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# On the modeling of elasto-acoustic network of pipes composed of straight waveguides and curved joints

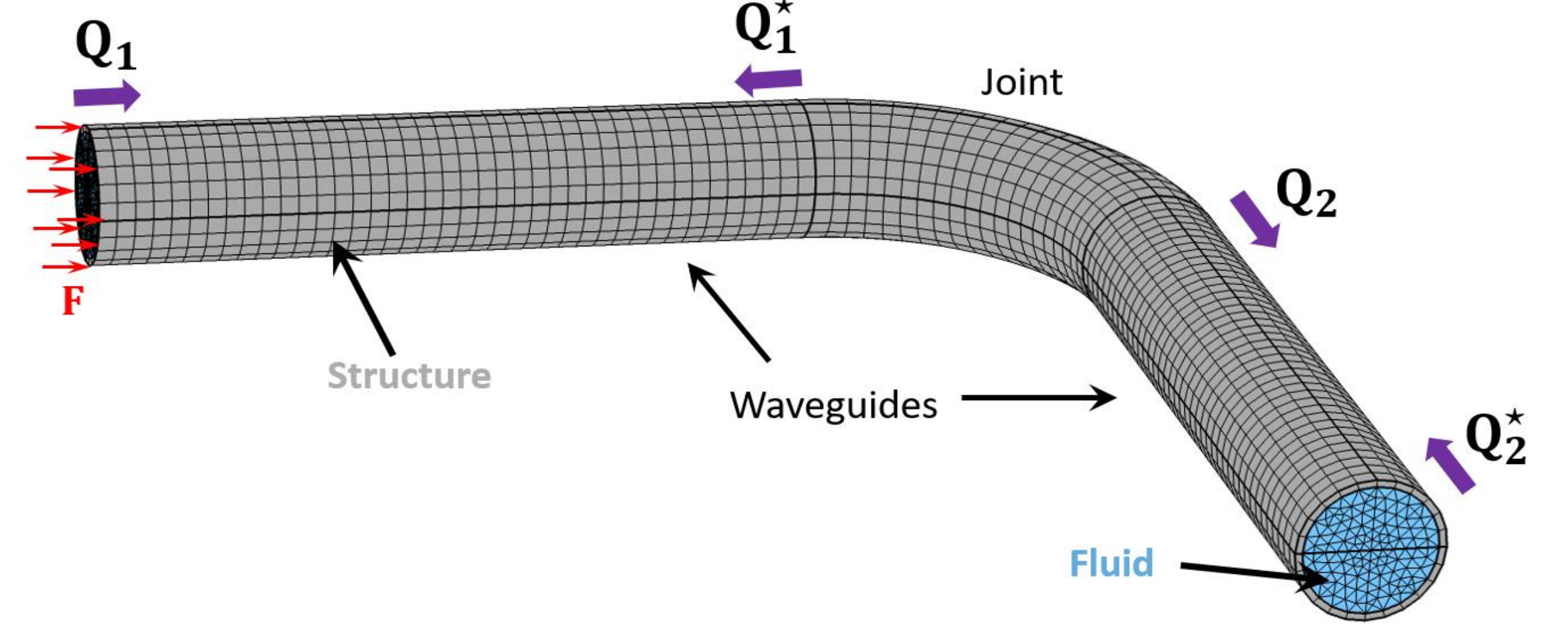
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## Objectives

- Modeling elasto-acoustic networks of pipes:
  - Wave modes and dispersion curves
  - Wave scattering at coupling elements
- Using model reduction for coupling elements:
  - Craig-Bampton method with basis enrichment
- Assessing transmission and reflection coefficients for the torsional mode  $T(0,1)$  at a curved joint
- Assessing fluid loading effect (transmission and reflection coefficients)

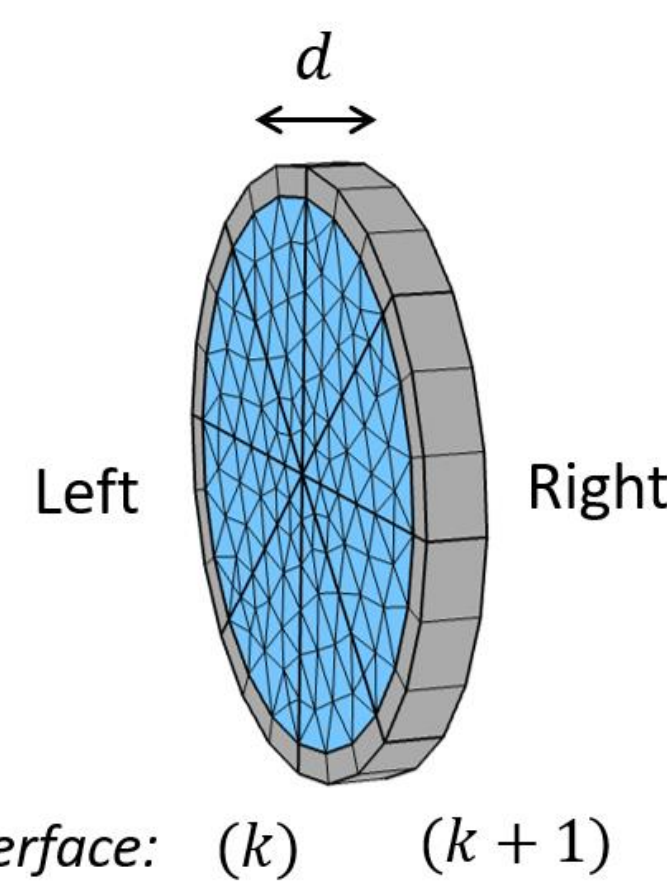


## Wave Finite Element (WFE) method

- Periodic systems  $\rightarrow N$  identical substructures (length  $d$ )
- FE model of a substructure  $\rightarrow$  wave mode basis

- Transfer matrix:

$$\begin{bmatrix} \mathbf{q}_L^{(k+1)} \\ -\mathbf{F}_L^{(k+1)} \end{bmatrix} = \mathbf{S} \begin{bmatrix} \mathbf{q}_L^{(k)} \\ -\mathbf{F}_L^{(k)} \end{bmatrix}$$



$\mathbf{q}$  : displacement (solid) and velocity potential (acoustic)  
 $\mathbf{F}$  : elastic and acoustic forces

- $\mathbf{S} + \mathbf{S}^{-1}$  transformation  $\rightarrow 2n$  eigenvalues  $\mu_j$  and eigenvectors  $\phi_j$  of  $\mathbf{S}$  :

- $n$  right-going wave modes  $(\mu, \Phi)$  and  $n$  left-going wave modes  $(\mu^*, \Phi^*)$
- Wavenumber  $\beta_j$  :  $\mu_j = \exp(-i\beta_j d)$

- Waveshapes :  $\Phi = \begin{bmatrix} \Phi_q \\ \Phi_F \end{bmatrix}$  and  $\Phi^* = \begin{bmatrix} \Phi_q^* \\ \Phi_F^* \end{bmatrix}$

- Wave mode expansion for a substructure boundary ( $k$ )

$$\begin{aligned} \mathbf{q}_L^{(k)} &= \mathbf{q}_R^{(k)} = \Phi_q \mu^{k-1} \mathbf{Q} + \Phi_q^* \mu^{N+k-1} \mathbf{Q}^* \\ -\mathbf{F}_L^{(k)} &= \mathbf{F}_R^{(k)} = \Phi_F \mu^{k-1} \mathbf{Q} + \Phi_F^* \mu^{N+k-1} \mathbf{Q}^* \end{aligned}$$

$\mathbf{Q}, \mathbf{Q}^*$  : wave amplitude vectors at the left and right ends of the pipe

## Joint modeling: Craig-Bampton (CB) method with basis enrichment

- Large-sized FE model  $\rightarrow$  Reduction is needed !

$$\mathbb{D}^j = -\omega^2 \mathbb{M}^j + i\omega \mathbb{G}^j + \mathbb{K}^j$$

$\mathbb{D}^j$  : dynamic stiffness matrix

- Transformation matrices:

Elastic part:

$$\Gamma_{CB}^E = \begin{bmatrix} \tilde{\mathbf{X}}_{el}^E & \tilde{\mathbf{u}}^E & \mathbf{X}_{st}^E \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

Acoustic part:

$$\Gamma_{CB}^A = \begin{bmatrix} \tilde{\mathbf{X}}_{el}^A & \tilde{\mathbf{u}}^A & \mathbf{X}_{st}^A \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

$$\mathbf{T}_{CB} = \begin{bmatrix} \Gamma_{CB}^E & \mathbf{0} \\ \mathbf{0} & \Gamma_{CB}^A \end{bmatrix}$$

$\mathbf{X}_{st}^{E,A}$  : matrix of static modes

$\tilde{\mathbf{X}}_{el}^{E,A}$  : matrix of fixed-interface modes

$\tilde{\mathbf{u}}^{E,A}$  : enrichment vectors

$$\Rightarrow \tilde{\mathbb{D}}^j = (\mathbf{T}_{CB})^T \mathbb{D}^j \mathbf{T}_{CB} \text{ where } \text{size}(\tilde{\mathbb{D}}^j) \ll \text{size}(\mathbb{D}^j)$$

## Scattering matrix

- Scattering matrix of the joint ( $\mathbb{C}^j$ ) (depends on  $\mathbb{D}^j$  and wave modes):

$$\begin{bmatrix} \mathbf{Q}_1^* \\ \mathbf{Q}_2 \end{bmatrix} = \begin{bmatrix} \mathbb{C}_{11}^j & \mathbb{C}_{12}^j \\ \mathbb{C}_{21}^j & \mathbb{C}_{22}^j \end{bmatrix} \begin{bmatrix} \mu^{N_1} \mathbf{Q}_1 \\ \mu^{N_2} \mathbf{Q}_2 \end{bmatrix}$$

$N_1$  : number of substructures in waveguide 1

$N_2$  : number of substructures in waveguide 2

$\mathbb{C}_{11}^j$  : transmission coefficients between waveguides 1 and 2

## Numerical results

- Material properties:

- Elastic domain: steel
- Acoustic domain: water
- Rayleigh damping

- Geometrical parameters:

- Pipe radius: 0.05 m
- Pipe thickness: 0.005 m
- Substructure length:  $d = 2 \times 10^{-3}$  m
- Joint radius:  $R_j = 0.20$  m

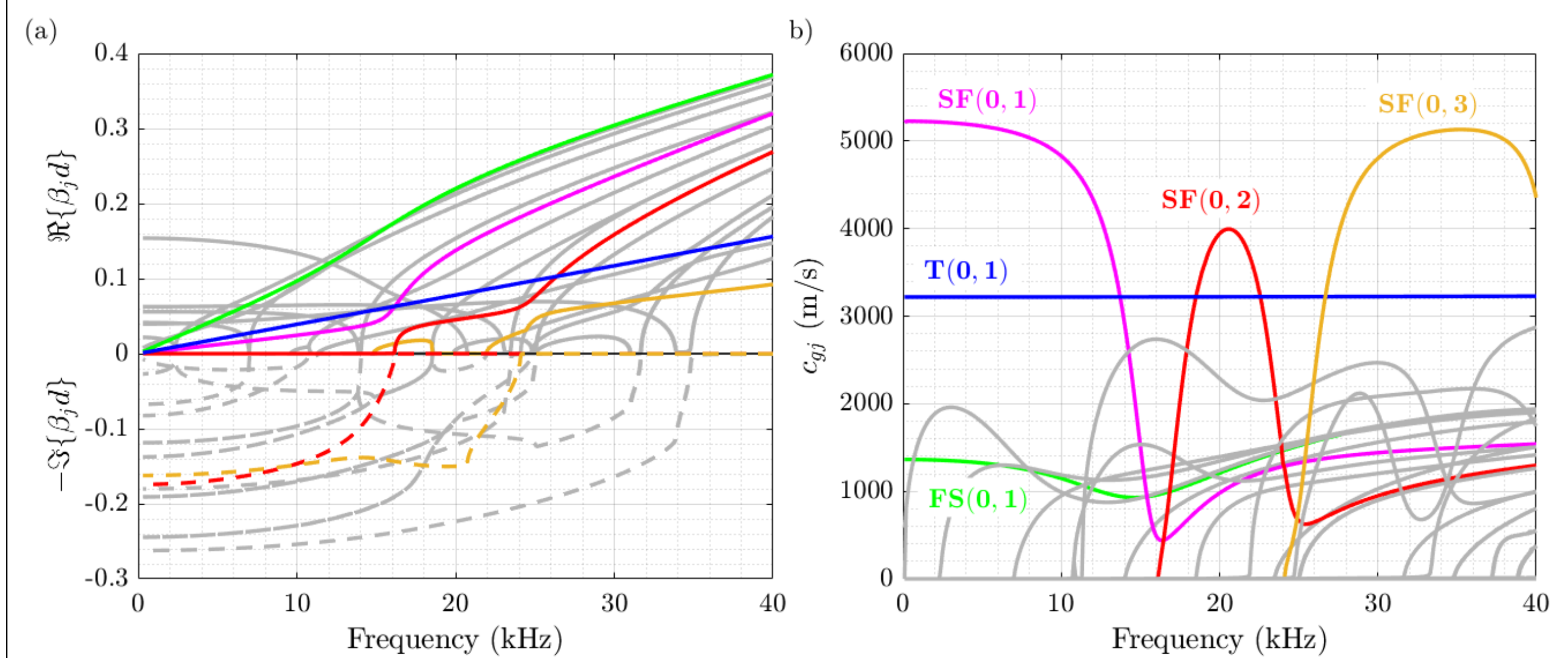
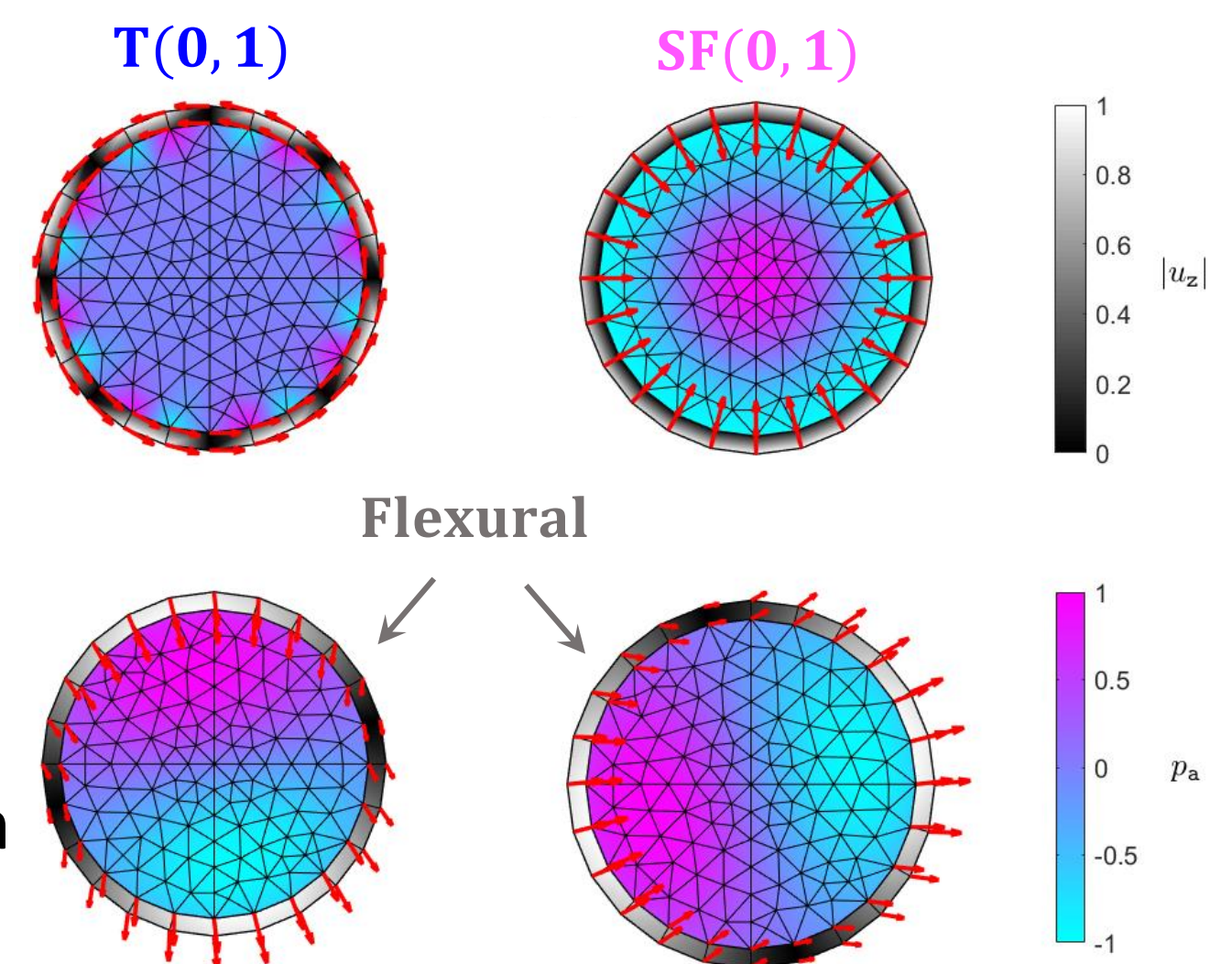


Figure : (a) Dispersion curves (real and imaginary parts of  $\beta_j d$  are plotted in continuous and dashed lines, respectively) and (b) group velocities of modes

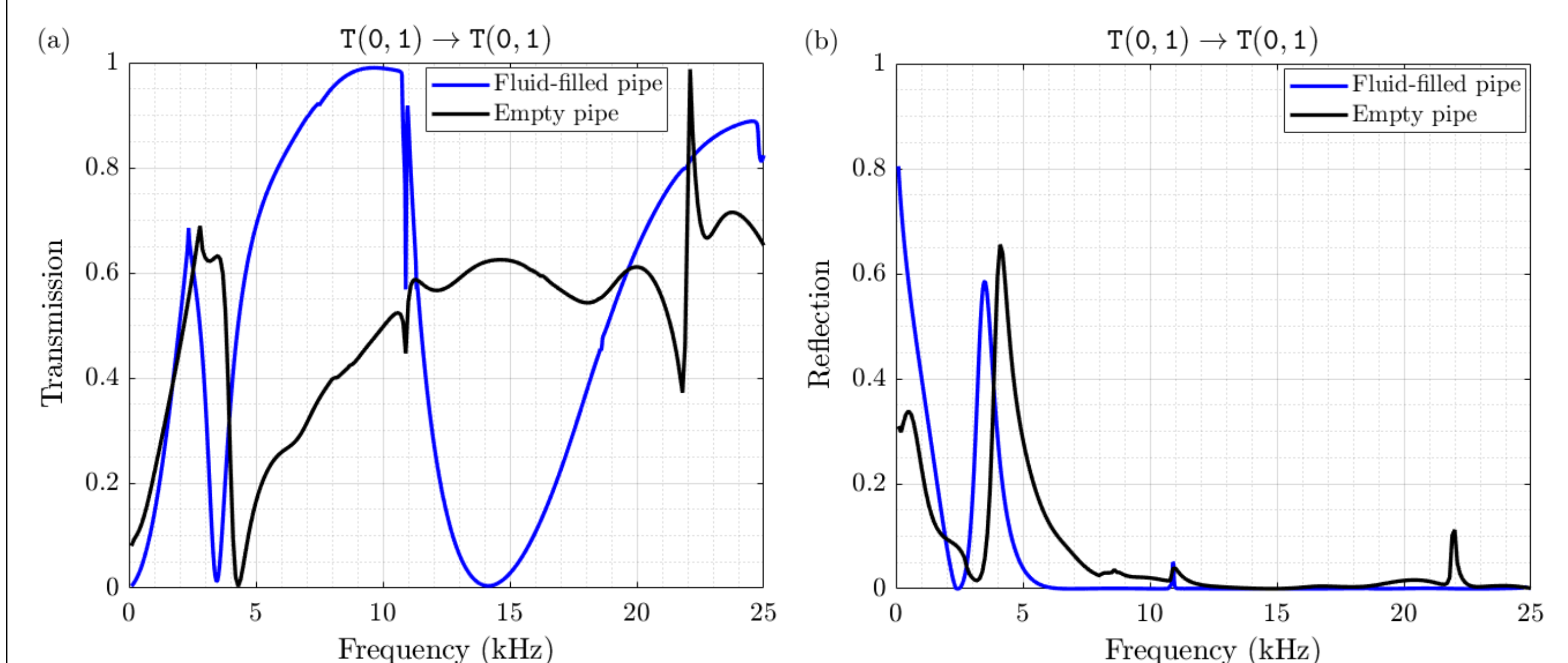


Figure : Power scattering coefficients for the joint: (a) transmission and (b) reflection  $T(0,1) \rightarrow T(0,1)$ . Fluid-filled pipe in blue lines, and empty pipe in black lines.

## Reduction strategy

Table : Elapsed times to compute the solutions using the FE method, the full WFE, and the WFE method with CB reduction for the joint

Method	Time (s)	Reduction (%)
FE	8711	---
WFE	6904	20.75
WFE + reduction	2704	68.95

- Computation of wave modes (WFE) only takes 265s

## Conclusions

- Elasto-acoustic responses of networks of pipes can be quickly computed using the WFE method and the CB reduction with basis enrichment.
- Fluid loading effect affects the scattering of the torsional mode at a curved joint.

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