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1 **Multifractal analysis for spatial characterization of high resolution Sentinel-** 2 **2/MAJA products in Southwestern France**

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5 **Highlights**

- 6 ● Particular scale breaks were detected on Sentinel-2/MAJA products
- 7 ● Time variations in scaling regimes were noticed in series of surface reflectances
- 8 ● The large-scale break was related to the landscape spatial structure
- 9 ● The fine-scale break was explained by the effect of the optical acquisition system

10 **Abstract**

11 Multifractal analysis was applied to evidence spatial properties, in particular scaling behaviour, of
12 Sentinel-2 surface reflectances and optical indexes acquired over the Southwest region of France.
13 Considering the significant impact of clouds on the spatial properties of optical products, no study to
14 our knowledge has focused on the multifractal analysis of a series of images. Images with minimal
15 cloud cover were identified for the whole year 2016 based on the MACCS-ATCOR Joint Algorithm
16 (MAJA) cloud and shadow detection process. This allowed us to access the dynamics of the scaling
17 properties of surface reflectances, contributing to demonstrate the potential of multifractals for spatial
18 characterization. Different scaling regimes, separated by significant scale breaks, were evident in these
19 data. Through simulations from the Universal Multifractal model, we could explain the scaling
20 properties and especially the position of scale breaks observed in the images studied by two types of
21 phenomena. First, a transition scale located between hundreds of meters and one kilometre was
22 detected on power spectra, structure functions and statistical moments. This specific scale was related
23 to the average size of the agricultural plots of the studied region, revealing the direct link between the
24 scaling behaviour observed and the physical properties of the surface. Second, an instrumental
25 phenomenon was detected on the power spectra, resulting in a systematic transition scale observed
26 near the resolution of the sensor and revealing the impact of the acquisition system on the high
27 frequencies of optical images. In this research work, theoretical and/or empirical validation arguments

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28 were provided as to the origin of the scales detected, contributing to demonstrate the value of
29 multifractals to extract image texture from surface reflectance products.

30 **Keywords**

31 Surface reflectances; Optical indexes; Spatial characterization; Universal Multifractal; Sentinel-2;
32 MAJA

33

34 1 Introduction

35 The last generation of space-borne sensors enables the acquisition of Earth surface information
36 with both high temporal and spatial resolutions, showing good capabilities for studying environmental
37 and anthropogenic evolutions at local to global spatial scales (Gutman et al., 2008; Huang et al., 2010).
38 Among others, the most popular may be the U.S. Landsat-8 (Roy et al., 2014) and the recent ESA
39 Sentinel-2 mission (Martimort et al., 2007) satellites. They provide multispectral images at spatial
40 resolutions of tens of meters with a revisit time ranging between 5 and 16 days. Thanks to the benefits
41 of their thirteen spectral bands, Sentinel-2 satellites (S2) deliver surface reflectances acquired from
42 visible to short-wave infrared, allowing observation of a strong spatial variability in quite a wide range
43 of wavelengths (Sotian et al., 2019).

44 Spatial heterogeneity is often closely linked to the temporal dynamics which itself relies on both
45 natural phenomena associated with specific environmental conditions (meadows, forested areas, bare
46 soils, etc.), and anthropogenic phenomena resulting from urban activities, agricultural practices, etc.
47 Because these factors are present at different space scales and may have different spectral signatures,
48 the information contained in acquired surface reflectances can be entangled in a very complex way
49 and difficult to analyse from an image processing stand point (De Cola, 1989; Lam, 1990). Depending
50 on the targeted applications, different techniques have been developed and used so far, to classify,
51 model or characterize the land surface spatial and temporal properties with the aim to account for its
52 complexity. In image classification, commonly used methods are based on the spectral signature of the
53 observed scene (Lu and Weng, 2007), some of them involve complex data processing algorithms such
54 as decision trees or neural networks (Hansen et al., 1996; Pal, 2005; Paola and Schowengerdt, 1995).
55 In object-oriented classifiers, segmentation techniques make use of the spatial context of each pixel
56 to improve the classification performance (Benz et al., 2004; Zucker, 1976). An embedding approach
57 insensitive to the initial conditions was also introduced recently (Mangiarotti et al., 2018). For the
58 modelling of the land surface, most of the developments have been based on optimization and data-
59 assimilation techniques with the aim to improve the integration of remote sensed data into land
60 surface process models (Bach and Mauser, 2003; Mangiarotti et al., 2008; Rodriguez-Fernandez et al.,
61 2019). Using the global modelling technique, the first chaotic attractor directly issued from satellite
62 data was obtained that illustrates the high complexity of the surface dynamics at large scale
63 (Mangiarotti et al. 2012; 2014) and which itself relies on the level of spatial synchronization
64 (Mangiarotti et al. 2016). To have a better description of the landscape changes, various linear and
65 nonlinear characterization methods were introduced to describe the spatial variability of land surface
66 (Gustafson, 1998; Puissant et al., 2005), some of them taking the spatial scale into account in their
67 analyses (Mangiarotti et al., 2010, 2012). However, a more systematic spatial characterization requires

68 a statistical framework able to describe variability and image texture over a wide range of spatial
69 scales. Statistical approaches like the Fourier power spectrum and entropy (Weszka et al., 1976) or
70 variogram and fractal dimension (Ramstein and Raffly, 1989) can give relevant information on the
71 spatial texture of the image such as roughness, autocorrelation, or the noisy nature of the signal for
72 example.

73 Fractal analysis can be a relevant tool to quantitatively characterize the high degree of complexity
74 and the spatial structure of surface products (Verrier et al., 2020) such as new remote sensing
75 reflectances similar to S2 products (Alonso et al., 2017). Indeed, S2 products can be seen as images
76 that embed a hierarchy of spatial structures of different sizes and of different energy levels. Many
77 remote sensing images of natural variables show a fractal structure that can be demonstrated by
78 various geometrical and statistical tools (Lovejoy et al., 2001; Qiu et al., 1999; Schmitt et al., 1993;
79 Tessier et al., 1993). Studying such scaling features can contribute to identify specific set of scales on
80 which specific properties apply. Kim and Barros (2002a) used such scaling analysis to characterize
81 space-time variability of remotely sensed soil moisture and relate it to landscape characteristics
82 (terrain, soil, vegetation). Two scale ranges were identified, a large one related to the
83 hydrometeorological conditions (evapotranspiration, precipitation processes) and a fine one related
84 to the soil properties of the surface (texture, structure). Precisely knowing these sets of scales aims to
85 improve land surface modelling, considering both natural phenomena like topography, streams or
86 drainage basins (Aguado et al., 2014; Cheng et al., 2001; Lyu et al., 2017) and anthropogenic
87 phenomena like urban growth (Chen and Wang, 2013; Tan, 2021) or irrigation (Ko et al., 2016).
88 Specifically, in hydrology these scaling features led to the development of better calibrated
89 downscaling algorithms applied to surface soil moisture (Kim and Barros, 2002b; Ko et al., 2016;
90 Mascaro et al., 2010; Neuhauser et al., 2019) and precipitation (Gires et al., 2012; Reborá et al., 2006).

91 One of the major constraints when measuring surface reflectances from space is the atmosphere.
92 The latter is composed of different elements which have direct impact on reflectances. Indeed, clouds,
93 gas molecules and aerosols are responsible for scattering and absorbing effects on the sun light
94 reflected by the surface (Fraser et al., 1985; Vermote et al., 1997). These atmospheric components
95 must be taken into account when analysing optical images, especially since they can affect the scaling
96 properties of the signal measured from space. For example, several works have shown the fractal (and
97 multifractal) behaviour of clouds acquired from visible and infrared sensors (Lovejoy et al., 2001;
98 Siebesma and Jonker, 2000; Tessier et al., 1993).

99 Thus, in order to avoid the impact of clouds, most of the scaling studies of surface reflectances
100 have been applied to images specifically chosen to be cloud-free (Alonso et al., 2017; Lovejoy et al.,

101 [2008b](#)). While these works prove the ability of multifractal analysis to investigate the scaling properties
102 of images of surface reflectances, no seasonal properties/differences were really explored yet.
103 Moreover, multifractal models were initially developed for modelling turbulence and atmospheric
104 processes ([Parisi and Frisch, 1985](#); [Schertzer and Lovejoy, 1987](#)), leading to many studies on
105 atmospheric dynamics ([Lovejoy et al., 2008a](#); [Lovejoy and Schertzer, 2010c](#)) but many fewer on
106 continental surface characteristics.

107 Recently, with the increasing use of spaceborne optical sensors, several algorithms have been
108 developed to take into account the effects of the atmosphere on the reflectances measured by satellite
109 ([Hagolle et al., 2010](#); [King et al., 1999](#); [Liang et al, 2001](#); [Rahman and Dedieu, 1994](#)). One of the most
110 recent is the MACCS-ATCOR Joint Algorithm, called MAJA. It relies on the MACCS (Multi-sensor
111 Atmospheric Correction and Cloud Screening) method – dedicated to aerosol optical depth estimation,
112 clouds detection and atmospheric corrections ([Hagolle et al, 2015](#); [Lenoble et al., 2007](#); [Rouquié et al.,
113 2017](#)) – and includes characteristics from the ATCOR algorithm (Atmospheric and Topographic
114 Correction; [Richter et al., 2006](#)) such as the correction of slope effects. This processing chain is used
115 within the French THEIA land data center (<http://www.theia-land.fr>) on multispectral images from
116 Landsat-8, S2 and from the recent Venµs satellite ([Manivasagam et al., 2019](#)).

117 Considering this context, key scientific questions still need to be addressed in the field of spatial
118 characterization of surface reflectances. The research work described here was realized in order to
119 deepen our understanding of multifractal analysis and, above all, to demonstrate its value for
120 improving spatial characterization of land surfaces. A particular reflection was made on the physical
121 origin of the specific scales identified, relating them to image texture and asking whether the latter
122 were due to:

- 123 • The seasonal specificities of the surface?
- 124 • The geographic specificities of the surface?
- 125 • The signal acquisition system of the satellite sensor?

126 To answer these questions, we investigated the occurrence of scaling properties in surface images
127 collected by S2 sensors and corrected by MAJA. The dataset consists of S2-MAJA maps located in
128 France at different dates within 2016. This research work was first focused on the southwestern region
129 of France, on which a selection of particular dates was made to minimize the impact of clouds. In this
130 region, particular consideration was given to the temporal dynamics of scaling properties due for
131 example to seasonal effects. In addition, other regions of France with a different land use were then
132 selected to investigate the impact of geographic specificities on the scaling properties. For each
133 selected map, we analysed and compared the scaling properties of S2 images acquired in blue (B2),

134 green (B3), red (B4), near infrared (B8) and short-wave infrared (B11, B12) bands. The latter were
135 chosen considering their strong relation with Earth surface physical and biophysical processes and their
136 common use in previous studies involving multifractal analysis of surface reflectances ([Alonso et al.,](#)
137 [2017](#); [Lovejoy et al., 2008b](#)). The several bands of S2 sensors enable to explore the possible impact of
138 the acquisition wavelength on scaling behaviour. Moreover, vegetation and soil moisture indexes
139 obtained from these bands were also analysed to investigate how the combination of such bands can
140 impact the scaling laws. This research study was done by applying multifractal analysis methods in the
141 framework of the Universal Multifractal model ([Schertzer and Lovejoy, 1987](#)).

142

143 2 Case study and data

144 2.1 Study area

145 Southwestern France is a highly anthropized site, mainly composed of agricultural land. More than
146 half of the ground area is exclusively dedicated to the development and management of crops
147 throughout the year. Among the most common are sunflower, corn, wheat, barley, soybean and
148 rapeseed crops. Considering also the urban areas which constitute ~ 2% of the landscape, the
149 Southwest region has a spatial structure which is largely characterized by anthropic factors. These have
150 an impact on the spatial heterogeneity of physical and biophysical variables such as surface
151 temperature, soil moisture or vegetation. Natural factors also play a significant role in the spatial
152 structuring of this landscape and its related processes, such as, for example, natural grasslands and
153 forests (~ 10% of the surface) (Baup et al., 2015; <http://osr-cesbio.ups-tlse.fr/~oso>).

154 The climate in this region is temperate, inducing seasonal effects that impact significantly the
155 temporal dynamics of surface variables. Springs are characterized by heavy rain, and summers are hot
156 and dry with temperatures often reaching 35 °C (Battude, 2017). These climatic conditions have a
157 direct effect on the landscape structure, more particularly on the evolution of this structure over the
158 seasons. Indeed, the crop cycles – described by harvest and sowing periods – contribute to modify the
159 heterogeneity of the surface as the seasons go by. In addition, the differentiation between winter and
160 summer crops implies that not all crops have the same seasonal evolution and do not affect the
161 landscape structure in the same way over time.

162 The study area is part of a French observatory, named Regional Spatial Observatory (RSO) and
163 labelled by the French National Institute of Sciences of the Universe (INSU) since 2007. The purpose is
164 to answer scientific questions related to the functioning and evolution of continental surfaces, but also
165 to deal with more applied issues related to land use planning and sustainable management of
166 resources (Dejoux et al., 2012). In order to ensure this, both in-situ (Béziat et al., 2009; Burel, 2018;
167 Tallec et al., 2013) and remote sensing observations (Baup et al., 2012) are combined. Thanks to these
168 data, several works related to crop monitoring by remote sensing have been carried out in this region
169 such as the study of wheat and rapeseed (Fieuzal et al., 2013) or even the estimation by agro-
170 meteorological model of crop yields such as sunflower (Fieuzal and Baup, 2015), soybean (Baup et al.,
171 2015; Betbeder et al., 2016) and corn (Fieuzal et al., 2017). These studies also allowed the preparation
172 of recent space missions such as Venµs or S2, subsequently leading to high spatial resolution modelling
173 of biomass and crop yields by optical data (Ameline et al., 2018; Battude et al., 2016; Claverie et al.,
174 2012). The optical products from these new satellites now benefit from particular attention in this

175 region, since they are made available on the THEIA platform (<http://www.theia-land.fr>), corrected for
176 atmospheric effects by the MAJA processing chain.

177 Due to the heterogeneity of its surfaces and the growing number of data acquired at various
178 spatial scales, southwestern France is particularly well suited to the application of multifractal analysis
179 tools such as those we used in this study. In this context, the present work consists of the analysis of
180 MAJA-corrected surface reflectances acquired in the region of the 30TYP S2 tile (according to the tiles
181 format and nomenclature provided by ESA; [Gatti and Naud, 2017](#)) whose spatial extent is shown in
182 Figure 1. More specifically, our study was focused on a smaller study area of 40 x 40 km² covering part
183 of the Gers French department (red square in Fig.1) and located within the 30TYP tile. This area in the
184 Gers was chosen due to the predominant place of agricultural surfaces, limiting the impact of urban
185 areas (cities, roads) on the spatial variability of physical and biophysical processes. In addition, due to
186 certain constraints related to analysis tools (sample size, computation time) and cloud cover, we
187 deliberately chose our study area by selecting a smaller zone than the area corresponding to the 30TYP
188 tile which covers an area of 100 x 100 km².

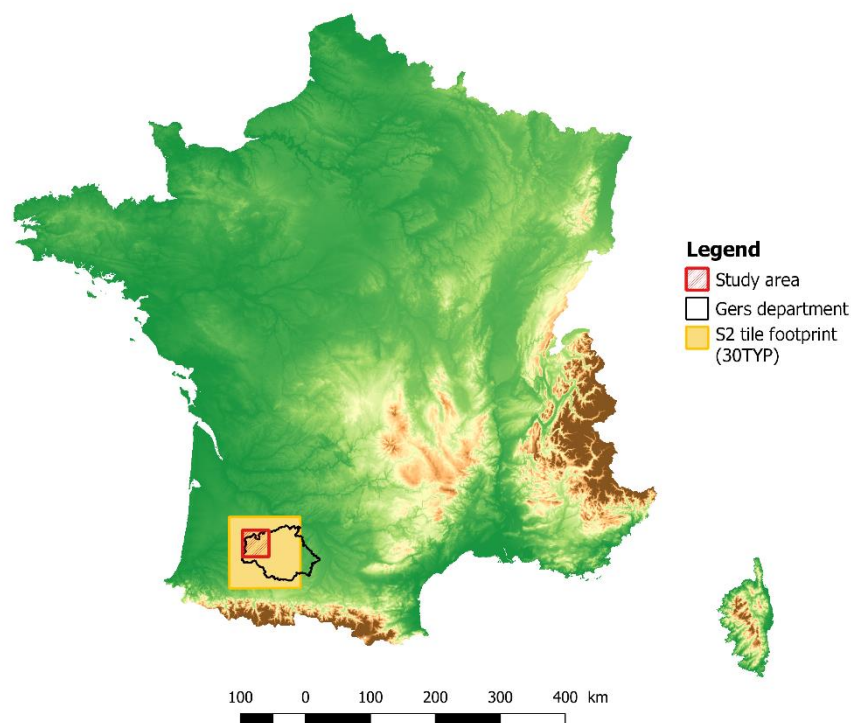


Figure 1 : Study area in the Southwest region of France.

189 This study investigates the period extending from January to December 2016, using the Sentinel-
190 2A satellite. With a ten-day revisit time, S2A provides a sufficient temporal frequency to analyze the
191 global temporal dynamics of the properties of continental surfaces over an entire year. Finally, we can
192 also point out that the Southwest region has a relatively low average cloud cover throughout the year

193 compared to other regions of France located further north and south. Considering the impact of clouds
194 on optical products, this criterion is not negligible in the choice of the study area.

195 2.2 Sentinel-2/MAJA products

196 We analyzed surface reflectances acquired in six spectral bands by the S2-A Multi-Spectral
197 Instrument (MSI; [Martimort et al., 2007](#)) and then corrected by the MAJA chain within the CNES
198 processing center (Centre National d'Etudes Spatiales; <https://theia.cnes.fr>). The selected bands are:
199 three bands in the visible domain – blue B2 (0.50 μm), green B3 (0.56 μm), red B4 (0.66 μm), one band
200 in near infrared B8 (0.84 μm) and two bands in the short-wave infrared B11 (1.6 μm) and B12 (2.2 μm).
201 Note that all these bands are defined at 10 m resolution (in terms of pixel size) except B11 and B12
202 which are defined at 20 m resolution. The six bands were chosen because of their important link with
203 the physical and biophysical processes of continental surfaces. The visible/near infrared has already
204 been the subject of studies aimed at transcribing the fractal ([De Cola, 1989](#); [Lam, 1990](#); [Qiu et al., 1999](#))
205 and multifractal ([Alonso et al., 2017](#); [Lovejoy et al., 2008b](#)) properties of continental surfaces.

206 In addition to reflectances, two optical indexes were also studied: the Normalized Difference
207 Vegetation Index (NDVI; [Tucker and Sellers, 1986](#)) and the Normalized Soil Moisture Index (NSMI;
208 [Lampkin and Yool, 2004](#)), which respectively account for the condition of the vegetation and the level
209 of soil moisture. Since these variables are frequently used to investigate the functioning of continental
210 surfaces, it is important to study their scaling properties. NDVI is constructed from surface reflectances
211 (ρ) in the red (band B4) and near infrared (B8) wavelengths. NSMI is obtained from the combination
212 of the bands in short-wave infrared B11 and B12:

$$214 \quad NDVI = \frac{\rho_{B8} - \rho_{B4}}{\rho_{B8} + \rho_{B4}} \quad (10 \text{ m}) \quad 1$$

$$216 \quad NSMI = \frac{\rho_{B11} - \rho_{B12}}{\rho_{B11} + \rho_{B12}} \quad (20 \text{ m}) \quad 2$$

217 These indexes are built in a non-linear way from reflectances in individual spectral bands, and it
218 is interesting to check if (and how) potential scaling properties of ρ could result in scaling properties
219 of the indexes.

220 Cloud masks from MAJA were used to identify images with less than 15% cloud cover. Fifteen
 221 dates were then selected over 2016 in order to limit the impact of clouds on surface reflectances and
 222 their scaling properties. These dates are shown in the chronogram in Figure 2, where we can see a
 223 distribution of dates that covers all seasons, with a larger number of images in summer. For the fifteen
 224 images, an average cloud cover of 2% was estimated over the period, with only 4 dates having more
 225 than 2% of clouds – the largest cloud cover being obtained on June 23 with 12.5%. Each of the
 226 remaining cloud pixels in the selected images were considered as missing data and were interpolated
 227 by bilinear interpolation. Note that no missing data other than that related to clouds was observed in
 228 our data, and that the corresponding size of the selected images are 4096 x 4096 pixels and 2084 x
 229 2048 pixels respectively for 10 m and 20 m resolution products.

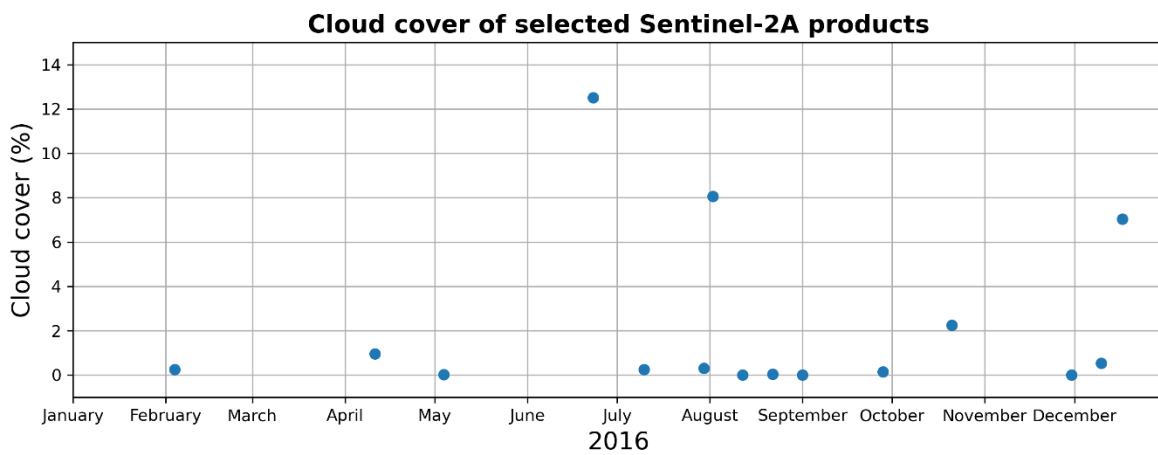


Figure 2 : Cloud cover of Sentinel-2A products selected over the January-December 2016 period, and corrected for atmospheric effects by MAJA. The corresponding dates were selected with less than 15% cloud cover.

230

231 **3 Multifractal analysis methodology**

232 During the last century, researchers realized that many geophysical processes could present scale
233 invariance or scaling properties (Kolmogorov, 1941; Mandelbrot, 1967; Richardson, 1922). Multifractal
234 analysis makes it possible to highlight these scaling behaviours, enabling spatial characterization of
235 surface processes. To do this, several scale sensitive statistical estimators are applied to the surface
236 reflectances acquired by satellite sensors. These estimators can be the power spectrum, the structure
237 functions or the statistical moments. Different multifractal models, like the Universal Multifractal
238 model (Schertzer and Lovejoy, 1987), are used to parametrize the scaling properties obtained from the
239 statistical estimators. In this paper, a focus was made on one estimator, the power spectrum, which
240 has the advantage to detect, in an easy and rapid way, some spatial properties of geophysical variables
241 over different space scales (Lovejoy et al., 2008b). Structure functions and statistical moments (in the
242 framework of Universal Model parametrization) were also investigated in this work. Since they confirm
243 the properties already observed by power spectra, and for sake of simplicity and clarity, the theory
244 and the methodology relative to these estimators are not presented in this part but are detailed
245 respectively in Appendix A and B. More generally, we invite the reader to refer to Appendix A for a
246 deeper understanding of the theoretical framework of fractals and Universal Multifractal model.

247 Because our study treats time series of satellite images, the statistical estimators will be computed
248 for each image available during the studied period. Therefore, from this analysis two types of spatial
249 characterization will be studied: (1) a mean characterization resulting from the average of individual
250 estimators computed within the period, and (2) a "date by date" characterization of each image,
251 allowing us to have access to the temporal evolution of the scaling properties of the products studied.
252 For the calculation of estimators, note that, since only satellite images will be treated in this study, we
253 will focus on the two-dimensional versions of the following techniques.

254 *3.1 Vocabulary : scale and resolution*

255 For the sake of clarity, we primarily want to define the terms "resolution" and "scale" within the
256 meaning of fractal mathematical formalism. Indeed, although their meaning is close to that usually
257 used by geophysicists, in particular in remote sensing, it can be appreciably different and could lead to
258 certain misinterpretations in the remainder of this paper.

259 If we consider a space of (geometric) dimension $D = 2$ discretized over a set of intervals or pixels,
260 the resolution λ of any fractal entity can be defined as the number of pixels present along a dimension
261 of space. In the case of a satellite image of size 4 x 4 pixels, the resolution of the fractal entity is
262 therefore equal to $\lambda = 4$ (Fig.3). Inversely proportional to the resolution, the scale l is associated with
263 the size of a pixel and is defined according to λ :

264

$$l = \frac{L}{\lambda}$$

265

3

266 with L the extent or domain size along one dimension and corresponding to the greatest value of scale
 267 l . Unlike λ , the scale l is a dimensional variable (meters for an image). In the case of our 4 x 4 satellite
 268 image, the scale is $l = L/4$, which is the size of an elementary pixel. If we now aggregate our image on
 269 grids made up of new pixels twice as wide (or with an area four times as large), the resolution of the
 270 image will be taken equal to $\lambda = 2$, and the scale will be $l = L/2$.

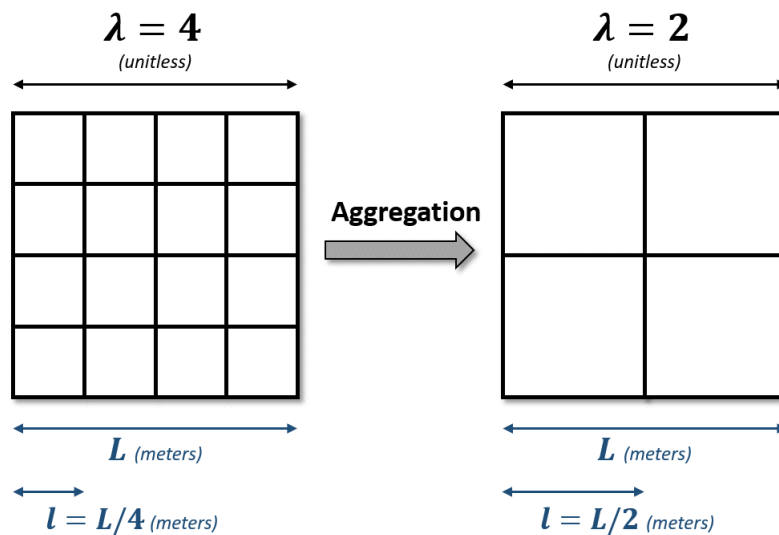


Figure 3 : Illustration of the concepts of resolution and scale from a two-dimensional fractal entity (satellite image).

271

272 In this way, any variable with fractal properties is defined on a continuum of resolutions (or scales)
 273 which extends from the finest resolution (to which is associated the smallest scale *i.e.* the size of an
 274 elementary pixel), to the coarsest resolution ($\lambda = 1$) corresponding to the largest scale ($l = L$).

275 In the following, scale sensitive statistical estimators will be called as “scaling estimators”, and will
 276 be computed on a spatial support represented either by resolutions λ , scales l , varying window size
 277 Δx or even spatial frequencies k , depending on the initial definitions of the estimators. Studying the
 278 evolution of surface products according to these spatial variables ($\lambda, l, \Delta x, k$) provides access to
 279 fractal/multifractal properties that will be called hereafter as “scaling properties”. A scale range,
 280 corresponding to a set of scales with similar scaling properties, will be called as “scaling regime”.

281 3.2 Power spectrum: evidence of spatial properties

282 Power spectra are highly sensitive to the transition scales (also called “scale breaks”) that are
283 located between scaling regimes characterized by different scaling properties. In a first step, the two-
284 dimensional power spectral density $P(k_x, k_y)$ of the data under analysis, X , is estimated:

$$285 \quad P(k_x, k_y) = |fft(X)|^2$$

286 4

287 with P the power spectral density (PSD) defined on both vertical and horizontal image axes,
288 corresponding respectively to k_x and k_y wavenumbers (spatial frequencies). Here, the estimation of
289 the PSD is done through a two-dimensional *fft* or Fast Fourier Transform. Then, the more convenient
290 one-dimensional isotropic angle-integrated power spectrum $E(k)$ is obtained (Lovejoy et al., 2008b;
291 §8), that provides a 1D representation of the 2D *fft* power spectra:

$$292 \quad E(k) = \int_{\|\vec{k}\|=k} P(\vec{k}) d\vec{k}$$

293 5

294 where k is the modulus of the wavenumber and $\|\cdot\|$ is the Euclidean norm. Since it expresses space
295 frequencies, k is directly related to the spatial resolution λ . If the process presents scaling properties,
296 the spectrum should follow a power law, where β is the negative slope of $E(k)$ on a log-log graph:

$$297 \quad E(k) \approx k^{-\beta}$$

298 6

299 The symbol “ \approx ” indicates an equality within the limits of slowly varying functions. This method makes
300 it possible to identify the range of scales (here the values of k) where there are scaling properties, and
301 this through a first parameter called the “spectral exponent”.

302 3.3 Scale break detection

303 When analysing the scaling estimators, a segmentation algorithm (D’Errico, 2017) was
304 systematically used to detect the scale breaks and thus the scale ranges in order to avoid subjective
305 detections. This algorithm is based on the non-linear regression of “spline” curves (piecewise
306 polynomial functions), making it possible to estimate the position of slope breaks within scaling
307 estimators. It requires three input parameters: (1) the position on the x-axis of each point of the cloud
308 to be segmented (support vector *i.e.* k , Δx or λ), (2) the value of each point on the y-axis (estimator
309 vector such as E) and (3) the number of breaks nb_knots to detect. Once applied, the function outputs
310 the position of the detected nb_knots breaks. Note that the number of breaks is a parameter which
311 is therefore defined by the user. In order to check if the position is reliable and independent of the

312 number of breaks proposed, the algorithm was applied for several values of *nb_knots* (*nb_knots* =
313 1, 2, 3), ensuring the same break was well detected in each case.

314

315 **4 Results**

316 **4.1 Mean characterization over the period**

317 Figure 4 shows the mean power spectra estimated over the full period (January-December 2016)
 318 of the different S2/MAJA products presented in Section 2. This represents an average spectrum based
 319 on individual spectra obtained within the period. Each spectrum is plotted in log-log coordinates, with
 320 the horizontal axis converted into spatial scale $l (= k^{-1})$ expressed in kilometres. The scale break
 321 detection algorithm identified two scale breaks common to all products: a first break l_1 located
 322 between a hundred meters and one kilometre, and a second break l_2 located on smaller scales. l_1 and
 323 l_2 positions are summarized in Table 1 for both surface reflectances and optical indexes.

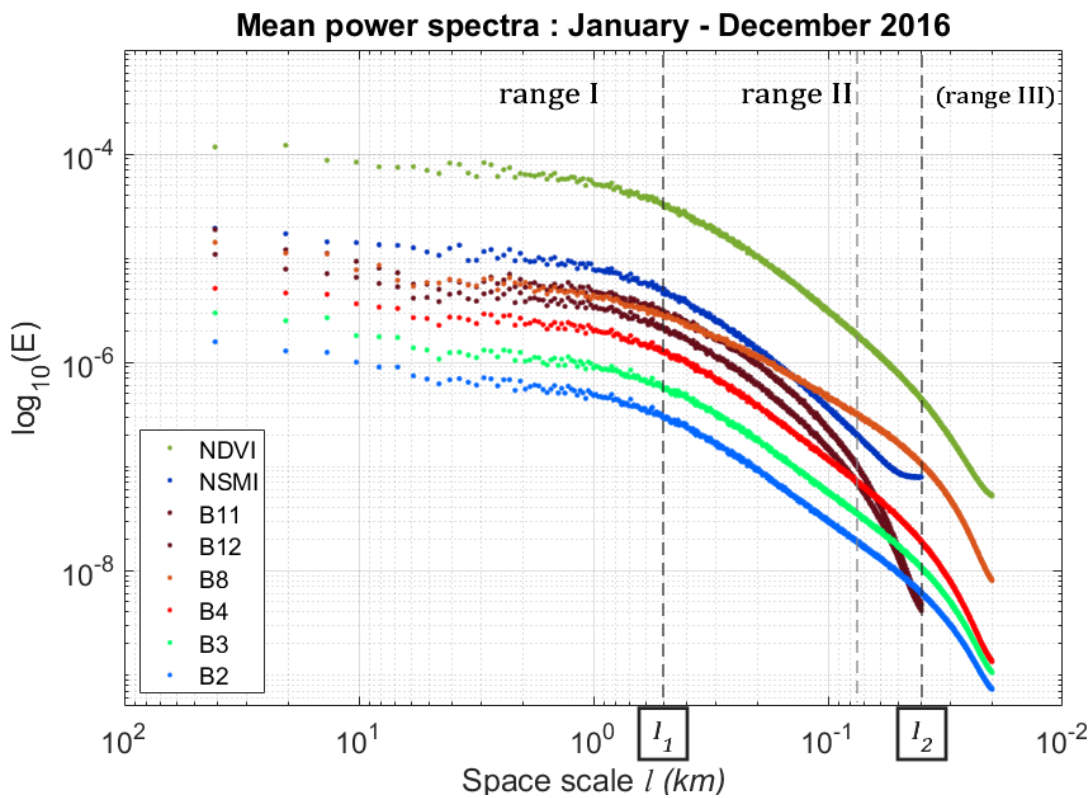


Figure 3 : Mean angle-integrated power spectra estimated from Sentinel-2 bands and indexes, over the period January-December 2016.

324

325 Considering all these products, the first scale break l_1 was estimated at about 400 m (computed
 326 from the geometric mean of individual breaks). The spectral slope β of each mean power spectrum
 327 (Table 2) was estimated for both ranges I ($l_1 > l$) and II ($l_1 > l > l_2$) (the last range (III) will be
 328 considered separately in Section 6). On range I, slopes are very close to zero and do not present
 329 significant differences within the different products ($0.2 < \beta < 0.3$). These results are in contradiction
 330 with those of [Lovejoy et al. \(2008b\)](#) who identified slopes greater than 1 ($\beta = 1.17 \pm 0.08$) on a similar

331 scale range (500 m – 25 km), from reflectances acquired in comparable spectral bands (MODIS sensor;
 332 [Justice et al., 1998](#)). Similarly, our results are also not comparable to those of [Alonso et al. \(2017\)](#) who
 333 observed only a single scaling regime over the range (120 m – 15 km), for reflectances from similar
 334 bands (Ikonos and Landsat-7 satellites). However, because of the different multifractal formalism used
 335 by [Alonso et al. \(2017\)](#), our spectral slopes and their results are difficult to compare. Nevertheless,
 336 [Alonso et al. \(2017\)](#) have indeed shown the presence of scaling properties over this range of scales.
 337 Thus, the differences between our results and those of [Lovejoy et al. \(2008b\)](#) and [Alonso et al. \(2017\)](#)
 338 can be linked to geographic specificities (central Spain in their cases) or to the fact that their analyzes
 339 were carried out on one or two particular dates, not allowing the acquisition of the scaling behaviour
 340 averaged over a period as we did here.

341

	Grid spacing (m)	SPECTRA		STRUCTURES	MOMENTS
		l_1 (m)	l_2 (m)	l_1 (m)	l_1 (m)
B2	10	420	37	580	360
B3		390	37	539	300
B4		390	39	555	360
B8		290	37	495	320
NDVI		380	48	575	340
B11	20	320	75	503	370
B12		370	75	565	390
NSMI		420	-	695	430

Table 1 : Positions of the scale breaks l_1 and l_2 estimated by segmentation (D’Errico, 2017) on mean Sentinel-2 power spectra, structure functions and statistical moments.

342

343 Regarding the behaviour of spectra on range II ($l_1 > l > l_2$), β values are greater than one and
 344 are relatively different between the products. The lowest slope value is estimated for the near infrared
 345 band ($\beta \approx 1.4$) and the greatest ones are found for the two short-wave infrared bands ($\beta \approx 2$).
 346 Considering visible bands (B2, B3, B4) and optical indexes (NDVI, NSMI), the β values obtained are
 347 between 1.6 and 1.8. We can also note in the spectra an increase of slope with the wavelength for
 348 reflectances acquired in visible domain.

349 As for the fine-scale break l_2 , the latter is located at around 40 m for products with a resolution
 350 of 10 m (visible bands, near infrared band and NDVI) and is shifted towards larger scales (75 m) for
 351 products with a 20 m grid resolution (short-wave infrared bands B11 and B12). It even disappears with
 352 regard to the NSMI soil moisture index (flat spectrum for the fine scales). If we analyze this last

353 spectrum in more detail, a rise in the curve is observed towards the points located at the finest scales
354 (~ 50 m). This rise reflects the presence of noise in the NSMI data which affects the highest frequencies
355 of the image, artificially creating an excess of energy at fine scales. Since bands B11 and B12, from
356 which the NSMI has been estimated, do not show such an increase in their spectra at fine scales, their
357 combination must be the source of the noise observed on the soil moisture index at high frequencies.
358 This phenomenon was expected since the ratio of two signals is known to contain more noise (due to
359 a reduction in the signal to noise ratio). Similarly, we can note that the NDVI is characterized by a l_2
360 scale break (at 48 m scale) that is less pronounced than for the bands B3 and B8 from which it was
361 generated, with a very slight rise of the spectrum for highest frequencies (scales close to 20 m).
362 Although the phenomenon is much less pronounced than for the NSMI, the vegetation index also
363 appears to be somewhat impacted by the combination of bands.

364 Continuing the comparison between bands and indexes, these results tend to show that on fine
365 scales (*range II*) the vegetation index is characterized by an average spectral behaviour comparable
366 to the band B4 (red) but different from band B8 (near infrared). However, the spectral slope of the soil
367 moisture index is found to be close to those obtained from both of the short-wave infrared bands B11
368 and B12. Another general remark on the spectra can be made on the quality of the scaling law which
369 is very good on *range II*, considering all products ($R^2 \approx 0.98$), while it is strongly reduced on *range I*
370 ($R^2 \approx 0.6$). This can partly be explained by the sampling of power spectra which is sparser on low
371 frequencies (large scales) than on high frequencies (fine scales).

372 The results presented above were compared with the two other approaches (structure functions
373 and statistical moments presented in Appendix B) and their results (figures in Appendix C) were found
374 to be consistent with those of the power spectra. The positions of l_1 scale break obtained from these
375 approaches are presented in Table 1, with the corresponding parameters detailed in Table 2 for both
376 ranges.

377

(a)	Grid spacing	Scale range	β	R^2_{β}	α	C_1	$R^2_{K(q)}$	H	R^2_H
B2	10 m	range I	0,30	0,62	1,96	0,007	0,89	-0,27	0,98
B3			0,31	0,64	1,86	0,006	0,91	-0,28	0,99
B4			0,24	0,55	1,81	0,007	0,92	-0,30	0,99
B8			0,29	0,60	1,80	0,004	0,89	-0,27	0,99
NDVI			0,20	0,50	1,91	0,005	0,88	-0,31	0,99
B11	20 m	range I	0,28	0,56	1,83	0,004	0,87	-0,29	0,99
B12			0,25	0,53	1,81	0,005	0,88	-0,29	0,99
NSMI			0,22	0,55	2,23	0,005	0,83	-0,29	0,99
(b)	Grid spacing	Scale range	β	R^2_{β}	α	C_1	$R^2_{K(q)}$	H	R^2_H
B2	10 m	range II	1,63	0,99	1,97	0,11	0,96	0,32	0,98
B3			1,67	0,99	1,78	0,11	0,96	0,32	0,98
B4			1,76	0,99	1,56	0,12	0,96	0,35	0,97
B8			1,37	0,98	1,53	0,09	0,96	0,23	0,93
NDVI			1,84	0,99	1,61	0,13	0,96	0,37	0,98
B11	20 m	range II	1,96	0,97	1,53	0,11	0,97	0,35	0,95
B12			1,96	0,98	1,53	0,12	0,96	0,37	0,97
NSMI			1,81	0,98	1,97	0,11	0,96	0,35	0,98

Table 2: Scaling parameters obtained from multifractal analysis on range I (a) and range II (b) of Sentinel-2 products, over the period January-December 2016. The values of β , α , C_1 and H were obtained from linear regressions applied on mean statistical estimators. However, R^2 coefficients presented here corresponds to the average of all coefficients obtained on each date. Note that $R^2_{K(q)}$ is the average of the coefficients obtained on every moment curves. Similarly, H is obtained from the average of H_x and H_y , as R^2_H is obtained from $R^2_{H_x}$ and $R^2_{H_y}$

379

380 4.2 Temporal and spatial variations of the scale breaks

381 Different scaling regimes were observed on the S2/MAJA products, separated by specific scale
382 breaks: a first break l_1 detected on the scaling estimators between hundreds of meters and one
383 kilometre, and a second break l_2 detected on the high frequencies of the mean power spectra. To
384 explain the origins of these observations and their link with the spatial texture of the data, the
385 evolution of these breaks was analysed according to the seasonal and spatial evolution of the surface.

386 To explore the seasonal evolution of the scale breaks l_1 and l_2 , the scale break detection algorithm
387 was applied on each individual spectrum obtained between January and December 2016 (*i.e.* spectra
388 computed for each available image). The time series of l_1 (Fig.5.a) is found to be largely variable for all
389 bands, whereas l_2 (Fig.5.b) presents a quite constant evolution during the period. Therefore, these

390 two scale breaks detected on power spectra seem to be of a different nature: on the one hand the
 391 large-scale break l_1 seems to be related to the seasonal evolution of surface processes, on the other
 392 hand the fine-scale break l_2 position is found to be relatively independent of time. In addition, we note
 393 that the ratio between l_2 and the minimum scale (that is to say the grid resolution of the image for a
 394 given band) is comparable considering both 10 m and 20 m bands: $l_2 \approx 4 \times l_{min}$. This relationship
 395 appears to be constantly present throughout the period.

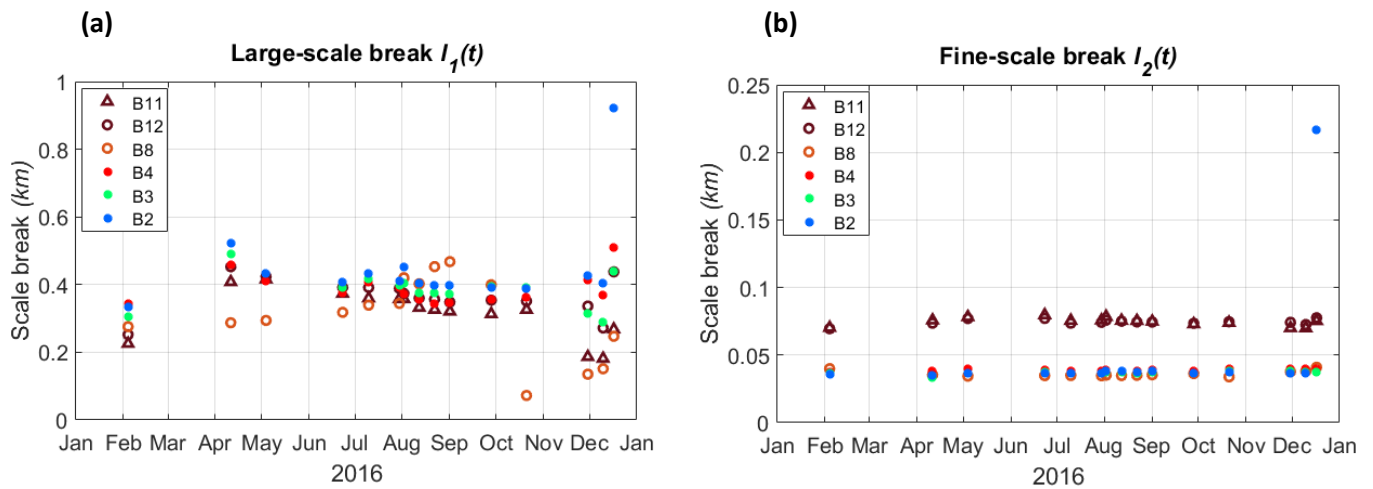


Figure 5: Time series of the position of the scale breaks l_1 (a) and l_2 (b) estimated from the power spectra of the Sentinel-2 bands, over the period January-December 2016.

396 To understand the origin of these two scales, it was investigated whether the geographic
 397 specificities related to the study area could have an impact on the position of these scale breaks. Since
 398 l_1 relies on the seasonal evolution of the surface, changes in land use should have an effect on the
 399 position of l_1 , whereas they should not on l_2 . To so investigate this question, several regions of France
 400 with a different land use were selected. These regions were identified using the land use products
 401 supplied by the Scientific Expertise Center for Land Use CES OSO (<http://osr-cesbio.ups-tlse.fr/~oso>).
 402 This analysis was done from the land use map for 2016, obtained from S2 products, and presented in
 403 Figure 6. In addition to the original tile (30TYP), five new S2 tiles were added to our study,
 404 corresponding to five types of surface or classes: *crops*, *forest*, *urban*, *vineyards* and *grasslands*. The
 405 areas were chosen so that the extracted S2 images contain for each class a majority of pixels belonging
 406 to the class in question. For the six zones, we extracted the surface reflectances acquired in two
 407 spectral bands defined at 10 m (B2, B8) at two dates in the year 2016 (summer, autumn). Table 3
 408 specifies the position of the breaks l_1 and l_2 detected in the power spectra. In general, considering all
 409 the zones, bands and dates provided, these results show a break l_2 which varies between 32 m and 42
 410 m, and a break l_1 which is between 72 m and 1 km. Therefore, the position of the large-scale break l_1
 411 seems much more impacted by the type of surface, the date of acquisition and the spectral band (when
 412 it is defined according to the same spatial resolution) than the fine-scale break l_2 .

413 These temporal and spatial behaviours confirm the different nature of the transition scales. In the
 414 following sections, demonstrations are made to relate these scale breaks with (1) the seasonal and
 415 geographic specificities of the surface in the case of the scale l_1 and (2) with the acquisition system of
 416 optical images for l_2 .

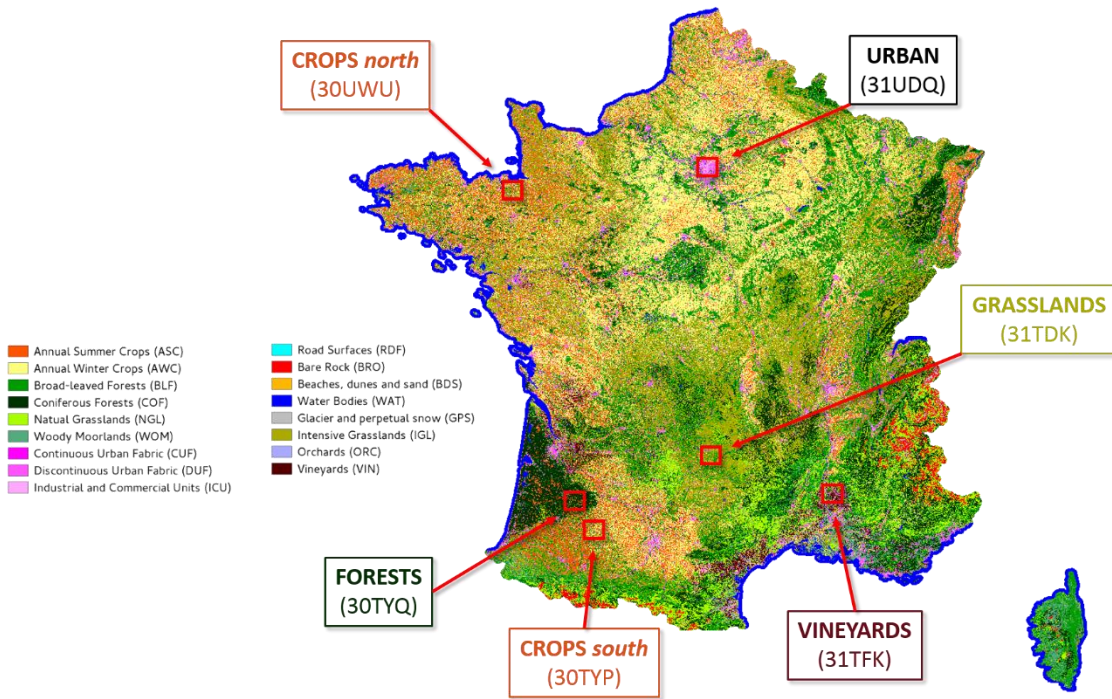


Figure 6: Six study areas corresponding to the five types of surface (crops, forest, urban, vineyards, grasslands) identified from the 2016 land use map of France CES OSO.

	Summer				Autumn			
	Band B2		Band B8		Band B2		Band B8	
	l_1	l_2	l_1	l_2	l_1	l_2	l_1	l_2
crops south	440	36	340	35	390	37	72	34
forests	860	32	1000	39	910	34	210	38
crops north	310	36	380	34	380	33	460	41
urban	-	40	-	42	-	39	-	40
vineyards	150	34	180	35	150	34	130	35
grasslands	280	37	350	34	140	37	250	37

Table 3: Positions (in meters) of the scale breaks l_1 and l_2 estimated from the power spectra corresponding to six types of surface, observed in bands B2 and B8, in summer and autumn 2016.

417 **5 A large-scale break related to the seasonal and geographic specificities of the surface**

418 **5.1 Seasonal dynamics linked to crop cycle**

419 To propose an explanation to the large-scale break l_1 between ranges I and II, first the temporal
 420 evolution of the scaling properties of the S2 products can be observed on the two ranges, for the
 421 studied period. The time evolution of the spectral exponent β estimated for the six bands (Fig.7)
 422 exhibits on the fine scale range (*range II*) a seasonal cycle of the spectral slopes, while no particular
 423 behaviour is detected on large scales (*range I*). In addition, the dynamics of the β values are much
 424 higher on fine scales ($0.8 < \beta < 2.4$) than on large scales ($0 < \beta < 0.9$).

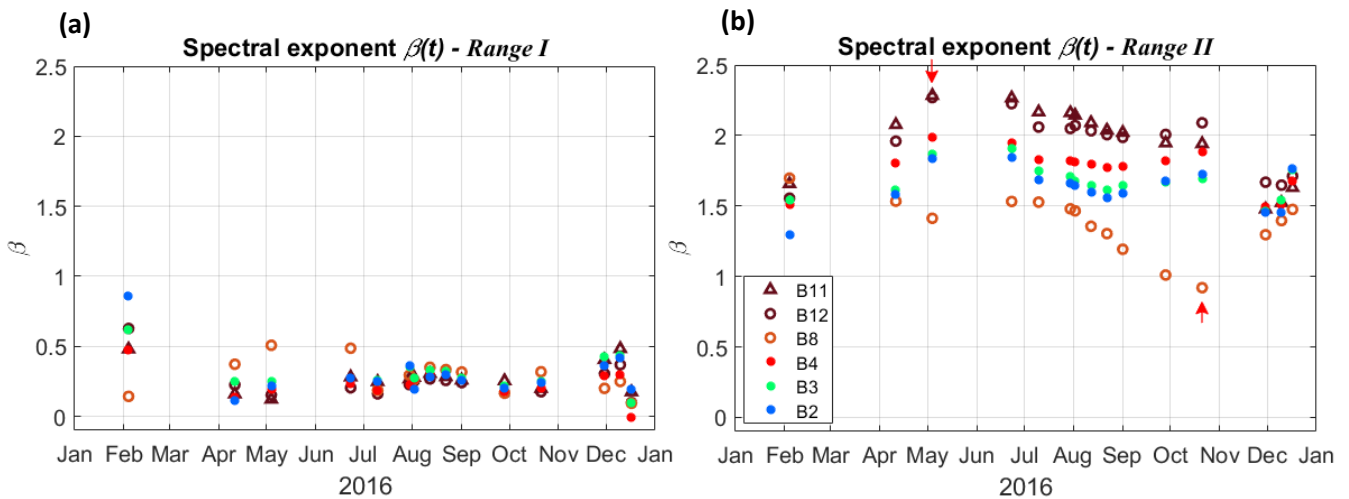


Figure 7: Time series of spectral exponents of the Sentinel-2 bands, over the period January-December 2016 for range I (a) and range II (b).

425 If we analyze in more detail the cycle observed on fine scales (Fig.11.b), the dynamic detected on
 426 the visible bands (B2, B3, B4) is characterized by minimum values in February, August and then
 427 December ($\beta \approx 1.5$) and peaks of values around May-June then October ($\beta \approx 1.9$). For the short-
 428 wave infrared bands (B11, B12), the maxima are placed at the same dates as for the reflectances
 429 acquired in the visible domain but they do not present the same intensity: the peak observed in May-
 430 June is very marked – with values reaching 2.3 (arrow on the graph) – while that of October is weaker
 431 (even almost non-existent for B11). Considering the near infrared B8 band, a reversed cycle is found
 432 on *range II* (compared to the other bands), characterized by minimum values for the months of May-
 433 June and then October. In addition, the spectral exponent of B8 shows a much more marked dynamic
 434 in the second half of the period – values of β falling from 1.5 (July) to 0.9 (October, arrow on the graph)
 435 – than in the first half of the period ($1.4 < \beta < 1.7$). Thus, for the month of October, the images
 436 acquired in the near infrared seem to lose spatial autocorrelation, with a spectral slope approaching
 437 zero. This temporal evolution of scaling properties is illustrated on Figure 8 where the individual power
 438 spectra of four near infrared images are compared: two images acquired in summer (July 10 and July

30) and two images acquired in autumn (September 28, October 21). For these spectra, the fine-scale break l_2 was estimated at around 35 m in all four cases, confirming its "instrumental" origin (see Section 6). On the other hand, two different l_1 breaks were obtained: at around 350 m for the July spectra (dark red and light red curves) and the September spectrum (light green curve), and at 72 m for October (dark green curve). It appears that the l_1 break in July moves towards fine scales in October, combining with l_2 . Thus, these results show that a switch takes place from two scaling regimes in summer and early autumn to a single one in autumn. On this period, we observe the decrease of the spectral slope with time, confirming the evolution presented in Figure 7.b. For October 21, the regime seems to be found on a scale range close to the global range provided by our data (72 m – 41 km), and presenting a spectral slope less than 1.

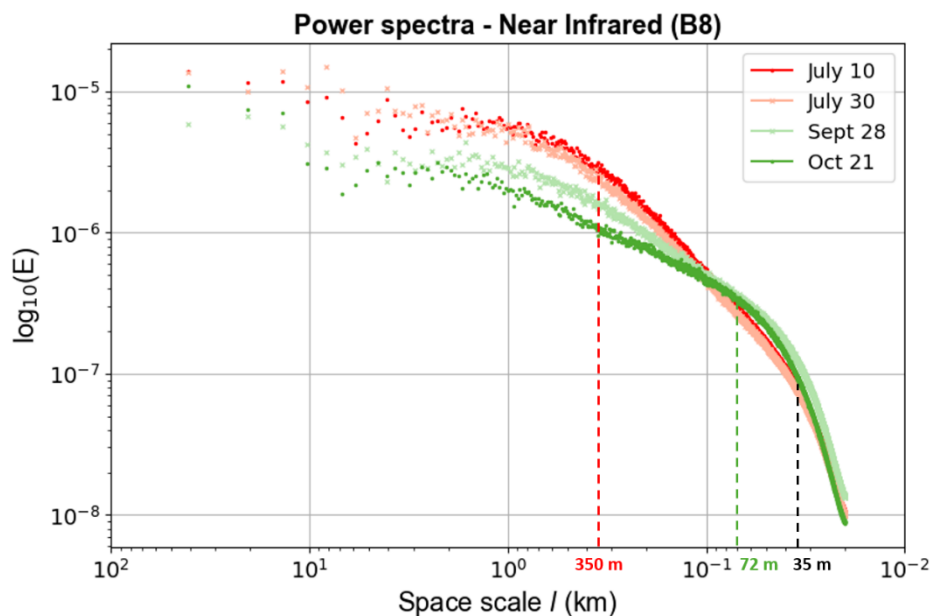


Figure 8: Power spectra of surface reflectances acquired in summer and autumn 2016 by the near infrared band B8.

449

These results confirm the link between the fine-scale break l_1 and the temporal and spatial variations of the surface. Therefore, considering the predominant place of agricultural surfaces in the study area, our hypothesis is that this time variation is related to the crop cycle (with the more abrupt changes associated with harvesting and sowing), taking place during the mid-season periods (autumn, spring). The evolution of crops, including the transitions between winter and summer crops, would explain the cycle that we detect here. This hypothesis can also explain the reverse nature of the cycles between near infrared and visible/short-wave infrared behaviours. Indeed, the reflectances acquired in the near infrared, being very sensitive to vegetation, should exhibit more spatial variability when

458 the crops are well developed, therefore for periods between the harvest and sowing phases. If we take
459 the example of corn, which is one of the main crops grown in Southwestern France, the harvesting
460 phase usually occurs between August and November. This corresponds to the fall observed on the β
461 parameter estimated from the band B8. Thus, a harvest phase would correspond to a loss of spatial
462 heterogeneity in these wavelengths, revealing a poorer and less structured signal than during a
463 vegetation growth phase. In addition, the gradual drop in spectral slope (beginning of decrease
464 observed from August) can be explained on the one hand by the staggering of the sowing phases
465 between August and November (according to the calendar of each farmer), and on the other hand by
466 the senescence phase occurring before harvest and helping to reduce the ability of vegetation to
467 reflect solar radiation (less chlorophyll). Conversely, visible reflectances behave differently by being
468 sensitive to other elements of the surface such as roads, buildings, or even the bare soil that appears
469 once harvesting is complete. Thus, at mid-season, when the vegetation has just been sown, the surface
470 shows more spatial variability in the visible than in the near infrared domain.

471 To develop the interpretation proposed above, our results were compared with some
472 characteristics of the surface, and more particularly with its spatial structure. The study area is mainly
473 composed of agricultural plots of which the majority reveals an area between 0.3 and 130 ha,
474 corresponding to a plot width (assuming square areas) of around 50 to 1000 m. This could be identified
475 thanks to the Graphic Parcel Register (GPR; <https://geoservices.ign.fr>) for the year 2016, allowing
476 access to the delimitation, the surface area and the type of crops of all the agricultural plots in the
477 department of Gers. An average plot width of around 170 m was deduced from this data. Figure 9
478 below shows a part of the GPR for our study area, for which the plots were classified according to their
479 extent. We observe a large number of plots whose characteristic widths are between 100 m and 1 km
480 (areas in light blue, green and yellow). Although the agricultural plots reveal quite different structures,
481 of more or less elongated shapes, even with rounded "edges", the more or less systematic cutting that
482 they generate on the surface could be at the origin of the large-scale break l_1 observed in our results.
483 In this way, the fine-scale regime observed on *range II* would reflect the *intra-plot* variability
484 governed by multifractal physical and biophysical processes linked to crops (soil moisture, vegetation,
485 etc.), while the large-scale regime on *range I* would represent the *extra-plot* variability resulting from
486 the aggregation of several fine-scale processes which together would not have particular multifractal
487 properties. This interpretation would explain the uncorrelated and less complex nature of the
488 reflectances on large scales (very low values of β and C_1 parameters on *range I*), whereas more
489 complex fractal and multifractal properties were obtained on *range II* tending to justify the interest
490 of applying a multifractal model like that of Universal Multifractals on these fine scales.

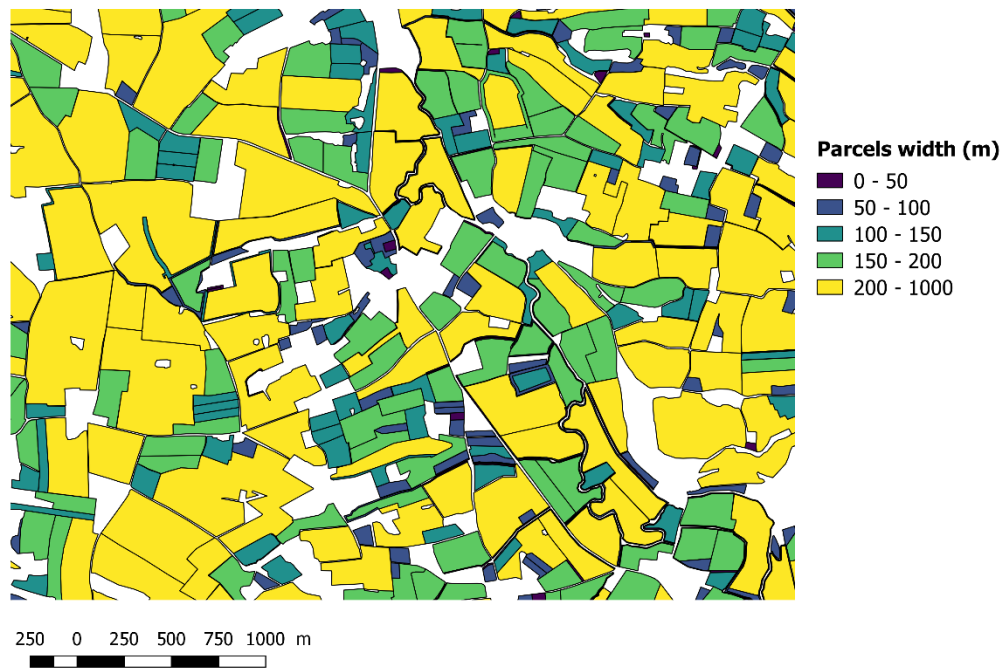


Figure 9: Classification of agricultural plots (parcels) according to their surface area estimated from the Graphic Parcel Register for the year 2016. The data represented here correspond to an area of $\sim 3 \times 3 \text{ km}^2$ extracted in our main study area. The plot widths were calculated from the surface areas by approximating them to square areas.

491

492 **5.2 Simulation of the impact of agricultural plots on the scaling properties of surface**
 493 **reflectances**

494 In order to check if the previous explanations are realistic, an example of agricultural area was
 495 simulated with the same scaling properties as the S2 reflectances. To do this, a synthetic multifractal
 496 field was simulated by applying the Universal Multifractal model (see Appendix D), to which we
 497 combined an image containing the delimitations of agricultural plots obtained from the GPR-2016 on
 498 our study area (Fig.10.a). The multifractal field was generated by defining the values of the universal
 499 parameters of the model at $\alpha = 2$, $C_1 = 0.1$ and $H = 0.3$, corresponding to the values obtained for
 500 the band B2 on *range II* (mean behaviour, Table 2.b). The combination method simply consists in
 501 adding the spatial texture generated by the agricultural plots to the image of the simulated multifractal
 502 field. To reproduce a more realistic texture, the entities ("plot" objects) have been previously grouped
 503 into six classes: the same pixel value between the min and max values of the initial field, *i.e.* 0 and 1,
 504 has been assigned to each of these classes. These have been defined according to the area of the
 505 entities: the plots have been grouped into object classes whose area is within a predefined interval.
 506 Here, these intervals have been defined from the area quantiles, so as to have the same number of
 507 objects in each class. Although this classification does not correspond in any way to reality, it allows to
 508 reproduce artificially the differences in intensity of reflectances according to the type of soil (different

509 types of crop, bare soil, forest, etc.) which can be observed on the real data (Fig.10.b). The final surface
 510 generated (i_{agri}) is based on an integrated multiplicative cascade (i_{MF}) defined at all scales of the
 511 image, but to which is added the spatial texture of agricultural plots ($offset_{parcels}$) such as:

512
 514
 513

$$i_{agri} = i_{MF} + offset_{parcels}$$

7

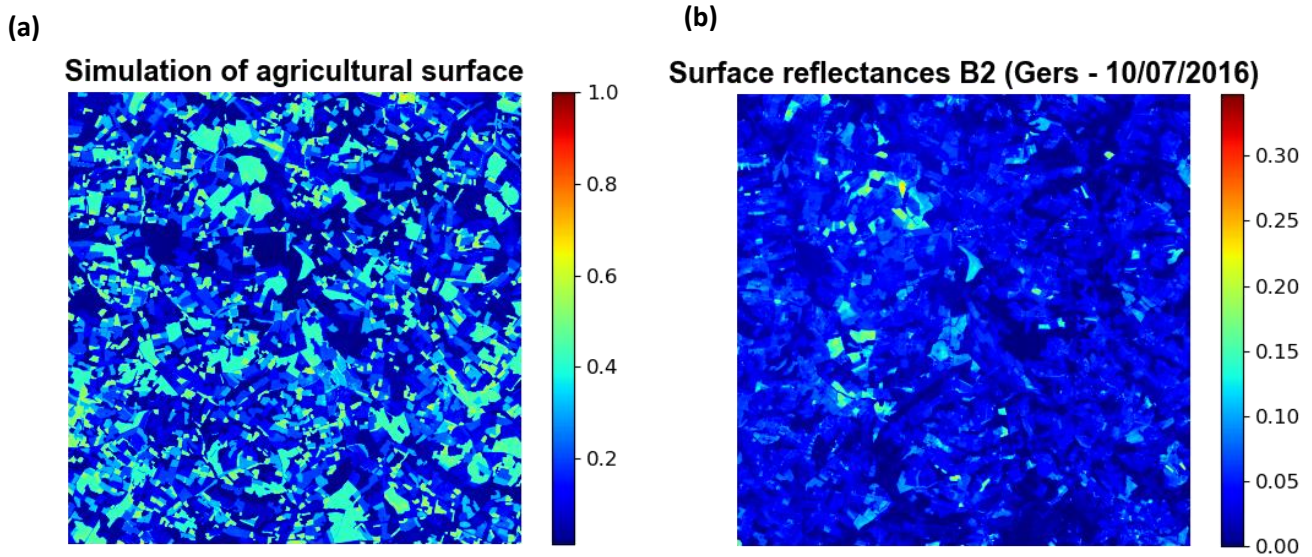


Figure 10: Simulated and real surface reflectances corresponding to the study area (Gers, 30TYP tile). The simulation was obtained by combining a multifractal field and the GPR of the year 2016.

515

516 The graph in Figure 11 represents the power spectrum obtained for the simulated agricultural
 517 surface i_{agri} (green curve), compared to that obtained from the original simulated multifractal field
 518 i_{MF} (blue curve). The power spectrum estimated from an image i_{B2} of real surface reflectances (gray
 519 curve) is also presented. i_{B2} is the image acquired in B2 band on July 10 (image presented in Fig.10.b).
 520 For the sake of clarity, the finest scale of the simulated fields was set to the size of a S2 pixel (band B2)
 521 so that $l_{min} = 10 m$.

522 As expected, a single scaling regime is observed on the original multifractal i_{MF} with no scale break
 523 detected, whereas two scaling regimes are indeed observed for i_{agri} , with a scale break l_1 detected at
 524 around 200 m. This break noticed on i_{agri} is located a few dozen scales higher than the finest scale,
 525 which is comparable to the results obtained from S2 data. The two regimes exhibit behaviours close
 526 to those observed on the ranges I and II of i_{B2} , at least in terms of the shape of the curves. Indeed, on
 527 large scales (*range I*) a spectral slope smaller than 1 is obtained, while a spectral slope greater than 1
 528 is estimated on fine scales (*range II*). Therefore, the twofold scaling behaviour of S2 products is clearly
 529 transcribed by our simulation : it was evidenced here that a definite link exists between the division of

530 the area by agricultural plots and the presence of two scaling regimes – with a transition scale which
 531 is effectively close to the average plot scale. We note a flattening at the finest scales of the i_{agri} power
 532 spectrum, which reveals the presence of noise localized on the last octave (20 m – 40 m). A similar
 533 flattening was observed on the NSMI spectrum (Fig. 4) due to the creation of noise by the application
 534 of the NSMI formula. For the i_{agri} spectrum, this phenomenon is probably due to the "local" effect of
 535 the boundaries of agricultural plots at resolutions close to the pixel size. Indeed, these boundaries
 536 generate abrupt transitions from one pixel of the image to its neighbour, leading to an increase of the
 537 energy at this specific scale. The fact that this flattening is observed on the simulated agriculture
 538 spectrum (i_{agri}) but is not on the real spectrum (i_{B2}) may be due to (1) the impact of our combination
 539 method creating artificial boundaries that are more abrupt on the simulation than on the real case,
 540 and/or (2) the impact of the optical acquisition system (Section 6) on the real spectrum which inversely
 541 affects the fine scales (slope increased) but that is not considered in this simulation.

542

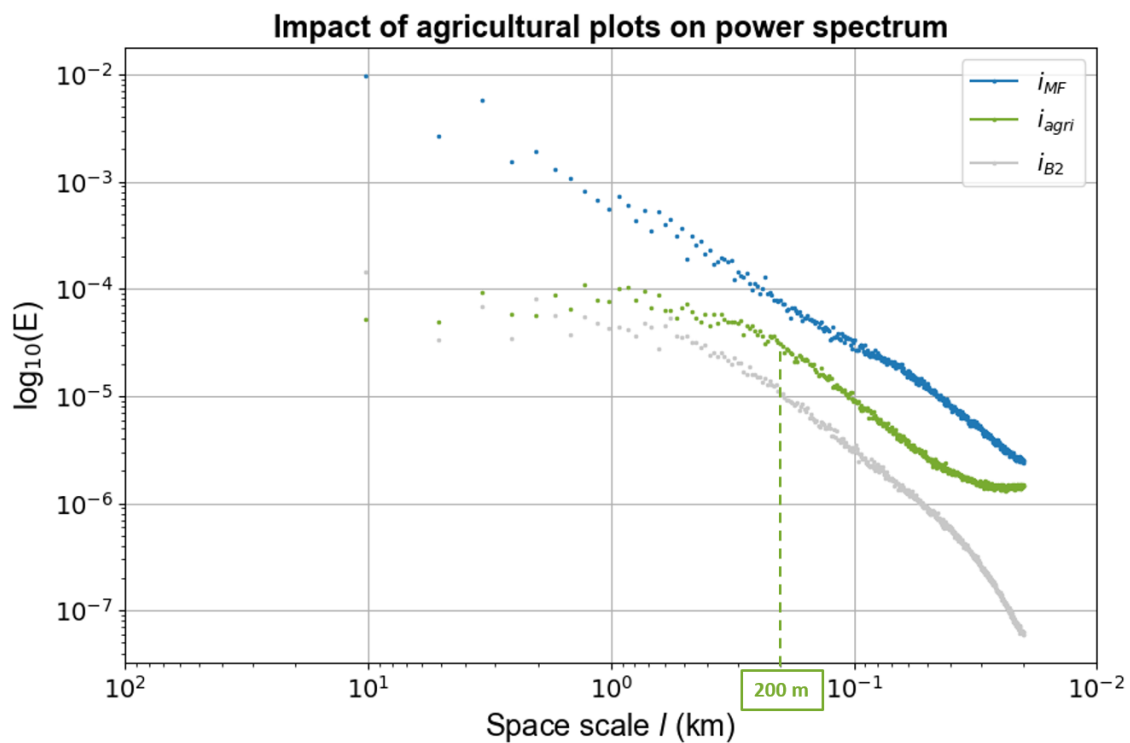


Figure 11: Impact of agricultural plots on power spectrum of simulated surface reflectances (Fig.10.a).

543

544

545 **6 A fine-scale break related to the optical acquisition system of the satellite sensor**

546 This section is dedicated to the interpretation of the scale break l_2 observed on power spectra
547 between range II and III.

548 *6.1 The effect of the Modulation Transfer Function*

549 Considering the relatively small variations of the position of the fine-scale break l_2 with time and
550 land use, this break could be a consequence of the acquisition system which affects the high spatial
551 frequencies of images. In any optical system, this phenomenon is modelled by a function called MTF
552 (Modulation Transfer Function; [Norton et al., 1977](#)). It defines the system ability to transmit the spatial
553 frequencies of the observed scene. It is affected by several components of the system such as the
554 objective (the lens) and detectors, but also by external factors such as temperature or atmospheric
555 diffusion. During the satellite launch in space, these last factors can modify the MTF. However, the
556 latter varies very little over time once the satellite is finally placed in its orbit. In this way, it could be
557 the reason for the systematic break observed at the maximum frequencies acquired by the MSI-S2
558 sensor.

559 As part of the validation and calibration procedures for S2 products, [Gascon et al. \(2017\)](#)
560 determined the modulation transfer function of the MSI imager after the launching of the S2-A
561 satellite. This has been estimated from an image acquired in the B2 band on Arizona (USA), applying
562 the "slanted-edge" method ([Viallefont-Robinet and Léger, 2010](#); [Reichenbach et al., 1991](#)) which has
563 often been used to determine the MTF of space sensors like Landsat ([Carnahan and Zhou, 1986](#)) or
564 Ikonos ([Choi, 2002](#)).

565 *6.2 Simulation of the impact of optical acquisition system on scaling properties*

566 Given what has been described above, we made the hypothesis that the scale break l_2 detected
567 on the power spectra is directly related to the impact of the MTF on the scaling properties of the
568 "original" signal acquired by the MSI sensor.

569 To demonstrate this, a multifractal field was simulated (according to the method presented in
570 Appendix D which is based on the Universal Multifractal model) representing the scaling behaviour of
571 a hypothetical original signal, *i.e.* with fractal/multifractal properties not impacted by the optical
572 system. Here, we suppose that the original signal should have the same scaling properties as those
573 obtained on *range II* (Figure 4), with parameters set as $\alpha = 2$, $C_1 = 0.1$, $H = 0.3$ (values obtained
574 from band B2, Table 2.b). Note that we did not consider in this simulation the impact of agricultural
575 plots (Section 5.2).

576 Then, to simulate the impact of optical acquisition system, we generated an MTF according to the
 577 model of [Viallefont-Robinet and Léger \(2010\)](#). The latter proposed a parametric model taking into
 578 account various characteristics of the system such as the diffraction effects of the lens and the
 579 sampling effects of the detector, as well as the impact of other factors such as the displacement of
 580 electronic charges or correction of the focus that can be achieved after launching the satellite. Here,
 581 for the sake of simplicity, only the major effects on MTF have been taken into account, namely
 582 diffraction and sampling. Finally, this function was applied to the simulated multifractal field. The
 583 operation was carried out in the Fourier domain by applying the following relation:

$$I_{filtered} = MTF \times I_{simu}$$

8

584 with $I_{filtered}$ and I_{simu} the Fourier transform modules of the images corresponding respectively to
 585 the field filtered by MTF and to the simulated original field.

588 For the calculation and generation of the MTF, the equations used are detailed in Appendix E. Two
 589 parameters are to be defined in the model: (1) the sampling frequency f_s of the detector which was
 590 fixed here at the resolution of the simulated multifractal field, that is, at 1 pixel^{-1} (here, the scale is
 591 expressed in pixels, the corresponding unit is thus "pixel⁻¹" for frequency), (2) the cut-off frequency f_c
 592 of the lens (also defined at 1 pixel^{-1} for the sake of simplicity) which is responsible for the diffraction
 593 effects. In reality, the value of f_c depends on the characteristics of the lens (diameter, focal length) and
 594 on the wavelength of the incident radiation. However, since we did not have access to the value of the
 595 MSI sensor focal length, we were unable to determine the actual cut-off frequency. According to
 596 [Gascon et al. \(2017\)](#), the MTF they measured for band B2 has a value close to 0 at the sampling
 597 frequency. This shows that, for this band, f_c should be positioned close to f_s , at slightly higher
 598 frequencies. According to this assumption, we will suppose thereafter that $f_s \approx f_c = 1 \text{ pixel}^{-1}$.

599 Figure 12 presents the power spectra of the original field (blue curve), of the field affected by the
 600 MTF by considering only the sampling effects of the detector (dark brown curve), only the diffraction
 601 effects of the lens (light brown curve), and finally by considering both the sampling and diffraction
 602 effects (red curve). For the sake of clarity, the scale axis of the figure was adapted to correspond to
 603 that of S2 spectra: the finest scale of the fields was set to the size of a S2 pixel (band B2) so that $l_s =$
 604 $\frac{1}{f_s} = 10 \text{ m}$. The fine scales are found to be impacted by the MTF the more significantly: the energy of
 605 the signal is attenuated for the high frequencies, resulting in an increase in the spectral slope for scales
 606 lower than 100 m. This behaviour is obviously more pronounced when the two effects (diffraction and
 607 sampling) are combined (spectrum in red). The scale break detection algorithm (Section 3.3) was
 608 applied on this last spectrum, which revealed a scale break located at 48 m ($= 4.8 l_s$). The relationship

609 found here between the position of the break and the resolution of the image is comparable to that
610 deduced from our S2 products, for which the coefficient linking the two scales was close to 4 over the
611 entire period (Fig.5.b).

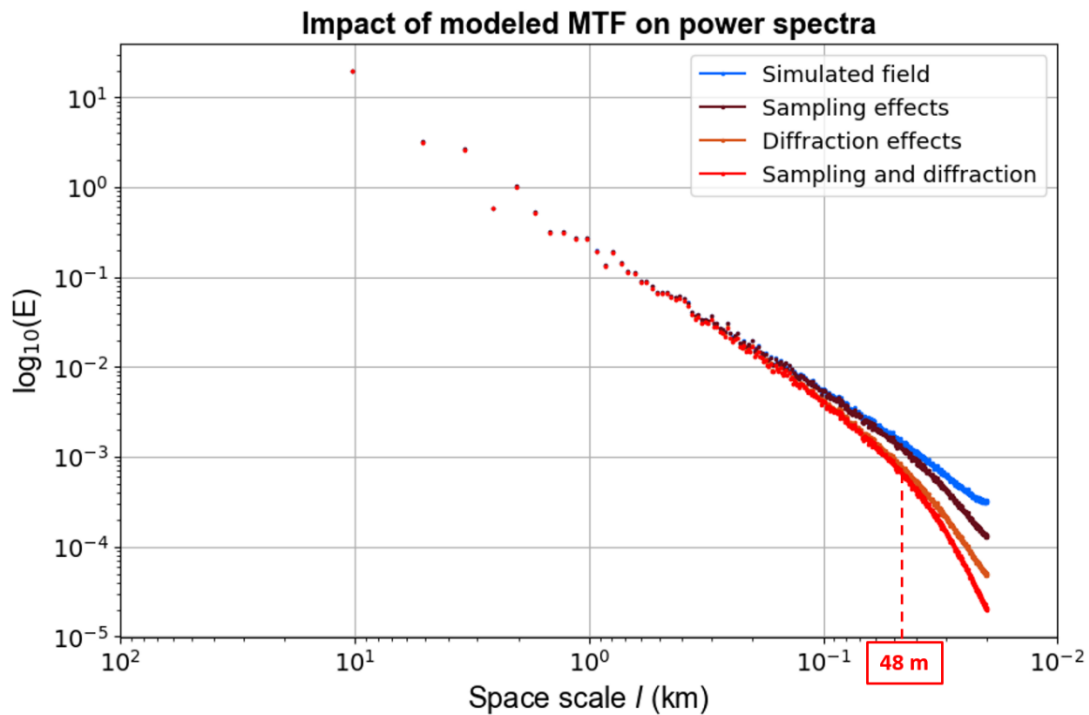


Figure 12: Impact of the Modulation Transfer Function on the power spectra of simulated multifractal fields. The MTF was generated from the analytical model described in [Viallefont-Robinet and Léger \(2010\)](#).

612

613 The MTF model used in this study enables to reproduce the impact of the acquisition system on
614 the high frequencies of the image. Although the value of the cut-off frequency was chosen in a more
615 or less arbitrary way, the scale break generated on the original spectrum is similar to that of the fine-
616 scale break l_2 observed on the spectra of our S2 bands. These results provide a physical explanation
617 as to the origin of this fine-scale break, which is the result of the acquisition system, *i.e.* MTF, that
618 reduces spatial variations (smooths the image) at high frequencies inducing a decrease of energy at
619 the finest scales. The strong similarity of the red and light brown curves in Figure 12 suggests that the
620 rupture is mainly related to the diffraction effects of the lens. Further research may be needed to be
621 conclusive of the latter findings by integrating other factors influencing the MTF (electric charge
622 displacements, temperature, etc.). However, the interpretations given here show the potential of
623 power spectra estimators to easily detect the on-orbit effective resolution of the optical sensors, *i.e.*
624 the actual resolution below which any signal will not correctly reflect the spatial properties of the
625 surface.

626

627 7 Conclusion and perspectives

628 In this study, the Universal Multifractal model was applied to analyze the scaling behaviour of
629 Sentinel-2 surface reflectances and optical indexes acquired over the Southwest region of France, and
630 corrected for atmospheric effects by the MAJA processing chain. Images with minimal cloud cover
631 were identified for the whole year 2016 based on the MAJA cloud and shadow detection process. This
632 allowed us to access the dynamics of the scaling properties of surface reflectances, which, to our
633 knowledge, had not yet been explored to date due to the significant impact of clouds on optical data.
634 Different scaling regimes, separated by significant scale breaks, were evidenced from these data.
635 Through the use of multifractal simulations, we could explain the scaling properties and especially the
636 position of scale breaks observed in the images studied by two types of phenomena.

637 First, a scale break between hundreds of meters and one kilometre was detected that was found
638 to be impacted mostly by the seasonal and geographic specificities of the surface. From this specific
639 break, two scaling regimes with distinct scaling properties were evidenced. Over large extents,
640 reflectances were found to be poorly autocorrelated, revealing spatial properties quite distant from
641 those commonly detected in the phenomena such as topography, rain and clouds. Over small extents,
642 higher autocorrelation were observed in our data, reflecting the presence of multifractal processes
643 comparable to those in turbulence. In addition, a seasonal cycle of scaling parameters was revealed
644 for the fine-scale regime, while no particular temporal behaviour was noticed for the large-scale
645 regime. We showed that this twofold scaling behaviour is directly linked to the physical properties of
646 the surface, and in particular to the agricultural plots that impact its spatial structure. In this way, the
647 fine-scale regime would reflect the *intra-plot* variability governed by multifractal physical and
648 biophysical processes related to crops (soil moisture, vegetation, etc.), while the large-scale regime
649 would represent the *extra-plot* variability resulting from the aggregation of several fine-scale processes
650 which together would not have particular multifractal properties. These interpretations were validated
651 by combining our knowledge of the surface and multifractal simulations, which confirmed that the
652 transition scale between the two observed regimes corresponds to the average size of the agricultural
653 plots for this region.

654 Second, a singular scaling behaviour was systematically detected at high frequencies on the
655 various optical products: a systematic spectral drop was observed near the resolution of the sensor,
656 reflecting an alteration of the signal measured on fine scales, independently of the observed surface
657 type. This phenomenon was found to be an instrumental artefact generated by the Modulation
658 Transfer Function of the optical system which affects the high frequencies of the acquired image.
659 Therefore, this analysis highlighted the performance limits of the Sentinel-2 optical sensor MSI. The

660 particular break detected at fine scales (*i.e.* about 40 m scale) would correspond to the *effective*
661 *resolution* of the sensor, beyond which the fine-scale fractal properties of the observed surface cannot
662 be detected.

663 To complete the interpretation of the spectral drop artifact observed near the resolution of the
664 sensor, the multi-scale model proposed here could be deepened by integrating other factors
665 influencing the Modulation Transfer Function such as electric charge displacements or temperature.
666 In order to be in the real conditions of Sentinel-2 data acquisition, the real value of cut-off frequency
667 of the MSI sensor could be used in the MTF model. It would also be interesting to establish precisely
668 why the fine-scale break is located at $\sim 4 l_s$ (mathematical relation) and possibly propose a method to
669 correct this artefact statistically. Here, a fractal stochastic disaggregation could be applied to replace
670 the "impacted" field at fine scales ($l < 4 l_s$) by a simulated multiplicative cascade that would have the
671 scaling properties of the large-scale field ($l > 4 l_s$). Based on scaling properties, this type of method
672 preserves the probability distribution from large to fine scales. Such fractal-based techniques have
673 been applied on geophysical variables such as rainfall ([Rebora et al., 2006](#); [Sharma et al., 2007](#); [Deidda,](#)
674 [2000](#); [Gires et al., 2012](#)) or soil moisture ([Bindlish and Barros, 2002a](#); [Kim and Barros, 2002b](#); [Mascaro](#)
675 [et al., 2010](#)). Therefore, it would be interesting to apply this "stochastic" correction to the S2/MAJA
676 surface reflectances and to verify if the spectral drop is still observed near the resolution of the sensor.
677 However, since the disaggregation is based on a random generator, an ensemble of possible fields can
678 be proposed from just one set of scaling parameters. Therefore, this kind of methodology may not be
679 fully suitable in the case of operational hydro-agricultural applications, in particular for determining
680 the position of the extremes. Another possibility to correct the effects of the optical system would be
681 to estimate the original "non-impacted" image from the MTF model proposed in this study. It would
682 simply consist of reversing equation 8, and then estimating the inverse Fourier transform of the field.
683 This solution would respect the deterministic behaviour of the original signal, giving the correct
684 position of the extremes not only statistically valid but also punctually. The challenge would be to get
685 the exact MTF model corresponding to the MSI sensor. Although several methods have been proposed
686 so far to estimate on-orbit MTF ([Lei and Tiziani, 1988](#); [Viallefont-Robinet and Léger, 2010](#); [Xu et al.,](#)
687 [2014](#)), differences persist between empirical and modelled MTF ([Gascon et al., 2017](#)). In the case of an
688 improperly parametrized MTF, this methodology could further alter the signal and generate more
689 scaling artifacts.

690 This research work demonstrated the potential of the multifractal analysis for spatial
691 characterization of surface reflectances. The theoretical background on which this analysis is based
692 makes it possible to provide an interpretation that is both different and complementary to the more
693 usual texture analysis methods, thus improving our understanding of surface processes and their

694 spatial heterogeneity. Thanks to the Universal Multifractal model, we were able to identify specific
695 sets of scales ranging from tens of meters to tens of kilometres and on which specific statistical
696 properties were detected. The observed scaling regimes were found to be the results of the
697 combination at different scales of both natural and anthropogenic factors. This spatial characterization
698 could be useful to precisely define the set of scales to consider in information extraction from remote
699 sensing images and help calibrating current surface models which take little account of scaling
700 behaviour, operating on either local or global scales (Ko et al., 2016). It would be possible to verify
701 whether the products synthesized by these models comply with the expected scaling regimes, and if
702 not to constrain the statistics of these models on certain scale ranges. In addition, the present analysis
703 showed that the particular scale ranges detected are directly related to the seasonal (and geographic)
704 specificities of the surface and that these variations should be taken into account in this type of spatial
705 analysis.

706 Finally, this study showed the potential of multifractal analysis to assess the reliability of satellite
707 products to transcribe what was observed by the sensor. Due to its genericity, the method presented
708 here could be applied to other optical satellite data (Landsat-8, Venµs), even defined at very high
709 resolution (Ikonos). It could also be used to analyze products from different technologies such as radar
710 (Sentinel-1) or passive microwaves (Neuhauser et al., 2019). From a more operational point of view,
711 this could be useful either for the preparation of future space missions in order to determine the
712 capacity of the sensors to detect the inherent scaling properties of the surface processes, or for post-
713 launch assessment to detect in an easy and rapid way the effective resolution of satellite sensors. This
714 methodology complements more usual validation methods enabling a better estimation and
715 understanding of the effective resolution of different products derived from satellite acquisitions.

716

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723 from the French THEIA land data center.

724 Appendix A: Theory of Multifractals

725 During the last century researchers realized that many geophysical processes could present scale
726 invariance properties. This was first anticipated by Richardson (1922) in the case of turbulence: he
727 described turbulent flows as cascade processes that transfer kinetic energy from large to small scales.
728 Based on this approach, statistical models of turbulence were proposed such as the famous
729 Kolmogorov law (1941) to describe velocity increments. Later research generalized the study to take
730 into account the heterogeneity of the energy flux (Kolmogorov, 1962; Oboukhov, 1962; Yaglom, 1966).
731 Multi-scale models such as multiplicative cascades were proposed to reproduce scale invariance
732 properties through the use of fractal geometry. Later, scale invariance was noticed in other geophysical
733 fields. For example, in his study of the coast of Britain, Mandelbrot (1967) revealed the presence of
734 fractal properties in topography.

735 1) *From fractal sets to multifractal fields*

736 The concept of fractal has been used in many works related to multi-scale analysis and geophysical
737 modelling. Indeed, the term “fractal” refers to any entity (time series or 2D/3D random field) in which
738 each part presents similar properties, geometrically or statistically, to the ensemble. In this manner,
739 the structure of a fractal entity is characterized by scale invariance.

740 Monofractal stochastic models aim to represent the simple scaling behaviour of geophysical
741 processes. In this context, the fractal dimension or scaling parameter is assumed to be unique,
742 restricting multi-scale modelling to a specific class of variability. However, most geophysical processes
743 are characterized by more complex statistics. In case of operational hydrology, rare and extreme
744 events, present in precipitation or soil moisture for example, correspond to high-order statistics and
745 need to be detected (Hubert et al., 1993). Therefore, multifractal models, characterized by an infinite
746 spectrum of fractal dimensions, have been proposed to account for a more exhaustive set of statistics.
747 Schertzer and Lovejoy (1987), based on the findings of Parisi and Frisch (1985), initially established the
748 multifractal formalism through the fundamental equation:

$$749 \Pr(\Phi_\lambda > \lambda^\nu) \approx \lambda^{-c(\nu)}$$

750 9

751 where Φ_λ is a positive normalized random scalar process, time series or random field defined on R^2
752 or R^3 . The symbol “ \approx ” indicates an equality within the limits of slowly varying functions. The mean of
753 the process is assumed to be statistically conserved across scales ($\langle \Phi_\lambda \rangle = const$). λ is the observation
754 resolution, here defined as the number of intervals (or pixels) along a dimension of space. Note that
755 the definition given here is different from that usually encountered in remote sensing. In our case, it is

756 a dimensionless factor that is inversely proportional to the scale l . The latter can be seen as the
 757 sampling time or pixel size for time and space domain processes respectively (see Section 3.1 for a
 758 more detailed definition of scale and resolution concepts).

759 Eq.9 expresses the fact that for a multifractal process, the probability of exceeding a threshold
 760 varies as a power law of the resolution with exponent $c(\gamma)$. This exponent is called as fractal
 761 codimension of the process, depending on the amplitude of thresholds. The thresholds are defined by
 762 the following power law:

$$T_\lambda = \lambda^\gamma$$

10

765 with γ the notion of singularity, characterizing the amplitude of the process independently of the
 766 resolution. Each singularity is associated with a fractal codimension $c(\gamma)$, corresponding to a measure
 767 of the sparseness of the area where the process exceeds the thresholds defined by γ . From a more
 768 physical point of view, high singularities (detected by high thresholds) are related to rare and extreme
 769 events, with high fractal codimensions and inversely low (box-counting) fractal dimensions $D_f(\gamma)$
 770 (Mandelbrot, 1967). Indeed, the latter are related to the dimension of space D through the relation
 771 $c(\gamma) = D - D_f(\gamma)$. Therefore, $c(\gamma)$ can be described as a codimension function, increasing with γ ,
 772 which completely characterizes the multi-scale statistical properties of the field Φ_λ . In general, if the
 773 field is multifractal, $c(\gamma)$ is found to be a convex and positive function (with a fixed point C_1 imposed
 774 by the condition of canonical conservation $\langle \Phi_\lambda \rangle = const$), whereas monofractality is associated to
 775 the trivial case $c(\gamma) = const$.

776 Since probability distributions and statistical moments are related by a Mellin transform,
 777 Schertzer and Lovejoy (1987) proposed an equivalent equation to Eq.9:

$$\langle \Phi_\lambda^q \rangle \approx \lambda^{K(q)}$$

11

780 where $\langle \cdot \rangle$ is the statistical averaging operator, q is the order of the moment ($q \geq 0$), and $K(q)$ is the
 781 moment scaling function. Eq.11 expresses that, for any fixed moment order, statistical moments and
 782 resolution are linked through a power law. Singularities and moment orders are directly linked, since
 783 the moment scaling function $K(q)$ is the Legendre transform of the codimension function $c(\gamma)$.
 784 Similarly to $c(\gamma)$, $K(q)$ is a convex function (with the special case $K(1) = 0$ related to the conservation
 785 of the mean across scales), which entirely characterizes the multifractal field.

786 2) *Multiplicative cascades*

787 Multiplicative cascades are stochastic models that can be used to build multifractal fields.
788 Cascades are multiplicative processes because they are defined by an iterative multiplicative
789 construction, from larger scales to smaller scales. Considering a two dimensional random signal (field),
790 the cascade consists of dividing each pixel at resolution λ_n (with n the construction level of the
791 cascade) into sub-pixels at finer resolution λ_{n+1} , and then to multiply the latter by a random variable
792 $\mu\varepsilon$. This is described by the following equation:

$$\Phi_{\lambda_{n+1}} = \mu\varepsilon \times \Phi_{\lambda_n}$$

793

794

12

795 In this manner, the statistical properties of the field $\Phi_{\lambda_{n+1}}$ are directly related to the statistical
796 properties of the coarser field Φ_{λ_n} . If all the multiplicative random variables used for each step of the
797 iterative construction are independent and identically distributed, and distributed independently of
798 the scale, the final random field (obtained after multiple iterations) presents scale invariant properties.

799 Several models of cascades have been developed so far. First models were built within the
800 framework of turbulence, such as the α -model (Schertzer and Lovejoy, 1984) which corresponds to
801 discrete construction of cascades: the multiplicative random variables are limited to two possible fixed
802 values, respectively leading to increasing or decreasing pixel value when the resolution is refined.
803 Later, more elaborate models were constructed generalizing the discrete case to continuous cascades
804 (Dubrulle, 1994; Schertzer and Lovejoy, 1987, 1991, 1997; She and Levêque, 1994). The latter are based
805 on an infinite number of steps between any pair of resolutions, leading to continuity in scale. The
806 benefit of continuous cascades is twofold. First, they can represent possibly more realistic structures
807 by avoiding any arbitrary discretization of scales. Moreover, they often converge towards random
808 processes that are characterized by a small number of degrees of freedom (special cases of log-
809 infinitely divisible distributions). This is interesting considering that multifractal fields built by
810 multiplicative cascade processes would otherwise need an infinite number of scaling parameters (one
811 for each fractal dimension). For example, She and Levêque (1994) proposed the Log-Poisson model
812 which constructs multifractal fields from the generation of the random variable $\mu\varepsilon$ whose distribution
813 law is of log-Poisson type. The (log-) Poisson laws are part of the (log-) infinitely-divisible laws, just as
814 the (log-) stable laws (including the Gaussian case). The Universal Multifractal (UM) model proposed
815 by Schertzer and Lovejoy (1987) uses random variables $\mu\varepsilon$ of log-stable laws to generate the
816 multifractal field (see Appendix D for more details on the generation of such a field). In both models,
817 only two fundamental parameters are needed to fully define multifractality, *i.e.* to fully characterize
818 the $c(\gamma)$, $K(q)$ functions.

851 words, the larger is H , the smoother is the field. The integrated flux is noted R_λ and is characterized
852 by a power law variation of its stationary increments:

$$\Delta R_\lambda \approx \Phi_\lambda \Delta x^H$$

14

855 where ΔR_λ are the increments (fluctuations of the flux) estimated over a varying window Δx , which is
856 equivalent to the spatial scale l . Note that when $H = 0$, the equation corresponds to the conservative
857 case Φ_λ , and that when $H < 0$ the equation corresponds to the specific “fractional derivation” non-
858 conservative case. Additionally, in the case of two dimensional fluxes, Eq.14 also applies for other
859 directions (*i.e.* Δy increments), with the same exponent H if the process is isotropic.

860 **Appendix B: Estimation of Structure Functions and Statistical Moments**

861 1) *Structure functions: some evidence of non-conservativity*

862 The structure functions were initially used in turbulence by [Kolmogorov \(1941\)](#) to highlight the
 863 scaling behaviour of velocity increments. Then, these methods were widely applied in geophysics, in
 864 particular thanks to their simplicity of construction and interpretation. Indeed, unlike the spectrum
 865 which implies a transform in the Fourier domain, the structure functions simply represent the
 866 fluctuations of the observations in real space, as a function of their distance. In addition, these
 867 methods allow direct access to the degree of smoothness (or degree of integration) of the data, also
 868 called in the multifractal formalism the “non-conservative” nature of the data studied. This is possible
 869 through the estimation of the parameter H that characterizes any integrated multifractal data R_λ . In
 870 the multifractal formalism, R_λ is defined by the power law variation of its stationary increments
 871 described in Appendix A (Eq.14).

872 There are several ways to estimate these fluctuations. The simplest is to calculate the average of
 873 the absolute differences (or gradient) of R_λ . In general, this method is very suitable for processes
 874 characterized by a parameter H between 0 and 1 ([Lovejoy and Schertzer, 2012a](#)), thus corresponding
 875 to increasing fluctuations with the distance Δx . However, one constraint with this definition is that it
 876 does not allow decreasing fluctuations to be represented with distance: for multifractal data defined
 877 by $H < 0$, the structure function converges to a constant "parasitic" value, which gives an inaccurate
 878 estimate of H (which would be considered equal to 0). To be able to extend the range of H to which
 879 the structure functions have access, one solution is to change the shape of the wave estimating the
 880 fluctuations. [Lovejoy and Schertzer \(2012b\)](#) proposed a simple formalism of definitions of fluctuations
 881 which is based on that of wavelets. In particular, Haar's wavelet is a definition that turns out to be
 882 simple to calculate and interpret, while allowing access to negative H values. The fluctuations
 883 estimated from this method are calculated according to the following equation:

884
$$\Delta R_\lambda(\Delta x) = \left\langle \frac{2}{\Delta x} \left| S(x) + S(x + \Delta x) - 2S\left(x + \frac{\Delta x}{2}\right) \right| \right\rangle$$

885 15

886 where x and Δx are vectors and $\langle \cdot \rangle$ corresponds to the empirical average. S is the cumulative sum of
 887 the integrated data R_λ from which the global average has been subtracted. Thus, although this
 888 definition seems complex at first glance, it simply consists in estimating the average fluctuations from
 889 x to $x + \frac{\Delta x}{2}$ and from $x + \frac{\Delta x}{2}$ to $x + \Delta x$. We can note that, from an algorithmic point of view, equation
 890 8 involves aggregating the field at half the resolution as to allow the calculation of $\frac{\Delta x}{2}$. This implies that,
 891 in the case of Haar wavelets, the structure function will not be defined for the finest observation scale.

892 This technique was used in this study to estimate the structure functions and the corresponding H
 893 parameters of our surface S2 products. In particular, in the case of 2D processes, fluctuations were
 894 estimated on vertical Δx and horizontal Δy distances separately, allowing us to obtain H_x and H_y
 895 parameters respectively. If the observations present indeed multifractal properties and are non-
 896 conservative in nature, the order of integration H_x (H_y) should be the slope of the increments ΔR_λ ,
 897 plotted in a log-log graph as a function of space scale Δx (Δy).

898 2) Statistical moments: multifractal properties

899 Statistical moments analysis deepens and extends spectral or structure functions analysis by
 900 focusing on the extreme variability of the signal. By comparison with the spectral analysis which
 901 represents the evolution of a single order of statistics, this method analyzes the behaviour of a set of
 902 statistical moments on the range of scales studied.

903 To test the presence of multifractal properties in the data (Eq.11), statistical moments and
 904 moment scaling function need to be estimated. To do this, different steps must be followed. First, the
 905 underlying flux $\Phi_{\lambda_{max}}$ has to be reconstructed from the data, *i.e.* random field R , at the maximum
 906 observation resolution λ_{max} . Because of the possible existence of a fractional integration of order H
 907 (Eq.14), a fractional derivative of the same order should be done. In this study, the modulus of the
 908 gradient was applied to the data. Indeed, this operator provides a simple and good numerical
 909 approximation of the fractional derivation without prior knowledge of H order (Lavallée et al., 1993;
 910 see also Renosh, 2015 for discussion about this approximation):

$$911 \quad \Phi_{\lambda_{max}} = \sqrt{\left(\frac{\partial R_{\lambda_{max}}}{\partial x}\right)^2 + \left(\frac{\partial R_{\lambda_{max}}}{\partial y}\right)^2}$$

912 16

913 Once the flux (conservative field) is retrieved, $\Phi_{\lambda_{max}}$ is normalized by its mean.

914 The second step involves the degradation of the flux at lower resolutions $\lambda < \lambda_{max}$. It aims to
 915 approximate the inversion of the stochastic multiplicative cascade by iteratively averaging the flux at
 916 coarser scales: each coarse pixel (level n of the cascade) is obtained by a simple average of neighboring
 917 finer pixels (level $n + 1$). Note that each roughened pixel size is a power of two multiplied greater than
 918 the observation scale $l_{min} (= \frac{L}{\lambda_{max}})$.

919 Finally, empirical moments (*i.e.* computing q -th order moments in Eq.11 while replacing statistical
 920 averages by empirical averages) are then computed for various orders and all accessible resolutions.
 921 Because of sample size limitations and/or possible divergence of high-order moments (Hubert et al.,

922 [2007](#)), in this study moments were computed for orders set from 0 to 3, by steps of 0.1. On a log-log
923 graph, the different moments are plotted as a function of the resolution. If linearity is observed for
924 each moment curve, at least over a significant range of resolutions, Eq.11 is therefore verified, which
925 is the signature of multifractality. The empirical moment scaling function can be estimated, with $K(q)$
926 corresponding to the slope of each linear fit of q th order moment. Afterwards, the universal
927 parameters α and C_1 may be obtained by optimization: here the Nelder-Mead method is used to find
928 the couple of parameters minimizing the (quadratic) distance between the empirical function $K(q)$
929 and the UM model form (Eq.13).

930

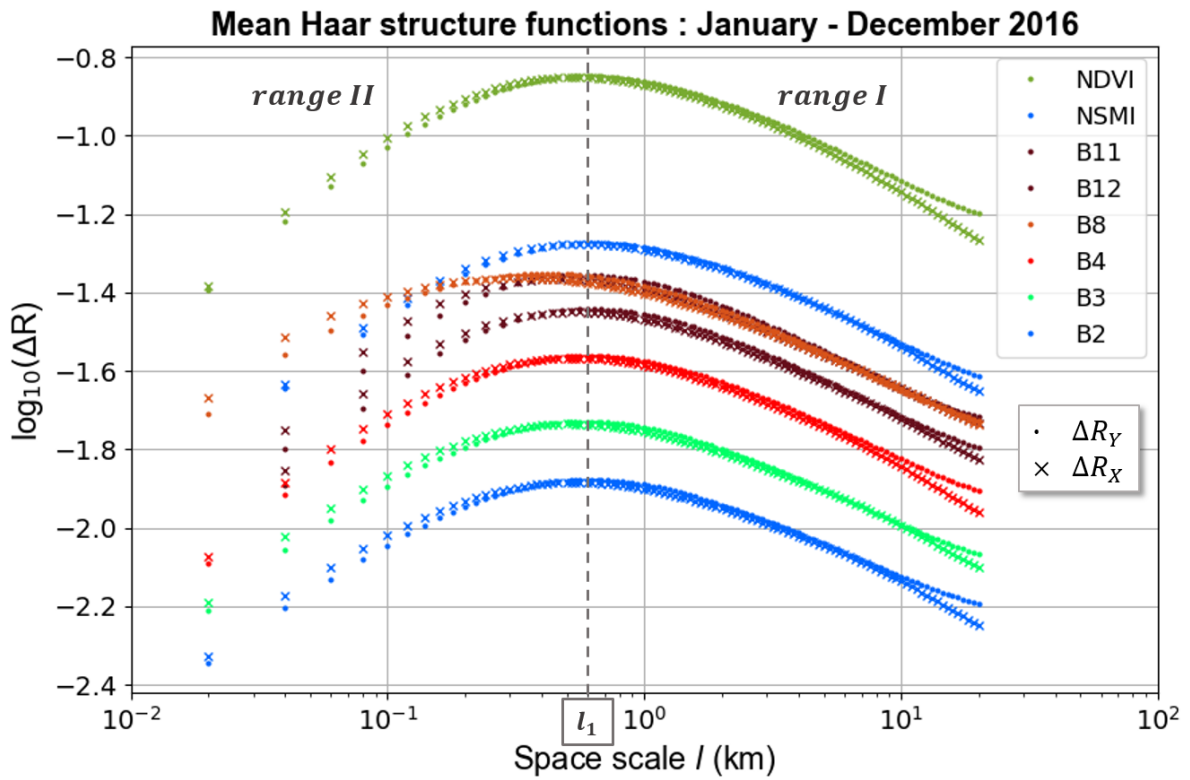


Figure 13: Mean Haar structure functions estimated from Sentinel-2 bands and indexes, over the period January-December 2016. For each band, structure functions are computed in two orthogonal directions (X and Y axis of the image).

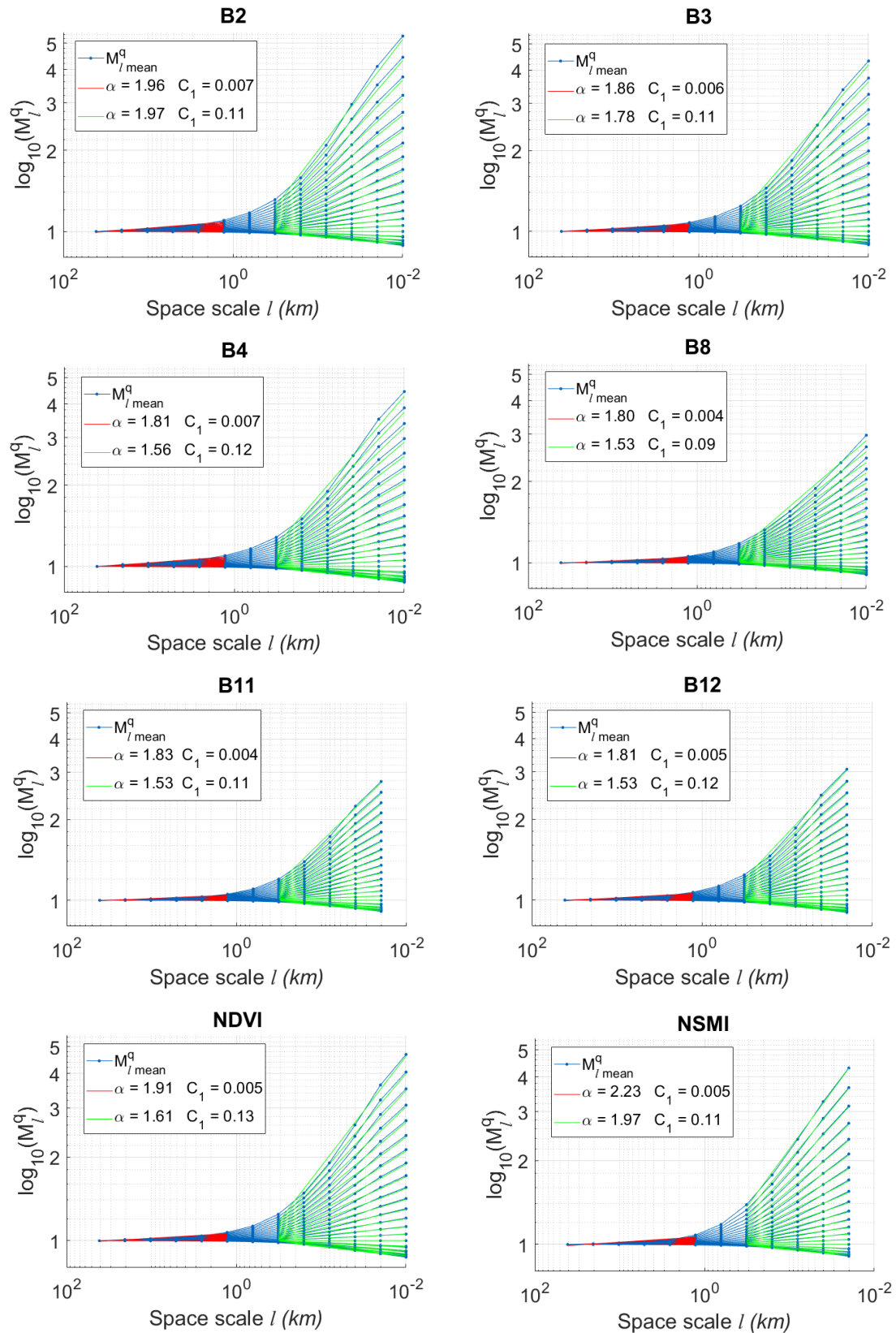


Figure 14: Mean moments estimated from Sentinel-2 bands and indexes, over the period January-December 2016. The straight lines in red and green correspond to the linear regressions carried out respectively on range I (large scales) and range II (small scales).

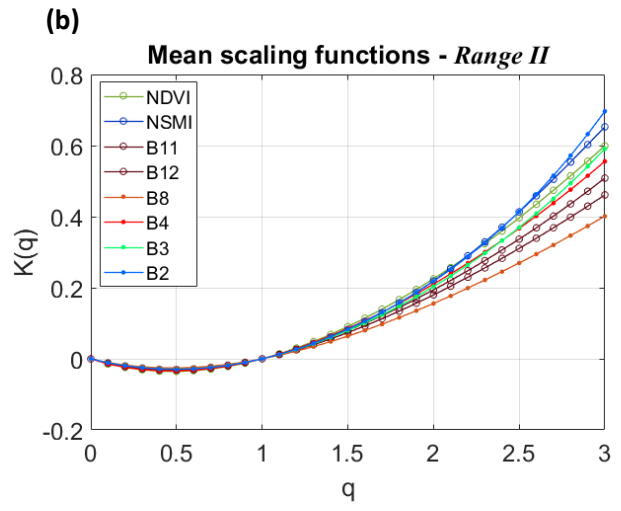
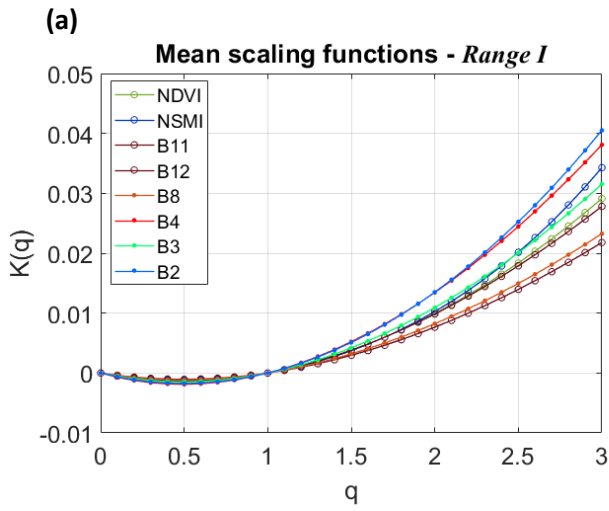


Figure 15: Scaling functions estimated from the mean statistical moments in figure 14, for range I (a) and range II (b). Note the different range of values on the vertical axes.

935 **Appendix D: Simulation of multifractal random fields**

936 The simulator that we present here is used to generate multifractal random fields knowing the
937 three parameters α , C_1 and H of the UM model (FIF version) as well as the size of the simulation
938 domain corresponding to the maximum resolution. This simulator includes several steps, which are
939 described in detail in various works on multifractal processes such as [Lovejoy and Schertzer \(2010b,](#)
940 [2010c\)](#), [Pecknold et al. \(1993\)](#), and [Verrier \(2011\)](#). We therefore invite the reader to refer to this work
941 for more information on these processing steps.

942 In general, this simulator aims to reproduce the mathematical construction of the multiplicative
943 cascades corresponding to the UM/FIF model, defined in Appendix A. This construction is based on five
944 main steps that we describe hereafter:

945 1) *Generation of the log-stable random variable Y_α :*

946 The construction of the UM model is based on the generation of a log-stable random variable,
947 corresponding here to an α -stable white noise Y_α . The parameter α designates the degree of
948 multifractality of the UM model, and also corresponds to the stability index relating to stable laws
949 ([Schertzer et al., 2002](#)). Thus, a first step consists in generating Y_α thanks to the algorithm of [Chambers](#)
950 [et al. \(1976\)](#). This algorithm makes it possible to simulate stable variables quickly and efficiently, simply
951 from realizations of independent random variables and according to uniform and exponential laws.
952 Note here that some additional constraints must be considered to generate Y_α , relatively to its
953 normalisation (unit asymmetrical α -stable white noise) and its asymmetry (asymmetry parameter set
954 to -1).

955 2) *Multiplication by the amplitude factor σ :*

956 The random variable Y_α is then multiplied by an amplitude factor σ allowing to integrate the
957 parameter C_1 in the construction of the cascade:

$$958 \quad \sigma = \left(\frac{c_1}{|\alpha - 1|} \right)^{\frac{1}{\alpha}}$$

959 17

960 3) *Generator creation r_λ :*

961 In order to obtain the multifractal field, we must first construct the random variable r_λ called
962 "generator". This is obtained by convolution of the log-stable variable $\sigma \cdot Y_\alpha$ and a filter g_α (in negative
963 power law of the distance) such that:

964

$$g_{\alpha}(\vec{x}) = \frac{1}{\|\vec{x}\|^{\frac{D}{\alpha}}}$$

965

18

966 where $\|\cdot\|$ designates the Euclidian norm, *i.e.* in the 2D case $\|\vec{x}\| = \|(x, y)\| = \sqrt{x^2 + y^2}$. For the sake
 967 of speed, the convolution operations present in this simulator are performed numerically in the Fourier
 968 domain by means of the two-dimensional *fft* or “Fast Fourier Transform”. D is here taken equal to 2
 969 for a two-dimensional simulation.

970

4) *Generation of the conservative process Φ_{λ}* :

971

The conservative process (or flux) is finally obtained by exponentiation of the generator, then
 972 normalization:

973

$$\Phi_{\lambda} = \frac{e^{r_{\lambda}}}{\langle e^{r_{\lambda}} \rangle}$$

974

19

975

5) *Generation of the integrated process R_{λ}* :

976

In the case of the simulation of a non-conservative process (integrated process named here as
 977 “field”), a last step consists in applying a convolution to Φ_{λ} by the filter in decreasing power law g_H ,
 978 defined by :

979

$$g_H(\vec{x}) = \frac{1}{\|\vec{x}\|^{D-H}}$$

980

20

981 This convolution amounts to carrying out a fractional integration of order H (less than D) ([Gagnon et](#)
 982 [al., 2006](#)). In the particular case where $H = 0$, step 5 is omitted since here the random process to be
 983 generated is conservative, that is to say $R_{\lambda} = \Phi_{\lambda}$.

984

985 **Appendix E: Modelling of the Modulation Transfer Function of an optical sensor from the**
986 **characteristics of the lens (diffraction) and detector (sampling)**

987

988 $MTF = MTF_{diffract} \times MTF_{sampling}$

989

990 $MTF_{diffract} = \frac{2}{\pi} \left\{ \cos^{-1} \left(\frac{f}{f_c} \right) - \left(\frac{f}{f_c} \right) \sqrt{1 - \left(\frac{f}{f_c} \right)^2} \right\}$

$$f = \sqrt{f_x^2 + f_y^2}$$

991

992

$$f_c = \frac{a}{\lambda D} \text{ (cut-off frequency)}$$

993

994

a : lens diameter

995

996

λ : radiation wavelength

997

998

D : focal length

999

1000

1001 $MTF_{sampling} = \text{sinc} \left(\pi \frac{f_x}{f_s} \right) \cdot \text{sinc} \left(\pi \frac{f_y}{f_s} \right)$

f_s : sampling frequency

1002

1003

1004

1005 **References**

- 1006 Aguado, P. L., Del Monte, J. P., Moratiel, R., & Tarquis, A. M. (2014). Spatial Characterization of Landscapes
1007 through Multifractal Analysis of DEM. *The Scientific World Journal*, 2014, 1-9.
1008 <https://doi.org/10.1155/2014/563038>
- 1009 Alonso, C., Tarquis, A. M., Zúñiga, I., & Benito, R. M. (2017). Spatial and radiometric characterization of multi-
1010 spectrum satellite images through multi-fractal analysis. *Nonlinear Processes in Geophysics*, 24(2),
1011 141-155. <https://doi.org/10.5194/npg-24-141-2017>
- 1012 Ameline, M., Fieuzal, R., Betbeder, J., Berthoumieu, J.-F., & Baup, F. (2018). Estimation of Corn Yield by
1013 Assimilating SAR and Optical Time Series Into a Simplified Agro-Meteorological Model : From
1014 Diagnostic to Forecast. *IEEE Journal of Selected Topics in Applied Earth Observations and Remote
1015 Sensing*, 11(12), 4747-4760. <https://doi.org/10.1109/JSTARS.2018.2878502>
- 1016 Bach, H., & Mauser, W. (2003). Methods and examples for remote sensing data assimilation in land surface
1017 process modeling. *IEEE Transactions on Geoscience and Remote Sensing*, 41(7), 1629-1637.
1018 <https://doi.org/10.1109/TGRS.2003.813270>
- 1019 Battude, M. (2017). *Estimation des rendements, des besoins et consommations en eau du maïs dans le Sud-
1020 Ouest de la France : Apport de la télédétection à hautes résolutions spatiale et temporelle*. Université
1021 Toulouse 3 Paul Sabatier.
- 1022 Battude, M., Al Bitar, A., Morin, D., Cros, J., Huc, M., Marais Sicre, C., Le Dantec, V., & Demarez, V. (2016).
1023 Estimating maize biomass and yield over large areas using high spatial and temporal resolution
1024 Sentinel-2 like remote sensing data. *Remote Sensing of Environment*, 184, 668-681.
1025 <https://doi.org/10.1016/j.rse.2016.07.030>
- 1026 Baup, F., Fieuzal, R., Marais-Sicre, C., Dejoux, J.-F., le Dantec, V., Mordelet, P., Claverie, M., Hagolle, O., Lopes,
1027 A., Keravec, P., Ceschia, E., Mialon, A., & Kidd, R. (2012). MCM'10 : An experiment for satellite multi-
1028 sensors crop monitoring from high to low resolution observations. *2012 IEEE International Geoscience
1029 and Remote Sensing Symposium*, 4849-4852. <https://doi.org/10.1109/IGARSS.2012.6352527>
- 1030 Baup, Frédéric, Fieuzal, R., & Betbeder, J. (2015). Estimation of soybean yield from assimilated optical and radar
1031 data into a simplified agrometeorological model. *Geoscience and Remote Sensing Symposium
1032 (IGARSS), 2015 IEEE International*, 3961–3964.
1033 <http://ieeexplore.ieee.org/abstract/document/7326692/>
- 1034 Benz, U. C., Hofmann, P., Willhauck, G., Lingenfelder, I., & Heynen, M. (2004). Multi-resolution, object-oriented
1035 fuzzy analysis of remote sensing data for GIS-ready information. *ISPRS Journal of Photogrammetry and
1036 Remote Sensing*, 58(3-4), 239-258. <https://doi.org/10.1016/j.isprsjprs.2003.10.002>
- 1037 Betbeder, J., Fieuzal, R., & Baup, F. (2016). Assimilation of LAI and Dry Biomass Data From Optical and SAR
1038 Images Into an Agro-Meteorological Model to Estimate Soybean Yield. *IEEE Journal of Selected Topics
1039 in Applied Earth Observations and Remote Sensing*, 9(6), 2540-2553.
1040 <https://doi.org/10.1109/JSTARS.2016.2541169>
- 1041 Béziat, P., Ceschia, E., & Dedieu, G. (2009). Carbon balance of a three crop succession over two cropland sites in
1042 South West France. *Agricultural and Forest Meteorology*, 149(10), 1628-1645.
1043 <https://doi.org/10.1016/j.agrformet.2009.05.004>
- 1044 Bindlish, R., & Barros, A. P. (2002). Subpixel variability of remotely sensed soil moisture : An inter-comparison
1045 study of SAR and ESTAR. *IEEE Transactions on Geoscience and Remote Sensing*, 40(2), 326-337.
1046 <https://doi.org/10.1109/36.992792>

- 1047 Burel, E. (2018). *Modélisation des hétérogénéités de la réserve utile et du développement des cultures au sein*
1048 *d'un sous-bassin versant en Midi-Pyrénées*. Université Toulouse 3 Paul Sabatier.
- 1049 Carnahan, W. H., & Zhou, G. (1986). *Fourier transform techniques for the evaluation of the Thematic Mapper*
1050 *line spread function*.
- 1051 Chambers, J. M., Mallows, C. L., & Stuck, B. W. (1976). A method for simulating stable random variables. *Journal*
1052 *of the american statistical association*, 71(354), 340-344.
- 1053 Chen, Y., & Wang, J. (2013). Multifractal characterization of urban form and growth : The case of Beijing.
1054 *Environment and Planning B: Planning and Design*, 40(5), 884-904. <https://doi.org/10.1068/b36155>
- 1055 Cheng, Q., Russell, H., Sharpe, D., Kenny, F., & Qin, P. (2001). GIS-based statistical and fractal/multifractal
1056 analysis of surface stream patterns in the Oak Ridges Moraine. *Computers & Geosciences*, 27(5),
1057 513-526. [https://doi.org/10.1016/S0098-3004\(00\)00112-6](https://doi.org/10.1016/S0098-3004(00)00112-6)
- 1058 Choi, T. (2002). *KONOS satellite on orbit modulation transfer function (MTF) measurement using edge and pulse*
1059 *method* [Electrical Engineering Department]. South Dakota State University.
- 1060 Claverie, M., Demarez, V., Duchemin, B., Hagolle, O., Ducrot, D., Marais-Sicre, C., Dejoux, J.-F., Huc, M.,
1061 Keravec, P., Béziat, P., Fieuzal, R., Ceschia, E., & Dedieu, G. (2012). Maize and sunflower biomass
1062 estimation in southwest France using high spatial and temporal resolution remote sensing data.
1063 *Remote Sensing of Environment*, 124, 844-857. <https://doi.org/10.1016/j.rse.2012.04.005>
- 1064 De Cola, L. (1989). Fractal Analysis of a Classified Landscape Scene. *Photogrammetric Engineering and Remote*
1065 *Sensing*, 55(5), 601-610.
- 1066 Deidda, R. (2000). Rainfall downscaling in a space-time multifractal framework. *Water Resources Research*,
1067 36(7), 1779-1794. <https://doi.org/10.1029/2000WR900038>
- 1068 Dejoux, J.-F., Dedieu, G., Hagolle, O., Ducrot, D., Menaut, J.-C., Ceschia, E., Baup, F., Demarez, V., Marais-Sicre,
1069 C., Kadiri, M., & Gascoin, S. (2012). *Kalideos OSR MiPy : Un observatoire pour la recherche et la*
1070 *démonstration des applications de la télédétection à la gestion des territoires*. 16.
- 1071 D'Errico, J. (2017). *Shape Language Modelling*. [https://fr.mathworks.com/matlabcentral/fileexchange/24443-](https://fr.mathworks.com/matlabcentral/fileexchange/24443-slm-shape-language-modeling)
1072 [slm-shape-language-modeling](https://fr.mathworks.com/matlabcentral/fileexchange/24443-slm-shape-language-modeling)
- 1073 Dubrulle, B. (1994). Intermittency in fully developed turbulence : Log-Poisson statistics and generalized scale
1074 covariance. *Physical review letters*, 73(7), 959.
- 1075 Fieuzal, R., Marais Sicre, C., & Baup, F. (2017). Estimation of corn yield using multi-temporal optical and radar
1076 satellite data and artificial neural networks. *International Journal of Applied Earth Observation and*
1077 *Geoinformation*, 57, 14-23. <https://doi.org/10.1016/j.jag.2016.12.011>
- 1078 Fieuzal, Rémy, & Baup, F. (2015). Estimation of sunflower yield using multi-spectral satellite data (optical or
1079 radar) in a simplified agro-meteorological model. *Geoscience and Remote Sensing Symposium*
1080 *(IGARSS), 2015 IEEE International*, 4001–4004.
1081 <http://ieeexplore.ieee.org/abstract/document/7326702/>
- 1082 Fieuzal, Rémy, Baup, F., & Marais-Sicre, C. (2013). Monitoring Wheat and Rapeseed by Using Synchronous
1083 Optical and Radar Satellite Data—From Temporal Signatures to Crop Parameters Estimation. *Advances*
1084 *in Remote Sensing*, 02(02), 162-180. <https://doi.org/10.4236/ars.2013.22020>
- 1085 Fraser, R., & Kaufman, Y. (1985). The Relative Importance of Aerosol Scattering and Absorption in Remote
1086 Sensing. *IEEE Transactions on Geoscience and Remote Sensing*, GE-23(5), 625-633.
1087 <https://doi.org/10.1109/TGRS.1985.289380>
- 1088 Gagnon, J.-S., Lovejoy, S., & Schertzer, D. (2006). Multifractal earth topography. *Nonlinear Processes in*
1089 *Geophysics*, 13(5), 541–570.

- 1090 Gascon, F., Bouzinac, C., Thépaut, O., Jung, M., Francesconi, B., Louis, J., Lonjou, V., Lafrance, B., Massera, S.,
 1091 Gaudel-Vacaresse, A., Languille, F., Alhammoud, B., Viallefont, F., Pflug, B., Bieniarz, J., Clerc, S.,
 1092 Pessiot, L., Trémas, T., Cadau, E., ... Fernandez, V. (2017). Copernicus Sentinel-2A Calibration and
 1093 Products Validation Status. *Remote Sensing*, 9(6), 584. <https://doi.org/10.3390/rs9060584>
- 1094 Gatti, A., & Naud, C. (2017). *Sentinel-2 Products Specification Document*. 487.
- 1095 Gires, A., Onof, C., Maksimovic, C., Schertzer, D., Tchiguirinskaia, I., & Simoes, N. (2012). Quantifying the impact
 1096 of small scale unmeasured rainfall variability on urban runoff through multifractal downscaling : A case
 1097 study. *Journal of Hydrology*, 442-443, 117-128. <https://doi.org/10.1016/j.jhydrol.2012.04.005>
- 1098 Gupta, V. K., & Waymire, E. C. (1997). Reply-Universal Multifractals Do Exist! : Comments on A Statistical
 1099 Analysis of Mesoscale Rainfall as a Random Cascade. *Journal of Applied Meteorology*, 36(9),
 1100 1304-1307.
- 1101 Gustafson, E. J. (1998). Quantifying Landscape Spatial Pattern : What Is the State of the Art? *Ecosystems*, 1(2),
 1102 143-156.
- 1103 Gutman, G., Byrnes, R., Masek, J., Covington, S., Justice, C., Franks, S., & Headley, R. (2008). Towards
 1104 monitoring Land-cover and land-use changes at a global scale : The global land survey.
 1105 *Photogrammetric Engineering & Remote Sensing*, 74, 6-10.
- 1106 Hagolle, O., Huc, M., Pascual, D. V., & Dedieu, G. (2010). A multi-temporal method for cloud detection, applied
 1107 to FORMOSAT-2, VEN μ S, LANDSAT and SENTINEL-2 images. *Remote Sensing of Environment*, 114(8),
 1108 1747-1755. <https://doi.org/10.1016/j.rse.2010.03.002>
- 1109 Hagolle, Olivier, Huc, M., Villa Pascual, D., & Dedieu, G. (2015). A Multi-Temporal and Multi-Spectral Method to
 1110 Estimate Aerosol Optical Thickness over Land, for the Atmospheric Correction of FormoSat-2, LandSat,
 1111 VEN μ S and Sentinel-2 Images. *Remote Sensing*, 7(3), 2668-2691. <https://doi.org/10.3390/rs70302668>
- 1112 Hansen, M., Dubayah, R., & Defries, R. (1996). Classification trees : An alternative to traditional land cover
 1113 classifiers. *International Journal of Remote Sensing*, 17(5), 1075-1081.
 1114 <https://doi.org/10.1080/01431169608949069>
- 1115 Huang, C., Goward, S. N., Masek, J. G., Thomas, N., Zhu, Z., & Vogelmann, J. E. (2010). An automated approach
 1116 for reconstructing recent forest disturbance history using dense Landsat time series stacks. *Remote
 1117 Sensing of Environment*, 114(1), 183-198. <https://doi.org/10.1016/j.rse.2009.08.017>
- 1118 Hubert, P., Tchiguirinskaia, I., Schertzer, D., Bendjoudi, H., & Lovejoy, S. (2007). Predetermination of floods.
 1119 *Extreme Hydrological EventsEvents: New Concepts for Security*, 78, 185-198.
- 1120 Hubert, P., Tessier, Y., Lovejoy, S., Schertzer, D., Schmitt, F., Ladoy, P., Carbonnel, J. P., Violette, S., &
 1121 Desurose, I. (1993). Multifractals and Extreme Rainfall Events. *Geophysical Research Letters*, 20(10),
 1122 931-934. <https://doi.org/10.1029/93GL01245>
- 1123 Justice, C. O., Vermote, E., Townshend, J. R. G., Defries, R., Roy, D. P., Hall, D. K., Salomonson, V. V., Privette, J.
 1124 L., Riggs, G., Strahler, A., Lucht, W., Myneni, R. B., Knyazikhin, Y., Running, S. W., Nemani, R. R.,
 1125 Zhengming Wan, Huete, A. R., van Leeuwen, W., Wolfe, R. E., ... Barnsley, M. J. (1998). The Moderate
 1126 Resolution Imaging Spectroradiometer (MODIS) : Land remote sensing for global change research. *IEEE
 1127 Transactions on Geoscience and Remote Sensing*, 36(4), 1228-1249.
 1128 <https://doi.org/10.1109/36.701075>
- 1129 Kim, G., & Barros, A. P. (2002a). Space-time characterization of soil moisture from passive microwave remotely
 1130 sensed imagery and ancillary data. *Remote Sensing of Environment*, 81(2-3), 393-403.
 1131 [https://doi.org/10.1016/S0034-4257\(02\)00014-7](https://doi.org/10.1016/S0034-4257(02)00014-7)

- 1132 Kim, G., & Barros, A. P. (2002b). Downscaling of remotely sensed soil moisture with a modified fractal
1133 interpolation method using contraction mapping and ancillary data. *Remote Sensing of Environment*,
1134 83(3), 400–413.
- 1135 King, M. D., Kaufman, Y. J., Tanré, D., & Nakajima, T. (1999). Remote Sensing of Tropospheric Aerosols from
1136 Space : Past, Present, and Future. *Bulletin of the American Meteorological society*, 80(11), 2229-2260.
- 1137 Ko, A., Mascaro, G., & Vivoni, E. R. (2016). Irrigation Impacts on Scaling Properties of Soil Moisture and the
1138 Calibration of a Multifractal Downscaling Model. *IEEE Transactions on Geoscience and Remote Sensing*,
1139 54(6), 3128-3142. <https://doi.org/10.1109/TGRS.2015.2511628>
- 1140 Kolmogorov, A. N. (1962). A refinement of previous hypotheses concerning the local structure of turbulence in
1141 a viscous incompressible fluid at high Reynolds number. *Journal of Fluid Mechanics*, 13(01), 82.
1142 <https://doi.org/10.1017/S0022112062000518>
- 1143 Kolmogorov, A. N. (1941). The local structure of turbulence in incompressible viscous fluid for very large
1144 Reynolds numbers. *Dokl. Akad. Nauk SSSR*, 30, 299–303.
- 1145 Lam, N. S.-N. (1990). Description and Measurement of Landsat TM Images Using Fractals. *Photogrammetric*
1146 *Engineering and Remote Sensing*, 56, 187-195.
- 1147 Lampkin, D. J., & Yool, S. R. (2004). Monitoring mountain snowpack evolution using near-surface optical and
1148 thermal properties. *Hydrological Processes*, 18(18), 3527-3542. <https://doi.org/10.1002/hyp.5797>
- 1149 Lavallée, D., Lovejoy, S., Schertzer, D., & Ladoy, P. (1993). Nonlinear variability of landscape topography :
1150 Multifractal analysis and simulation. In *Fractals in Geography* (Prentice Hall, p. 158-192). Eds. Nina Siu-
1151 Ngan Lam and Lee De Cola.
- 1152 Lei, F., & Tiziani, H. J. (1988). A Comparison of Methods to Measure the Modulation Transfer Function of Aerial
1153 Survey Lens Systems from the Image Structures. *PHOTOGRAMMETRIC ENGINEERING*, 6.
- 1154 Lenoble, J., Herman, M., Deuzé, J. L., Lafrance, B., Santer, R., & Tanré, D. (2007). A successive order of
1155 scattering code for solving the vector equation of transfer in the earth's atmosphere with aerosols.
1156 *Journal of Quantitative Spectroscopy and Radiative Transfer*, 107(3), 479-507.
1157 <https://doi.org/10.1016/j.jqsrt.2007.03.010>
- 1158 Liang, S., Fang, H., & Chen, M. (2001). Atmospheric correction of Landsat ETM+ land surface imagery. I.
1159 Methods. *IEEE Transactions on Geoscience and Remote Sensing*, 39(11), 2490-2498.
1160 <https://doi.org/10.1109/36.964986>
- 1161 Lovejoy, S., & Schertzer, D. (2010a). Towards a new synthesis for atmospheric dynamics : Space–time cascades.
1162 *Atmospheric Research*, 96(1), 1-52. <https://doi.org/10.1016/j.atmosres.2010.01.004>
- 1163 Lovejoy, S., & Schertzer, D. (2010b). On the simulation of continuous in scale universal multifractals, part I :
1164 Spatially continuous processes. *Computers & Geosciences*, 36(11), 1393-1403.
1165 <https://doi.org/10.1016/j.cageo.2010.04.010>
- 1166 Lovejoy, S., & Schertzer, D. (2010c). On the simulation of continuous in scale universal multifractals, Part II :
1167 Space–time processes and finite size corrections. *Computers & Geosciences*, 36(11), 1404-1413.
1168 <https://doi.org/10.1016/j.cageo.2010.07.001>
- 1169 Lovejoy, S., & Schertzer, D. (2012a). Low-Frequency Weather and the Emergence of the Climate. In A. S.
1170 Sharma, A. Bunde, V. P. Dimri, & D. N. Baker (Éds.), *Geophysical Monograph Series* (Vol. 196, p.
1171 231-254). American Geophysical Union. <https://doi.org/10.1029/2011GM001087>
- 1172 Lovejoy, S., & Schertzer, D. (2012b). Haar wavelets, fluctuations and structure functions : Convenient choices
1173 for geophysics. *Nonlinear Processes in Geophysics*, 19(5), 513-527. [https://doi.org/10.5194/npg-19-](https://doi.org/10.5194/npg-19-513-2012)
1174 513-2012

- 1175 Lovejoy, S., Schertzer, D., Lilley, M., Strawbridge, K. B., & Radkevich, A. (2008a). Scaling turbulent atmospheric
1176 stratification. I : Turbulence and waves. *Quarterly Journal of the Royal Meteorological Society*,
1177 *134*(631), 277-300. <https://doi.org/10.1002/qj.201>
- 1178 Lovejoy, S., Schertzer, D., & Stanway, J. D. (2001). Direct Evidence of Multifractal Atmospheric Cascades from
1179 Planetary Scales down to 1 km. *Physical Review Letters*, *86*(22), 5200-5203.
1180 <https://doi.org/10.1103/PhysRevLett.86.5200>
- 1181 Lovejoy, S., Tarquis, A. M., Gaonac'h, H., & Schertzer, D. (2008b). Single-and multiscale remote sensing
1182 techniques, multifractals, and MODIS-derived vegetation and soil moisture. *Vadose Zone Journal*, *7*(2),
1183 533–546.
- 1184 Lu, D., & Weng, Q. (2007). A survey of image classification methods and techniques for improving classification
1185 performance. *International Journal of Remote Sensing*, *28*(5), 823-870.
1186 <https://doi.org/10.1080/01431160600746456>
- 1187 Lyu, C., Cheng, Q., Zuo, R., & Wang, X. (2017). Mapping spatial distribution characteristics of lineaments
1188 extracted from remote sensing image using fractal and multifractal models. *Journal of Earth Science*,
1189 *28*(3), 507-515.
- 1190 Mandelbrot, B. (1967). How long is the coast of Britain ? Statistical self-similarity and fractional dimension.
1191 *Science*, *156*(3775), 636-638.
- 1192 Mandelbrot, B. B. (1975). Stochastic models for the Earth's relief, the shape and the fractal dimension of the
1193 coastlines, and the number-area rule for islands. *Proceedings of the National Academy of Sciences*,
1194 *72*(10), 3825-3828. <https://doi.org/10.1073/pnas.72.10.3825>
- 1195 Mandelbrot, BENOIT B., & Van Ness, J. W. (1968). Fractional Brownian motions, fractional. *Geophys. Res. Lett*,
1196 *24*, 2099–2102.
- 1197 Mangiarotti, S., Drapeau, L., & Letellier, C. (2014). Two chaotic global models for cereal crops cycles observed
1198 from satellite in northern Morocco. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *24*(2),
1199 023130.
- 1200 Mangiarotti, S., Mazzega, P., Hiernaux, P., & Mougin, E. (2010). The Vegetation dynamics in West Africa from
1201 AVHRR-NDVI data : Horizons of predictability versus spatial scales. *Remote Sensing of Environment*,
1202 *114*(9), 2036-2047.
- 1203 Mangiarotti, S., Mazzega, P., Hiernaux, P., & Mougin, E. (2012). Predictability of vegetation cycles over the
1204 semi-arid region of Gourma (Mali) from forecasts of AVHRR-NDVI signals. *Remote Sensing of
1205 Environment*, *123*, 246-257. <https://doi.org/10.1016/j.rse.2012.03.011>
- 1206 Mangiarotti, S., Mazzega, P., Jarlan, L., Mougin, E., Baup, F., & Demarty, J. (2008). Evolutionary Bi-objective
1207 Optimization of a Semi-Arid Vegetation Model with Satellite Data. *Remote Sensing of Environment*,
1208 *112*(4), 1365-1380.
- 1209 Mangiarotti, S., Sharma, A. K., Corgne, S., Hubert-Moy, L., Ruiz, L., Sekhar, M., & Kerr, Y. (2018). Can the global
1210 modeling technique be used for crop classification? *Chaos, Solitons & Fractals*, *106*, 363-378.
1211 <https://doi.org/10.1016/j.chaos.2017.12.003>
- 1212 Manivasagam, V. S., Kaplan, G., & Rozenstein, O. (2019). Developing Transformation Functions for VENμS and
1213 Sentinel-2 Surface Reflectance over Israel. *Remote Sensing*, *11*(14), 1710.
1214 <https://doi.org/10.3390/rs11141710>
- 1215 Martimort, P., Arino, O., Berger, M., Biasutti, R., Carnicero, B., Del Bello, U., Fernandez, V., Gascon, F., Greco, B.,
1216 Silvestrin, P., Spoto, F., & Sy, O. (2007). Sentinel-2 optical high resolution mission for GMES operational
1217 services. *2007 IEEE International Geoscience and Remote Sensing Symposium*, 2677-2680.
1218 <https://doi.org/10.1109/IGARSS.2007.4423394>

- 1219 Mascaro, G., Vivoni, E. R., & Deidda, R. (2010). Downscaling soil moisture in the southern Great Plains through a
 1220 calibrated multifractal model for land surface modeling applications : DOWNSCALING SOIL MOISTURE
 1221 IN THE GREAT PLAINS. *Water Resources Research*, 46(8). <https://doi.org/10.1029/2009WR008855>
- 1222 Neuhauser, M., Verrier, S., Merlin, O., Molero, B., Suere, C., & Mangiarotti, S. (2019). Multi-scale statistical
 1223 properties of disaggregated SMOS soil moisture products in Australia. *Advances in Water Resources*,
 1224 18. <https://doi.org/10.1016/j.advwatres.2019.103426>
- 1225 Norton, C. L., Brooks, G. C., & Welch, R. (1977). Optical and Modulation Transfer Function. *Photogrammetric*
 1226 *Engineering and Remote Sensing*, 43, 613-636.
- 1227 Oboukhov, A. M. (1962). Some specific features of atmospheric turbulence. *Journal of Fluid Mechanics*, 13(01),
 1228 77. <https://doi.org/10.1017/S0022112062000506>
- 1229 Pal, M. (2005). Random forest classifier for remote sensing classification. *International Journal of Remote*
 1230 *Sensing*, 26(1), 217-222. <https://doi.org/10.1080/01431160412331269698>
- 1231 Paola, J. D., & Schowengerdt, R. A. (1995). A review and analysis of backpropagation neural networks for
 1232 classification of remotely-sensed multi-spectral imagery. *International Journal of Remote Sensing*,
 1233 16(16), 3033-3058. <https://doi.org/10.1080/01431169508954607>
- 1234 Parisi, G., & Frisch, U. (1985). A multifractal model of intermittency. *Turbulence and Predictability in*
 1235 *Geophysical Fluid Dynamics and Climate Dynamics*, 84-88.
- 1236 Pecknold, S., Lovejoy, S., Schertzer, D., Hooge, C., & Malouin, J. F. (1993). *The simulation of universal*
 1237 *multifractals*.
- 1238 Puissant, A., Hirsch, J., & Weber, C. (2005). The utility of texture analysis to improve per-pixel classification for
 1239 high to very high spatial resolution imagery. *International Journal of Remote Sensing*, 26(4), 733-745.
 1240 <https://doi.org/10.1080/01431160512331316838>
- 1241 Qiu, H.-L., Lam, N. S.-N., Quattrochi, D., & Gamon, A. J. (1999). Fractal characterization of hyperspectral
 1242 imagery. *Photogrammetric Engineering and Remote Sensing*, 65, 63-71.
- 1243 Rahman, H., & Dedieu, G. (1994). SMAC : A simplified method for the atmospheric correction of satellite
 1244 measurements in the solar spectrum. *International Journal of Remote Sensing*, 15(1), 123-143.
 1245 <https://doi.org/10.1080/01431169408954055>
- 1246 Ramstein, G., & Raffy, M. (1989). Analysis of the structure of radiometric remotely-sensed images. *International*
 1247 *Journal of Remote Sensing*, 10(6), 1049-1073. <https://doi.org/10.1080/01431168908903944>
- 1248 Reborá, N., Ferraris, L., Von Hardenberg, J., & Provenzale, A. (2006). Rainfall downscaling and flood
 1249 forecasting : A case study in the Mediterranean area. *Natural Hazards and Earth System Science*, 6(4),
 1250 611-619.
- 1251 Reichenbach, S. E. (1991). Characterizing digital image acquisition devices. *Optical Engineering*, 30(2), 170.
 1252 <https://doi.org/10.1117/12.55783>
- 1253 Renosh, P. R. (2015). *Characterisation of the coupling between oceanic turbulence and the variability of the*
 1254 *coastal waters optical properties, using in situ measurements and satellite data* [Université Lille 1
 1255 Sciences et Technologies].
 1256 [http://search.proquest.com/openview/e82d9e0f7d9042460290440064b0c107/1?pq-](http://search.proquest.com/openview/e82d9e0f7d9042460290440064b0c107/1?pq-origsite=gscholar&cbl=54809)
 1257 [origsite=gscholar&cbl=54809](http://search.proquest.com/openview/e82d9e0f7d9042460290440064b0c107/1?pq-origsite=gscholar&cbl=54809)
- 1258 Richardson, L. F. (1922). Weather prediction by numerical process. *Cambridge Univ. Press*.
- 1259 Richter, R., Schläpfer, D., & Müller, A. (2006). An automatic atmospheric correction algorithm for visible/NIR
 1260 imagery. *International Journal of Remote Sensing*, 27(10), 2077-2085.
 1261 <https://doi.org/10.1080/01431160500486690>

- 1262 Rodríguez-Fernández, N., de Rosnay, P., Albergel, C., Richaume, P., Aires, F., Prigent, C., & Kerr, Y. (2019). *SMOS*
1263 *Neural Network Soil Moisture Data Assimilation in a Land Surface Model and Atmospheric Impact*. 23.
- 1264 Rouquié, B., Hagolle, O., Bréon, F.-M., Boucher, O., Desjardins, C., & Rémy, S. (2017). Using Copernicus
1265 Atmosphere Monitoring Service Products to Constrain the Aerosol Type in the Atmospheric Correction
1266 Processor MAJA. *Remote Sensing*, 9(12), 1230. <https://doi.org/10.3390/rs9121230>
- 1267 Roy, D. P., Wulder, M. A., Loveland, T. R., C.E., W., Allen, R. G., Anderson, M. C., Helder, D., Irons, J. R., Johnson,
1268 D. M., Kennedy, R., Scambos, T. A., Schaaf, C. B., Schott, J. R., Sheng, Y., Vermote, E. F., Belward, A. S.,
1269 Bindschadler, R., Cohen, W. B., Gao, F., ... Zhu, Z. (2014). Landsat-8 : Science and product vision for
1270 terrestrial global change research. *Remote Sensing of Environment*, 145, 154-172.
1271 <https://doi.org/10.1016/j.rse.2014.02.001>
- 1272 Schertzer, D., & Lovejoy, S. (1984). On the Dimension of Atmospheric motions. *Turbulence and Chaotic*
1273 *phenomena in Fluids*, 505-512.
- 1274 Schertzer, D., & Lovejoy, S. (1987). Physical modeling and analysis of rain and clouds by anisotropic scaling
1275 multiplicative processes. *Journal of Geophysical Research*, 92(D8), 9693.
1276 <https://doi.org/10.1029/JD092iD08p09693>
- 1277 Schertzer, D., & Lovejoy, S. (1991). Nonlinear geodynamical variability : Multiple singularities, universality and
1278 observables. *Non-Linear Variability in Geophysics*, 41-82.
- 1279 Schertzer, D., & Lovejoy, S. (1997). Universal multifractals do exist! : Comments on "A statistical analysis of
1280 mesoscale rainfall as a random cascade". *Journal of Applied Meteorology*, 36(9), 1296–1303.
- 1281 Schertzer, D., Lovejoy, S., & Hubert, P. (2002). An introduction to stochastic multifractal fields. *ISFMA*
1282 *Symposium on Environmental Science and Engineering with related Mathematical Problems*, 106–179.
1283 <http://132.206.6.11/~gang/eprints/Paris.eprints/Stochastic.multi.eprint2002.pdf>
- 1284 Schmitt, F. (1993). Estimation of universal multifractal indices for atmospheric turbulent velocity fields.
1285 *Fractals*, 1(3), 568-575.
- 1286 Sharma, D. (2007). Spatial disaggregation of bias-corrected GCM precipitation for improved hydrologic
1287 simulation : Ping River Basin, Thailand. *Hydrol. Earth Syst. Sci.*, 19.
- 1288 She, Z.-S., & Leveque, E. (1994). Universal Scaling Laws in Fully Developed Turbulence. *Physical review letters*,
1289 72(3).
- 1290 Siebesma, A. P., & Jonker, H. J. J. (2000). Anomalous Scaling of Cumulus Cloud Boundaries. *Physical Review*
1291 *Letters*, 85(1), 214-217. <https://doi.org/10.1103/PhysRevLett.85.214>
- 1292 Stoian, A., Poulain, V., Inglada, J., Poughon, V., & Derksen, D. (2019). Land Cover Maps Production with High
1293 Resolution Satellite Image Time Series and Convolutional Neural Networks : Adaptations and Limits for
1294 Operational Systems. *Remote Sensing*, 11(17), 1986. <https://doi.org/10.3390/rs11171986>
- 1295 Tallec, T., Béziat, P., Jarosz, N., Rivalland, V., & Ceschia, E. (2013). Crops' water use efficiencies in temperate
1296 climate : Comparison of stand, ecosystem and agronomical approaches. *Agricultural and Forest*
1297 *Meteorology*, 168, 69-81. <https://doi.org/10.1016/j.agrformet.2012.07.008>
- 1298 Tan, X., Huang, B., Batty, M., & Li, J. (2021). Urban Spatial Organization, Multifractals, and Evolutionary Patterns
1299 in Large Cities. *Annals of the American Association of Geographers*, 111(5), 1539-1558.
- 1300 Tessier, Y., Lovejoy, S., & Schertzer, D. (1993). *Universal Multifractals : Theory and Observations for Rain and*
1301 *Clouds*. <https://doi.org/10.1175/1520-0450>
- 1302 Tucker, C. J., & Sellers, P. J. (1986). Satellite remote sensing of primary production. *International Journal of*
1303 *Remote Sensing*, 7(11), 1395-1416. <https://doi.org/10.1080/01431168608948944>

- 1304 Vermote, E. F., Tanre, D., Deuze, J. L., Herman, M., & Morcette, J.-J. (1997). Second Simulation of the Satellite
1305 Signal in the Solar Spectrum, 6S : An overview. *IEEE Transactions on Geoscience and Remote Sensing*,
1306 35(3), 675-686. <https://doi.org/10.1109/36.581987>
- 1307 Verrier, S. (2011). *Modélisation de la variabilité spatiale et temporelle des précipitations à la sub-mésoéchelle*
1308 *par une approche multifractale* [Université de Versailles-Saint Quentin en Yvelines]. [https://hal-](https://hal-sde.archives-ouvertes.fr/tel-00734327/)
1309 [sde.archives-ouvertes.fr/tel-00734327/](https://hal-sde.archives-ouvertes.fr/tel-00734327/)
- 1310 Verrier, S. (2020). Multifractal and multiscale entropy scaling of in-situ soil moisture time series : Study of
1311 SMOSMANIA network data, southwestern France. *Journal of Hydrology*, 585, 124821.
1312 <https://doi.org/10.1016/j.jhydrol.2020.124821>
- 1313 Viallefont-Robinet, F., & Léger, D. (2010). Improvement of the edge method for on-orbit MTF measurement.
1314 *Optics Express*, 18(4), 3531. <https://doi.org/10.1364/OE.18.003531>
- 1315 Weszka, J. S., Dyer, C. R., & Rosenfeld, A. (1976). A Comparative Study of Texture Measures for Terrain
1316 Classification. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-6(4), 269-285.
1317 <https://doi.org/10.1109/TSMC.1976.5408777>
- 1318 Yaglom, A. M. (1966). The influence of fluctuations in energy dissipation on the shape of turbulence
1319 characteristics in the inertial interval. *Soviet Physics Doklady*, 11, 26.
- 1320 Zucker, S. W. (1976). Region growing : Childhood and adolescence. *Computer Graphics and Image Processing*,
1321 5(3), 382-399. [https://doi.org/10.1016/S0146-664X\(76\)80014-7](https://doi.org/10.1016/S0146-664X(76)80014-7)
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