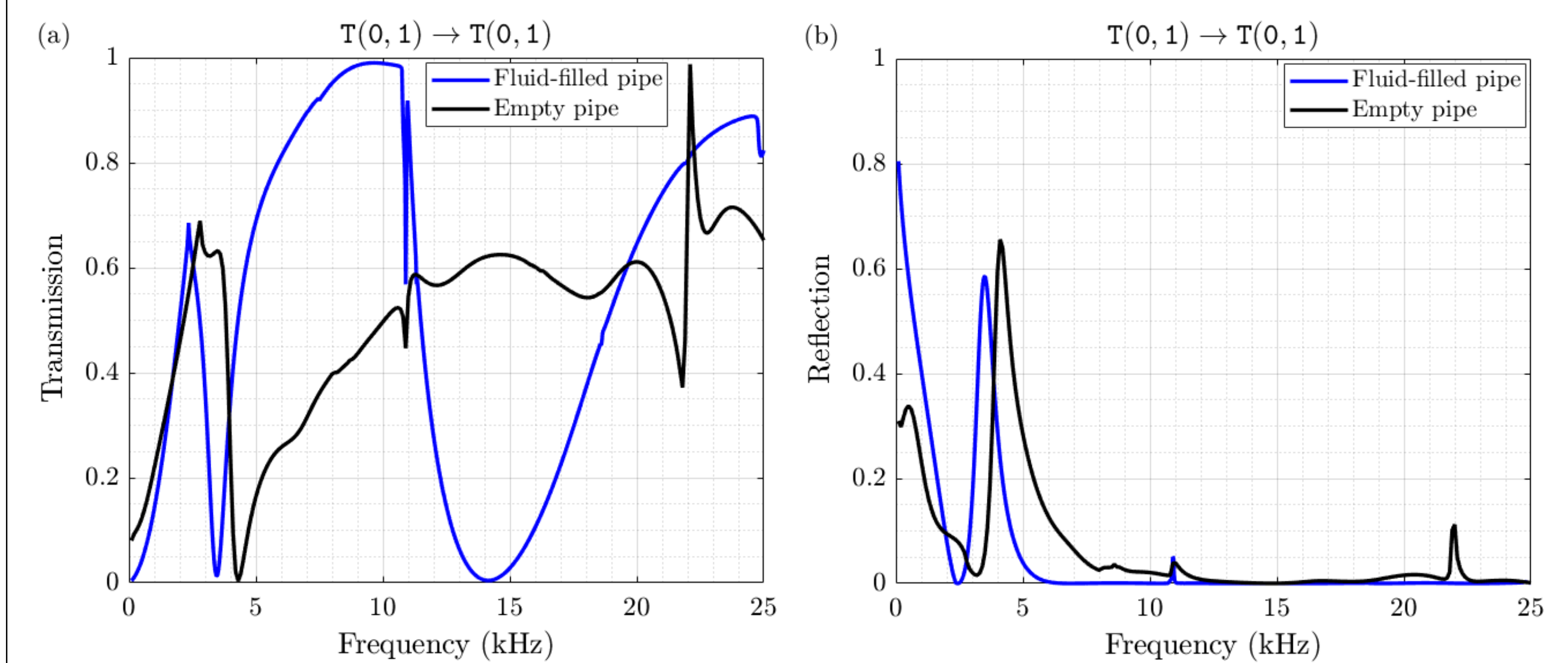


Figure : Power scattering coefficients for the joint: (a) transmission and (b) reflection $T(0,1) \rightarrow T(0,1)$. Fluid-filled pipe in blue lines, and empty pipe in black lines.

Reduction strategy

Table : Elapsed times to compute the solutions using the FE method, the full WFE, and the WFE method with CB reduction for the joint

Method	Time (s)	Reduction (%)
FE	8711	---
WFE	6904	20.75
WFE + reduction	2704	68.95

- Computation of wave modes (WFE) only takes 265s

Conclusions

- Elasto-acoustic responses of networks of pipes can be quickly computed using the WFE method and the CB reduction with basis enrichment.
- Fluid loading effect affects the scattering of the torsional mode at a curved joint.

References:

- J.-M. Mencik and M. N. Ichchou. Wave finite elements in guided elastodynamics with internal fluid. International Journal of Solids and Structures, 44(7):2148–2167, April 2007.
- Q. H. Tran, M. Ouisse, and N. Bouhaddi. A robust component mode synthesis method for stochastic damped vibroacoustics. Mechanical Systems and Signal Processing, 24(1):164–181, January 2010.